Reference B-1
Pages 55 through 62, 165 and 168

AN INVESTIGATION OF LAMINAR FLOW IN FRACTURED POROUS ROCKS

by

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November 1970

$$\frac{\partial J}{\partial \phi_m} = \frac{1}{2D}\Bigg[\left\{K_{xx}b_m b_i + K_{xz}(c_m b_i + b_m c_i) + K_{zz}c_m c_i\right\}\phi_i$$

$$+ \left\{K_{xx}b_m b_j + K_{xz}(c_m b_j + b_m c_j) + K_{zz}c_m c_j\right\}\phi_j$$

$$+ \left\{K_{xx}b_m b_m + K_{xz}(c_m b_m + b_m c_m) + K_{zz}c_m c_m\right\}\phi_m\Bigg]$$

These solutions are valid only for elements whose nodes both i and j lie on a boundary, and node m does not lie on a boundary. For an element with only one node on the boundary, the integration along the length of the boundary is zero, the velocity term drops out, and the standard internal equation results. For nodes along boundaries of zero flow, V will be zero and the equations reduce to the standard internal equation. A copy of the finite element computer program produced from these theoretical considerations is presented in Appendix A.

2. Verification and Use of the Triangular Element Program

This program was verified by creating several simple problems which could be checked by hand. First, flow in a straight parallel-wall fracture was calculated, and the results were in good agreement with hand calculations. Next, a system with one fracture sloping upward at +4.5°, and a second fracture intersecting near the midpoint and branching down at -9.0° was checked (Fig. II-10).

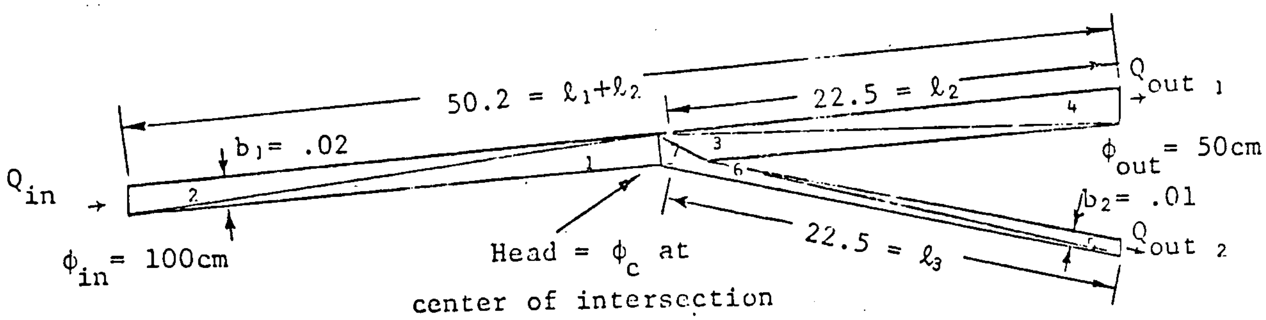

Fig. II-10: Plan for second check of computer program. Elements are numbered 1 to 7. Lengths and heads ϕ in cm units.

is less than about 200. Laminar flow in any conduit is always characterized by a linear relationship between velocity and gradient:

$$V = c \frac{\partial \phi}{\partial \ell}$$

where ℓ is measured along the path and the proportionality factor c, called the hydraulic conductivity, is a function of conduit geometry and fluid properties. For smooth wall parallel plate flow the hydraulic conductivity is equal to $b^2\gamma/12\mu$. Hence for a flow channel of any shape, if velocities are in the laminar range an aperture can be calculated for a smooth parallel plate conduit which will offer the same resistance to flow as the arbitrarily shaped channel. Therefore, in the laminar regime, any real fracture with its many contortions and wall asperities will behave in an overall sense, but not in terms of internal detail, as a parallel plate with a correctly chosen effective aperture b_{eff}.

A simple model of a fracture with varying aperture is presented in Fig. II-11 where three separate apertures are encountered along the length of the fracture. From continuity considerations similar to those used in the calculations accompanying Fig. II-10, the total flow was calculated by hand to be 0.0207 cm^3/sec. By assigning element apertures equal to the local width of the fracture, and setting the permeability ellipsoid orientation angle $\alpha = 0$ in all elements, the finite element result of 0.0201 cm^3/sec compares well with the hand calculated value.

From these same continuity considerations, the effective permeability for a fracture of three different apertures may be expressed as:

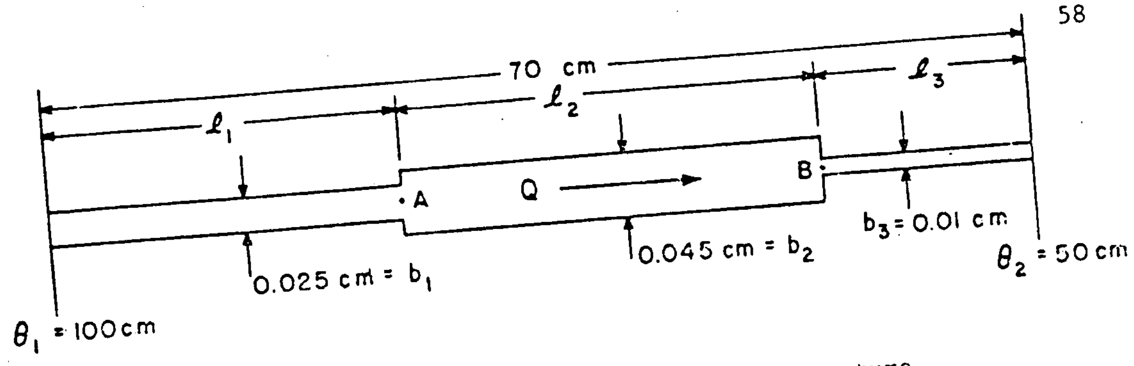

Fig. II-11. Fracture with irregular aperture.

