APPENDIX 3.6c

BEND FORCE CALCULATION IN A PIPING SYSTEM USING CALPLOTF2421

3.6C.1 DERIVATION OF EQUATIONS

A postprocessor computer code called CALPLOTF2421 has been written for the RELAP computer code to enable the calculation of piping forces on each bend following a flow transient and to act as an interface for input into any pipe stress analysis code. The forces on each control volume in the piping system are calculated using the momentum equation:

→(DRN 00-1032)

$$\overline{\mathrm{F}}_{\mathrm{S}} + \iiint_{cv} \overline{\mathrm{B}}\ \mathrm{p\ d\ v} = \oiint_{cs} \overline{\mathrm{V}} \left(\mathrm{p}\overline{\mathrm{V}} \cdot \mathrm{d}\overline{\mathrm{A}}\right) + \frac{\partial}{\partial t} \iiint \overline{\mathrm{V}} \left(\mathrm{p\ d\ v}\right) \tag{1}$$

If gravity is assumed negligible, the following equation results:

$$\overline{\mathrm{F}}\mathrm{s} = \oiint \overline{\mathrm{V}} \left(\mathrm{p}\ \overline{\mathrm{V}} \cdot \mathrm{d}\overline{\mathrm{A}}\right) + \frac{\partial}{\partial\, \mathrm{t}} \iiint_{cv} \overline{\mathrm{V}}\, (\mathrm{p\, d\, v}) \tag{2}$$

Since a pipe bend can only exist in two dimensions, the above equation can be further divided into its two scalar components, x and y:

$$\left(F_S\right)_x = \oiint_{cs} V_x \left(p\,\overline{V} \cdot d\overline{A}\right) + \frac{\partial}{\partial t} \iiint_{cv} V_x\ \mathrm{p\ d\ v} \tag{3}$$

$$\left(F_S\right)_y = \oiint_{cs} V_y \left(p\,\overline{V} \cdot d\ A\right) + \frac{\partial}{\partial t} \oiint_{cv} V_y\ \mathrm{p\ d\ v} \tag{4}$$

←(DRN 00-1032)

The surface forces across volume 1 (Figure 3.6C-1) can be written as:

$$(F_s)_y = (\mathrm{P_2} - \mathrm{P_a})\ \mathrm{A} \sin \Phi + \mathrm{R_y} \tag{5}$$

$$(F_s)_x = (\mathrm{P_o} - \mathrm{P_a})\ \ \mathrm{A} + (\mathrm{P_2} - \mathrm{P_a})\ \mathrm{A} \cos \Phi + \mathrm{R_x} \tag{6}$$

The following relationships can be derived for the bend volume 1:

→(DRN 00-1032)

$$K_y - R_y = (P_2 - P_a)\, A \sin \Phi + p_1' \frac{V_1'^2\, A \sin \Phi}{g} + \sum_k \frac{(\dot{M}_{kt+\Delta t} - \dot{M}_{kt})}{\Delta t g} \sin \theta_k\, \theta \Delta_k \qquad (7)$$

$$K_x = - R_x = (P_o - P_a)\, A + (P_2 - P_a)\, A \cos \Phi + P_1' \frac{V_1'^2\, A \cos \Phi}{g} +_{\psi} P_o \frac{V_o'^2\, A}{g} + \sum_k \frac{(\dot{M}_{kt+\Delta t} - \dot{M}_{kt}}{\Delta t\, g} \cos \theta_k\, R \Delta \theta_k \qquad (8)$$

←(DRN 00-1032)

defining:

$$\dot{M} = \rho A V$$

$$\Delta s_1 = R \Delta \theta$$

Equations 7 and 8 calculate the forces on a bend in a piping system. If a guillotine pipe break occurs directly following a pipe bend, the two equations must be modified to account for the forces developed at the pipe opening. Consequently using the momentum equation the force on control volume 2 (Figure 3.6C-2) can be written as:

→(DRN 00-1032)

$$K_y = \left\{ ^{-}(P_1 - P_a) A + \rho_2' \frac{V_2'^2\, A}{g} - \rho_1' \frac{V_1'^2\, A}{g} + \frac{(M_2' t + \Delta t\, M_2' t)}{\Delta t} \, ^{g} \frac{\Delta s_2}{g} \right\} \sin \Phi \qquad (9)$$

$$K_x = \left\{ (P_1 - P_a)\, A + \rho_2' \frac{V_2'^2}{g} A - \rho_1' \frac{V_1^2\, A}{g} + \frac{\dot{M}_2' t - \Delta t \dot{M}_{2T})}{\Delta t} \frac{\Delta\, s_2}{g} \right\} \cos \Phi \qquad (10)$$

←(DRN 00-1032)

Therefore, the total force on a bend just before a guillotine break is the sum of 7 and 9 for the y component and 8 and 10 for the x component.

