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At-Streamgage Flood Frequency Analyses for Very Low Annual Exceedance Probabilities from a Perspective of Multiple Distributions and Parameter Estimation Methods

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Very Low AEP Estimation:

- **Some Facts:**
  - The longest streamflow records are on the order of 120 years but often just a few decades of data are available.
  - Conventional flood frequency requires estimates for return periods of about 10–500 years. Common guidance in the U.S. is generally accepted as adequate (log-Pearson type III distribution; method of moments; Bulletins 17B and 17C).
- **Flood frequency for VL-AEPs (very low annual exceedance probabilities) *requires different approaches and considerations* than used conventionally.**
  - This work stresses the *communication of uncertainty* in VL-AEPs.
  - This study shows that choices of probability models and fitting methods can produce enormous ranges in estimates that are associated with large uncertainty.

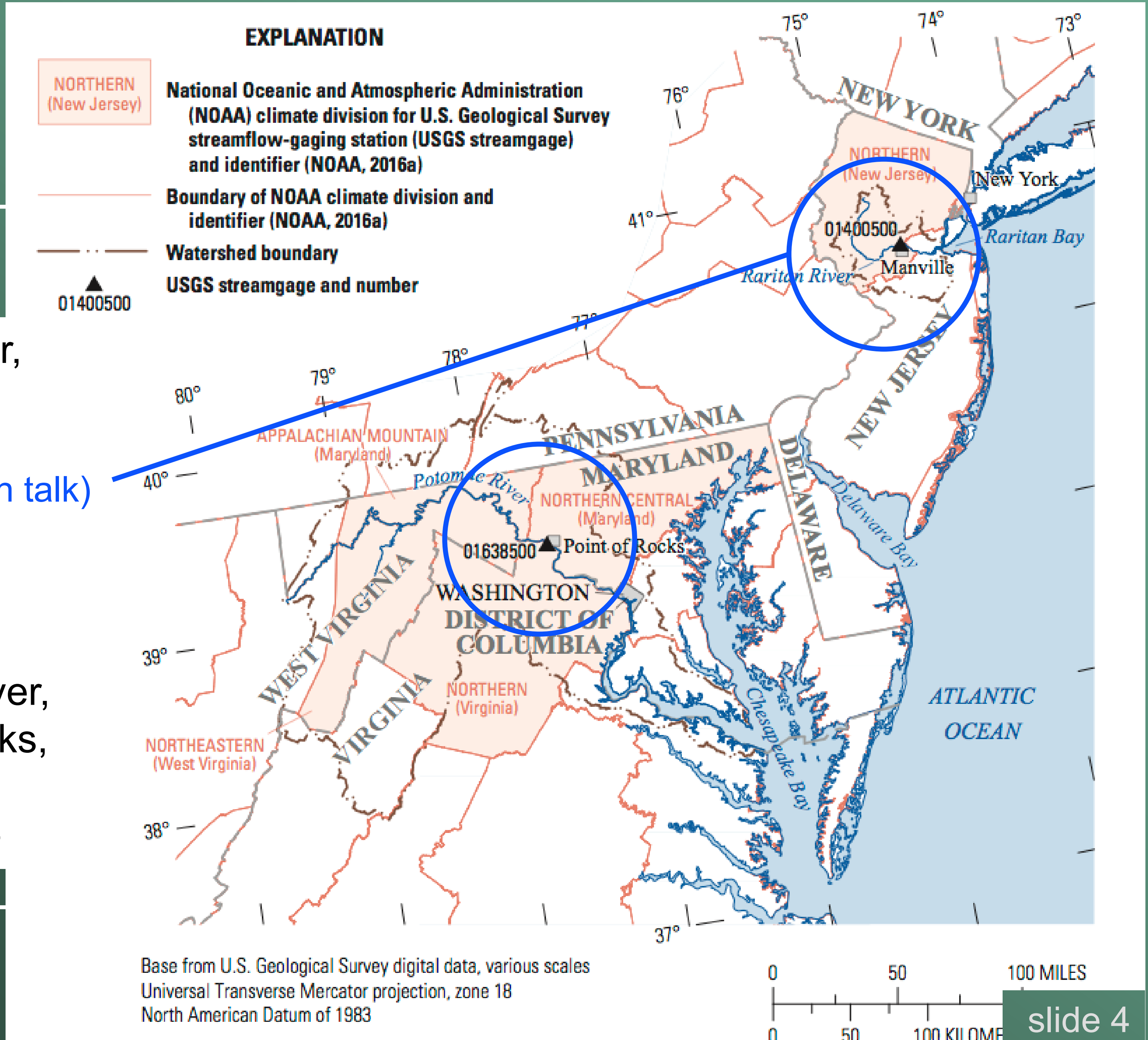
Overall Project Details

- **U.S. Geological Survey in cooperation with U.S. Nuclear Regulatory Commission (2015–2017)**
- **Magnitude and frequency of instantaneous peak streamflow**
  - **Task 1 (This talk and pending USGS Scientific Investigations Report [SIR])**
  - Tasks 2 and 3 (nonstandard flood information, nonstationarity, another USGS SIR)
  - Task 4 (USGS-led training seminar)
- **Task 1 concerns estimation at very low AEPs (VL-AEPs) and uncertainty (error) quantification.**
- DATA: annual peaks at two USGS long-term streamgages.
- **AEP: annual exceedance probability and VL-AEP < 0.001 or >1,000-year equivalent recurrence intervals [“AEP” preferred].**
- We might also say “distal tail estimation” when VL-AEP are sought.

Raritan River, Manville, NJ ≈1904–2014 (highlighted in talk)

and

Potomac River, Point of Rocks, MD ≈1895–2015



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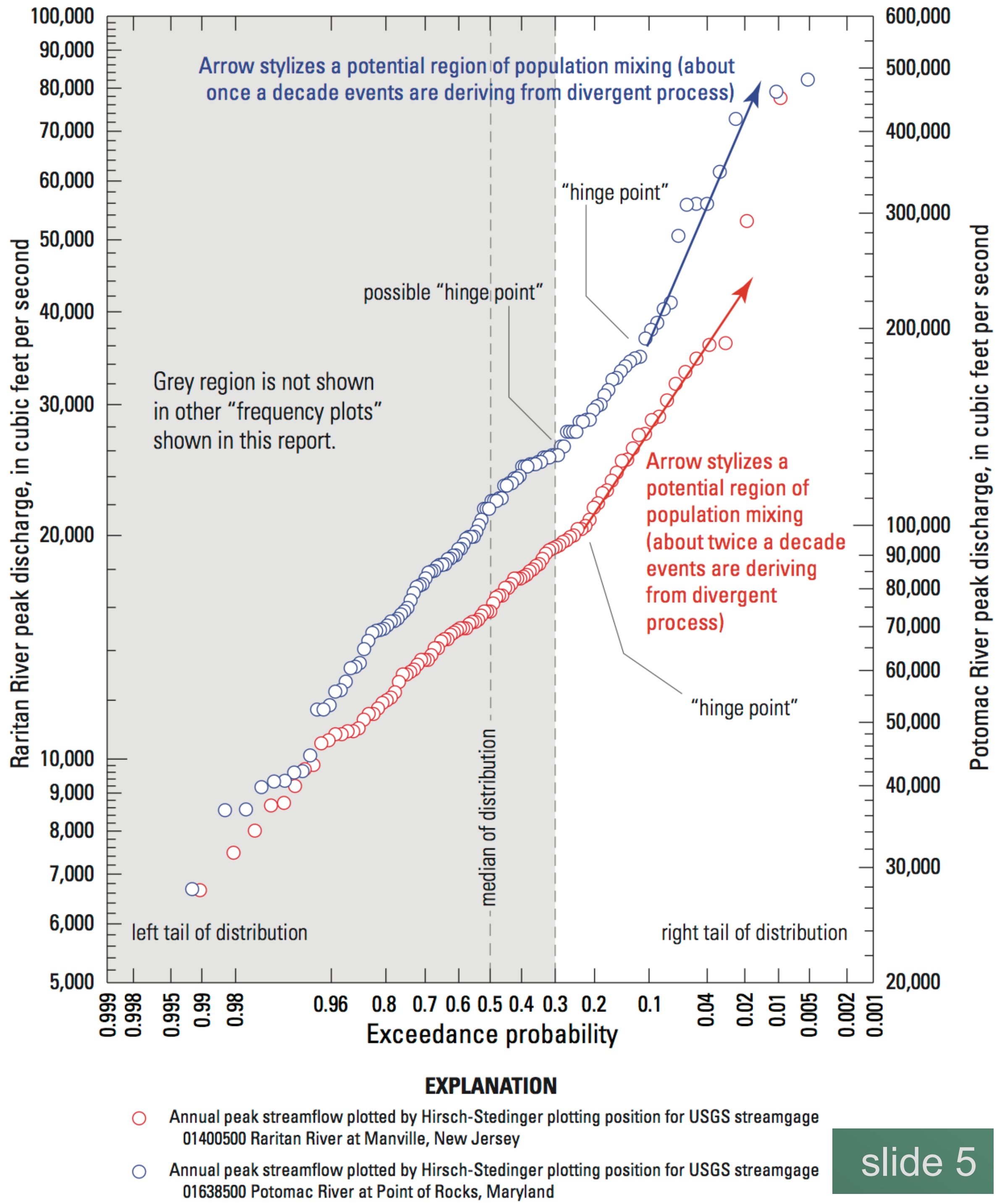
Frequency Analysis