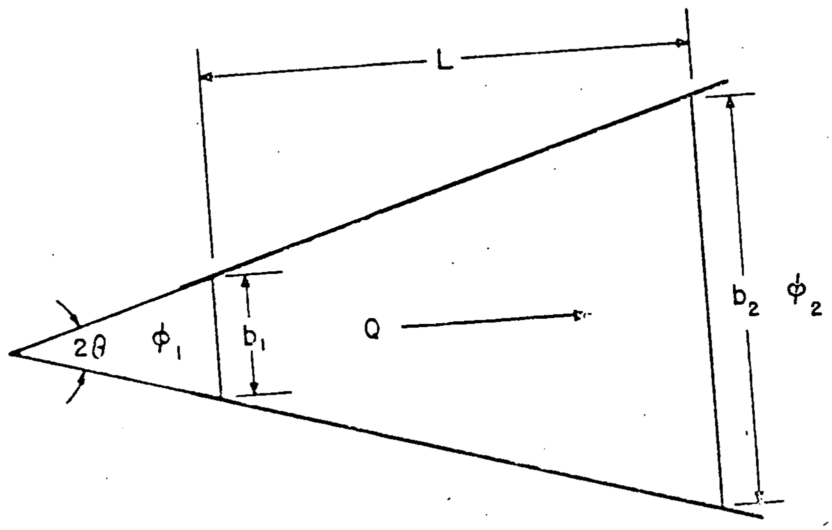

Fig. II-12. A wedge-shaped fracture.

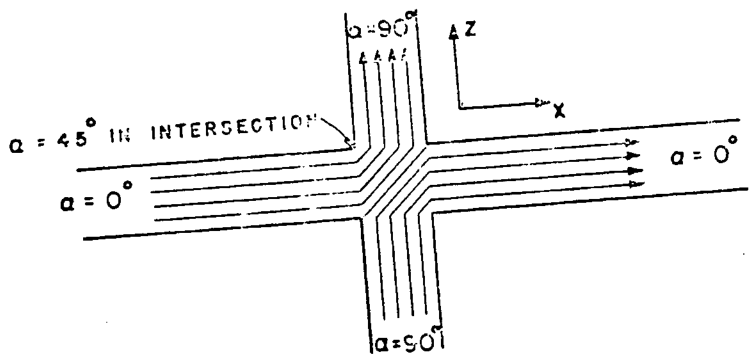

Fig. II-13. Idealized flow lines in intersecting fractures of identical aperture and flow. α gives orientation of K_x, in various regions.

$$b_{eff}^{3} = \frac{\ell_1 + \ell_2 + \ell_3}{\frac{\ell_1}{b_1^3} + \frac{\ell_2}{b_2^3} + \frac{\ell_3}{b_3^3}} \qquad \text{(II-28)}$$

whence $b_{eff} = 0.0161$ for the system of Fig. II-11. Calculating b_{eff} from the computer results yields:

$$b_{eff} = \left(\frac{12\mu Q_T L}{\gamma \Delta H}\right)^{\frac{1}{3}} = 0.0159 \text{ cm}$$

It is interesting to note that at point A in Fig. II-11, the head has dropped from 100 cm to 95.4 cm, and at point B it has further dropped to 94.5, leaving the remaining head of 44.5 cm to be lost in the short length of narrow channel. In this system 89% of the headloss occurs in only 21% of the length because of the smaller channel and the cubed relationship between aperture and flow volume.

It is evident from equation II-28 that for a flow channel of more than three apertures along its length, the effective aperture may be expressed as:

$$b_{eff}^{3} = \frac{\sum_{i=1}^{n} \ell_i}{\sum_{i=1}^{n} \frac{\ell_i}{b_i^3}} \qquad \text{(II-29)}$$

Or, if the aperture varies continuously along the length of the fracture, the effective aperture may be expressed as:

$$b_{eff}^3 = \frac{\int_0^L d\ell}{\int_0^L \frac{d\ell}{b(\ell)^3}} \tag{II-30}$$

The aperture b is now expressed as a function of path length ℓ along the fracture, where L is the total path length. This derivation assumes that locally the flow lines will be essentially horizontal and will not hold in those cases where apertures change abruptly along the length of the fracture.

