SAN ONOFRE NUCLEAR GENERATING STATION, UNIT 1

ENERGY BALANCE METHOD FOR PIPING SYSTEMS

Prepared by:

Southern California Edison Company

April 15, 1985

TABLE OF CONTENTS

1.0 INTRODUCTION

The energy balance method is used on a case-by-case basis to evaluate pipe functionality for isolated supports which are determined to yield or fail when performing support evaluation. It should be noted that failures are postulated when comparing the predicted stresses to the LTS stress limits, and do not represent actual failure. Since this is a hand evaluation procedure, conservative assumptions are made at each stage of the evaluation to ensure conservative results.

The energy approach compares the earthquake kinetic energy input to the piping system, versus the strain energy capacity of the piping at an acceptable deformation limit. If the earthquake energy input exceeds the strain energy capacity, the system experiences excessive deflection and is assumed to fail. However, if the allowable energy absorption of the piping system is greater than the energy provided by the earthquake phenomena, the piping system will remain functional. The strain energy capacity of a piping system is a direct function of maximum allowable response displacement. By accounting for the pipe's actual plastic deformation, it can be shown that piping systems are capable of absorbing a significant amount of input energy before any failure mechanism can occur.

2.0 ENERGY BALANCE METHOD--SDOF SYSTEM

The energy approach concept is first developed for a single-degree-of-freedom (SDOF) system. This procedure is then extended for multiple-degree-of-freedom (MDOF) systems. Consider a SDOF system of mass M and stiffness K. The maximum kinetic energy input to the system can be conservatively estimated from the maximum response of the system. The maximum kinetic energy input is given by:

$$KE = \frac{1}{2} MV^2$$

where V = maximum response velocity

For the same system, the strain (potential) energy capacity is given by:

$$PE = \frac{1}{2} KX^2$$

where X = maximum response displacement.

By equating the maximum kinetic energy input to the strain energy capacity of the system, the maximum response displacement X can be estimated as

$$X = \frac{V}{\omega}$$

where ω is the system's natural frequency in radians per second.

If X is less than some measure of displacement capacity (for example, an allowable strain), then the SDOF system is capable of absorbing the maximum energy input.

3.0 ENERGY BALANCE METHOD--PIPING SYSTEMS

Using the same analogy for piping systems (MDOF), an evaluation is performed where the energy input to the piping is equated to the pipe strain energy. The maximum strain in the system is then computed for the maximum deformation state defined by the absorption of the kinetic energy. The maximum strain, including elastic and plastic strains, is compared to the limiting allowable strains. If the calculated strains are below allowables, the piping remains functional.

A detailed procedure has been developed to evaluate the maximum strain. At each stage in the procedure, assumptions are made to ensure that an upper bound prediction of strain is obtained for all analysis applications. The approach is discussed below. The engineering basis for the selection of appropriate models which give conservative results at each stage in the evaluation procedure is described in detail.

3.1 Development of the Analysis Model

First, a simplified straight span analysis model is developed to represent the piping to be reevaluated as a result of failed or yielded supports. The analysis model is developed by reviewing the pipe span containing the weak support. The model is conservatively developed so as to explicitly include the effects of valve weights, elbow flexibilities and branch connections.

In predicting a lower bound frequency for the pipe span, the effects of large lumped masses (eg., valves, piping runs perpendicular to the span direction) are considered. It is important to calculate a lower bound frequency in order to obtain an upper bound spectral acceleration for the kinetic energy computation. This is further explained in Section 3.2. For spans with large lumped masses, the frequency of the span is evaluated using the formula given by Dunkerley [2]:

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}$$

where f_1 = frequency of beam with uniformly distributed mass
f_2 = frequency of beam with lumped mass
f = equivalent frequency of beam

The effect of elbow flexibility on the pipe span is accounted for. For all elbows which experience out-of-plane bending, the span length is appropriately increased.

For any branch connections, the mass of half the span of the branch is conservatively lumped in the analysis model. At the same time, the stiffness effect on the pipe span resulting from the branch connection is conservatively neglected.

3.2 Kinetic Energy Computation

The maximum kinetic energy, which is the maximum earthquake energy input to the system, is computed by integrating the maximum pipe velocity over the length of pipe:

$$KE = \frac{1}{2} \int_0^L MV^2 \; dL$$

where M = mass per unit length of pipe.

It should be noted that the maximum velocity can be determined in several ways: direct time integration, Fourier Transform, or response spectrum analysis. If the maximum velocity is conservatively computed, the upper bound of the maximum energy input is always obtained, regardless of the method of analysis. For large bore piping, the more conservative response spectra approach is used to compute the maximum velocity for computing the maximum energy input. As a result, the peak response analyses are simplified, since the floor spectra are readily available for all elevations. Additional conservatism is introduced by using the elastic response spectra with damping values corresponding to elastic systems, thereby overpredicting the piping maximum velocity. Thus, the computed kinetic energy is an upper bound on the earthquake energy input to the piping system.

