

**Attachment B**

**NWMI-2014-RPT-006, *MCNP 6.1 Validations with Continuous Energy ENDF/B-VII.1*  
*Cross-Sections* (Rev. 0) (Public Version)**

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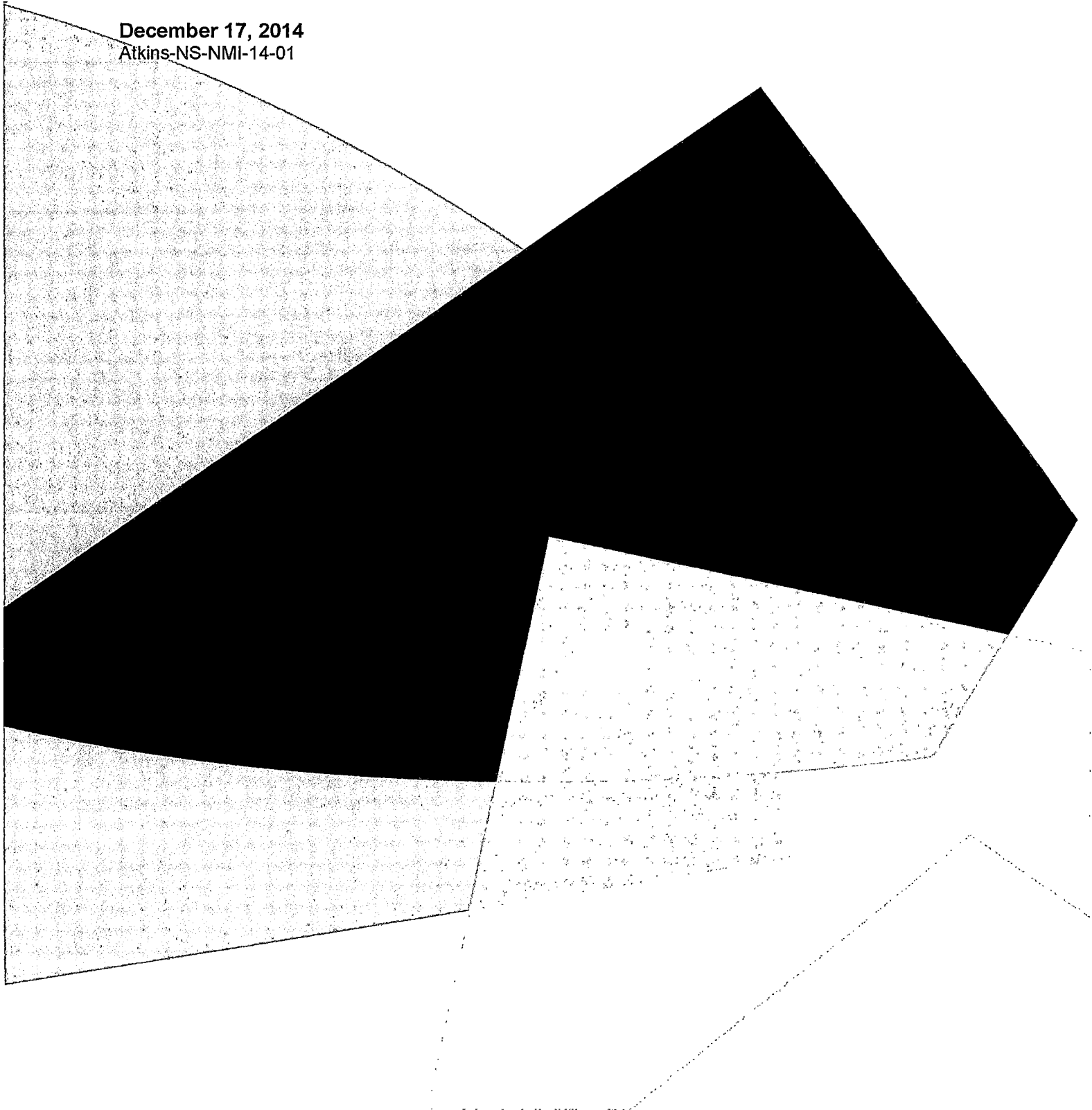
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# MCNP 6.1 Validations with Continuous Energy ENDF/B-VII.1 Cross-Sections

Northwest Medical Isotopes, LLC

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## List of Terms

ANECF	Average Neutron Energy Causing Fission [MeV]
AoA	Area of Applicability
cm	centimeter
ENDF	Evaluated Nuclear Data File
$H/^{235}U$	Ratio of hydrogen to $^{235}U$ number densities in a material
H/X	Ratio of hydrogen to fissile nuclide number densities in a material
HEU	High Enrichment Uranium
IEU	Intermediate Enrichment Uranium
in	inch
LANL	Los Alamos National Laboratory
LEU	Low Enrichment Uranium
LTL	Lower Tolerance Limit
MeV	Million Electron Volt
MoS	Margin of Subcriticality
NPM	Non-Parametric Margin
NWMI	Northwest Medical Isotopes, LLC
pcm	per cent mille
USL	Upper Subcritical Limit
VNIIEF	All Russian Research Institute of Experimental Physics (Russian)
ZAID	Nuclide identifier

# Executive Summary

This report documents the methodology and results for the MCNP 6.1 code system validation for its use with the Northwest Medical Isotopes, LLC (NWMI) applications. Criticality safety experiments were selected from the International Handbook of Evaluated Criticality Safety Benchmark Experiments that adequately match the uranium enrichment, geometry, moderator, reflector, and neutron energy relevant to the NWMI applications. The bias results demonstrate that the calculated values matched the reality of the experiments. The final validation is expressed as an Upper Subcritical Limit (USL) calculated using the statistical accumulation of the experiment's bias and bias uncertainty. MCNP 6.1 calculations of NWMI applications should result in values less than [Proprietary Information] to ensure criticality safety of the application.