As an example, the results of equation II-30 will be compared to a computer solution for the case of a wedge-shaped fracture (Fig. II-12). For this fracture, equation II-30 may be written:

$$b_{eff}^3 = \frac{\int_0^L d\ell}{\int_0^L \frac{d\ell}{(m\ell + c)^3}}$$

where if 2θ is the angle of opening, then $m = 2 \tan \theta$, $c = b_1$ = aperture at smaller end, and $mL + c = b_2$ = aperture at larger end. The solution to this equation is:

$$b_{eff}^3 = \frac{2mLb_1^2 b_2^2}{b_2^2 - b_1^2} = \frac{2b_1^2 b_2^2}{b_1 + b_2} \tag{II-31}$$

For example, if $b_1 = 0.205$ cm, $b_2 = 0.415$ cm, L = 20 cm, and $\theta = 0°18'$, then from equation II-31, $b_{eff} = 0.286$ cm. The computer, using 40 elements, obtained a value of $b_{eff} = 0.280$ cm. Even greater accuracy can be obtained from the finite element method if more elements are used. Note that while equation II-31 is independent of L and θ, in reality θ must be rather small for flow to remain essentially horizontal.

A laboratory study of flow in wedge-shaped parallel plate conduits was performed by G. M. Lomize[133]. In one of Lomize's experiments, utilizing water in a conduit with the same dimensions as in the previous example, a flow rate of approximately 3 cm^3/sec per unit plate width was measured under a gradient of 0.02 cm/cm. From these results an effective aperture of b_{eff} = 0.27 cm is calculated.

This conduit has been modeled using 40 triangular elements. Assigning the same gradient of 0.02 cm/cm, and assuming unit density and a viscosity of 1 centipoise for the water in the test, the flow rate was calculated to be 3.6 cm^3/sec per unit width. This compares well with the experimental value, the discrepancies being due in a large part to slight roughnesses in the plate walls in Lomize's experiments (they cannot be ideally smooth as is assumed in the mathematical model). Also error occurs because the water temperature used in the calculations was assumed to be 70°F, which is probably not identical to the experimental conditions. No information was given by Lomize concerning temperature.

Since errors due to effects of wall roughness and temperature can be accounted for mathematically if these effects are known, in general the results should be as good as or better than those obtained above. This comparison with Lomize's experimental results indicates that fractures with gradually varying apertures can be accurately modeled using the two-dimensional finite element program.

Fracture intersections present a difficulty in the application of the triangular element program because hydraulic conductivity is not easily defined in this space. The intersection belongs mutually to two separate fractures which may be of different aperture and hence different permeabilities, and at their juncture neither the magnitude nor the orientation of the permeability ellipsoid is defined.

The magnitude of the permeability which is mathematically identified with each triangular fracture element is related to an aperture assigned to that element. In an intersection or in other areas where the aperture is undefined the permeability is also undefined. Hence for elements in these regions it is necessary to arbitrarily select an "effective aperture" which will give a reasonable permeability value. By varying this effective aperture in intersection elements it is possible to introduce special headlosses into the model to account for interference effects which exist at intersections.

The magnitude of these effects was studied in the laboratory and the results are presented in Chapter III. They were found to be sufficiently small at low flow rates to be neglected, and it has been the practice in this paper to assign to intersection elements apertures equal to that of the larger fracture. However, the conductivity assigned to each fracture element is arbitrary and if it is desired, this model can be made to account for interference effects by assigning slightly smaller apertures to intersection elements.

The orientation of the permeability ellipsoid in each element is governed by the choice of angle α which determines specifically the orientation of the directional permeability $K_{x'}$ within the permeability ellipsoid. Within a fracture segment it is clear that α should equal the fracture orientation, and that $K_{x'}$ should equal $b^2\gamma/12\mu$, but within an intersection flow is undergoing abrupt changes in direction and also

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Appendix C. Back Up for Figure 4.6

Figure 4.6 represents simply a cross plotting of normalized flow $Q/\Delta H$ versus axial stress σ_e (Figure 1 of Reference) and aperture 2b versus normalized flow graphs (Figure 5 of Reference). For granite these curves are presented on Figures 1 and 5 in Witherspoon et al., (October 1979) attached as reference C-1 and for marble and basalt they are presented on Figures 4.40 and 4.45 attached from Iwai's thesis and Figures 7 and 8 in Witherspoon et al., (October 1979). These figures are labeled C-1 through C-6 and are used to explain the cross plotting completed to develop Figure 4.6. Specifically, for granite a range of axial stress values was obtained from the data on Figure C-1 for a selected value of $Q/\Delta h$. The value of aperature corresponding to the same value of $Q/\Delta h$ was then obtained from the 2b vs. $Q/\Delta h$ graph at the top of Fig. C-2. The value of 2b (aperture) and range of σ_e (axial strss) was then plotted with aperature as the ordinate and compressive stress (axial stress) as the abscissa. Similar cross plotting was completed for marble using Figures C-3 and C-4 and for basalt using Figures C-5 and C-6. The range of values shown on Figure 4-6 approximately envelopes the data resulting from this cross plotting.

