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MEETING SUMMARY DISTRIBUTION

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Memo4
ccp

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

DEC 28 1978

Docket Nos.: 50-358, 50-352/353, 50-367, 50-373/374, 50-387/388, 50-410, 50-322, 50-397

APPLICANT: Members of Mark II Owners Group

SUBJECT: MEETING WITH MARK II OWNERS GROUP TO DISCUSS THE STAFF'S MARK II CONTAINMENT ACCEPTANCE CRITERIA RELATED TO SUBMERGED STRUCTURE DRAG LOADS - NOVEMBER 15, 1978

Background

The Mark II Owners Group notified the staff at a meeting held on October 19, 1978 of certain exceptions they would propose with respect to our pool dynamic loads acceptance criteria. The purpose of the meeting on November 15, 1978 was to discuss their proposed exceptions to our criteria related to submerged structure drag loads and the bases for these exceptions.

An attendance list and meeting handouts are enclosed.

Summary

Exceptions to our acceptance criteria for submerged structure drag loads were proposed in a number of areas, including: 1) LOCA/SRV water jet loads; and 2) LOCA/SRV air bubble loads.

For our criteria related to LOCA water jet loads, the Mark II owners provided the results of their analyses to determine the significance of the acceleration drag loads. These analyses include a disturbance of the jet flow field by the target such that a zero normal boundary condition is satisfied. This is in contrast to the staff's criteria which assumes that the flow field is not disturbed by the target. Their treatment of the flow field shows a potential for a large reduction in the staff's criteria for LOCA jet induced acceleration drag loads. However, the staff raised a number of questions regarding assumptions implicit in this new methodology. We require that these questions be resolved before these loads can be considered negligible.

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The Mark II owners showed films of scaled tests to support their argument that the penetration of the water jet is limited, since the jet assumes a mushroom shape upon entrance into the pool. They maintain that a substantial fraction of the kinetic energy of the jet is converted into vorticity. Thus they conclude that a good representation of the flow field can be generated by an analytical model based on the movement of a vortex ring. This model is currently under development by the Mark II owners. The staff stated that this approach appears promising. However, it is doubtful that we would receive documentation describing the model in time for its use by the lead plant applicants.

The Mark II owners also proposed to take exception to several of our criteria associated with LOCA air bubble submerged structure drag loads. These included acceleration drag coefficients, the equivalent uniform flow velocity and modification of drag coefficients to account for interference effects. A summary of our related discussions is provided below.

Our criteria for acceleration drag coefficients used in the calculation of air bubble associated drag loads are based on a bounding approach. A value of three times the standard drag coefficient was chosen to bound both the situation of uniform flow characteristic of most pool swell phenomena and the oscillating flow that is characteristic of SRV actuation. The Mark II owners proposed a modification to our criteria wherein they would specify separate criteria for uniform and oscillating flow fields. For uniform flow fields, unpublished data of Sarpkaya was referenced which indicates that an upper bound of 1.4 can be justified for the standard drag multiplier. For oscillating flow fields, they propose direct application of the Keulegan-Carpenter corrections for standard drag coefficients. The staff stated that the proposed approach appeared reasonable and that the unpublished date of Sarpkaya should be submitted to substantiate their proposed uniform flow field criteria.

The staff's criteria specify that the maximum velocity "seen" by the structure should be used in submerged structure drag calculations. The Mark II owners proposed use of the velocity at the center of the structure. The results of their analyses were provided to support their view that this methodology satisfies our criteria. In addition, they discussed problems they would have in applying our criteria. The flow field may be very complicated due to the presence of multiple sources and sinks. Thus, determination of the point of maximum velocity may be very costly. They proposed that sensitivity studies be performed by each A/E to define a multiplier that may be easily applied to the velocity calculated at the target geometric center. The staff identified several problems associated with this approach. First, we stated that their argument for the velocity at the center of the target being a maximum did not cover the case of offset targets. The approach of establishing a simplified approach such as defining a multiplier to the velocity at the center of the target appears reasonable. However, we stated that this should be pursued generically instead of on a plant unique basis.

The staff criteria specifies that for certain conditions, a multiplier of 4 times the standard drag coefficient be used to account for interference of nearby structures. The Mark II owners proposed performing analysis on a plant unique basis. Data were referenced to substantiate their view that our criteria is unrealistically high. The staff stated that an alternate approach to our interference criteria, based on references to available data, appeared reasonable. However, the references should be clearly specified. In addition, generic guidelines should be developed to cover those cases which involve extrapolation to conditions outside those tested. Again we stated that exceptions to our criteria should be approached on a generic rather than plant unique basis.