- **What is meant by "frequency analysis"?**



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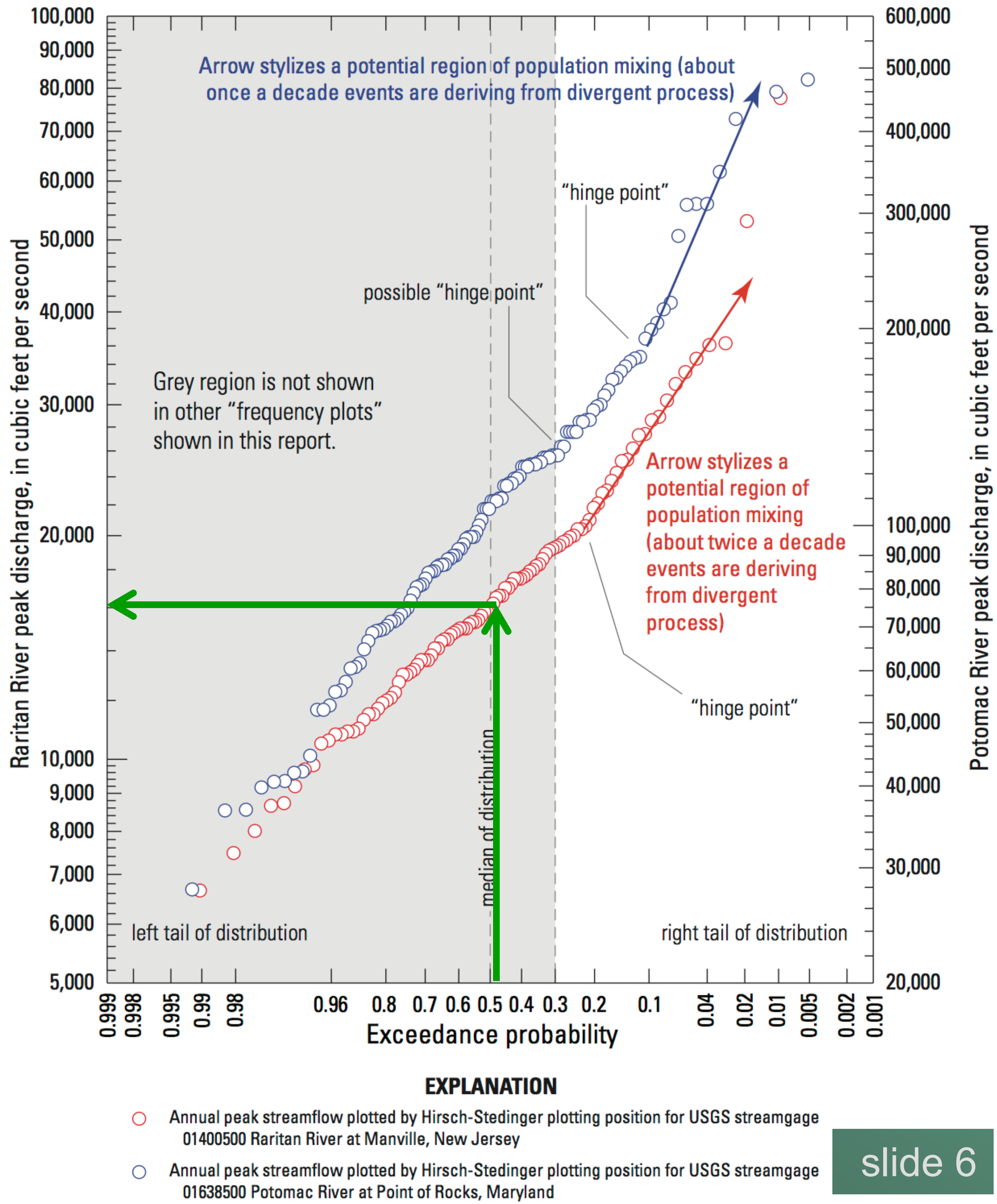
- **What is meant by the familiar “mean” or “median” statistics?**