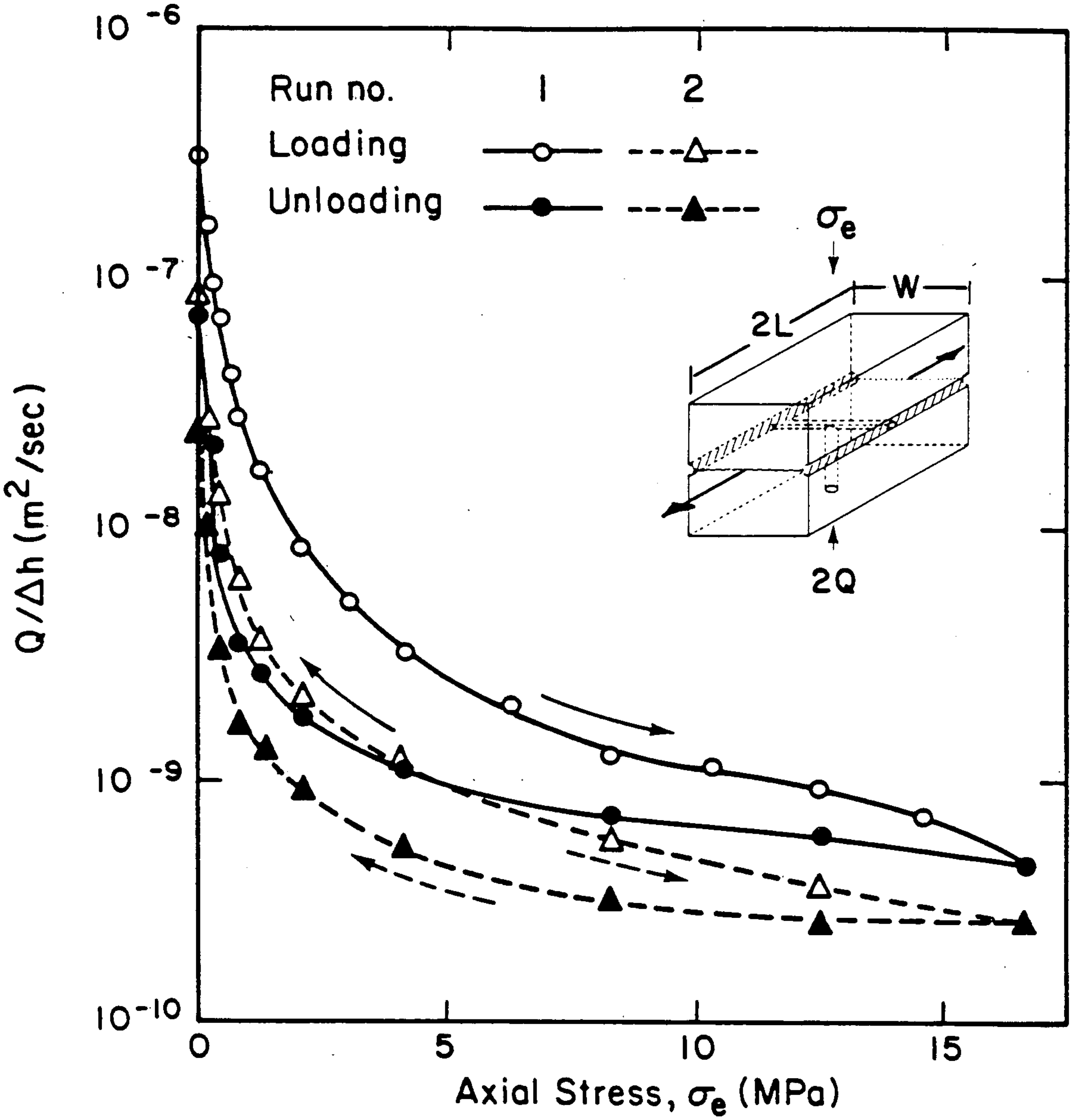

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Fig. 1. Effect of cyclic loading on permeability of tension fracture in granite with straight flow (after Iwai, 1976).

