**RE: EPRI Proposal for determining 95/95 confidence limits on HFP measured reactivity decrement**

This memo describes a proposed statistical approach to determine 95% confidence intervals for the measured Hot Full Power (HFP) PWR fuel reactivity depletion decrement errors that have been derived from Duke reactor data as documented in EPRI Technical Report[1] 1022909, entitled "Benchmarks for Quantifying Fuel Reactivity Depletion Uncertainty", issued in August of 2011.

The data derived from flux map measurements over 44 cycles of operation of four Duke Power reactors is summarized in the scatter plot of measured error of CASMO-5 (ENDF-B/VII) sub-batch reactivity versus sub-batch burnup as depicted in Fig.1. Also plotted are **red** curves that correspond approximately to +/- 5% reactivity decrements (e.g., ~Kopp bounds, albeit at HFP), a **black** curve that is a nonlinear quadratic regression to the data, **green** curves that correspond to 95% prediction intervals for the regression fit, and **blue** dotted curves that correspond to the 95% confidence interval for the regression fit.

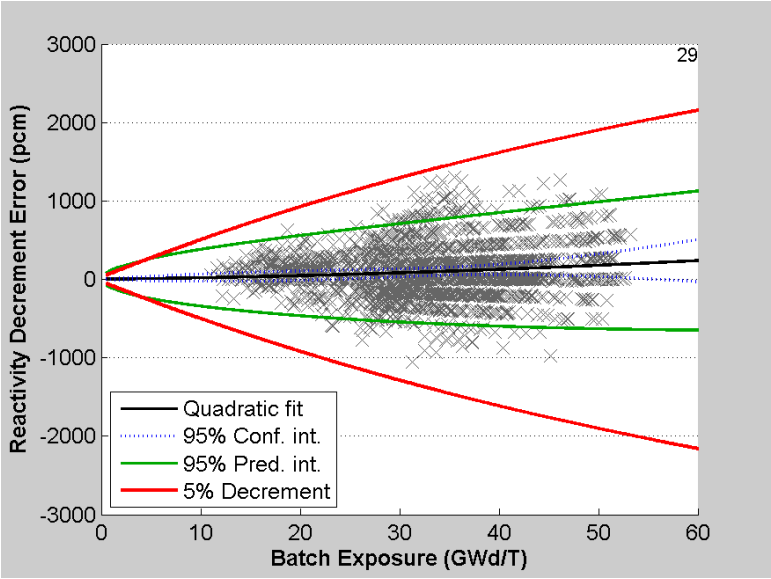


**Figure 1 CASMO-5 Reactivity Decrement Error vs. Sub-batch Burnup**

While it is tempting to interpret the very narrow confidence intervals in Fig. 1 to imply that the shape of the error in CASMO reactivity decrements is known quite precisely, the EPRI report briefly discusses why this is not an appropriate interpretation for the regression confidence intervals. The EPRI report contains results of an indirect (non-statistical) analysis that was used to approximate the uncertainty of the regression curve.

NRC has communicated that review of the EPRI report is currently at an impasse because assembly depletion reactivity decrement biases are desired to have uncertainties that satisfies the 95/95 criteria (95 percent probability, 95 percent confidence level) so that final SFP criticality uncertainty rollups will satisfy the 10 CFR 50.68 regulation that requires SFP assembly reactivity (k-effective) must not exceed 0.95 (at a 95 percent probability, 95 percent confidence level) when credit is not taken for soluble boron.

In order to better understand the requirements for applying purely statistical analysis to EPRI experimental reactivity decrement measurement data, it is important to understand exactly what the data represents. Each data point represents the change in CASMO fuel sub-batch reactivity (k-infinity to be precise) that produces the minimum deviation between measured and calculated (CASMO/SIMULATE) fission rate distributions at one flux-map (e.g., a specific time point in the cycle depletion of a reactor core). There are ~2500 sub-batch reactivity data points representing ~120 fuel batches (3.4% to 4.95% enrichment) in 44 operating cycles of four

Westinghouse 4-loop PWRs. It has been shown in the original EPRI report that the scatter in the data and regression fits are very independent of fuel enrichment, so for purposes here of discussing statistical methods, it is assumed for the moment that all data is for fuel of the same enrichment (without burnable absorbers). Each data point now represents one measurement of the error in fuel reactivity as a function of fuel burnup. If the data points are plotted for just **one sub-batch** of fuel, the reactivity decrement error as a function of burnup might behave as shown in Fig. 2.