The primary focus of this validation is to determine the bias and bias uncertainty for intermediate enriched uranium (IEU) systems (about 20%), however sufficient experiments for low enriched uranium (LEU) and high enriched uranium (HEU) are included to demonstrate that there is no variation in the USL with varying enrichment. Similarly, the primary focus of this validation is upon thermal neutron energy systems, however sufficient experiments for intermediate and fast energy experiments are included to demonstrate that there is no variation in the USL with increasing neutron energy.

The recommended USL for the NWMI applications with approximately 20% wt. %  $^{235}\text{U}$  material is [Proprietary Information].

	Bias	USL
Low Enriched Uranium	[Proprietary Information]	[Proprietary Information]
Intermediate Enriched Uranium	[Proprietary Information]	[Proprietary Information]
High Enriched Uranium	[Proprietary Information]	[Proprietary Information]
Combined Enrichments	[Proprietary Information]	[Proprietary Information]

The physical parameters of the NWMI applications bounded by this validation are shown below.

Parameter	Area of Applicability
Fissile Material	[Proprietary Information]
Fissile Material Form	[Proprietary Information]
H/ $^{235}\text{U}$ ratio	[Proprietary Information]
Average Neutron Energy Causing Fission (MeV)	[Proprietary Information]
Enrichment*	[Proprietary Information]
Moderating Materials	[Proprietary Information]



Parameter	Area of Applicability
Reflecting Materials	[Proprietary Information]
Absorber Materials	[Proprietary Information]
Geometry	[Proprietary Information]

# 1. Introduction

Nuclear criticality safety analysis is performed for fissile material systems for the Northwest Medical Isotopes, LLC facility. The nuclear criticality safety analysis establishes the nuclear safety operating limits for the systems and operations. Calculation methods are used to provide an estimate of criticality conditions and the margin of subcriticality for the systems and operations under evaluation. The computational methods predict the neutronic behavior of the system and operation. However, certain approximations are inherent in the computer code used including inexact neutron cross section data and statistical uncertainty.

Validation compares the computational method with documented critical experiments to determine any bias that might exist between the calculated reactivity of a given system and the actual conditions. Validation is a process that determines and establishes computational method applicability, adequacy, and uncertainty.

This report documents the MCNP 6.1 validation. This report includes discussions of input files that model the critical experiments chosen for validation of the MCNP 6.1 computer code system for Northwest Medical Isotopes, LLC operations, statistical evaluation of the calculation results, and the code bias and bias uncertainty. The validation is conducted using the ENDF/B-VII.1 continuous energy group cross section library. The validation is for use by Nuclear Criticality Safety personnel in performing analysis and evaluation of various facility / site activities involving enriched uranium. Through the selection and validation of appropriate benchmark critical experiments and analysis, this report will validate the computational methods for an entire range of normal and off-normal operating conditions involving heterogeneous and homogeneous fissile material. Toward that end, critical experiments are modeled as reported in NEA/NCS/DOC (95)03 (Reference 1).

## 1.1. Limits of Applicability

The parameters associated with the critical experiments documented in this report will be used to set the Area of Applicability (AoA) for applications modeling fissile material operations. Applications using the bias and bias uncertainty established for this experiment data set must use the modeling conventions described in Table 1 or have the Upper Subcritical Limit (USL) reduced. The benchmark calculations were performed on the Atkins Linux computer cluster (Reference 3), therefore the validation conclusions herein are applicable to this computer.

## 2. MCNP 6.1 Code

The verification of MCNP 6.1 has been completed on the Atkins' system. The Atkins' system is a computer cluster composed of several servers that use Intel processors running the Fedora Linux operating system. MCNP 6.1 has been installed in the read only disk area; the installation has been verified with the execution of the sample problems (Reference 2).

### 2.1. MCNP Summary

MCNP is a general-purpose Monte Carlo N-Particle code that can be used for neutron, photon, electron, or coupled neutron/photon/electron transport, including the capability to calculate eigenvalues for critical systems. The code treats an arbitrary three-dimensional configuration of materials in geometric cells bounded by first- and second-degree surfaces and fourth-degree elliptical tori.

Pointwise cross-section data are used. For neutrons, all reactions given in a particular cross-section evaluation (such as ENDF/B-VII.1) are accounted for. Thermal neutrons are described by both the free gas and  $S(\alpha,\beta)$  models. For photons, the code accounts for incoherent and coherent scattering, the possibility of fluorescent emission after photoelectric absorption, absorption in pair production with local emission of annihilation radiation, and bremsstrahlung. A continuous-slowing-down model is used for electron transport that includes positrons, k x-rays, and bremsstrahlung, but does not include external or self-induced fields.

Important standard features that make MCNP very versatile and easy to use include a powerful general source, criticality source, and surface source; both geometry and output tally plotters; a rich collection of variance reduction techniques; a flexible tally structure; and an extensive collection of cross-section data.

### 2.2. ENDF/B-VII.1 Cross Section Library

The ENDF/B-VII.1 cross-section library is used for the critical experiment calculations in this validation analysis. This library contains data for all nuclides (more than 300). A list of the elements used in this evaluation is provided in Table 1. Where the library does not contain a "natural" mixture of isotopes, the isotopic fractions are included. All of these isotopes were identified with the .80c extension in the cases executed for the validation. The graphite (grph.20t) light water (lwtr.20t) and poly (poly.20t)  $S(\alpha,\beta)$  correction are used for graphite, water and hydrocarbon materials respectively.