The staff stated the need for a follow up meeting on this topic of submerged structure drag loads, to enable us to resolve some of the concerns raised in this meeting.

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Clifford Anderson, A-8 Task Manager
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November 15, 1978
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G. Bienkowski	Princeton-(BNL)
C. Anderson	NRC/DSS/CSB
T. M. Su	NRC/DSS/CSB
J. A. Kudrick	NRC/DSS/CSB
L. E. Lasher	GE
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P. D. Hedgecock	WPPSS
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W. M. Davis	GE
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T. G. Peterson	GE
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D. Toner	Lilco
W. R. Butler	NRC/DSS/CSB

NRC MARK II

LEAD PLANT

ACCEPTANCE CRITERIA

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SUBMERGED STRUCTURE LOADS

DISCUSSION OUTLINE

- LOCA WATER JET
- LOCA AIR BUBBLE
- SRV AIR BUBBLE

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LOCA WATER JET

NRC CRITERIA	LEAD PLANT POSITION
III A.1.	
(A) ACCELERATION DRAG IMPINGMENT FACTOR. $R_{A/S} \sim 6$	(A) MORE REALISTIC BOUNDARY CONDITIONS GIVE $R_{A/S} \sim -3/64$
(B), (C) POTENTIAL TO ACCOUNT FOR MOVING JET FRONT	(B), (C) POTENTIAL FROM RING VORTEX MODEL

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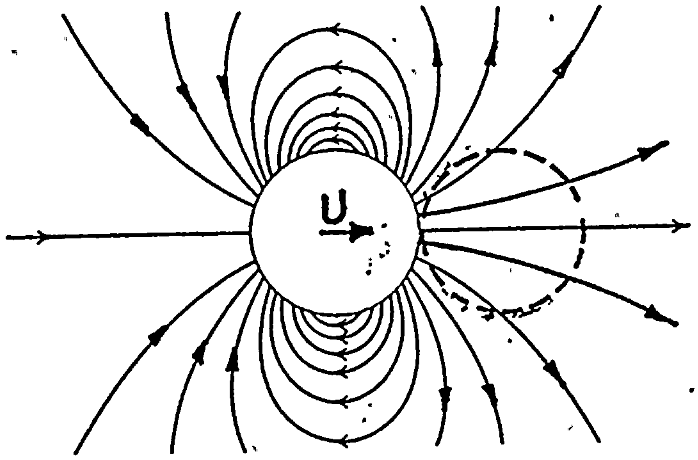
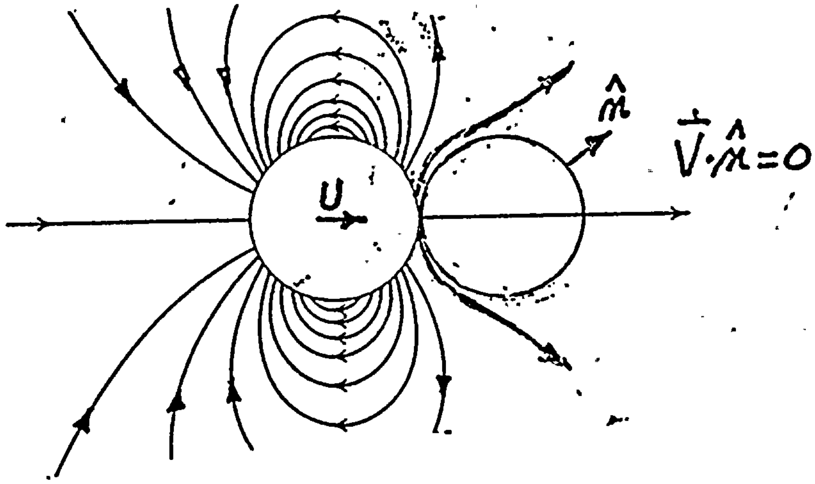
ACCELERATION DRAG PRIOR TO IMPACT

OF A STRUCTURE AND TRANSLATING

SPHERE



FLOW FIELD UNDISTURBED BY STRUCTURE: $R_{A/S} \sim 6$



FLOW FIELD SATISFIES ZERO NORMAL VELOCITY BOUNDARY CONDITIONS: $R_{A/S} \sim -3/64$

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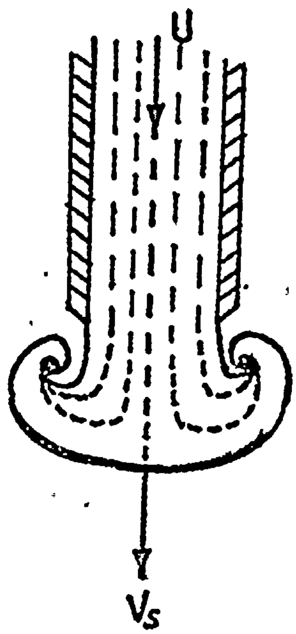
WATER JET EXPERIMENTS

- FILMS OF 1/13.3 AND 1/4 SCALE TESTS SHOW MUSHROOM NOT BULLET.