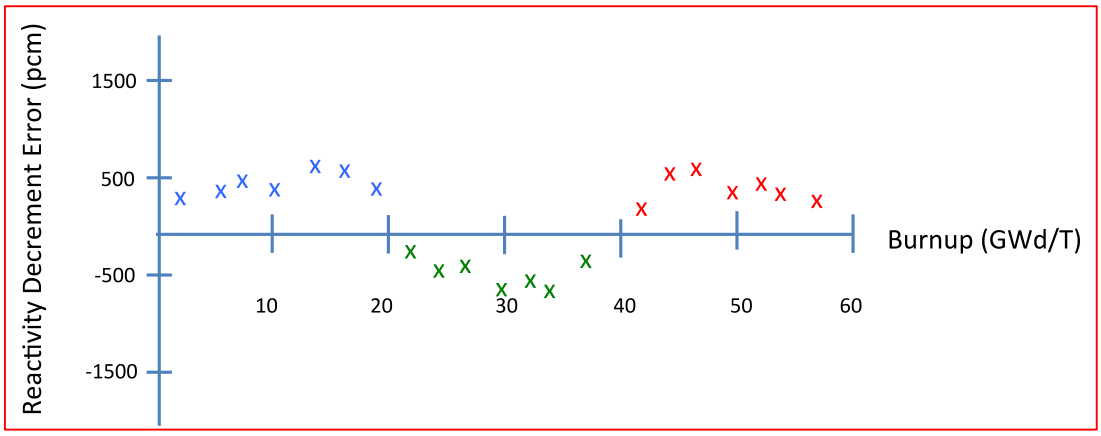


**Figure 2 Sub-Batch Reactivity Decrement Error vs. Sub-batch Burnup**

**Data Covariance**

The reactivity decrement errors within each cycle (depicted by different colors) are correlated. This is because the fission rates are measured very accurately (purely random measurement errors are small), and the data within a cycle represent a **time series of measurements, not random independent measurements**. This is demonstrated by considering the case of two flux maps being taken on the same day; other than reducing the purely random uncertainty in the experimental reactivity decrement error, no new knowledge is obtained from the second measurement. Even when the real data (flux maps taken at 30-day intervals) is considered, the data at successive flux maps is still highly correlated because the core flux distribution is evolving slowly - as are differences between CASMO/SIMULATE calculations and the measurement. Therefore, all sub-batch data points within a cycle are highly correlated. When fuel is shuffled between cycles, sub-batch reactivity decrement errors change significantly (because new fuel assemblies have been loaded, the relative positions of fuel sub-batches in the core have changed, and core power distributions are dramatically altered), and the time series is broken.

**In order to apply formal regression statistics, we need to know the co-variances between measured data points.** Note that in the limit of infinitely many data points, **data correlations have no impact on the regression fit – only on the confidence intervals associated with the fit.** In the limit of complete correlation within a sub-batch/cycle, one can think of the data as simply representing a single measurement of reactivity decrement error (the cycle-averaged error). Given that we have ~120 sub-batches from which we can fit the curve of reactivity decrement error versus sub-batch burnup, one conservative way to treat the correlation is to simply average sub-batch errors within each cycle and treat each sub-batch/cycle as a single measurement. This would make confidence intervals wider than those in Fig. 1, and it would avoid the need to explicitly approximate the data covariance matrix.