**Table 1 - Library Definitions for Various Elements**

Element	ZAID	Isotopic Fraction*
Hydrogen	1001	
Boron	5010	0.199
	5011	0.801
Carbon	6000	
Nitrogen	7014	0.9963
	7015	0.0037
Oxygen	8016	

Element	ZAID	Isotopic Fraction*
Sodium	11023	
Magnesium	12024	0.78990
	12025	0.10000
	12026	0.11010
Aluminum	13027	
Silicon	14028	0.92223
	14029	0.04685
	14030	0.03092
Phosphorus	15031	
Sulfur	16032	0.9502
	16033	0.0075
	16034	0.0421
Chlorine	17035	0.7576
	17037	0.2424
Potassium	19039	0.92223
	19040	0.04685
	19041	0.03092
Calcium	20040	0.96940
	20042	0.00647
	20043	0.00135
	20044	0.02087
	20046	0.00004
	20048	0.00187
Titanium	22046	0.08250
	22047	0.07440

Element	ZAID	Isotopic Fraction*
	22048	0.73720
	22049	0.05410
	22050	0.05400
Chromium	24050	0.04345
	24052	0.83789
	24053	0.09501
	24054	0.02365
Manganese	25055	
Iron	26054	0.05845
	26056	0.91754
	26057	0.02119
	26058	0.00282
Cobalt	27059	
Nickel	28058	0.68077
	28060	0.26223
	28061	0.011399
	28062	0.036346
	28064	0.009255
Copper	29063	0.6915
	29065	0.3085
Zinc	30064	0.4917
	30066	0.2773
	30067	0.0404
	30068	0.1845
	30070	0.0061

Element	ZAID	Isotopic Fraction*
Zirconium	40090	0.51450
	40091	0.11220
	40092	0.17150
	40094	0.17380
	40096	0.02800
Niobium	41093	
Molybdenum	42092	0.14530
	42094	0.09150
	42095	0.15840
	42096	0.16670
	42097	0.09600
	42098	0.24390
	42100	0.09820
Silver	47107	0.51839
	47109	0.48161
Cadmium	48106	0.0125
	48108	0.0089
	48110	0.1249
	48111	0.1280
	48112	0.2413
	48113	0.1222
	48114	0.2873
	48116	0.0749
Indium	49113	0.0429
	49115	0.9571

Element	ZAID	Isotopic Fraction*
Gadolinium	64152	0.002
	64154	0.0218
	64155	0.1480
	64156	0.2047
	64157	0.1565
	64158	0.2484
	64160	0.2186
Tantalum	73181	
Tungsten	74180	0.0012
	74182	0.265
	74183	0.1431
	74184	0.3064
	74185	0.2843
Gold	79197	
Uranium	92234	Specified by individual experiments.
	92235	
	92236	
	92238	
*From Chart of the Nuclides, <a href="http://www.nndc.bnl.gov/chart">www.nndc.bnl.gov/chart</a>		

### 3. Validation Methodology

ANSI/ANS-8.1 (Reference 4) requires that calculational methods used for nuclear criticality safety (e.g., determining  $k_{eff}$  of a system or deriving subcritical limits) be validated to determine the appropriate biases and uncertainties for the areas of applicability. The bias and uncertainty represent the numerical difference between the results of modeling critical benchmark experiments with a computer code and the experimental  $k_{eff}$ . These biases may result in either under- or over-predictions of criticality. The bias may be reported as

either a positive or negative bias. A positive bias occurs when the computations tend to report a higher  $k_{\text{eff}}$  than the benchmark experiments (i.e.,  $k_{\text{eff}} > 1.0$ ). A negative bias occurs when the calculated results tend to report a lower  $k_{\text{eff}}$  than the benchmark experiments (i.e.,  $k_{\text{eff}} < 1.0$ ).

ANSI/ANS-8.24 (Reference 6) outlines the validation methodology and documentation used herein while NUREG/CR-6698 (Reference 10) details the calculation algorithms. Biases (and their associated uncertainties) are determined through statistical treatment of the calculated results from criticality benchmark experiments. Weighted single sided lower tolerance limits are used for statistical calculations in this validation report when the calculated results data are normally distributed. A non-parametric method is used when the calculated results data are not from a normal statistical distribution.

When performing calculations to assess the subcriticality of a system or operation, a limit must be established on the calculated  $k_{\text{eff}}$  to ensure that subcriticality is achieved. This limit is defined for the purposes of this validation as the Upper Subcritical Limit (USL). In this validation, the USL is determined by statistical analysis of the calculated  $k_{\text{eff}}$ s from the benchmark critical experiments.

### 3.1. Establishment of an Upper Subcritical Limit (USL)

The purpose of a computer code validation is to determine values of  $k_{\text{eff}}$  that are demonstrated to be subcritical (at or below the USL) for areas of applicability similar to systems or operations being analyzed. The USL is defined as follows:

$$\text{USL} = 1.0 - \text{Bias} - \text{Bias Uncertainty} - \text{Margin of Subcriticality (MoS)}$$

setting  $\bar{k}_{\text{eff}} = 1.0 - \text{Bias}$  and  $K \cdot S_t = \text{Bias Uncertainty}$

gives:

$$\text{USL} = \bar{k}_{\text{eff}} - K \cdot S_t - \text{MoS}$$

where: USL = Maximum subcritical value of  $k_{\text{eff}}$

$\bar{k}_{\text{eff}}$  = weighted mean  $k_{\text{eff}}$  value of the benchmark experiments

$K^*$  = tolerance factor for 95% confidence that 95% of the population is bound

$S_t$  = square root of the pooled variance

$S^2$  = variance about the mean

MoS = margin of subcriticality (0.05 for NWMI)

From this, a  $k_{\text{eff}}$  calculated by the analysis is required to meet the following condition:

$$\text{calculated } k_{\text{eff}} + 2\sigma \leq \text{USL}$$

where  $\sigma$  is the Monte Carlo statistical uncertainty associated with the analysis.

As defined, the USL explicitly incorporates a Margin of Subcriticality, which is required per ANSI/ANS-8.1. The Margin of Subcriticality is an additional safety factor which is applied to the statistically calculated limit (e.g., a lower tolerance limit).

The bias and its associated uncertainty may be represented by one of several statistical methods:

- a weighted, single sided, lower tolerance limit,
- a weighted confidence interval, or
- a non-parametric statistical analysis.

### **3.2. Margin of Subcriticality**

The margin of subcriticality is an administrative addition in  $\Delta k$  applied to nuclear criticality safety calculations. The Margin of Subcriticality (MoS) is site specific and usually contained in the fuel facility license or other regulatory authorization basis. The MoS value for NWMI applications is 0.05.