- JET KINETIC ENERGY CONVERTED INTO VORTICITY.
- MOVEMENT OF VORTEX RING PRODUCES FLOW FIELD IN POOL.

CONCLUSION: RING VORTEX MODEL CAN BE EXPECTED TO PREDICT VENT CLEARING.

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WATER JET CONCLUSIONS

- ACCELERATION DRAG FACTOR AT IMPINGEMENT IS 128 SMALLER THAN CRITERION AND IS ATTRACTIVE.

- FLOW FIELD GENERATED BY RING VORTEX MODEL APPROPRIATE FOR LOAD CALCULATIONS.

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LOCA AIR BUBBLE (I)

NRC CRITERIA	LEAD PLANT POSITION
(A) INCREASED LOAD FROM BUBBLE ASYMMETRY.	(A) CONSENT
(B) MODIFY DRAG COEFFICIENT BY $C_D'/C_D = 3$	(B) USE EXPERIMENTAL LITERATURE TO DETERMINE APPROPRIATE VALUE OF C_D'/C_D

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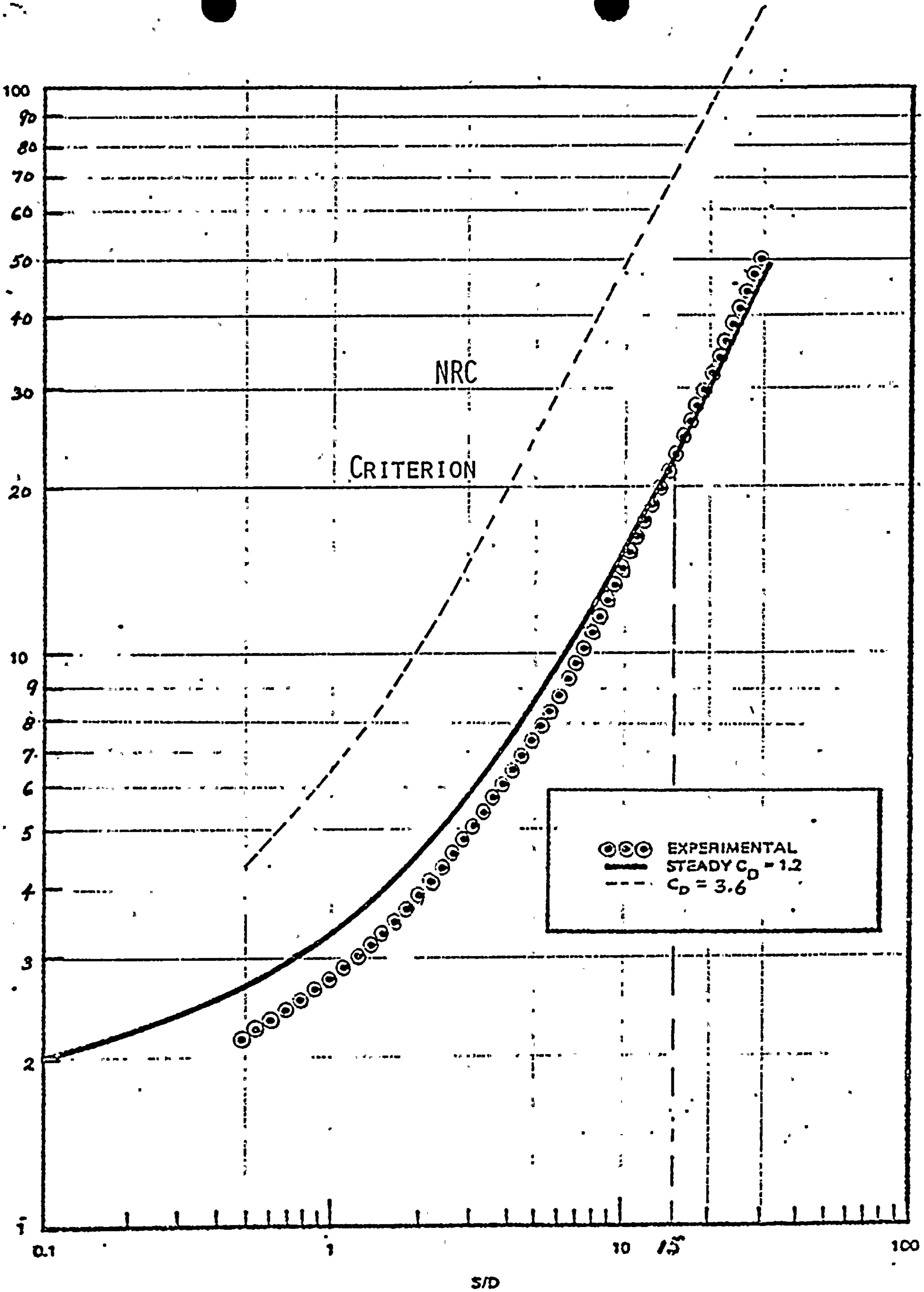