Figure C-1 Granite

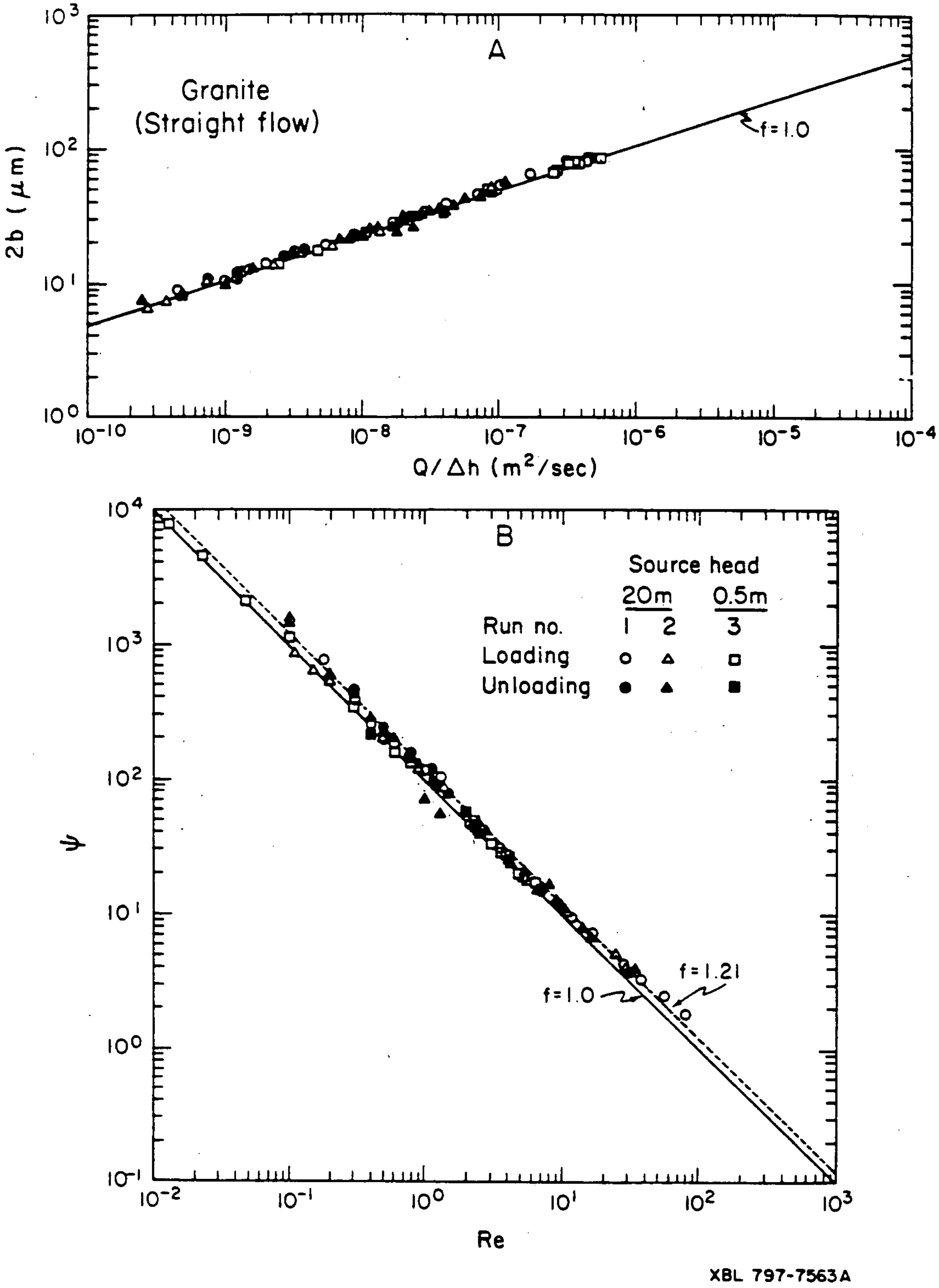

Fig. 5. Comparison of experiment for straight flow through tension fracture in granite with cubic law.