For systems which are outside the validation area of applicability, an increased margin of subcriticality may be warranted, depending on the specific problem being analyzed. The analyst must document any extrapolation beyond the validation area of applicability and justification must be made for no adjustments to the MoS when extrapolations are made.

### **3.3. Determination of the Area of Applicability**

The area of applicability determination quantifies parameters potentially important to the computational calculation of  $k_{\text{eff}}$ . An area of applicability determination should be performed as a part of every calculation done and compared to the area of applicability of the benchmark experiments used for the code validation. This comparison insures that appropriate benchmark experiments have been selected to determine the USL for the calculation. The area of applicability determination for the benchmark experiments used in this validation has been performed using guidelines consistent with LA-12683 (Reference 5), specifically Appendix E of that report.

### **3.4. Discussion of Statistical Analysis**

A weighted, single sided lower tolerance limit is a single lower limit above which a defined fraction of the true population of  $k_{\text{eff}}$  is expected to lie, with a proscribed confidence and with the defined area of applicability. A lower tolerance limit should be used when there are no apparent trends in the benchmark results. Use of this limit requires the benchmark results to have a normal statistical distribution. If the data does not have a normal statistical distribution, a non-parametric statistical treatment must be used. The method used for analysis of data with a non-normal distribution in this validation is taken from NUREG/CR-6698 (Reference 10).

#### **3.4.1. Normality Testing of Data**

There are several tests which can be performed to determine if data follows a normal distribution. Depending on the size of the data sets used in establishing the areas of applicability, the modified Chi Square test, Kolmogorov-Smirnov test, Lilliefors test or the Shapiro-Wilk test may be utilized. The modified Chi Square and Kolmogorov-Smirnov tests may be used to test for normality regardless of the number of data points. The Lilliefors test for normality is performed for cases with greater than 50 data points, while the Shapiro-Wilk test for normality is performed for cases with less than or equal to 50 data points. The methodology for these tests can be found in NUREG/CR-4604 (Reference 8) and Natrella (Reference 9).

For the modified Chi Square test, the critical experiment data are ordered and grouped into classes. For each class, the data range midpoint ( $m_i$ ) and data point frequency ( $O_i$ ) are recorded. The method of moments is used to estimate the mean ( $\bar{\mu}$ ) and the variance ( $\bar{\sigma}^2$ ):



$$\bar{\mu} = \sum_{j=1}^c \frac{O_j m_j}{n}$$

$$\bar{\sigma}^2 = \left[ \sum_{j=1}^c \frac{O_j m_j^2}{n} \right] - \bar{\mu}^2$$

Where  $c$  is the number of classes in which the experimental data are grouped. The expected values  $E_j$  are then computed for a normal distribution with mean  $\bar{\mu}$  and variance  $\bar{\sigma}^2$ . The test statistic is computed using the following:

$$\chi^* = \left[ \sum_{j=1}^c \frac{O_j^2}{E_j} \right] - n$$

$\chi^*$  is compared to  $\chi_{1-\alpha}^2(c-k-1)$  obtained from Table A4 of Reference 8;  $k$  is the number of unspecified parameters and  $\alpha$  is 0.05. If  $\chi^*$  is less than  $\chi_{1-\alpha}^2(c-k-1)$ , the hypothesis that the data is from a normal distribution is supported.

For the Kolmogorov-Smirnov test, the empirical cumulative distribution function (cdf)  $G(x)$  from the random sample is compared with the hypothesized cdf  $F^*(x)$ . The empirical cdf is a function of  $x$ , which equals the fractions of the observations  $x_i$  that are less than or equal to  $x$  for each  $x$ ,  $-\infty < x < \infty$ . The test statistic is calculated as follows:

$$T^* = \sup_x |F^*(x) - G(x)|$$

The supremum requires comparing  $F^*(x)$  to  $G(x)$  both just before and just after each step in  $G(x)$ . Both  $|F^*(x_i) - G(x_i)|$  and  $|F^*(x_i) - G(x_{i-1})|$  are calculated and  $T^*$  is the largest of the absolute differences over all  $i$ . If  $T^*$  is less than  $w_{1-\alpha}$  (determined from Table A17 in Reference 8), the hypothesis that the data are from a normal distribution is supported.

For the Lilliefors test, the standardized sample values are calculated:

$$z_i = \frac{x_i - \bar{x}}{s}$$

where:  $\bar{x}$  = sample mean  
 $s$  = sample standard deviation

The test consists of letting  $F^*(z)$  be the standard normal cumulative distribution function (cdf) and then comparing it to the empirical cdf of the  $z$ s, denoted by  $G(z)$ . The Lilliefors test statistic is the greatest difference between  $F^*(z)$  and  $G(z)$ , i.e.

$$T^* = \sup_z |F^*(z) - G(z)|$$

$T^*$  is the largest of all values  $|F^*(z_i) - G(z_i)|$  or  $|F^*(z_i) - G(z_{i-1})|$ . Values of  $F^*(z_i)$  are obtained from Table A3 in Reference 8. The table consists of values for the cumulative standard normal distribution.  $T^*$  is compared to  $w_{1-\alpha}$ , obtained from Table A18 of Reference 8. The variable  $w_{1-\alpha}$  is dependent on the sample size as well as the desired level of significance. If  $T^*$  is less than  $w_{1-\alpha}$ , the data are probably from a normal distribution.

For the Shapiro-Wilk test, the sample observations are ordered from smallest to largest. The test statistic is given by:

$$W^* = \frac{b^2}{(n-1)s^2}$$

where:  $b = \sum_{i=1}^k a_i (x^{(n-i+1)} - x^i)$

$s$  = the sample standard deviation

$k = n/2$  if  $n$  is even or  $(n-1)/2$  if  $n$  is odd

$a_i$  = coefficients (which depend on  $n$ ) obtained from Table A19 of Reference 8

$W^*$  is compared to  $w_\alpha$ , obtained from Table A20 of Reference 8. The variable  $w_\alpha$  is dependent on the number of sample observations and the desired level of significance. If  $W^*$  is greater than  $w_\alpha$ , the data are probably from a normal distribution.