Figure 1. Total Force on Cylinders, Constant Acceleration (Sarpkaya and Garrison, 1963)

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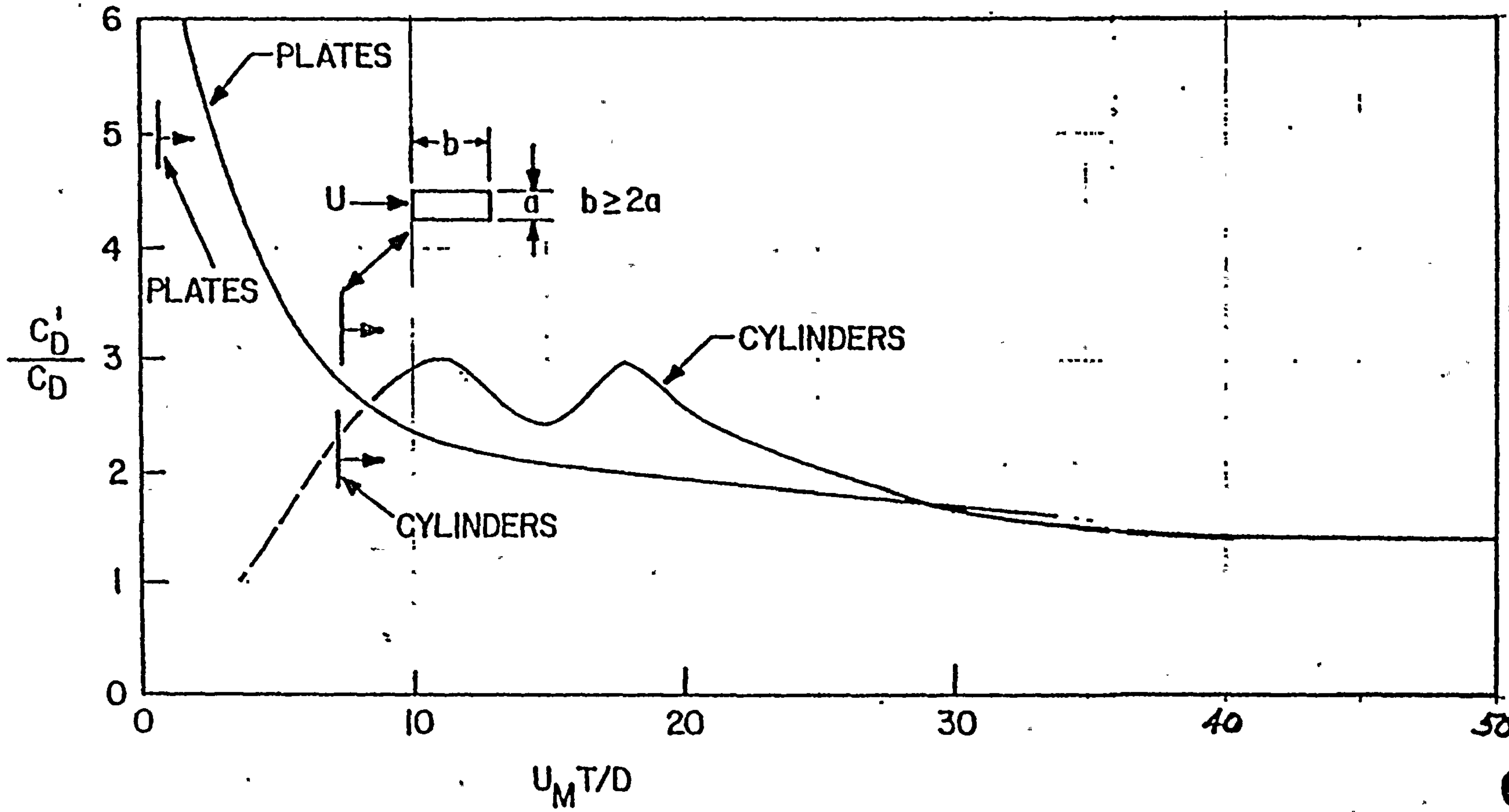
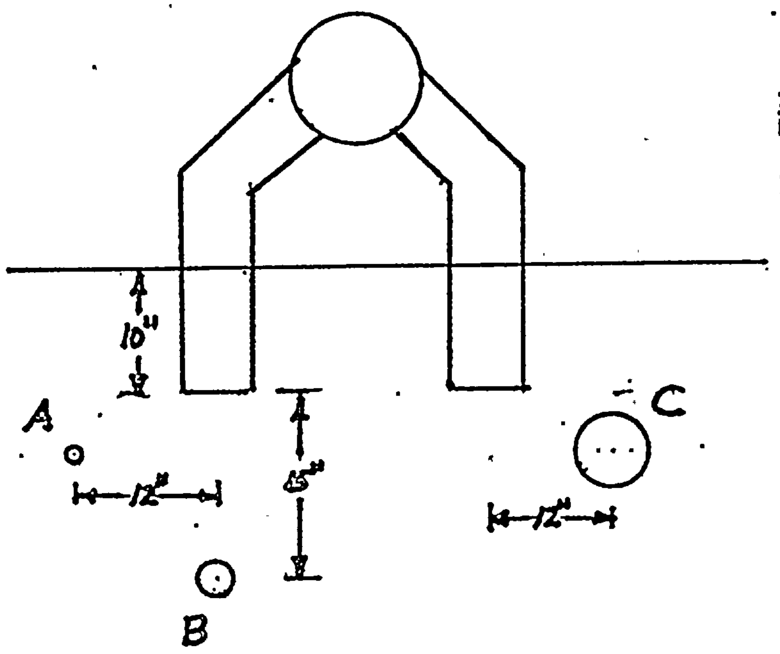