The response spectrum maximum acceleration (a_{max}) is used for the velocity determination:

$$V_{max} = \frac{a_{max}}{\omega}$$

where a_{max} = the maximum spectral acceleration at the frequency of the pipe span using the PVRC - recommended damping value. For pipe span frequencies on the flexible side of the spectral peak, the peak spectral acceleration is conservatively used.

To further maximize the maximum velocity, a lower bound fundamental frequency (ω) of the beam span is used. For continuous spans, a conservative lower bound frequency is obtained by assuming pin-ended boundary conditions for the analysis model. To determine the average span velocity, a sine-distribution is assumed, with V_{max} at midspan. The sine-distribution assumes a critical velocity field at the incipient hinge formation at mid-span.

$$V_{avg} = \int_0^L \frac{V_{max}}{L} \sin \frac{\pi x}{L} dx = \frac{a_{max}}{\pi} \frac{2}{\omega}$$

$$KE = \frac{1}{2} M \left(\frac{a_{max}}{\pi} \frac{2}{\omega}\right)^2$$

In summary, the following conservatisms maximize the computation of the input kinetic energy:

- use of an upper bound spectral acceleration from elastic spectra,
- use of a lower bound span frequency, and
- use of a conservative velocity distribution.

3.3 Strain Energy Computation

The pipe strain (potential) energy is computed by integrating the strain energy per unit volume over the entire volume of the pipe material:

$$U = \int \sigma \epsilon \quad dv$$

where σ = stress per unit volume
ϵ = strain per unit volume

Since a beam absorbs more energy in plastic deformation than in elastic deformation, the span is assumed to form point hinges, which greatly limits the plastic deformation and the evaluated strain energy absorption capacity of the beam. Thus, except for plastic strain energy in the point hinges, elastic strain energy is calculated for the rest of the beam.

The strain energy capacity of the pipe span using both pin-ended and fixed-ended beams is evaluated to determine a conservative analysis model for the computation purpose.

3.3.1 Pin-Ended Beam

For a pin-ended beam, the formation of a hinge in the center of the beam is the limiting condition. The analysis model is shown in Figure 1(a). For this case, the strain energy capacity is given by:

$$U_{emax} = \int_0^L M^2 / (2EI) \, dx$$

$$= W^2_{max} \; L^5/(240 \text{ EI}) \qquad \text{(see Ref. 1)}$$

At the formation of the hinge, the maximum equivalent load is:

$$W_{max} = 8\, M_p / L^2 = 4Sy(D^3 - d^3)/(3L^2)$$

where M_p = plastic moment = $S_y Z$
S_y = yield stress of pipe material
Z = plastic modulus = $(D^3 - d^3)/6$
D = outer pipe diameter
d = inner pipe diameter

If the input kinetic energy is less than or equal to the elastic strain energy required to form the hinge, the resulting strain is computed by equating the two energies. The equivalent load used to evaluate this strain is

$$W_{eq} = \sqrt{\frac{240\, EI(KE)}{L^5}}$$

If the input kinetic energy is greater than the strain energy, a collapse mechanism is postulated.

3.3.2 Fixed-Ended Beam

To calculate the strain energy in the beam, two different models are considered. The first model assumes fixed-ends and computes elastic strain energy until formation of the first set of hinges at the ends. The second model considers new boundary conditions which limit the end moments to the plastic moment of the section. This model is then used to compute the elastic energy in the beam and the plastic energy in the point hinges up to the formation of a collapse mechanism.

Model 1

This model is schematically shown in Figure 1(b). The maximum elastic strain energy to the formation of two hinges at the ends is given by

$$U_{elmax} = \int_0^L M^2 / (2EI)\, dx$$

$$= W_{1max}^2\, L^5/(1440\, EI) \qquad \text{(See Ref. [1])}$$

At the formation of two hinges,

$$W_{1max} = 12\, M_p / L^2 = 2S_y(D^3 - d^3)/ L^2 \quad \text{(for a pipe section)}$$

If the input kinetic energy (KE) is less than or equal to the elastic strain energy required to form two hinges, the resulting strain is computed by equating the two energies. The equivalent load used to evaluate this strain is

$$W_{eq} = \sqrt{\frac{1440\ EI(KE)}{L^5}}$$

If the input kinetic energy is greater than the elastic strain energy to form two hinges, a second model is used to predict the strain energy capacity of the pipe.