If the data does not have a normal statistical distribution, a non-parametric statistical treatment must be used. The method used for analysis of data with a non-normal distribution is taken from NUREG/CR-6698. It should be noted that this approach is more conservative than other methods for dealing with non-normal data distribution, for example calculating a distribution-free confidence interval based on the sign test (Thompson, Savur) as presented in Hollander and Wolfe (Reference 7).

### 3.4.2. Weighted Single Sided Lower Tolerance Limits

If the benchmark experiment results are verified to be part of a normal distribution, a weighted, single sided lower tolerance limit technique may be used to construct an Upper Subcritical Limit (USL) for criticality. The weighted, single sided lower tolerance limit is calculated with a 95% confidence that 95% of the benchmark data lies above it. Thus, a calculation involving a subcritical system would have a 95% confidence that 95% of all calculations performed on it would yield a result less than the tolerance limit. The weighted, single sided lower tolerance limit is calculated using the method presented in NUREG/CR-6698 (Reference 10). The weighted, single sided lower tolerance limit is adjusted by applying a margin of subcriticality to define the USL. The USL is defined by the following:

$$USL = k_{eff} - K^* S_t - MoS$$

where: USL = Maximum subcritical value of  $k_{eff}$

$\bar{k}_{eff}$  = weighted mean  $k_{eff}$  value of the benchmark experiments

$K^*$  = tolerance factor

$S_t$  = square root of the pooled variance

$S^2$  = variance about the mean

MoS = margin of subcriticality (0.05)

$$\bar{k}_{eff} = \frac{\sum \frac{1}{\sigma_i} k_{eff_i}}{\sum \frac{1}{\sigma_i}}$$

and

$$S^2 = \frac{(\frac{1}{n-1}) \sum \frac{1}{\sigma_i^2} (k_{eff_i} - \bar{k}_{eff})^2}{\frac{1}{n} \sum \frac{1}{\sigma_i^2}}$$

$$\sigma_i = \sqrt{(\sigma_s^2 + \sigma_e^2)}$$

$$\bar{\sigma}^2 = \frac{n}{\sum \frac{1}{\sigma_i^2}}$$

$$S_t = \sqrt{(s^2 + \bar{\sigma}^2)}$$

where:  $\sigma_s$  = Monte Carlo statistical uncertainty associated with the calculation,  
 $\sigma_e$  = experimental uncertainty associated with the benchmark experiment,  
 $\bar{\sigma}^2$  = average uncertainty

The statistical uncertainty,  $\sigma_s$ , is the standard deviation calculated by the code and reported in the output for each benchmark experiment. If available, the experimental uncertainty,  $\sigma_e$ , is determined through rigorous evaluation of each benchmark experiment. NEA/NCS/DOC (95)03 documents such evaluations and thus reports an experimental uncertainty.

The tabulated lower tolerance factors (Reference 10) are listed for a maximum of 50 data items, however the evaluation herein uses more data points. Therefore, the lower tolerance factors ( $K^*$ ) for data collections greater than 50 items are derived from Reference 9:

$$K^* = \frac{z_p + \sqrt{z_p^2 - ab}}{a}$$

where

$$a = 1 - \frac{z_\gamma^2}{2(N-1)}$$

$$b = z_P^2 - \frac{z_\gamma^2}{N}$$

And  $z_P$  and  $z_\gamma$  values are the critical values from the normal distribution that is exceeded with specified probability ( $P = 95\%$  and  $\gamma = 95\%$ ) and are both 1.645.

### 3.4.3. Non-Parametric Analysis

Data that do not follow a normal distribution curve can be analyzed using non-parametric techniques. The method used for this validation is taken from NUREG/CR-6698. As stated previously, this approach is more conservative than other non-parametric techniques available to determine distribution-free confidence interval (e.g., one based on the sign test as presented in Hollander and Wolfe, Reference 7). This method results in a determination of the degree of confidence that a fraction of the true population of data lies above the smallest observed value. This determination is calculated as follows:

$$\beta = 1 - \sum_{j=0}^{m-1} \frac{n!}{j!(n-j)!} (1-q)^j q^{n-j}$$

where:

- $\beta$  = level of confidence,
- $q$  = the desired population fraction (0.95 for this validation),
- $n$  = the number of data in one data sample,
- $m$  = the rank order indexing from the smallest sample to the largest ( $m=1$  for the smallest sample). Non-parametric techniques do not require reliance upon distributions, but are rather an analysis of ranks,
- $j$  = the ranked sample in the sample population being evaluated.

As stated in NUREG/CR-6698, for a desired population fraction of 95% and a rank order of 1 (the smallest data sample) the equation simplifies to:

$$\beta = 1 - 0.95^n$$

For a non-parametric set of data, the USL is determined as follows:

$$\text{USL} = \text{Smallest } k_{\text{eff}} \text{ value in the data set} - S_t - \text{NPM} - \text{MoS}$$

- Where:  $S_t$  = standard deviation corresponding to the smallest  $k_{\text{eff}}$  value in the data set
- NPM = non-parametric margin, determined from  $\beta$
- MoS = margin of subcriticality (0.05)

The non-parametric margin is an additional amount subtracted from the lowest data point to account for the small sample size and non-normal distribution of the data. Recommended values for the non-parametric margin are established in NUREG/CR-6698.