Figure C-2 Granite

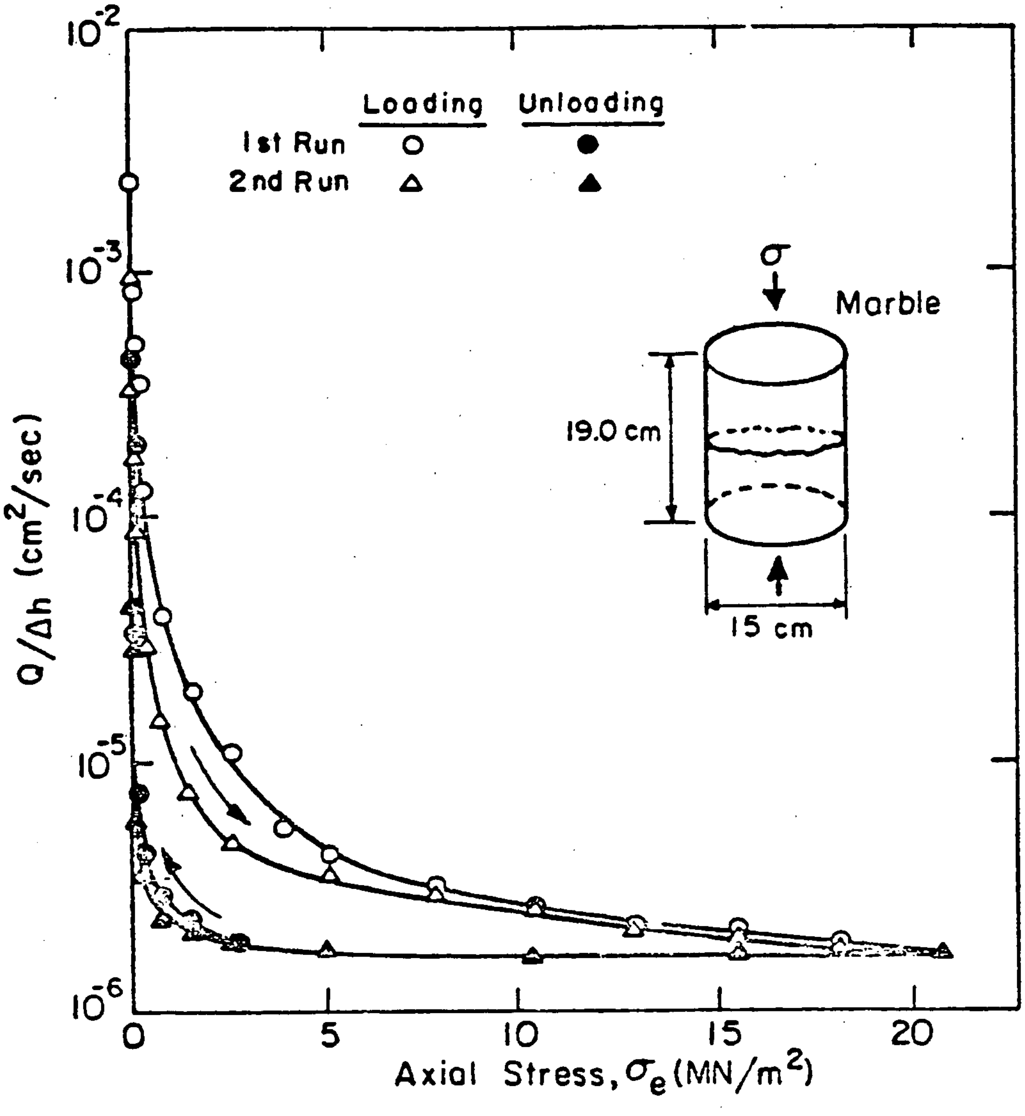

Fig. 4.45. Effect of cyclic loading on permeability of tension fracture in marble with radial flow.

Figure C-3 Marble

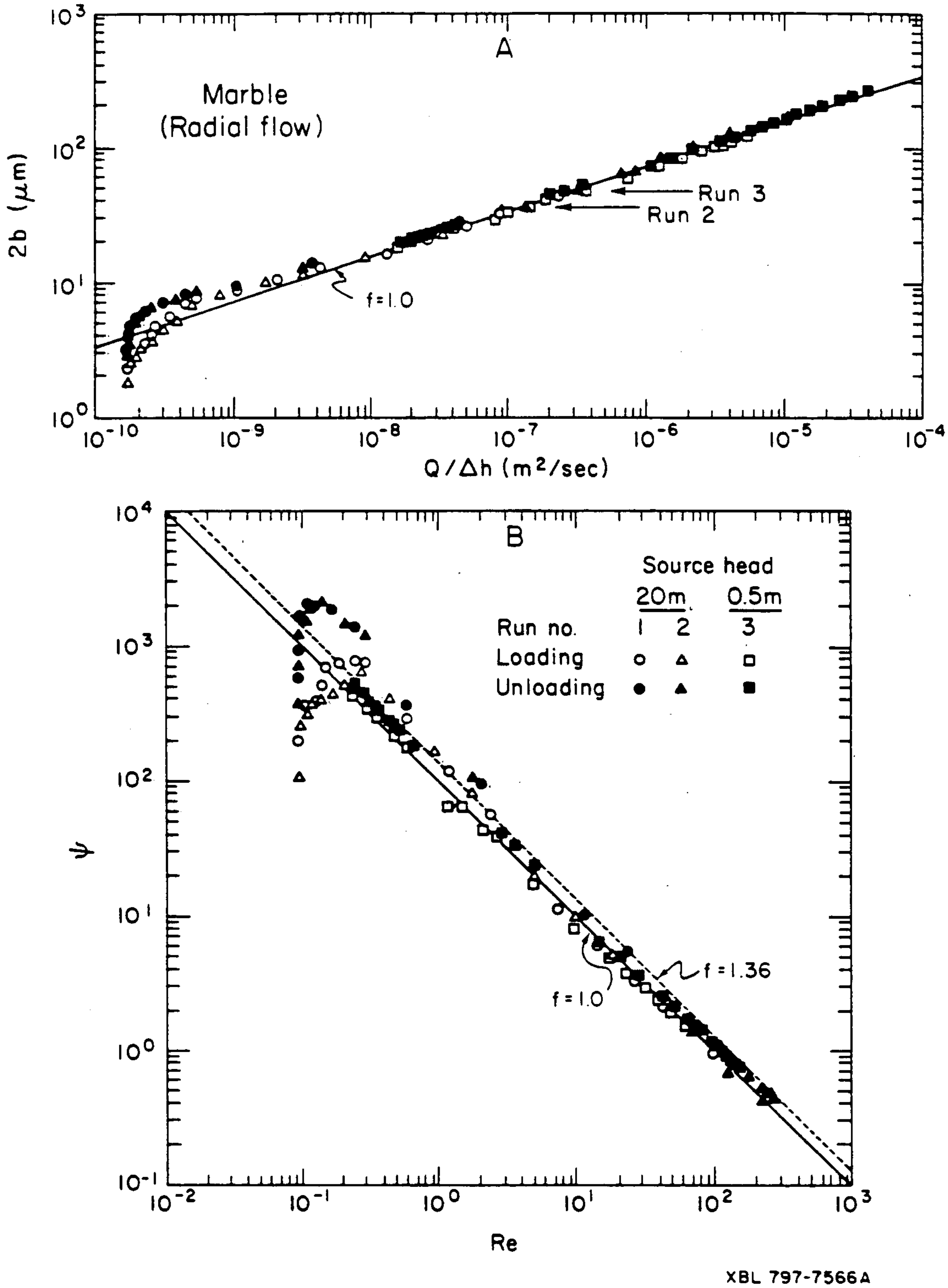

Fig. 8. Comparison of experimental results for radial flow through tension fracture in marble with cubic law. In Runs 2 and 3, fracture surfaces were no longer in contact during unloading when aperture exceeded value indicated by arrow.