FIGURE C-1 ACCELERATION CORRECTION FOR STANDARD DRAG COEFFICIENT

QSTF RESULTS FOR LOCA AIR BUBBLE

TARGET	MODEL PREDICTION(LBF)	QSTF MEASUREMENTS(LBF)
A	50	3
B	30	5
C	290	65

TARGET LOCATIONS IN QSTF



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LOCA AIR BUBBLE LOADS CONCLUSIONS

- O ANALYTICAL MODEL VERY CONSERVATIVE

- O ANALYSIS OF APPLICABLE EXPERIMENTAL DATA INDICATES THAT 1.4 IS AS UPPER BOUND FOR C_D'/C_D (RATHER THAN 3)*. APPROPRIATE FACTORS WILL BE CALCULATED DEPENDENT UPON INDIVIDUAL PLANT GEOMETRY.

- O USE OSCILLATING FLOW RESULTS AT CORRESPONDING KEULEGAN-CARPENTER NUMBER FOR POOL-SWELL.

* PUBLISHED AND UNPUBLISHED WORK OF T. SARPKAYA.

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LOCA AIR BUBBLE (II)

NRC CRITERIA	LEAD PLANT POSITION
III B.1.	
(c) TAKE MAXIMUM VALUES OF FLOW FIELD "SEEN" BY STRUCTURE	(c) USE CENTER LOCATION FOR ACCELERATION. DEFINE MULTIPLIER TO BE APPLIED TO VELOCITY AT CENTER