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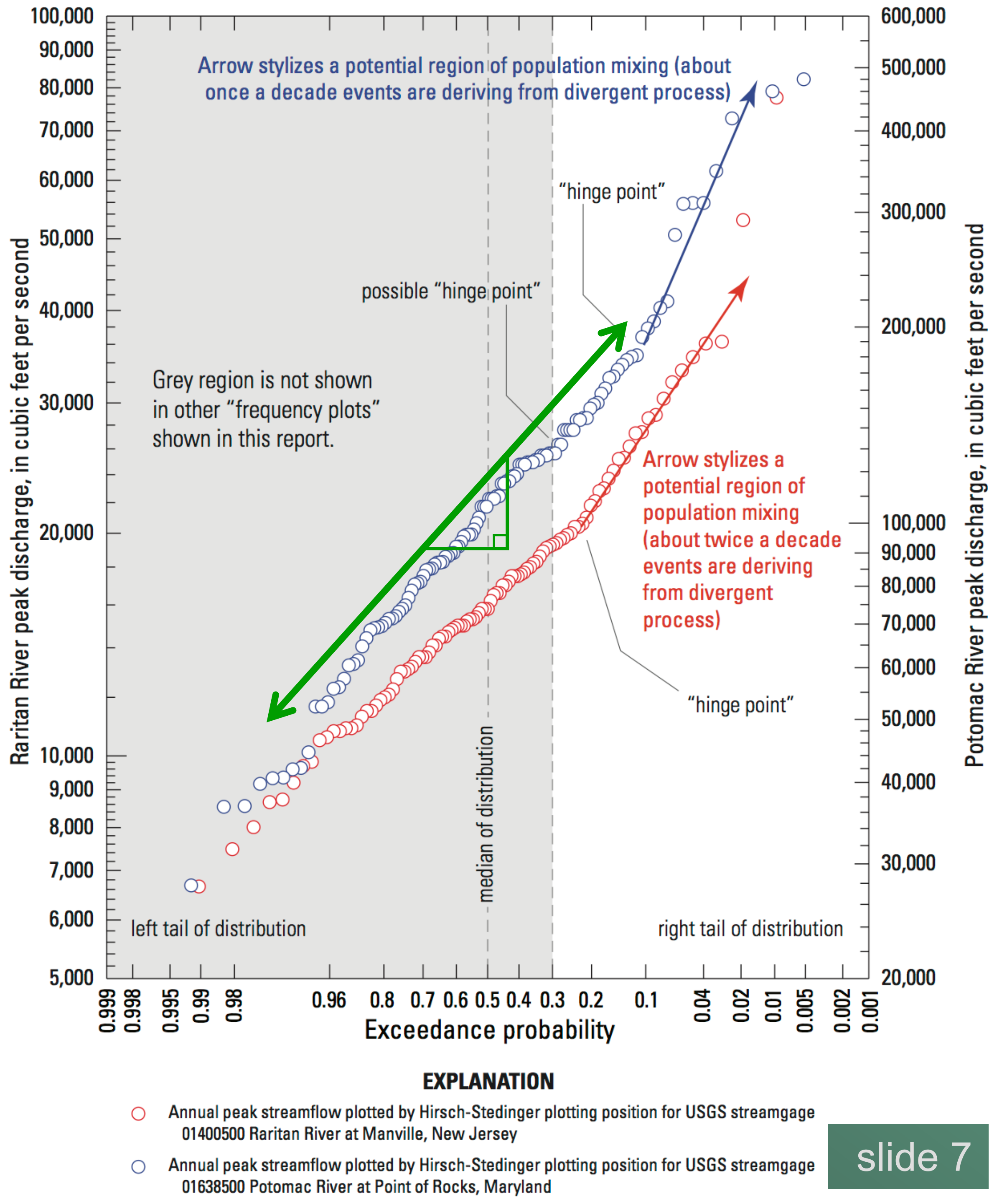

Frequency Analysis

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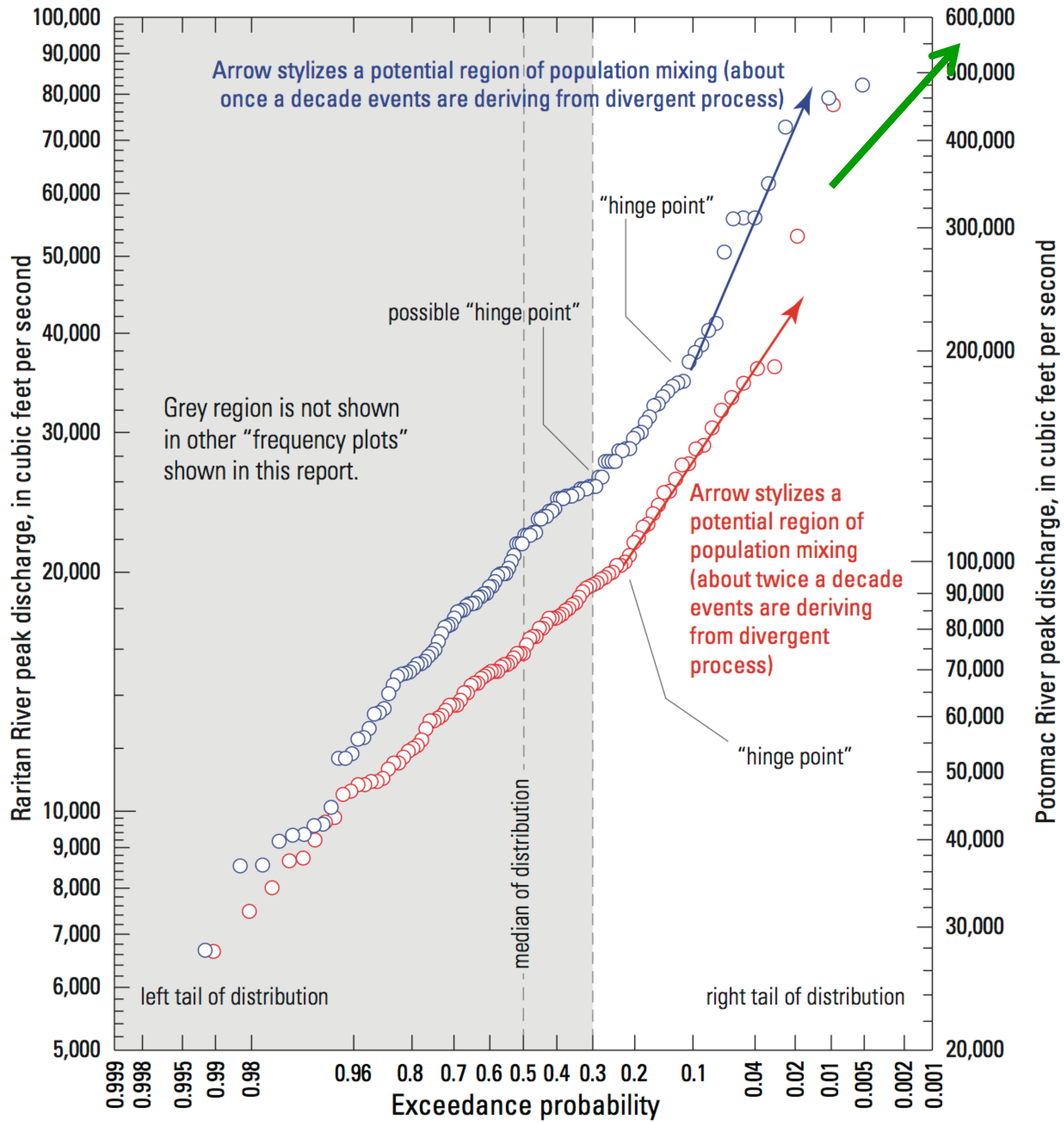
- What is meant by “variation” or “dispersion” of the data mean?
- **What is meant by “distal tail”?**



EXPLANATION

○ Annual peak streamflow plotted by Hirsch-Stedinger plotting position for USGS streamgage 01400500 Raritan River at Manville, New Jersey

○ Annual peak streamflow plotted by Hirsch-Stedinger plotting position for USGS streamgage 01638500 Potomac River at Point of Rocks, Maryland

Task 1 Details — Uncertainty

- **At-streamgage analysis (single site data):**
  - **Exclusion of covariates (conditional probabilities) influencing distal tails (*quantile dependency* [e.g. Tropical Cyclones as possible trigger for highest magnitude peaks]). This could be thought of population mixing.**
- **Quantification of uncertainty into two forms:**
  - ***Sampling uncertainty* (aleatoric, random chance [stochastic])**
    - This is a <u>sampling error</u> related to variances-covariances of either sample moments or parameters. This uncertainty can be reduced by including more data.

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  - ***Distribution choice uncertainty* (epistemic, model error)**
    - True probability model unknown, semi-quantitative, dependent on choices. This uncertainty can possibly be reduced by regional study of distribution tails and goodness-of-fit evaluations.

Both uncertainties increase as AEP decreases, and both are relatively large for very low AEP estimation.