Figure C- 4 Marble

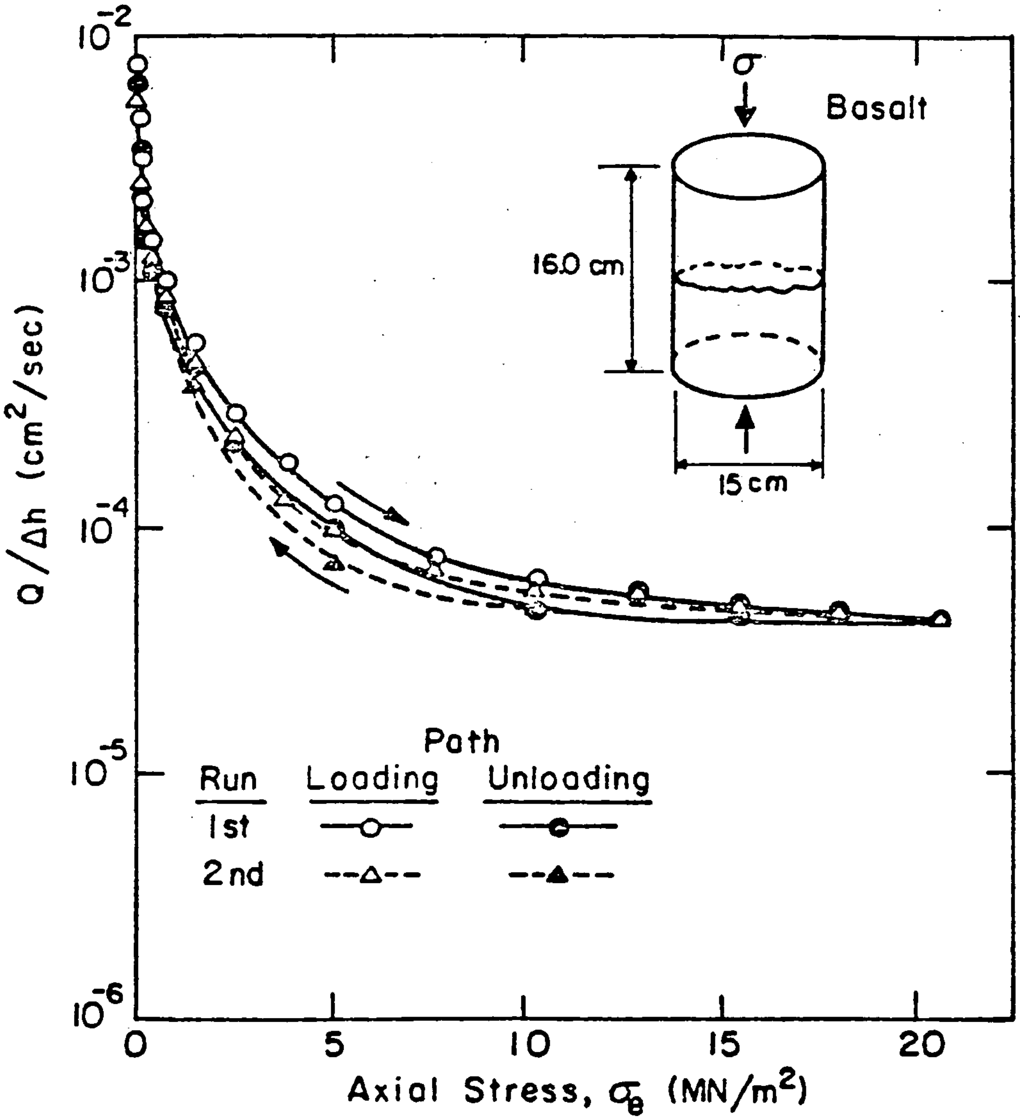

Fig. 4.40. Effect of cyclic loading on permeability of tension fracture in basalt with radial flow.