→(DRN 00-1032)

$$K_{Ty} = \left\{ (P_2 - P_1)\, A + \rho_2' \frac{V_2'^2 A}{g} + \frac{(\dot{M}'_{2t+\Delta t} - \dot{M}'_{2t})}{\Delta t} \frac{\Delta s_2}{g} - \right\} \sin\Phi \tag{11}$$

$$+ \sum_k \frac{(\dot{M}_{kt+\Delta t} - \dot{M}_{kt})}{\Delta t\, g} (\sin\theta_k) R \Delta\theta_k$$

$$K_{Tx} = \left\{ (P_2 - P_1) A + \rho_2' \frac{V_2'^2 A}{g} + \frac{\dot{M}'_{2t+\Delta t} - \dot{M}'_{2t})}{\Delta t} \frac{\Delta s_2}{g} \right\} \cos\Phi \tag{12}$$

$$+ (P_o - P_a)\, A + \rho_o' \frac{V_o'^2 A}{g} + \sum_k \frac{(\dot{M}_{kt+\Delta t} - \dot{M}_{kt})}{\Delta t g} (\cos\theta_k)\; R\Delta\theta_k$$

←(DRN 00-1032)

If two-phase choked flow occurs at the break exit of the pipe, equations 11 and 12 must be modified to account for the pressure unbalance that occurs at the pipe exit plane. A rederivation for the choked flow case results in the following relations:

$$K_{T_Y} \text{ chocked } = K_{t_y} \text{ unchocked } + (P_{th} - P_a)\ a \sin\phi \tag{13}$$

$$K_{T_x} \text{ chocked } = K_{t_x} \text{ unchocked } + (P_{th} - P_a)\ a \cos\phi \tag{14}$$

3.6C.2 EXPERIMENTAL COMPARISON

→(DRN 00-1032)

The results of the modified RELAP calculated forces for a test geometry have been compared with the experimental results. Figures 3.6C-3 and 3.6C-4 show the experimental setup, and the RELAP model. Figure 3.6C-5 compares the RELAP calculations to the experimental measurements. This figure also shows the effects of varying the Moody multiplier to choked flow at the break plane. Moody multipliers of 1.0, 0.8 and 0.6 were inputed into their RELAP computer runs and their calculated bend forces are indicated on the graph.

←(DRN 00-1032)

3.6C.3 MATHEMATICAL MODEL FOR PIPING LEG FORCE CALCULATION

The CALPLOTFII computer code is the latest version of CALPLOTF which has been written to convert the transient flow conditions calculated in a piping system by the WHAMMOCII (a method of characteristics computer code) or the RELAP series of computer codes into transient forces on the piping system. Specifically, CALPLOTFII calculates and plots the forces on straight lengths of pipe between changes in pipe direction (bends), or between a change in direction and pipe break. The derivation of the equations used in the code are given below.

3.6C.4 STRAIGHT LENGTHS OF PIPES BETWEEN DIRECTIONAL CHANGES

The force on a straight length of pipe between direction changes (Figure 3.6C-6) is calculated using the momentum equation:

→(DRN 00-1032)

$$\overline{F_s} + \iiint_{cv} \overline{B}\ \rho dv = \oiint_{cs} \overline{V}\ (\rho \overline{V} \cdot d\overline{A}) + \frac{\partial}{\partial t}\left(\iiint_{cv} \overline{V}\,(\rho dv)\right) \tag{15}$$

←(DRN 00-1032)

If the gravity term is assumed negligible, the following equation results:

→(DRN 00-1032)

$$\overline{F_s} = \oiint_{cs} \mathrm{V}\ (\rho\overline{\mathrm{V}}\cdot \mathrm{d}\overline{\mathrm{A}}) + \frac{\partial}{\partial \mathrm{t}}\left(\iiint_{\partial t} \overline{V}\ (\rho \mathrm{dv})\right) \tag{16}$$

Since the force on the straight pipe length only exists in one dimension, the above equation can be written in a scalar form:

$$F_s = \oiint_{cs} \mathrm{V}\ (\rho\overline{\mathrm{V}}\cdot \mathrm{d}\overline{\mathrm{A}}) + \frac{\partial}{\partial \mathrm{t}}\left(\iiint_{cv} V\rho d\mathrm{v}\right) \tag{17}$$

←(DRN 00-1032)

Since both the WHAMMOCII and the RELAP computer codes calculate the pressures and the flowrates at different physical positions in the piping system, it is necessary to subdivide a piping length into two control volume types for application of the momentum equation. The first division creates the pressure control volumes. The divisions for the pressure control volumes are the positions in the pipe length where the pressures are calculated by the computer code, and serve as the the boundaries across which the control volume surface forces are calculated. The second control volume divisions are due to flow conditions. The boundaries of the flow control volumes are located at the pipe length locations where flows are calculated by the computer code. The forces in the pipe length which are due to the rate of efflux of momentum across a control volume and the change of momentum in a control volume are calculated using the flow boundaries as flow control volume divisions.