Task 1 Details — Distributions

- **Logarithmic transformation of annual peaks used, and the adjective “log-” (e.g. log-Pearson type III) implied in talk.**
- **Nine probability distributions:**
  - Generalized Extreme Value (GEV, three parameter)
  - Generalized Logistic (GLO, three parameter)
  - Generalized (“skew”) Normal (GNO, three parameter; log-Normal3)
  - Generalized Pareto (GPA, three parameter)
  - Pearson type III (PE3, three parameter; a standard choice in U.S.)
  - Weibull (WEI, three parameter; reversed GEV)
  - Kappa (KAP, four parameters; common in regional L-moments)
  - Asymmetric Exponential Power (AEP4, four parameters, attractive tails)
  - Wakeby (five parameters; very flexible)

Task 1 Details — Parameter Estimation

- **Four methods of parameter estimation are used:**
  - Expected Moments Algorithm: (EMA, product moments) though restricted to PE3 (Pearson type III). "Bulletin 17C" publication pending from USGS.
    - Special "Extended Output" option added to USGS-PeakFQ software for <0.001 AEP estimation and on out to AEP = 10^{-6}.

$$M_r = \mathrm{E}[(X-\mu)^r] = \int_{-\infty}^{\infty} (x-\mu)^r\, f(x)\,\mathrm{d}x$$

  - L-moments (LMR): linear combinations of the *quantile function*

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{n!}{(j-1)!(n-j)!}\int_0^1 x(F)\times F^{j-1}\times(1-F)^{n-j}\,\mathrm{d}F,$$

  - Maximum Likelihood (MLE): maximization of sum of logarithmic densities via the *probability density function*

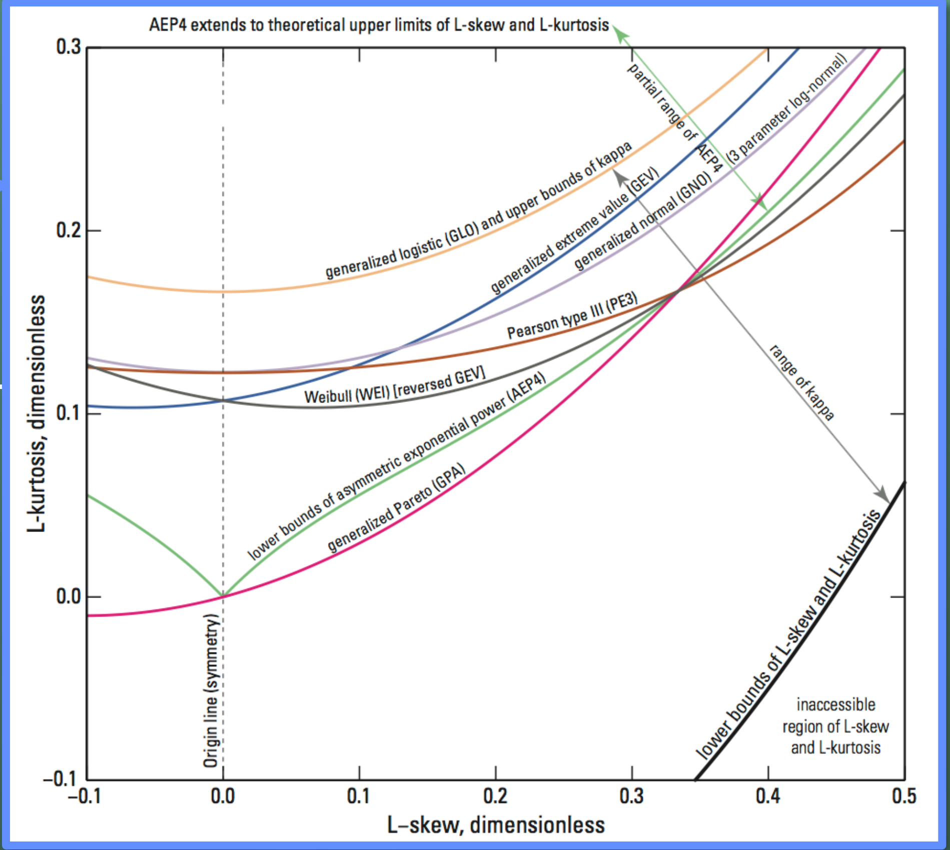
$$\log(L_n) = \sum_{i=1}^{n}\log(f(x_i;\theta)),$$

  - Maximum Product of Spacings (MPS): maximization of sum of U-statistic increments via the *cumulative distribution function*

$$M_n(\theta) = \sum_{i=1}^{n+1}\log\left[U_i(\theta)-U_{i-1}(\theta)\right] \text{ for } U_i(\theta) = F(x_{i:n};\theta)$$

Task 1 Details — Goodness-of-Fit

- **Goodness-of-Fit measures considered for the distributions:**
  - **Akaike Information Criterion (AIC)**
  - **Cramér–von Mises**
  - **Moran's M**
  - **Kolmogorov–Smirnov**
- **L-moment ratio diagram**
  - Delta L-kurtosis — The difference between L-kurtosis of a fitted distribution and the sample L-kurtosis.
  - Three-parameter distributions have their own unique L-kurtosis once fit to the mean, variation, and L-skew.