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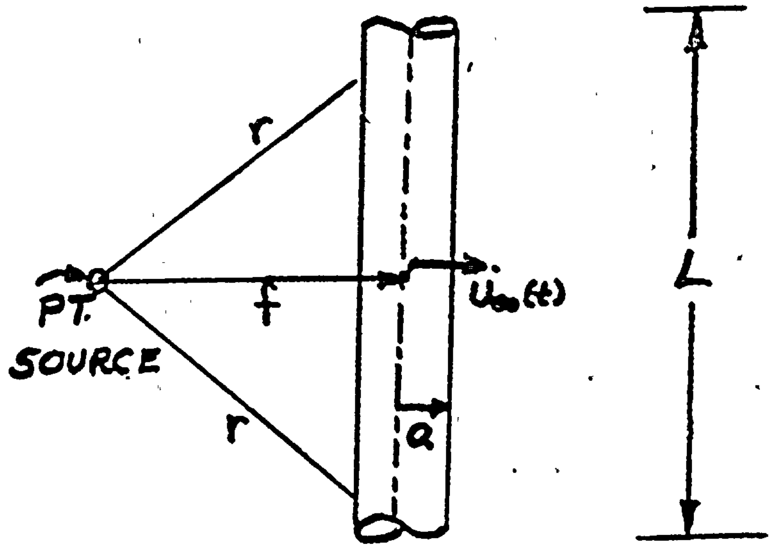
EQUIVALENT UNIFORM FLOW ACCELERATION



$$\frac{F_A}{\rho \cdot 2\pi a^2 \dot{U}_\infty} = \left\{ \frac{L}{r} f \right.$$

- $\rightarrow L$ (MOODY) — $[f/r < 1 \text{ for } L < \infty]$
- $\frac{L}{r} f$ (EXACT)
- $\rightarrow 2F$ (TUNG EXACT FOR $L \rightarrow \infty$) — $[\frac{L}{r} \rightarrow 2 \text{ as } L \rightarrow \infty]$

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EQUIVALENT UNIFORM FLOW VELOCITY

- USE OF MAXIMUM VELOCITY "SEEN" BY STRUCTURE IS IMPRACTICABLE.

  - FLOW FIELD MAY BE VERY COMPLICATED CONTAINS MULTIPLE SOURCES AND SINKS
  - DETERMINATION OF POINT OF MAXIMUM VELOCITY MAY BE VERY COSTLY (SCHEDULE AND RESOURCES).

- EACH A/E TO DO SENSITIVITY STUDY TO DEFINE A MULTIPLIER THAT MAY BE APPLIED TO THE VELOCITY CALCULATED AT GEOMETRIC CENTER AND TO ASSESS THE IMPORTANCE OF THIS EFFECT.

EQUIVALENT UNIFORM FLOW FIELD CONCLUSIONS

- Acceleration Drag Load is Conservatively Estimated by Using Acceleration at Center of Structure.

- Impracticable to Identify Maximum Velocity Point for Each Structure.

- Sensitivity Study to Define.
  Multiplier to Apply to Center Velocity.

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LOCA AIR BUBBLE (III)

NRC CRITERIA	LEAD PLANT POSITION
III. B. 1	
(D) To Account for (E) Interference of Nearby Structures Multiply C_D and Va by 4	(D) Detailed Analysis of (E) Interference Effects is Being Performed.
(F) Change Coeffienct for Fall Back Load	(F) Consent

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INTERFERENCE EFFECTS

- STANDARD DRAG
  - DALTON AND SZABO EXPERIMENTS ON GROUP OF 3 CYLINDERS AT VARIOUS SPACINGS AND ORIENTATIONS SHOW C_D ALWAYS BOUNDED BY 1.2.
  - ZDRAVKOVICH COMPREHENSIVE HISTORICAL REVIEW SHOWS THAT IN MOST CASES INTERFERENCE REDUCES STANDARD DRAG. IN NO CASE IS THE STANDARD DRAG INCREASED MORE THAN 30% EXCEPT FOR SIDE BY SIDE ARRANGEMENT OF CYLINDERS ALMOST TOUCHING ONE ANOTHER, WHEN THERE IS A 95% INCREASE.
- ACCELERATION DRAG
  - INTERTIA COEFFICIENT CAN BE INCREASED DUE TO INTERFERENCE.
  - EACH A/E IS DETERMINING APPROPRIATE FACTORS.

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INTERFERENCE EFFECTS CONCLUSION

- A DETAILED ANALYSIS OF INTERFERENCE EFFECTS IS BEING DONE AS SUGGESTED IN THE ACCEPTANCE CRITERION.

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SRV AIR BUBBLE

NRC CRITERIA	LEAD PLANT POSITION
III. B. 2.	
(A) Include Standard Drag Magnitude Check	(A) Consent
(B) Apply Appropriate Constraints Discussed With Loca Air Bubble	(B) Use Oscillating Flow Results to Obtain C_D'. Position is Same On Other Constraints as in Loca Air Bubble Discussion.