The resultant force on the fluid across the boundary of the pressure control volumes 1, 2 and 3, shown in Figure 3.6C-6 are:

$$F_{S1} = -(P_A - P_a)\,A_A + R_1 \tag{18}$$

$$F_{S2} = P_A\,A_A - P_B\,A_B + P_a\,(A_B - A_A) + R_2 \tag{19}$$

$$F_{S3} = (P_B - P_a)\,A_B + R_3 \tag{20}$$

The net surface force on the straight pipe length is obtained by summing equations 18, 19 and 20:

$$F_{S1} + F_{S2} + F_{S3} = R_1 + R_2 + R_3 \tag{21}$$

$$F_S = R \tag{22}$$

Therefore, the force on the straight pipe length due to surface forces is equal to the net normal and shear stresses on the pipe wall length.

The right side of equation 17 can now be evaluated for each of the flow control volumes A and B:

→(DRN 00-1032)

$$F_{S1} = \frac{\rho_2 V_2^2 A_A}{g} + \frac{\partial \dot{\mathrm{M}}_{\mathrm{A}}}{\partial \mathrm{t}} \Delta\mathrm{A} \tag{23}$$

$$F_{S2} = \frac{-\rho_2 V_2{}^2 A_A}{g} + \frac{\partial \dot{\mathrm{M}}_{\mathrm{B}}}{\partial \mathrm{t}} \Delta\mathrm{B} \tag{24}$$

←(DRN 00-1032)

Summing equations 23 and 24, and using equation 22, the net fluid force on the pipe length can be obtained:

→(DRN 00-1032)

$$K = -\mathrm{F_S} = -\mathrm{R} = \frac{-\partial \dot{\mathrm{M}}_{\mathrm{A}}}{\partial \mathrm{t}} \Delta\mathrm{A} - \frac{\partial \dot{\mathrm{M}}_{\mathrm{B}}}{\partial \mathrm{t}} \Delta\mathrm{B} \tag{25}$$

If the straight length of pipe considered is bounded by a directional change and an open end, a break, the forces obtained using equation 25 must be modified to account for the force developed at the pipe exit plane. Consequently, using the momentum equation, the force on the straight pipe length shown on Figure 3.6C-7, for unchoked break flow, can be written as:

$$K_{unc} = \frac{-\rho_2 V_2{}^2 \mathrm{A}_2}{\mathrm{g}} - \frac{\partial \dot{\mathrm{M}}_{\mathrm{A}}}{\partial \mathrm{t}} \Delta\mathrm{A} \tag{26}$$

←(DRN 00-1032)

If choked break flow is determined to exist by the fluid transient computer code, then equation 26 must be modified to account for the pressure unbalance that occurs at the pipe exit plane. A rederivation of the equation for the straight pipe length for this case results in the following relation:

→(DRN 00-1032)

$$K_{ch} = -(P_2 - P_a)\ A_2 - \frac{p_2 V_2^{\,2}}{g}\ A_2 - \frac{\partial\, \dot{M}_A}{\partial t}\ \Delta A \tag{27}$$

←(DRN 00-1032)

or

$$K_{ch} = K_{unc} - (P_2 - P_a)\ A_2 \tag{28}$$

where

$$P_2 = P_A + \frac{p_A V_A^{\,2}}{2g} - \frac{p_2 V_2^{\,2}}{2g} - \Delta P_f - \Delta P_{acc} - \Delta P_{el} \tag{29}$$

3.6c.5 EXPERIMENTAL COMPARISON

The results of the force calculated using the WHAMMOCII and CALPLOTFII computer codes are compared to the force measured by Hanson(1) for a pressurized, non-flashing tank containing a rupture disk. Figure 3.6C-8 contains a sketch of the test section and the comparisons between the experimental and calculated values. The initial temperature of the tank water was assumed to be 60F since no measured initial temperature was indicated. Since no rupture disk opening rupture disk, the travel time for the acoustic wave was underpredicted. Thus, the importance of the opening time of the rupture disk was shown. For the second calculation, the break opening time plotted on Figure 3.6C-9, was assumed. The results for this case closely predicted the measured data up to 3 msec. After that time, when the acoustic effects no longer dominated, it seemed that the rupture disk opening time and frictional losses were still not sufficiently modeled in order to accurately predict the experimental results. For the acoustic portion of the transient, however, the CALPLOTFII computer code closely predicts the experimental data.

APPENDIX 3.6C REFERENCE

1. Hanson, G.H., Subcooled Blowdown Forces on Reactor System Components: Calculational Method and Experimental Confirmation, Idaho Nuclear Corporation IN-1354, June, 1970.

Nomenclature

A	flow area
B	body force of a control volume
F_s	surface force resultant on a control volume
g	gravitational constant
K	force of fluid on piping
M	control volume flowrate
P	pressure
P_a	pressure outside pipe control volumes
R	normal and shear stresses in a control volume
t	time
v	volume of a control volume
V	velocity of fluid in a control volume

Greek Letters

ρ	density in control volume

Subscripts

acc	acceleration
f	friction
ch	choked flow
cs	control surface
cv	control volume
el	elevation
unc	unchoked