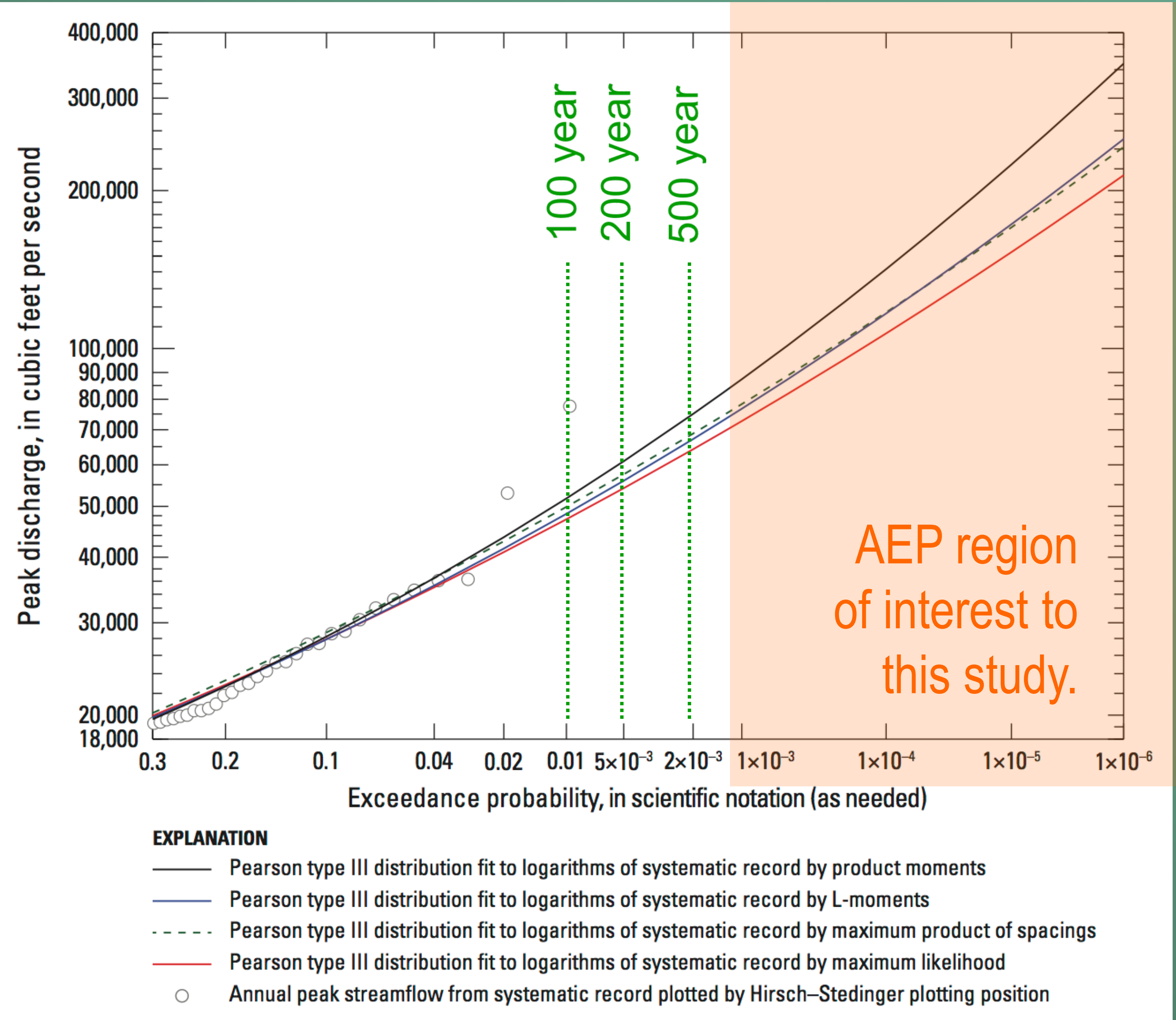


Task 1 Results — Raritan River, Manville, NJ

Four PE3 fit by EMA, LMR, MLE, and MPS:

The four methods estimate similarly for AEPs of interest to transportation design and flood plain management (AEP < 0.002).

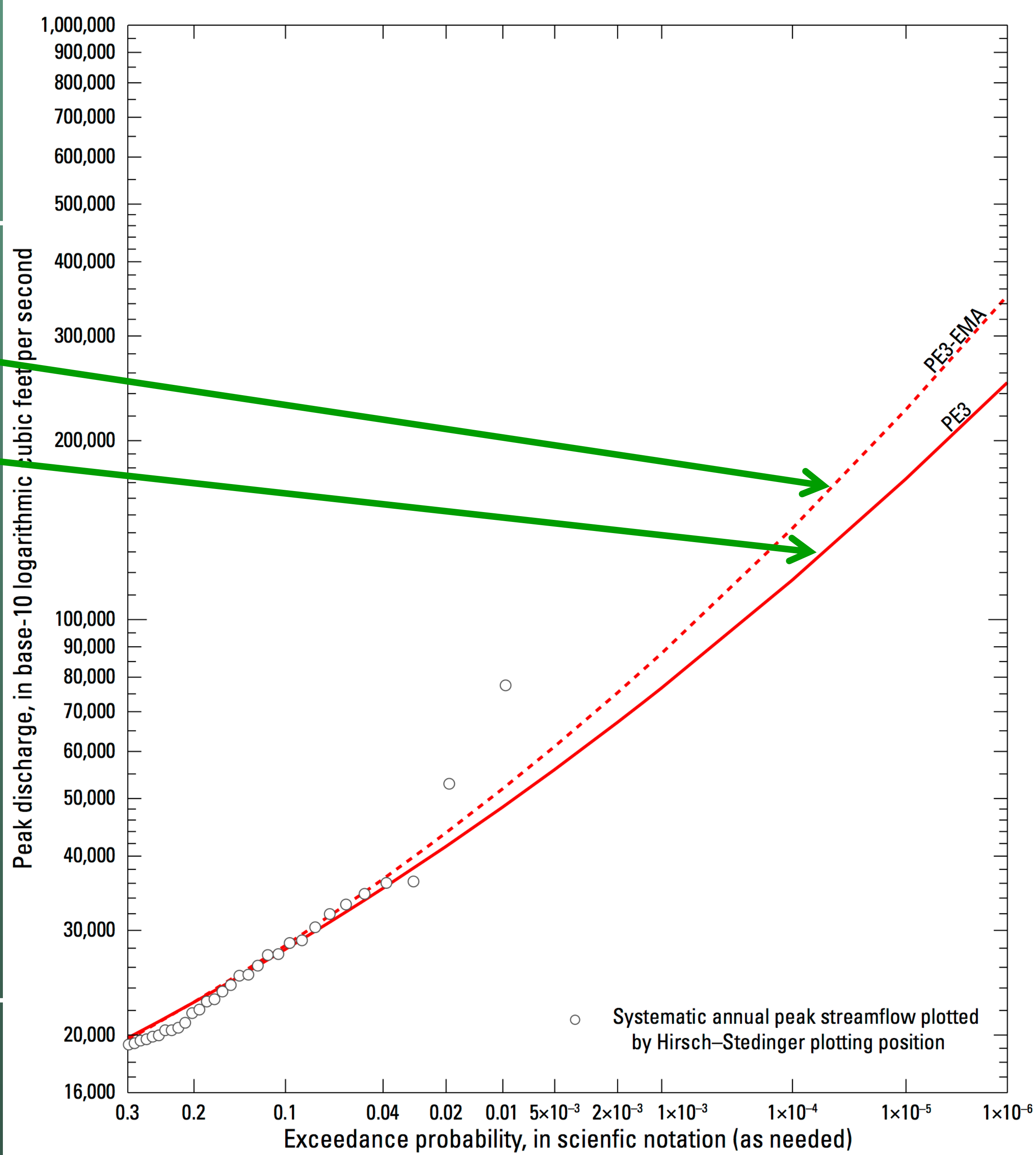
We do not quantify this concept (differing est. methods) as another type of uncertainty, but we acknowledge it.



Raritan River

Pearson type III fits

PE3 EMA and by L-moments (LMR)