**Burnup Uncertainty**

Most statistical methods require that the independent variable (or at least its uncertainty) be known precisely in order to satisfy the regression mathematical assumptions. However, in this case, the ordinate (Burnup) is actually a quantity computed by CASMO/SIMULATE. Often, sub-batch burnup has been assigned a 95/95 uncertainty of 5.0%[2], so considering the shaded box depicted in Fig. 3, it is easy to see that that the vertical data spread is much larger than the horizontal spread represented by a 5.0% burnup uncertainty. Actual quantifications of PWR burnup uncertainty[3, 4] have shown that 95/95 uncertainties are much closer to 2.0%.

The burnup uncertainty principally affects where the data points are positioned horizontally and the regression fit is certainly not very sensitive to this positioning. This situation is similar to the case of regression fits in Spent Fuel Pool (SFP) analyses that fit biases versus boron concentration, enrichment, etc. Such independent variables are not known precisely - it has been accepted that the independent ordinate values are well known relative to the uncertainty of the dependent variable being fitted.

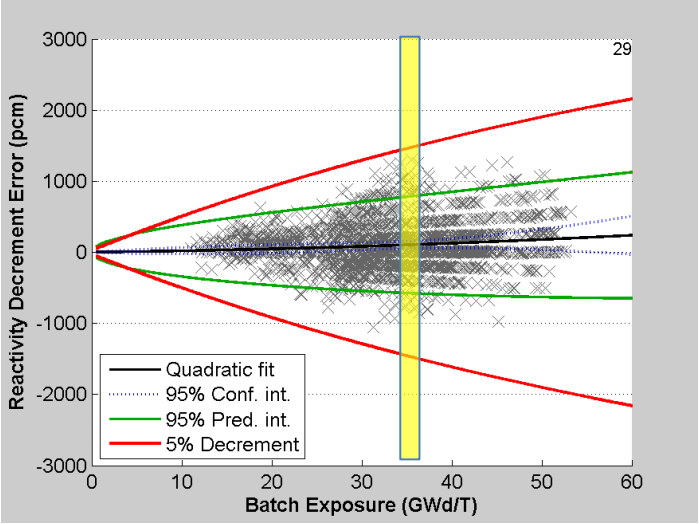


**Figure 3 CASMO-5 Reactivity Decrement Error vs. Sub-batch Burnup**

Also formally, the independent variable is required to be independent of fitted variable (reactivity decrement error). Since the CASMO/SIMULATE models contribute to both the decrement error and the Burnup, the two are not totally independent. The reactivity decrement errors and the sub-batch burnups have a small correlation because they both are partially derived from analytical models, but this is unlikely to be statistically significant for our application. This is similar to other criticality analyses[5], where regression variables are computed for EALF or burnup that have been inferred using computational models.

**Experimental Data Uncertainties**

It is apparent from Fig. 1 that the spread in the data grows with respect to burnup. This is a characteristic of heteroskedastic data in which the uncertainties of data points differ. This is common in fitting of data that represents functions that are monotonically increasing. If the measurement uncertainty of some signal were a fixed percentage of the signal, the absolute magnitude of the uncertainty in data would be increased for larger values of measured data.

Typically, when applying Ordinary Least Squares (OLS) data regressions, it is implicitly assumed that all data points have the same uncertainty. However, the data points in Fig. 1 do not all have the same uncertainty. In the EPRI report, the sensitivities of the regression fits (and confidence intervals) to assumptions about how data uncertainty grows with burnup were presented. It was shown that there is little difference in the regression fits obtained by hypothesizing either a linear or quadratic variation of the data uncertainty with burnup. Therefore, the quadratic model is used, as it is more realistic since we expect increased uncertainty in model predictions of fuel reactivity as higher actinides and fission products are produced.

The data of Fig. 1 has an additional complexity. This can be seen by examining four of the figures from the NRC RAI Responses for report 1022909, which are reproduced here as Fig. 4.