## 4. Benchmark Experiment Descriptions

The input files are specifically developed for MCNP 6.1 and the continuous energy ENDF/B-VII.1 cross section library. Ninety-two benchmark cases are modeled. The majority are in the thermal neutron energy range, however some bridge into intermediate and fast energy ranges. The ANECF ranges from 0.0043 to 1.46 MeV. The majority of the experiments are intermediate  $^{235}\text{U}$  enrichment; however sufficient low and high enriched experiments are included to allow these to be included in the AoA. A broad range of chemical forms and metal are included to evaluate potential bias from the physical form. Additionally, the cases are fairly evenly split between homogeneous and heterogeneous physical forms. Hydrogen identified in water is modeled with the water lwtr.20t S( $\alpha,\beta$ ) while hydrogen in hydrocarbon materials is modeled with the poly.20t S( $\alpha,\beta$ ). Graphite is modeled with the grph.20t S( $\alpha,\beta$ ).

### 4.1. [Proprietary Information]

#### 4.1.1. [Proprietary Information]

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#### 4.1.2. [Proprietary Information]

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## **4.2. [Proprietary Information]**

### **4.2.1. [Proprietary Information]**

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**4.2.2. [Proprietary Information]**

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### **4.3. [Proprietary Information]**

#### **4.3.1. [Proprietary Information]**

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	MCNP 6.1 Calculation			Benchmark Values		Normalized Results	
Exp Name	k-calc	$\sigma$ -calc	ANECF MeV	k-meas	$\sigma$ -exp	k-norm	$\sigma$ -tot
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]

The results of each enrichment group are examined for normality. The Shapiro-Wilk test (Section 3.4.1) is used to test the k-normalized values for normality within each subgroup of experiments. The null hypothesis is that the data are normally distributed, and 95% confidence is required to reject this assumption herein. The hypothesis of normality is accepted for all subgroups. The results of the normality testing of the pooled data are shown in Section 5.2.4.

## 5.1. Trend Evaluation

The  $k_{\text{eff}}$  results are also analyzed to determine if a trend exists with important validation parameters. A calculational methodology should have a bias that neither has dependence on a characteristic nor is a smooth function of a parameter. If a trend exists, the bias will vary as a function of that trend over the parameter range. If no trend exists, then the bias will be constant over the area of applicability. Critical experiment parameters examined include the hydrogen to fissile material ratio (H/X), the Average Neutron Energy Causing Fission (ANECF), the  $^{235}\text{U}$  enrichment, the moderator material, the reflector material and the chemical form of the fissile material. Graphs of the validation results for versus these parameters are shown. Where appropriate, the graphs of the results and the trending parameters also include a plotted trend line and the coefficient of determination value ( $R^2$ ) for the trend line. Note, an  $R^2$  value less than 0.3 is considered to indicate no data correlation, while an  $R^2$  value of 0.8 or greater is indicative of data correlation. It is concluded that no trend in the bias is observed.

### 5.1.1. Average Neutron Energy Causing Fission (ANECF)

The ANECF value is a calculated value used to characterize the system neutron energy. Consistent with the purpose of this validation, the majority of the experiments evaluated are in the thermal neutron range with ANECF values between [Proprietary Information]. However, sufficient higher energy experiments are included with ANECF values up to [Proprietary Information] to demonstrate that there is no trend in the  $k_{\text{eff}}$  values relative to the ANECF value. Figure 1 presents the MCNP 6.1 data. The very slight negative slope to the data's linear fit is judged to be insignificant as its change in value over the entire data range is on the order of the average total uncertainty.

[Proprietary Information]

**Figure 1 - ANECF Trend**

### 5.1.2. $H/^{235}\text{U}$ Values

The  $H/^{235}\text{U}$  value is a physical system parameter used to characterize the system neutron energy. Consistent with the purpose of this validation, the majority of the experiments evaluated are in the thermal neutron range and  $H/^{235}\text{U}$  values from [Proprietary Information] are well represented. However, sufficient higher energy experiments are included with  $H/^{235}\text{U}$  values as low as 3.1 to demonstrate that there is no trend in the  $k_{\text{eff}}$  values relative to the  $H/^{235}\text{U}$  value. Figure 2 presents the MCNP 6.1 data.

[Proprietary Information]

**Figure 2 -  $H/^{235}\text{U}$  Trend**

### 5.1.3. ANECF vs. $H/^{235}\text{U}$

Both the ANECF and the  $H/^{235}\text{U}$  values are measures of the system neutron energy. Note that the  $H/^{235}\text{U}$  value characterizes the system neutron moderation only as it is affected by the n-H scatter reaction while the ANECF is a summation of the effect of all n scatter reactions. Therefore, the MCNP calculated ANECF is judged to be a more useful parameter when comparing applications to the validation AoA. However, the  $H/^{235}\text{U}$  value is a physical property of the experiments. Thus, it is useful to observe the relationship between the two parameters.

Figure 3 shows this relationship for the MCNP data. Note that the high thermal neutron capture in HEU solution systems produces the only significant aberration in the relationship. From this relationship it is judged that the ANECF value accurately characterizes the system neutron energy in the absence of accurate  $H/^{235}\text{U}$  values and in the presence of other neutron scattering isotopes (e.g., carbon).

[Proprietary Information]

### Figure 3 - ANECF vs. $H/^{235}\text{U}$ Evaluation

#### 5.1.4. Enrichment

The system enrichment (wt. %  $^{235}\text{U}$ ) is a physical system parameter used to characterize the system. Consistent with the purpose of this validation, the majority of the experiments evaluated are [Proprietary Information] to demonstrate that there is no trend in the  $k_{\text{eff}}$  values relative to enrichment. Figure 4 presents the MCNP 6.1 data. The very slight positive slope to the data's linear fit is judged to be insignificant as its change in value over the entire data range is much less than the average total uncertainty.

[Proprietary Information]

**Figure 4 - Enrichment Trend**

### **5.1.5. Moderator**

The system neutron moderator material is a physical system parameter used to characterize the system. Figure 5 displays the normalized  $k_{\text{eff}}$  values for the various moderator materials used herein. As shown, there is no significant bias with the various moderator materials

[Proprietary Information]

### **Figure 5 - Moderator Evaluation**

### 5.1.6. Reflector

The system reflector material is a physical system parameter used to characterize the system. Figure 6 displays the normalized  $k_{\text{eff}}$  values for the various reflector materials used herein. As shown, there is no significant bias in the calculated  $k_{\text{eff}}$  values relative to the system reflector material.