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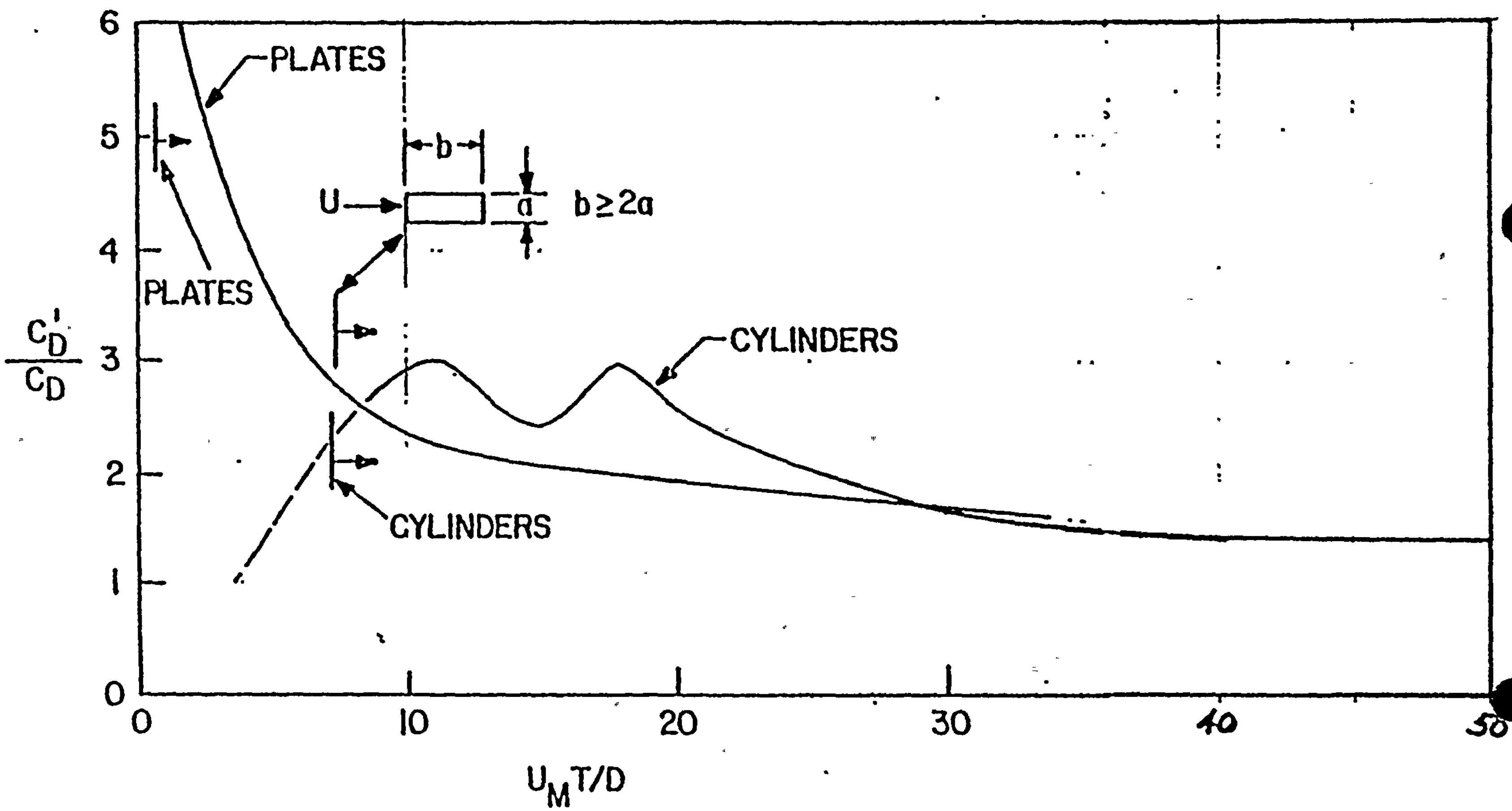

FIGURE C-1 ACCELERATION CORRECTION FOR STANDARD DRAG COEFFICIENT

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SRV AIR BUBBLE CONCLUSIONS

- USE OSCILLATING FLOW RESULTS AT APPROPRIATE KEULEGAN - CARPENTER NUMBER TO DETERMINE C_D/C_D.

- LOCA AIR BUBBLE CONCLUSIONS ON EQUIVALENT UNIFORM FLOW FIELD AND INTERFERENCE EFFECTS APPLY HERE ALSO.

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<u>RATIO OF ACCELERATION TO STANDARD DRAG FOR JETS</u>

It is evident from the quarter-scale model tests and the pictures taken at the Stanford Research Institute that the LOCA water jet does not travel as a bullet.