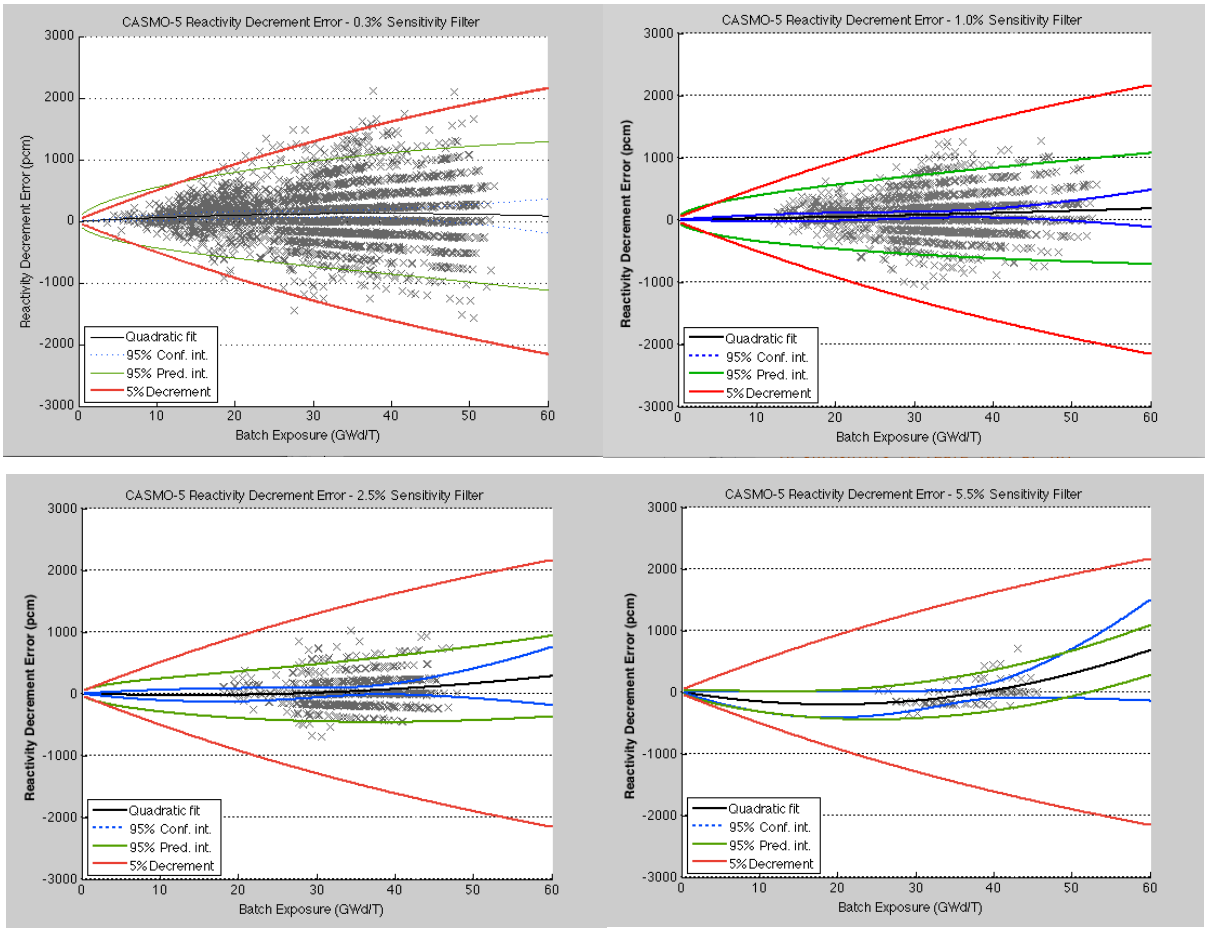


**Figure 4 Regressions vs. Sensitivity Screening Filter Values**

From these four figures, it can be seen that the spread in data is related to the sensitivity filter applied to the data. This sensitivity is better understood by examining Fig. 5, where typical flux map r.m.s differences (between calculated and measured node-wise fission rates) are depicted for two hypothetical fuel sub-batches. Perturbation of sub-batch "a" burnup drives the flux map r.m.s. through the range from ~5.0% to ~1.0% and sub-batch "b" through the range from ~1.3% to ~1.0%. The sub-batch sensitivity parameters are defined by this r.m.s. range, being 4.0% and 0.3% for these two sub-batches (depicted by the blue arrows), respectively. The magnitude of the burnup perturbation required to drive the r.m.s. from its unperturbed value (depicted by the circles) to its minimum value is converted to pcm reactivity decrement error, as depicted by the red arrows. Thus, it is easy to see from these two curves why the inferred reactivity decrement error is much more uncertain when the sub-batch