Model 2

This model is schematically shown in Figure 1(c). This model considers new boundary conditions (plastic hinges at the pipe span ends) which limit the end moments to the plastic moment of the section. The elastic strain energy in the beam for the formation of the third hinge in the center is evaluated here.

$$U_{e2} = \int_0^L M^2 / (2EI)\ dx$$

$$= U_{e2,\ 0\text{-}W_2} - U_{e2,0\text{-}W_{1max}}$$

$$= \frac{1}{2EI}\left[\frac{1}{120}\left(W_2^2 - W_{1max}^2\right) L^5 - \frac{1}{6} M_p \left(W_2 - W_{1max}\right) L^3\right] \quad \text{(see Ref. 1)}$$

For the formation of the third hinge,

$$W_{2max} = 16\ M_p/L^2 = 16\ Sy\ (D^3 - d^3)\ /(6L^2)$$

The analysis model for the prediction of the plastic strain energy in the point hinges is shown in Figure 1(c) and is given by

$$U_{p2} = 2\ M_p\theta$$

$$= \frac{M_p L^3}{12EI}\left(W_2 - W_{1max}\right)$$

$$\text{where}\quad \theta = \int_0^{L/2} M/(EI)dx$$

The total elastic and plastic strain energy for the formation of the third hinge is

$$U_2 = \frac{L^5}{240EI}(W_2^2 - W_{1max}^2)$$

Equating the strain and kinetic energy

$$KE = U_{e1} + U_2$$

Simplifying,

$$W_{eq} = \sqrt{\frac{4S_y^2(D^3-d^3)}{L^4} + \frac{240EI}{L^5}(KE - U_{e1})}$$

W_{eq} is limited to W_{2max}, which signifies the formation of a third hinge.

3.3.3 Selection of the Most Conservative Model

The equivalent load to cause the formation of first hinges in a fixed-ended model is:

$$W_{eq} = \sqrt{\frac{1440\ EI(KE)}{L^5}}, \qquad W_{eq} \leqslant W_{1max} = 2S_y\ (D^3 - d^3)/L^2$$

For the pin-ended model to the formation of first hinge,

$$W_{eq} = \sqrt{\frac{240\ EI(KE)}{L^5}}, \qquad W_{eq} \leqslant W_{max} = 4S_y\ (D^3 - d^3)/(3L^2)$$

This demonstration shows that the fixed-ended model always predicts a larger equivalent load for the formation of hinges and is the more conservative analysis model. Since the same model is used to determine the maximum rotations from the equivalent load (see Section 3.3), upper bound rotations and, as a result, higher strains) will be predicted using the fixed-ended model.

Restated, the fixed-ended model goes through less deformation than a pin-ended model before reaching a limiting strain allowable, when both are subjected to the same amount of input kinetic energy. Therefore, less potential (strain) energy gets absorbed in a fixed-ended model.

In summary, the following conservatisms minimize the strain energy absorption capacity of the beam:

- assumption of point hinges minimizes plastic strain energy,
- use of elastic energy equations to calculate energy in yielded portions of the beam is conservative, and
- a conservative model is selected to predict the strain energy in the beam.

3.4 Prediction of Maximum Rotations

Using the evaluated equivalent load W_{eq}, an upper bound rotation for the span is evaluated by assuming a simply supported span. The resulting rotation is:

$$\theta = W_{eq} L^3 / (24 EI)$$

This relationship is used to predict upper bound rotations for both fixed-ended and pin-ended beam models. On a case-by-case basis, more realistic rotations can be predicted by considering the stiffness of the adjacent spans.

3.5 Calculation of Strain

Functionality of the pipe span is ensured if the span rotations do not result in excessive strains. Experimental data is used to predict strains from calculated upper bound rotations. Elbow elements are assumed to be the critical components, and are assumed to be present at span ends and at midspan. Test results for 6 inch non-pressurized carbon and stainless steel elbow elements [3] are used to determine rotation-strain relationships. The test data which give the most conservative prediction of strain are used as the basis for predicting the maximum strain. The predicted strain is then compared to the allowable strain limits of 1 and 2 percent for carbon and stainless steel, respectively.

4.0 REFERENCES

1. "Energy Comparison Methodology Development," Impell Calculation No. EC-METH, Revision 2, Job No. 0310-028-1355, April 1984.

2. Marks' Standard Handbook for Mechanical Engineers, Seventh Edition, 1967.

3. Greestreet, W.L., "Experimental Study of Plastic Responses of Pipe Elbows," ORNL/NUREG 24, February 1978.

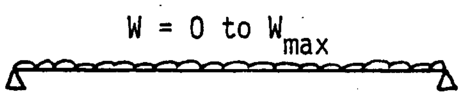

Figure 1(a) Pin-Ended Beam Model

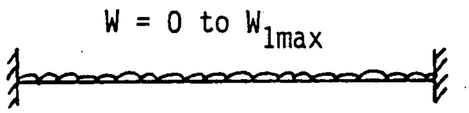

Figure 1(b) Fixed-Ended Beam - Model 1

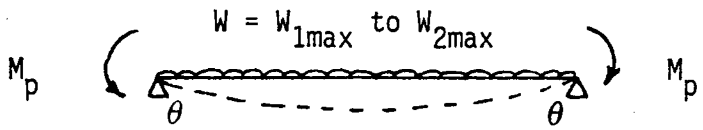

Figure 1(c) Fixed-Ended Beam - Model 2

FIGURE 1 Analytical Models for Evaluation of Pipe Strain Energy Capacity