[Proprietary Information]

### Figure 6 - Reflector Evaluation

### 5.1.7. Chemical Form

The chemical form of the system fissile material is a physical system parameter used to characterize the system. Figure 7 displays the normalized  $k_{eff}$  values for the various chemical forms of uranium materials used herein. As shown, there is no significant bias in the calculated  $k_{eff}$  values relative to the chemical form of the system fissile material.

[Proprietary Information]

**Figure 7 - Chemical Form Evaluation**

## 5.2. Normalcy Evaluation

The summary of the normalcy results is show in Table 4.

**Table 4 - Normalcy Results Summary**

<b>Low Enriched Uranium</b>	[Proprietary Information]
<b>Intermediate Enriched Uranium</b>	[Proprietary Information]
<b>High Enriched Uranium</b>	[Proprietary Information]



### 5.2.1. Low Enriched Uranium

The examination of these data includes 22 cases as shown in Table 5. As the number of cases (22) is less than the range of coefficients (50) for the Shapiro-Wilk test, this test was used to evaluate the distribution of the results. As shown in the data are normally distributed.

**Table 5 - LEU Results**

i	$X_i$	$An+1-i$	$X_{n+1-i} - X_i$	$An+1-i * (X_{n+1-i} - X_i)$	Results	
1	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
2	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
3	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
4	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
5	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
6	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
7	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
8	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
9	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
10	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
11	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
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18	[Proprietary Information]					
19	[Proprietary Information]					
20	[Proprietary Information]					
21	[Proprietary Information]					
22	[Proprietary Information]					

### 5.2.2. Intermediate Enriched Uranium

The examination of these data includes 50 cases as shown in Table 6 below. The Shapiro-Wilk test was used to evaluate the distribution of the results and the results are shown in Table 6. Therefore, the data are judged to be from a normal distribution.

**Table 6 - IEU Normalcy**

i	$X_i$	$A_{n+1-i}$	$X_{n+1-i} - X_i$	$A_{n+1-i} * (X_{n+1-i} - X_i)$	Results	
1	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
2	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
3	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
4	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
5	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
6	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
7	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
8	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
9	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
10	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
11	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
12	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
13	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
14	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
15	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
16	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
17	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
18	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
19	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
20	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
21	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		

i	$X_i$	$A_{n+1-i}$	$X_{n+1-i} - X_i$	$A_{n+1-i} * (X_{n+1-i} - X_i)$	Results	
22	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
23	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
24	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
25	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
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30	[Proprietary Information]					
31	[Proprietary Information]					
32	[Proprietary Information]					
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45	[Proprietary Information]					
46	[Proprietary Information]					

i	$X_i$	$An+1-i$	$X_{n+1-i} - X_i$	$An+1-i * (X_{n+1-i} - X_i)$	Results	
47	[Proprietary Information]					
48	[Proprietary Information]					
49	[Proprietary Information]					
50	[Proprietary Information]					

### 5.2.3. High Enriched Uranium

The examination of these data includes 20 cases as shown below. The Shapiro-Wilk test was used to evaluate the distribution of the results and the results are shown in Table 7. Therefore, the data are judged to be from a normal distribution.

**Table 7 - HEU Normalcy**

i	$X_i$	$An+1-i$	$X_{n+1-i} - X_i$	$An+1-i * (X_{n+1-i} - X_i)$	Results	
1	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
2	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
3	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
4	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
5	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
6	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
7	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
8	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
9	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
10	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
11	[Proprietary Information]					
12	[Proprietary Information]					
13	[Proprietary Information]					
14	[Proprietary Information]					

$i$	$X_i$	$A_{n+1-i}$	$X_{n+1-i} - X_i$	$A_{n+1-i} * (X_{n+1-i} - X_i)$	Results	
15	[Proprietary Information]					
16	[Proprietary Information]					
17	[Proprietary Information]					
18	[Proprietary Information]					
19	[Proprietary Information]					
20	[Proprietary Information]					

#### 5.2.4. Combined Data Set

As there is no trend associated with varying enrichment, the LEU, IEU and HEU groups can be combined and evaluated as a single data set. The examination of these data includes 92 cases as shown Appendix 1. Since the number of cases (92) exceeds the range of coefficients (50) for the Shapiro-Wilk test, the Modified Chi Square test, Kolmogorov-Smirnov test, and Lilliefors test (Section 3.4.1) were used to evaluate the distribution of the combined.

The results are:

- [Proprietary Information]
- [Proprietary Information]
- [Proprietary Information]

These calculations are shown in Appendix A and the comparison of the observed distribution vs. the expected distribution of a normal system is shown in Figure 8. The data are judged to be from a normal population because:

- [Proprietary Information]
- [Proprietary Information]
- [Proprietary Information]

[Proprietary Information]

**Figure 8 - Combined Group Distribution**

## 5.3. Bias and Bias Uncertainty Evaluation

### 5.3.1. Summary

A summary of the weighted bias and USL calculations is shown in Table 8. Per Reference 10 positive bias values are not used and the bias is set to unity for the USL calculation. The [Proprietary Information]. The USL value of [Proprietary Information] is recommended for use with the NWMI 19 wt. %  $^{235}\text{U}$  materials.

**Table 8 - USL Results Summary**

	Bias	USL
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]

### 5.3.2. Low Enriched Uranium

As the data are judged to be from a normal distribution the bias and bias uncertainty is represented with a Lower Tolerance Limit (LTL). The USL calculation for MCNP [Proprietary Information] is developed in Table 9. Notice that the bias [Proprietary Information]. However, per Reference 4 positive bias values are not used and the  $k_{\text{mean}}$  is set to unity for the USL calculation.