sensitivity is very small. Small sensitivity implies that the flux map is not sensitive to this sub-batch reactivity, and k-infinity of the sub-batch is not the dominant contributor to the deviation of the CASMO/SIMULATE calculation from the measurement. This is to be expected because there are sub-batches for which the fission rates have very little sensitivity, and our measurement process conservatively treats all errors in calculated flux maps as if they are due to fuel sub-batch reactivity errors – even when errors might be due to SIMULATE core modeling approximations, fuel manufacturing tolerances, core flow uncertainties, fuel bowing, crud deposition, etc.

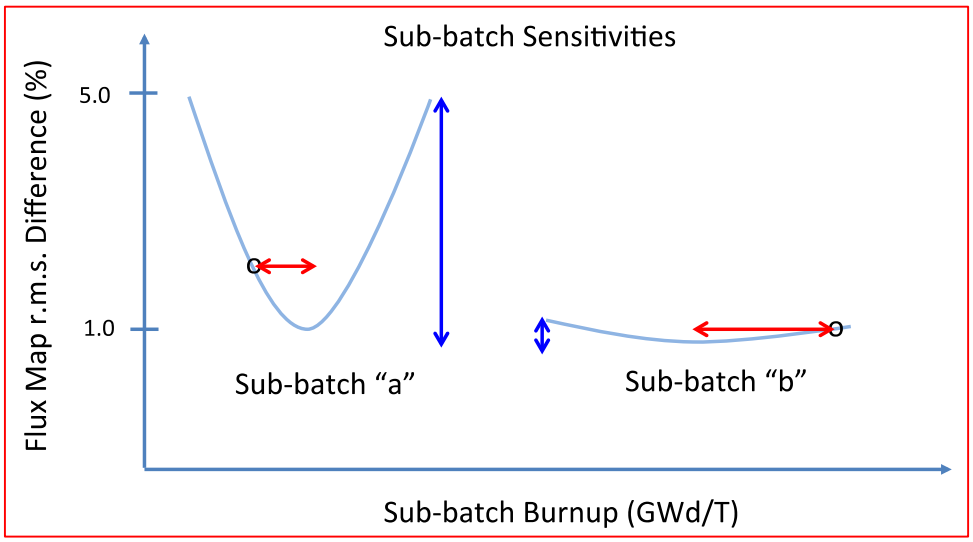


**Figure 5 Sub-batch Sensitivity Definition**

As the sensitivity filter is systematically increased, only those sub-batches that produce large changes in core flux distributions are retained, and the sub-batch reactivity becomes a much larger fraction of the desired signal. While the shapes of the regression fits in Fig. 4 do not change much with the applied sensitivity filter, the prediction intervals narrow considerably with increased sensitivity filters (e.g., the green curves narrow). This means that the measured reactivity decrement uncertainty is a significant function of the sensitivity, and this implies that statistical methods will require that data uncertainties be treated in the regression process.

**Mathematical derivations of nonlinear Weighted Least Squares (WLS) regression methods depend on data uncertainties being known precisely.** With real data, such knowledge is not possible; however, the uncertainties can be approximated. The results of regressions and the determinations of their confidence intervals will depend on the approximations that are made, and most statistical analyses simply rely on demonstrating that regression fits are not sensitive to these assumptions. While this is not mathematically satisfying, it is simply a reality of applying regression analysis to real data. Most SFP criticality analyses[5] implicitly make this assumption, whether explicitly stated or not.