For the sake of argument, if the jet were to be modelable by a moving source of varying intensity, then the equation (1) of the reviewer (Reference 1) is correct. However, the subsequent arguments lead to overly conservative results. The force on a target at the time the jet front touches the target, if calculated as proposed by the reviewer, does not account for the presence of the target. To incorporate the correct target boundary conditions, one must represent the target with appropriate singularities and take the mutual images of these singularities in the axisymmetric half body (representing the jet slug) and the target. Without such a procedure, the condition of no flow through the target boundary is not satisfied.

Let us demonstrate the use of a correct procedure for a doublet approaching a rigid sphere. This corresponds to the case of two spheres approaching each other along the lines joining their centers, or to the case of one sphere approaching the other.

This case has been treated in the literature (see e.g., References 2 and 3).

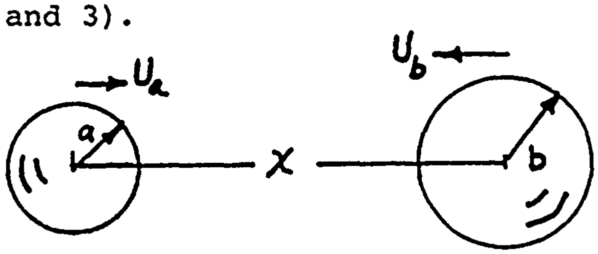

Figure 1

The correct fomulation of the potential function through the use of many image doublets yields the kinetic energy,

$$T = 2\pi\rho \frac{a^3 b^3}{x^3} U_a U_b + \frac{\pi\rho}{3}(a^3 U_a^2 + b^3 U_b^2)\left(1 + \frac{3a^3 b^3}{x^6}\right) \qquad (1)$$

Now assume that sphere "b" is at rest, i.e., $U_b = 0$ then

$$T = \frac{\pi\rho}{3} a^3 U_a^2 \left(1 + \frac{3a^3 b^3}{x^6}\right) \tag{2}$$

The use of the equation of Lagrange (Reference 3) together with Equation (2) yields the force at the time of the touching of two spheres, i.e., when x = a + b.

$$F_i = \frac{d}{dt}\left(\frac{\partial T}{\partial U_a}\right) - \frac{\partial T}{\partial x} \tag{3}$$

Assuming, for the sake of simplicity, a = b, one has

$$F = -\frac{3}{64}\pi\rho a^2 U_a^2 = -\frac{9}{256}\left(\frac{4}{3}\pi\rho a^3\right)\frac{U_a^2}{a} \tag{4}$$

In terms of the acceleration volume $\forall_A = 1.5 \cdot \frac{4}{3}\pi a^3$, one has

$$F_A = -\frac{9}{256} \cdot \frac{2}{3}\rho \forall_A \frac{U_a^2}{a} \tag{5}$$

The standard drag for a uniform velocity U_a past sphere"b", would have been

$$F_d = \frac{1}{2}\rho A_p C_D U_a^2 \tag{6}$$

The ratio of the forces is $R_{A/S}$

$$R_{A/S} = \frac{3}{64}\frac{\forall_A}{C_D A_p a} \tag{7}$$

It should be noted in passing that the sphere "a" is repelled by the sphere "b".

Let us now assume that the flow field created by the sphere "a" is not disturbed by sphere"b". Then the potential for a moving sphere "a" is given by

$$\phi = \frac{U_a a^3}{2r^2} \cos\theta \tag{8}$$

The velocity V_r is calculated from

$$V_r = -\frac{\partial\phi}{\partial r} = U_a \frac{a^3}{r^3} \cos\theta$$

Also

$$\frac{\partial V_r}{\partial t} = -\frac{\partial V_r}{\partial r} \cdot U_a = 3 U_a^2 \frac{a^3}{r^4} \cos\theta \tag{9}$$

At the time of contact $\frac{\partial V_r}{\partial t} = + 3\, U_a{}^2/a$ the force would have been

$$F_A = \rho \forall_A (3 U_a^2)/a \tag{10}$$

When compared with the standard drag, one would have

$$R_{A/s} = 6 \frac{\forall_A}{C_D A_p a} \tag{11}$$

Equation (11) is quite comparable to the Equation (5) of the reviewer, but it is overly conservative for the reasons cited above.

REFERENCES

1. Bienkowski's Review of General Electric Reports, NEDO-21730, NEDE-21472, and NEDO-21471.

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3. Milne-Thomson, Theoretical Hydrodynamics, MacMillan, 1960.