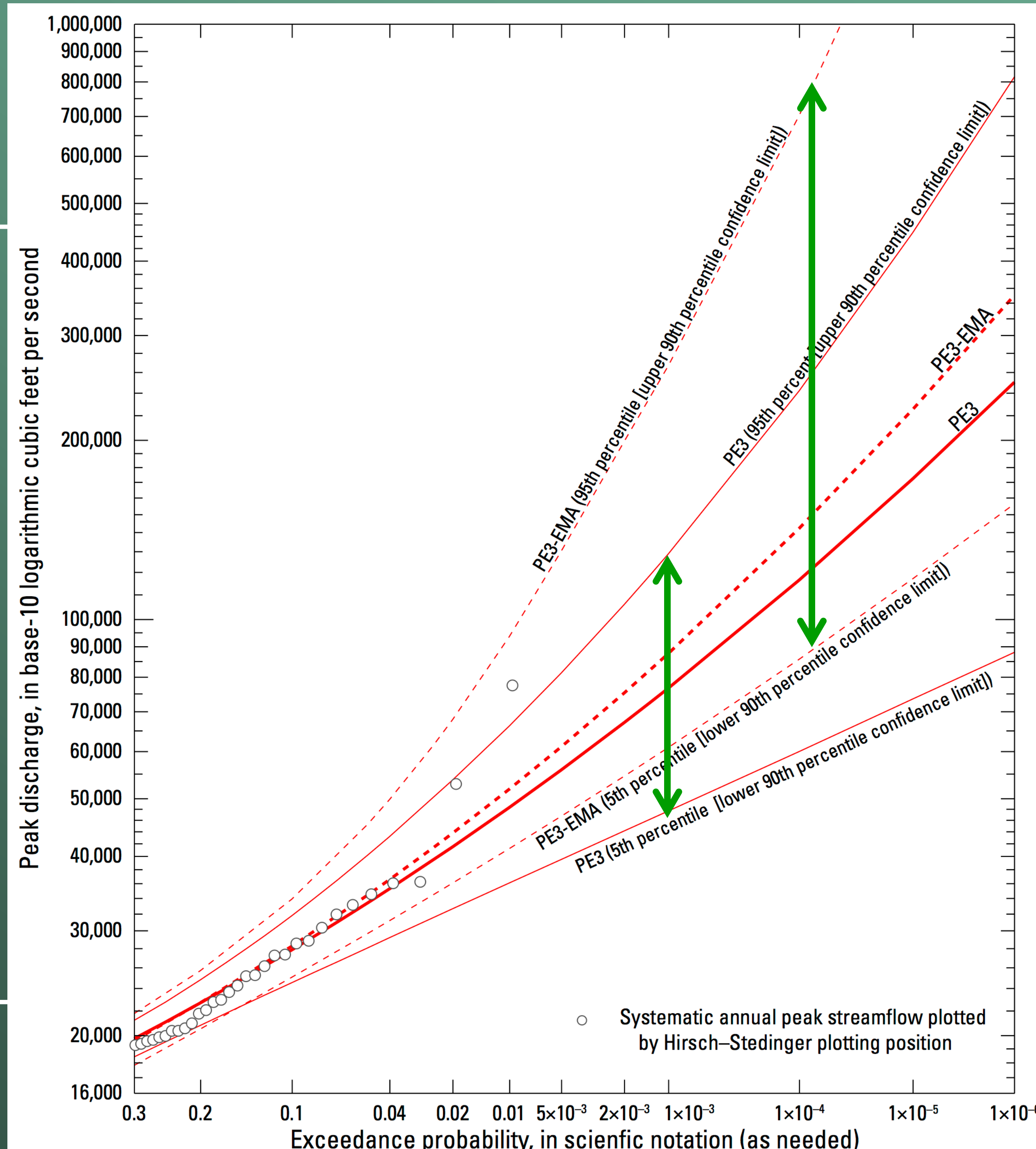


Raritan River
(Sampling Uncertainty)

PE3 EMA + LMR

PE3 confidence limits (*sampling uncertainty*) for an AEP

Confidence limits wide for VL-AEP

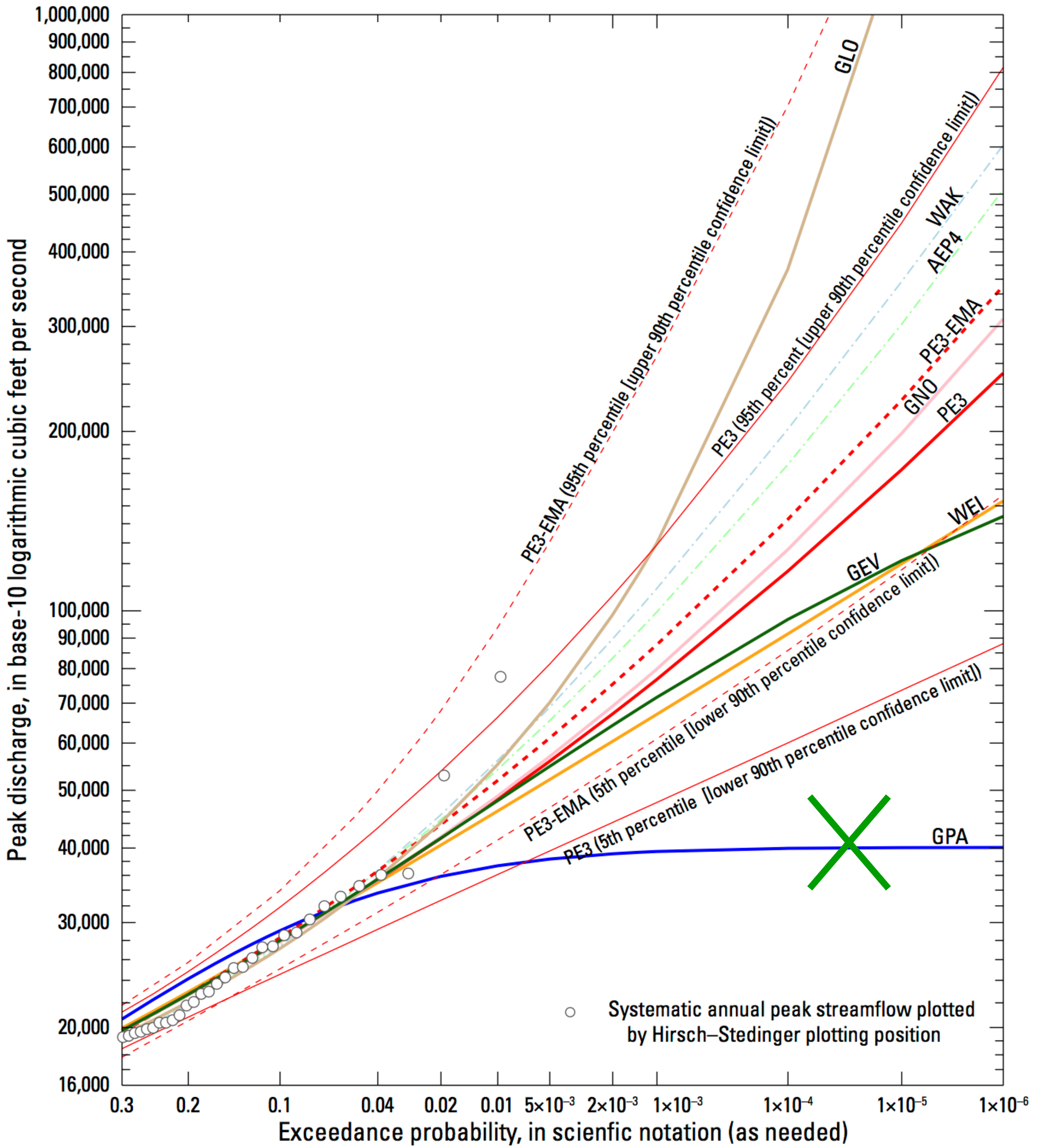


Raritan River
(Distribution Choice Uncertainty)

Nine fits for different distributions:

Some fits less more "suitable" than others; GPA is rejected.

Note: PE3 EMA + LMR confidence limits are still plotted.