For the complex reactivity decrement error uncertainties that exist in our data, it is proposed to plot the absolute value of the reactivity decrement errors versus the sub-batch sensitivity, perform a regression fit of the sub-batch errors vs. sensitivity, and use the resulting 95% prediction interval to approximate the two-sigma data uncertainty. By combining this sensitivity-based variance (assumed to be one-sigma squared) with the assumed quadratic burnup variance, we can obtain an estimate of the total variance for each sub-batch data point that can then be used in the WLS regression analysis. Note that the absolute value of the data variance is not important, only the relative variance between individual data points is significant in regression fitting.

**Experimental Data Sensitivity to Core Physics Methods**

The preceding discussion of the measured data uncertainty pointed out an assumption that has been made in the data reduction – namely that the measured reactivity decrements are assumed to be independent of the core analysis methods. At the time of the original EPRI report, this assumption was motivated by the expectation that each sub-batch would pass through a sequence of different core locations and any modeling sensitivity would tend to cancel out - because each flux map represents a "normalized shape". If the core were comprised of only one batch to fuel, each flux map would have single fuel type sub-batches at all core locations and modeling imperfections would tend to cancel because of the flux map normalization process (the sum of all deviations between measured and calculated fission rates is by definition zero). In the case of cores that contain many sub-batches of fuel, this argument is not strictly valid.

In order to be confident that results are not sensitive to the SIMULATE nodal method approximations (see original EPRI report for an enumeration of approximations), Gunow[6] compared experimentally reduced reactivity decrements using 2D full-core heterogeneous multi-group transport calculations (using CASMO) to eliminate the dependency on nodal core models. These results demonstrated that the mean difference between 2D nodal and multi-group transport inferred reactivity decrement errors was ~33 pcm (with a standard deviation of ~78 pcm). Since these differences are very small compared to the assigned HFP reactivity decrement uncertainty of 250 pcm, **it was concluded that approximations of the nodal methods themselves do not contribute significantly to biases and uncertainties reported in the original EPRI report.**

Another approximation made in the original EPRI report was that the sensitivity of the calculated core fission rate distribution to sub-batch reactivity could be computed by perturbing sub-batch reactivity by altering sub-batch burnup in the SIMULATE model. While this procedure does produce the desired effect of forcing sub-batch reactivity through the desired range of k-infinities, the perturbation method is not unique; as there are many ways to perturb fuel assembly reactivity (k-infinity). In order to test the significance of this approximation, Sykora[7] has now completed his MS thesis, in which the perturbations of reactivity are performed by artificially changing sub-batch fuel temperatures. While clearly unphysical, this perturbation does force sub-batch reactivity through the desired range of k-infinities. Results show that this perturbation produces almost the same measured reactivity decrement errors as the burnup perturbation method, having a mean difference of only 27 pcm and a standard deviation of 82 pcm from the burnup perturbation method. Since these differences are very small compared to the assigned HFP reactivity decrement uncertainty of 250 pcm, **it was concluded that the burnup perturbation technique does not contribute significantly to biases and uncertainties reported in the original EPRI report**.

**Proposal for determining 95/95 confidence intervals for the HFP measured reactivity decrement errors**

With this discussion in hand, a procedure is proposed to compute statistical confidence intervals for the reactivity decrement biases curves derived using the existing EPRI methodology and measured data:

1. Using all the >2500 HFP sub-batch points in the original EPRI report, generate a master database containing: a unique sub-batch/cycle index, the sub-batch burnup, the sub-batch sensitivity, and the sub-batch reactivity decrement error.

2. Plot the reactivity decrement errors vs. sub-batch sensitivity and perform a quadratic regression to determine the 95% prediction interval for the reactivity decrement error versus sub-batch sensitivity.

3. Approximate the individual sub-batch reactivity decrement error variances from the spread of the 95% prediction interval curves (of the fitted reactivity decrement error vs. sub-batch sensitivity) by using the individual sub-batch sensitivities.