Figure C-5 Basalt

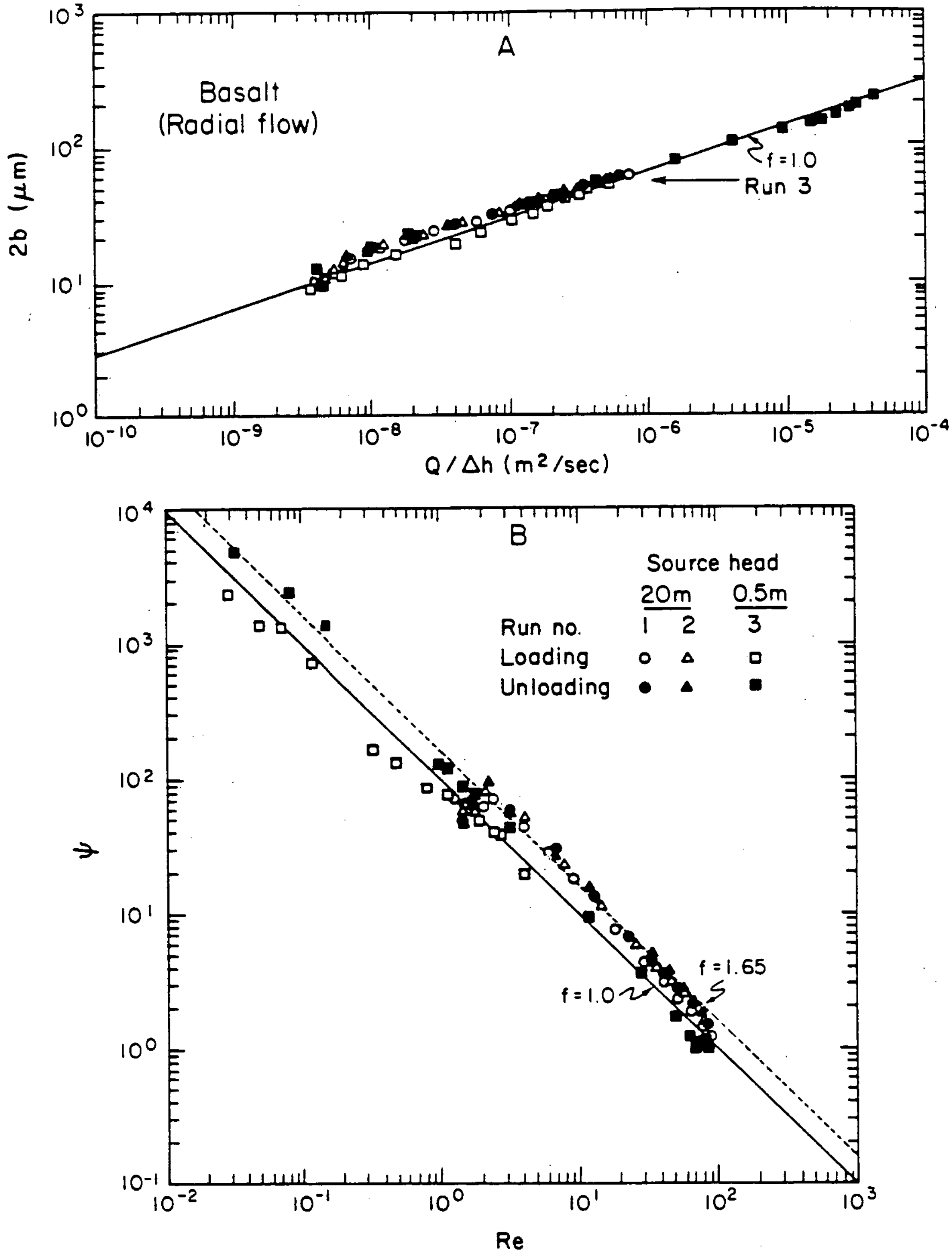

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Fig. 7. Comparison of experimental results for radial flow through tension fracture in basalt with cubic law. In Run 3, fracture surfaces were no longer in contact during unloading when aperture exceeded value indicated by arrow.

Figure C-6 Basalt

LBL-9557
SAC-23
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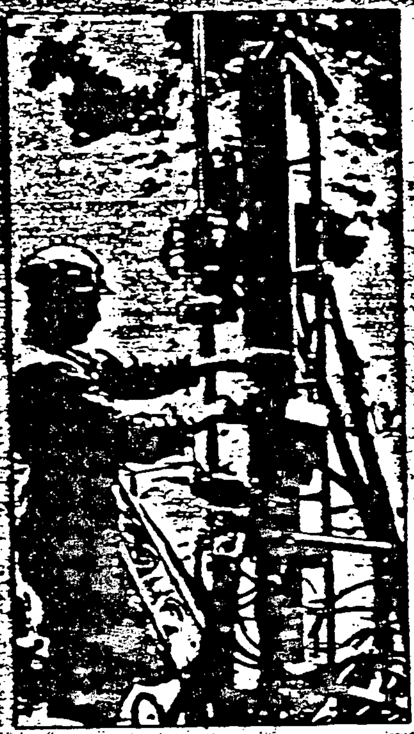
Reference C-1

SWEDISH-AMERICAN COOPERATIVE PROGRAM ON RADIOACTIVE WASTE STORAGE IN MINED CAVERNS IN CRYSTALLINE ROCK



Technical Information Report No. 23

VALIDITY OF CUBIC LAW FOR FLUID FLOW IN A DEFORMABLE ROCK FRACTURE

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October 1979

A Joint Project of

Swedish Nuclear Fuel Supply Co.
Fack 10240 Stockholm, Sweden
Operated for the Swedish
Nuclear Power Utility Industry

Lawrence Berkeley Laboratory
Earth Sciences Division
University of California
Berkeley, California 94720, USA
Operated for the U.S. Department of
Energy under Contract W-7405-ENG-48