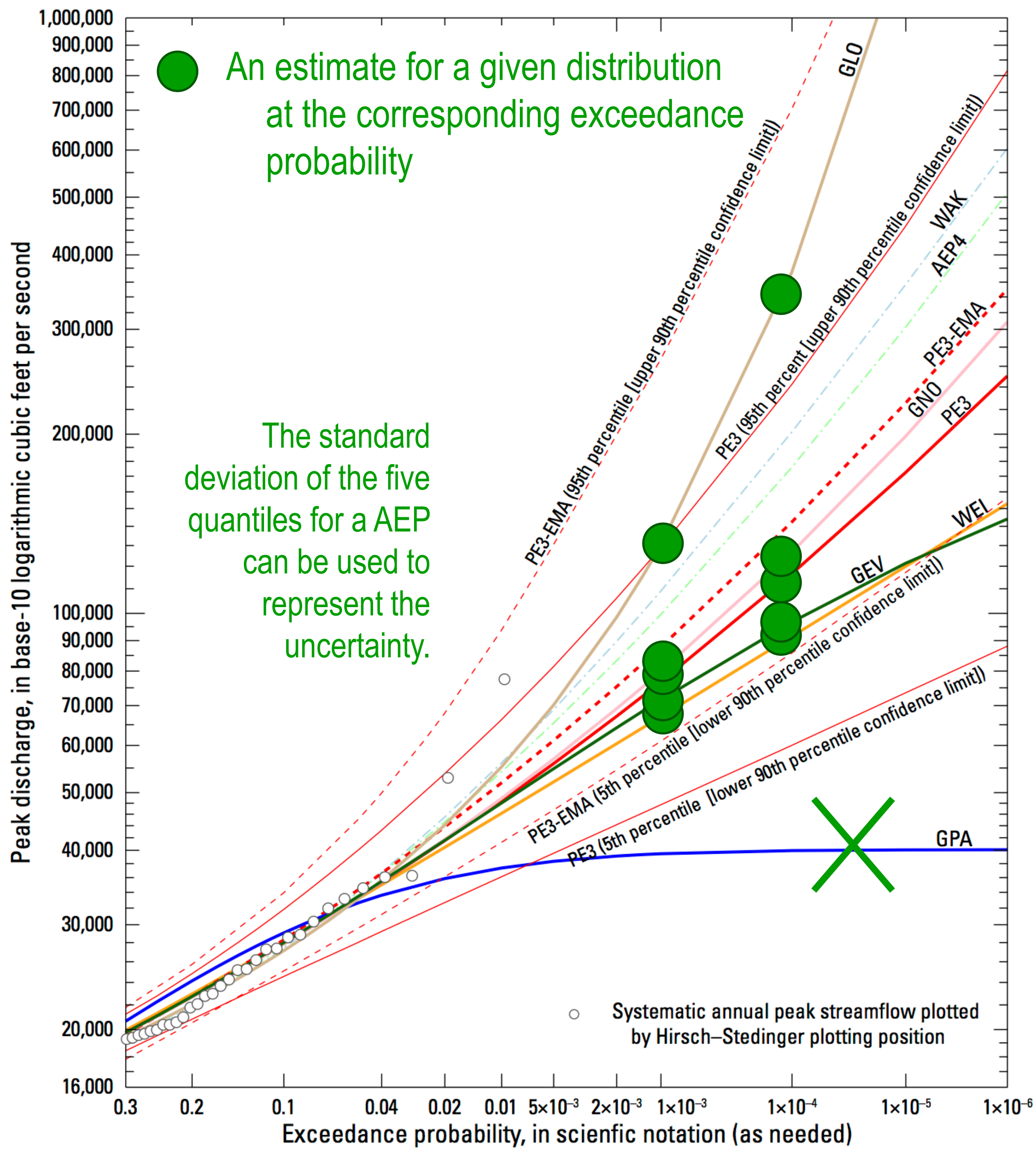


Raritan River
(Distribution Choice Uncertainty)

Distribution choice uncertainty **for an AEP (the dots)**

Distribution choice uncertainty **is** **<u>extremely large</u>** **for VL-AEP and is sensitive to analyst choices.**

Note: PE3 EMA + LMR confidence limits and GPA still plotted.

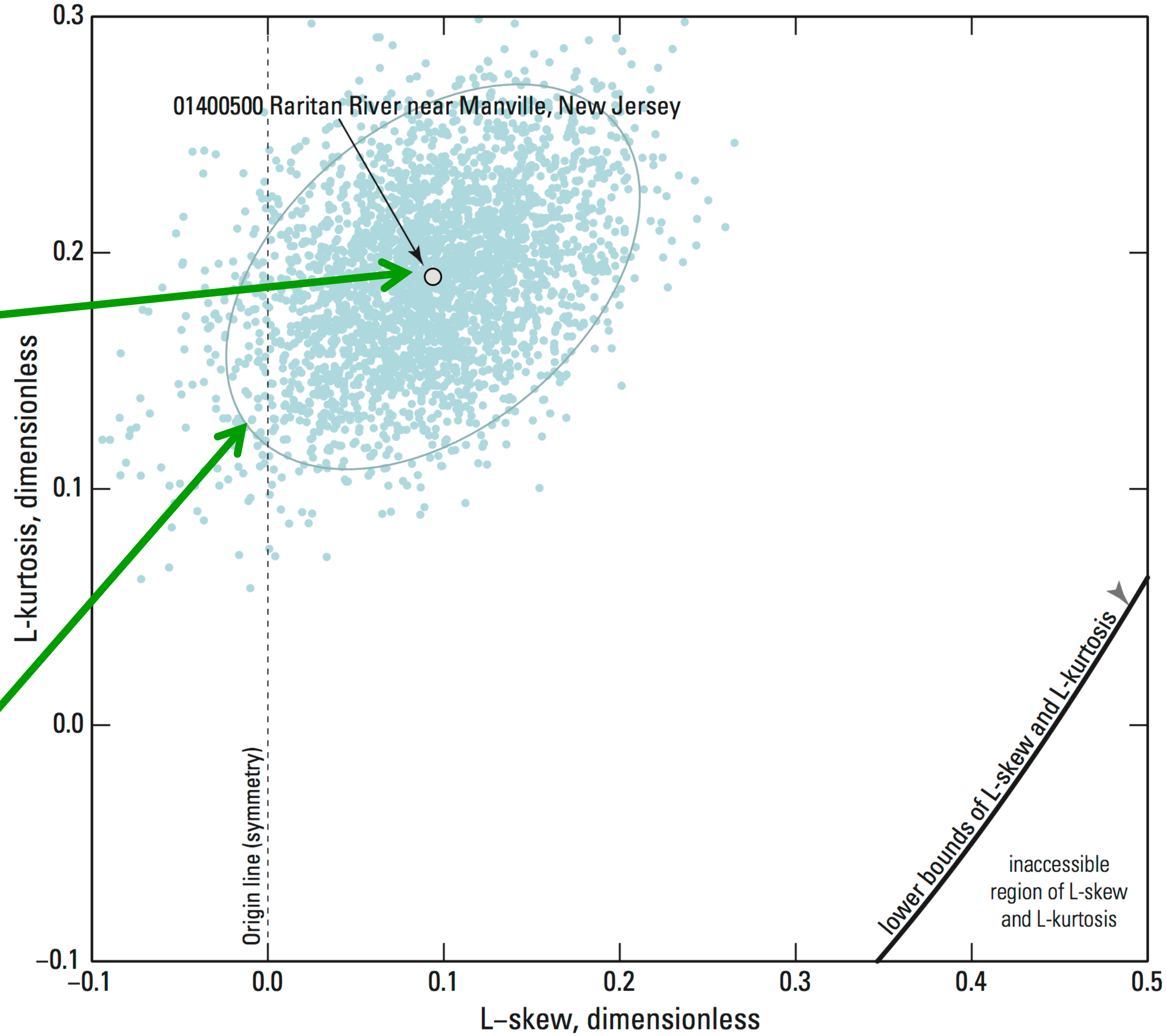


Raritan River LMR Diagram

Sample L-skew and L-kurtosis shown for Raritan River.

Monte Carlo simulation and ellipse for 90th percentile joint L-skew/ L-kurtosis domain.

We will see on next slide that distributions have distinguishably different appearance in the L-skew/L-kurtosis domain.