4. Determine the uncertainty of individual reactivity decrement errors data points from a quadratic function of sub-batch burnup, and combine this uncertainty with the sub-batch sensitivity variance to get a total variance estimate for each of the >2500 data points.

5. Collect all the >2500 HFP reactivity decrement error data, and collapse all data within an individual sub-batch/cycle to one average value of reactivity decrement error by statistically combining with the appropriate individual data variances. This will reduce the number of data points from >2500 to about ~120. (This is the assumption of 100% correlation within a sub-batch/cycle.)

6. Perform a Weighted Least Squares (WLS) nonlinear regression fit to the ~120 reactivity decrement error data points versus sub-batch burnup, and compute the 95% confidence intervals for the regression fit.

7. Perform a Shapiro-Wilk test to determine if the data "passes the normality test," so that the confidence limits can be correctly interpreted.

8. Translate the HFP regression 95% confidence interval into 95/95 confidence limits on the decrement bias by using the largest span of confidence interval observed between 10.0 and 50.0 GWd/T sub-batch burnup. (e.g., we want 95% probability of 95% confidence that the true data is within this confidence limit of the regression fit. (Note the 95/95 criteria should actually be applied to the final SFP criticality reactivity - not just this HFP reactivity decrement bias component.)

It should be noted that we have made the assumption that there is one reactivity decrement bias and uncertainty that can be applied to all fuel sub-batches – regardless of fuel enrichment. It would be possible to repeat the preceding analysis for small ranges of enrichment to further examine this approximation or to derive enrichment-dependent biases and uncertainties, but this might produce larger uncertainties simply because smaller numbers of measured data points would exist within each enrichment range. It is proposed to continue with the original analysis method, since the entire enrichments range of the data is narrow, spanning from 3.4% to 4.95% enrichment.

Naturally, if new uncertainties are larger that those contained in the original EPRI report, we will recombine the new uncertainties with the original TSUNAMI HFP-to-Cold additional uncertainties to get updated cold reactivity decrement uncertainties. The EPRI Report will then be updated with the new statistical analysis and re-issued.

This plan is a very reasonable statistical plan that can actually be implemented, and it addresses the identified major statistical issues. However, it is important that all parties agree on the minor deviations from the purely statistical ideals. We look forward to our discussions with all concerned parties to obtain a consensus on the statistical plan to be implemented and then to performing the subsequent analysis that is desired.

**References**


1. K. S. Smith et al., ‘‘Benchmarks for Quantifying Fuel Reactivity Depletion Uncertainty,’’ Technical Report 1022909, Electric Power Research Institute (2011).

2. J. M. Scaglione and J. C. Wagner, "Burnup Credit Approach for the Proposed United States Repository at Yucca Mountain," *Proceedings of the IAEA/CSN International Workshop on Advances in Applications of Burnup Credit for Spent Fuel Storage, Transport, Reprocessing, and Disposition*, Cordoba, Spain, October 27-30, 2009.

3. J. C. Wagner, M. D. DeHart, and C. V. Parks, “Recommendations for Addressing Axial Burnup in PWR Burnup Credit Analyses,” NUREG/CR-6801 (ORNL/TM-2001/273 (2003).

4. EPRI, “Determination of the Accuracy of Utility Spent Fuel Burnup Records – Interim Report,” Electric Power Research Institute, EPRI TR-109929, (May 1998).

5. G. Radulescu, et al., “Range of Applicability and Bias Determination for Postclosure Criticality of Commercial Spent Nuclear Fuel,” ORNL/TM-2007/127 (2007).

6. G.A. Gunow, “LWR Fuel Reactivity Depletion Verification Using 2D Full Core MOC and Flux Map Data,” M.S. Thesis, MIT (September, 2014). https://dspace.mit.edu/handle/1721.1/97963

7. E. Sykora, “Testing the EPRI Reactivity Depletion Decrement Uncertainty Methods,” M.S. Thesis, MIT (August, 2015). Pending MIT dspace.mit.edu/handle assignment.