Using Cooper-Jacob Approximation to Take Account of Pumping Well Pipe Storage Effects in Early Drawdown Data of a Confined Aquifer

by Robert P. Chapuis[a]

Abstract

Most hydrogeology textbooks warn the reader that in order to avoid large errors the Cooper-Jacob solution should only be applied when the u-parameter of the Theis solution is less than 0.01 or 0.02. This paper proposes a graphical representation for visualizing and quantifying the difference between the two solutions. The graph of drawdown vs log (time) may be divided into three zones, the early one being influenced by storativity, pumping well pipe capacity and skin effects (among others), the latest by boundary effects, and the intermediate one by the transmissivity and storativity of the aquifer. The differences between the theoretical solutions are maximal in the early data zone. Because both solutions consider a pumping well of infinitesimal diameter, the early time data graph may be distorted by the influence of real well pipe storage capacity and, consequently, may yield a poor estimate of the parameters of the aquifer. The method of Papadopulos and Cooper is already available to take pumping well pipe storage into account in interpreting the pumping well drawdown data, but may result in unreliable S values due to the difficulty in curve matching. A more practical solution, which is also applicable to observation wells, is proposed and demonstrated with a worked example. This solution can be used when the Cooper-Jacob approximation is valid and it does not require curve matching. Early drawdown data in the pumping well cannot provide a reliable estimate of storativity for many reasons. These early data can be used, however, to obtain a better estimate of storativity and transmissivity from the drawdown data of observation wells. The effect of pumping well pipe storage in the early drawdown data may be significant in cases of low transmissivity aquifers and low pumping rates, which are quite common in ground-water remediation.

Introduction

The problem of unsteady-state flow due to a constant pumping rate in a fully confined aquifer was solved by Theis (1935). An approximate solution was proposed by Cooper and Jacob (1946) who stated, "The approximation will be tolerable where u is less than about 0.02." The difference between the Theis and Cooper-Jacob solutions is important if the early time data of an aquifer test are to be used to determine the aquifer storativity, S. In addition, both solutions consider a pumping well of infinitesimal diameter. Actually, the storage capacity of a real well pipe will adversely affect the early measurements of drawdown which are essential in determining the storativity. The method of Papadopulos and Cooper (1967) considers the influence of pumping well pipe storage on time-drawdown relationships in large-diameter pumping wells. This curve-matching method applies only to the pumping well, and the authors stated that "a determination of S by this method has questionable reliability."

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Using the Cooper-Jacob approximation, this paper proposes a more practical method which is applicable both to the pumping well and to observation wells. First, a graphical representation is proposed for visualizing and explaining the nature of the Cooper-Jacob approximation, and for quantifying the difference between this and the Theis solution. Then, a worked example is provided to facilitate comparison of the proposed method to that of Papadopulos and Cooper (1967).

Mathematical Development

The Theis Solution

Theis (1935) presented a solution for a vertical well fully penetrating a fully confined horizontal isotropic aquifer of infinite areal extent. When this well is pumped at a constant rate, the influence of the hydraulic head discharge extends outward with time. The problem is axisymmetric around the well axis. The classical conservation equation for ground-water flow assuming isotropic permeability is:

$$\mathrm{div}(\mathrm{grad}\ h) = (S/T)(\partial h/\partial t) \qquad (1)$$

where h is the hydraulic potential (total head), S is the storativity, T is the transmissivity, and t is the time. In polar coordinates (r, θ), it gives:

$$\partial^2 h/\partial r^2 + (1/r)(\partial h/\partial r) = (S/T)(\partial h/\partial t) \qquad (2)$$