EXPLANATION

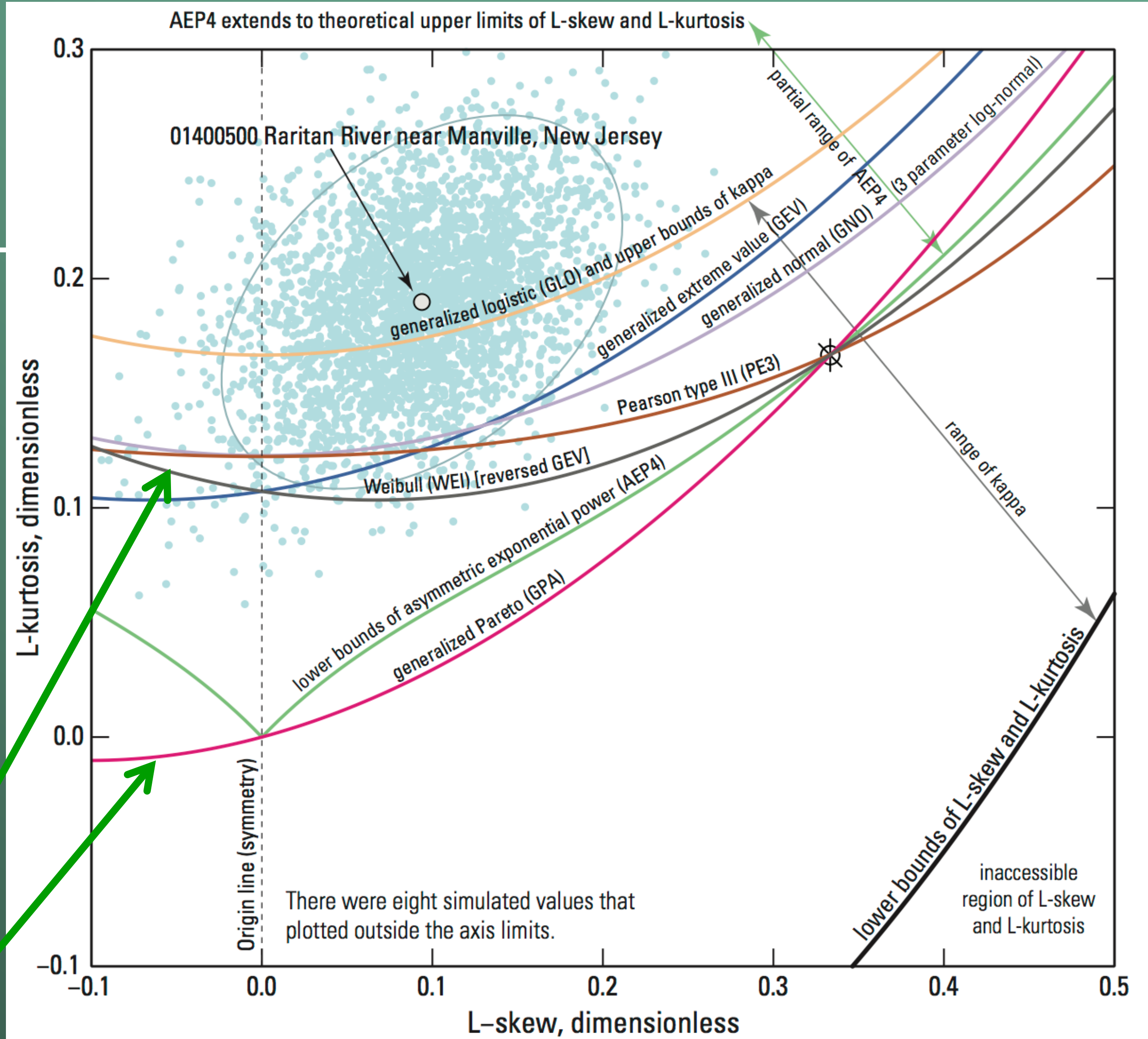
Simulated value based on systematic record—Sample variance-covariance matrix of L-moments used in multivariate-normal simulation of size 3,000 with some values not shown as indicated by note. The elliptical region demarks an approximate 90-percent confidence region based on covariance structure of the size 3,000 simulation.

Systematic record (1904–1906, 1909–1915, 1922–2014) — L-moments computed for t record in conventional approach.

Raritan River LMR Diagram

3-p distributions have unique trajectories of L-skew and L-kurtosis.

- GLO, GEV, GNO, PE3 pass by being inside the ellipse for the L-skew of the Raritan River.
- WEI is close but outside.
- **GPA is outside!**
- AEP4 and WAK pass because each fit to L-kurtosis.



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- **Systematic record (1904–1906, 1909–1915, 1922–2014)** — L-moments computed for t record in conventional approach.

Raritan River — Goodness-of-Fit (GoF)

- **GoF is immensely challenging with no optimality for VL-AEPs.**
  - Sample sizes involved nearly assure zero observations of the phenomena that the analyst is trying to predict.

Goodness-of-fit statistic	Conceptual under-pinning	AEP4	Three-parameter probability distribution type					
			GEV	GLO	GNO	GPA	PE3	WEI
01400500 Raritan River at Manville, New Jersey			Relative ranks amongst the statistics listed by statistic					
Cramér–von Mises statistic	CDF	2	4	1	3	7	5	6
Kolmogorov–Smirnov statistic	CDF	1–2	4	1–2	4	7	4	6
Moran–Darling statistic	CDF	2	5	1	3	--	4	--
Akaike Information Criterion (AIC)	PDF	2	5	1	3	--	4	--
Delta L-kurtosis	QDF[1]	1	5	2	3	7	4	6

- **Ranks for the six 3-parameter dists. + AEP4 (asym. exp. power).**
  - Most 3-parm+ distributions pass GoF hypothesis tests.
  - Delta L-kurtosis pushes the fit question to the next highest shape parameter. (Reason AEP4 ranks over GLO.)

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These metrics do not answer the fundamental question: Is a given fit inclusive of distribution form *good enough*?

Raritan River — Results in Plain Speech

- **The study is designed:**
  - To explore VL-AEP estimation from a perspective of multiple distributions and parameter fitting methods,
  - To quantify two uncertainties (**sampling uncertainty [σ_s]** and **distribution choice [σ_{dc}]** as standard deviations in $\log_{10}$), and
  - Not to recommend prescriptive flows for either the Raritan or Potomac Rivers.
- **Plain Speech Example of a VL-AEP Estimate:**
  - Of six three-parameter distributions, the GLO has best ‘fit.’ (*However, this statement implies little in terms of most suitable or good enough for VL-AEP.*)
  - “The 10^{-4} AEP estimate based on the GLO distribution is 373,600 ft^3/s (90-percent conf. interval 103,600 to 2,793,000 ft^3/s based on $\sigma_s = 0.442 \log_{10}$) with $\sigma_{dc} = 0.250 \log_{10}$.”

Future Tasks (2 – 4)

2. Nonstandard flood information (regional + paleo + climate + historical sources) use in PE3-EMA (expected moments algorithm).

3. Non-stationarity (land use, regulation, climate change).

4. Training seminar led by USGS at NRC HQ in late summer 2017 to review Tasks 1, 2, and 3.

Future Research Directions for VL-AEP

- **Regional skew update for Nation:**
  - Substantial non-USGS sponsorship needed.
  - PE3-EMA + vastly improved “low-outlier detection” + more peak data since late 1970s (Bulletin 17B).
  - Improved error estimates for weighted skew computations — critically important for short-record streamgages.
  - Include L-skew + L-kurtosis — Value added component to assess regional distribution forms and (or) strength of the Pearson type III for VL-AEP.

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Distribution shape parameters (skewness and kurtosis) control distal tail estimates for very low AEPs (VL-AEPs).

Future Research Directions for VL-AEP

- **EMA extension to other three-parameter distributions:**
  - Generalized Extreme Value (GEV) and thus Weibull is accessed
  - Generalized Normal and thus the log-Normal3 is accessed
- **Unification of theory for historical data (censoring) for L-moments.** (We use L-moments by left-censoring by indicator variable within this project.)
- **Method of MPS[1] needs further review.** (Sampling properties appear similar to L-moments.)
- **Further study of four-parameter distributions**
  - Kappa + Asymmetric Exponential Power distributions as a "joint family" canvasing the entire L-skew / L-kurtosis domain.

[1] Maximum product of spacings or "maximum spacing estimation."

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