**Table 9 - LEU USL**

File Name	$k_{\text{eff}}$	$\sigma_1$	$1/(\sigma_1)^2$	Weighted $k_{\text{eff}}$	Weighted Variance
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]

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### 5.3.3. Intermediate Enriched Uranium

As the data are judged to be from a normal distribution the bias and bias uncertainty is represented with a Lower Tolerance Limit (LTL). The USL calculation MCNP [Proprietary Information] is developed in Table 10.

### Table 10 - IEU USL

[illegible]





File Name		$k_{eff}$	$\sigma_t$	$1/(\sigma_t)^2$	Weighted $k_{eff}$	Weighted Variance
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
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[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
	[Proprietary Information]	[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	
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	[Proprietary Information]	[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]

#### 5.3.4. High Enriched Uranium

As the data are judged to be from a normal distribution the bias and bias uncertainty is represented with a Lower Tolerance Limit (LTL). The USL calculation for MCNP [Proprietary Information] is developed in Table 11.

### Table 11 - HEU USL

[illegible]

File Name		$k_{eff}$	$\sigma_i$	$1/(\sigma_i)^2$	Weighted $k_{eff}$	Weighted Variance
	[Proprietary Information]	[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	
				[Proprietary Information]	[Proprietary Information]	
	[Proprietary Information]	[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
	[Proprietary Information]	[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]

### 5.3.5. Combined LEU, IEU and HEU Enrichments

As the data are judged to be from a normal distribution the bias and bias uncertainty is represented with a Lower Tolerance Limit (LTL). The USL calculation for MCNP [Proprietary Information] is developed in Table 9. Notice that the bias [Proprietary Information]. However, per Reference 4 positive bias values are not used and the  $k_{mean}$  is set to unity for the USL calculation.

**Table 12 - Combined USL**

File Name		$k_{eff}$	$\sigma_i$	$1/(\sigma_i)^2$	Weighted $k_{eff}$	Weighted Variance
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
[Proprietary Information]		[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
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## 6. Area of Applicability

This validation is appropriate for homogeneous and heterogeneous intermediate enriched uranium systems. A summary of the area of applicability for these experiments is provided in Table 13. For systems outside the validation area of applicability, an increased MoS value may be warranted, depending on the specific problem being analyzed. The analyst must document any extrapolation beyond the validation area of applicability and justification must be made for no adjustments to the MoS when extrapolating.

**Table 13 - Area of Applicability Summary**

Parameter	Area of Applicability
Fissile Material	[Proprietary Information]
Fissile Material Form	[Proprietary Information]
H/ <sup>235</sup> U ratio*	[Proprietary Information]
Average Neutron Energy Causing Fission (MeV)	[Proprietary Information]
Enrichment*	[Proprietary Information]
Moderating Materials	[Proprietary Information]
Reflecting Materials	[Proprietary Information]
Absorber Materials	[Proprietary Information]
Geometry	[Proprietary Information]

\* See following text.

The H/<sup>235</sup>U ratio of the experiments has values ranging from 0 to 1611 and Figure 2 of Section 5.1.2 demonstrates that ratios up to an H/<sup>235</sup>U ratio of 1200 are well covered. However, above 1200 there is only one value and this one is somewhat an outlier. However, as described in Reference 5, a  $\pm 20\%$  interpolation is considered acceptable for the ratio of moderator to fissile material in an AOA. Adding 20% to the 1200 value yields 1440. Therefore, given that Section 5.1.2 demonstrates that there is no trend between H/<sup>235</sup>U ratio and bias, it is judged herein that this validation can be conservatively used for the H/<sup>235</sup>U ratio AOA range listed in Table 13.

The enrichment range for the data set experiments ranges from 9 to 94 wt.% <sup>235</sup>U, while the enrichment of greatest interest in NWMI criticality applications is 19 wt.% <sup>235</sup>U. Figure 4 of Section 5.1.4 shows the distribution of the bias as a function of enrichment and indicates that there is no trend and that values around 19% enrichment are well covered. Therefore, this validation can be conservatively used for the entire enrichment range listed in Table 13.

## 7. References

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# Appendices

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# Appendix A. Combined Data Normalcy Test Calculations

Tables A1 through Table A3 present the calculations for the normality tests for MCNP data from Section 7.5.2.4.

**Table A1 - Modified Chi Square Normality Test**

Ordered Data	Class	Class	Class Midpoint ( $M_i$ )	Occurrence Frequency ( $O_i$ )	$O_i \cdot M_i / n$	$O_i \cdot (M_i)^2 / n$		
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
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[Proprietary Information]	[Proprietary Information]							

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Ordered Data	Class	Class	Class Midpoint ( $M_j$ )	Occurrence Frequency ( $O_j$ )	$O_j \cdot M_j / n$	$O_j \cdot (M_j)^2 / n$		
[Proprietary Information]	[Proprietary Information]							
[Proprietary Information]	[Proprietary Information]							
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Table A2 - Kolmogorov-Smirnov Normality Test

Observation Number	Ordered Data	$G(x_i)$	$F^*(x_i)$	$[F^*(x_i) - G(x_i)]$	$[F^*(x_i) - G(x_{i-1})]$	$T^* = \sup[F^*(x) - G(x)]$	$w_{95}(92)$
1	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]
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Observation Number	Ordered Data	$G(x_i)$	$F^*(x_i)$	$[F^*(x_i) - G(x_i)]$	$[F^*(x_i) - G(x_{i-1})]$	$T^* = \sup[F^*(x) - G(x)]$	$w_{95}(92)$
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Observation Number	Ordered Data	$G(x_i)$	$F^*(x_i)$	$[F^*(x_i) - G(x_i)]$	$[F^*(x_i) - G(x_{i-1})]$	$T^* = \sup[F^*(x) - G(x)]$	$w_{95}(92)$
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91	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
92	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]	[Proprietary Information]		
$\bar{x}_i$	[Proprietary Information]						
$\sigma$	[Proprietary Information]						

[illegible]

[illegible]

[illegible]



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## **Appendix B. Electronic Copy of Input / Output Files**

A CD with all Input and output files is included with the original copy.

[Proprietary Information]

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