Basic Electric Circuit Analysis

SECOND EDITION

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tions, such as digital computers, the second is an impracticably large unit. As a result, times such as 1 nanosecond (1 ns or 10^{-9} s) are in common use. Another common example is 1 gram (g) = 10^{-3} kg.

Exercises

1.1.1 Find the number of microseconds in 20 s. ANS. 2×10^7

1.1.2 Find the number of miles in 100 km if a mile equals 5280 ft. ANS. 62.14

1.1.3 Find the constant force in μN which applied to a mass of 6 g for a distance of 20 m results in 4 mJ of work. ANS. 200

1.2 CHARGE AND CURRENT

We are familiar with gravitational forces of attraction between bodies, which are responsible for holding us on the earth and which cause an apple dislodged from a tree to fall to the ground rather than to soar upward into the sky. There are bodies, however, that attract each other by forces far out of proportion to their masses. Also, such forces are observed to be repulsive as well as attractive and are clearly not gravitational forces.

We explain these forces by saying that they are electrical in nature and caused by the presence of *electrical charges*. We explain the existence of forces of both attraction and repulsion by postulating that there are two kinds of charges, positive and negative, and that unlike charges attract and like charges repel.

As we know, according to modern theory, matter is made up of atoms, which are composed of a number of fundamental particles. The most important of these particles are protons (positive charges) and neutrons (neutral, with no charge) found in the nucleus of the atom and electrons (negative charges) moving in orbit about the nucleus. Normally the atom is electrically neutral, the negative charge of the electrons balancing the positive charge of the protons. Particles may become positively charged by losing electrons to other particles and become negatively charged by gaining electrons from other particles.

As an example, we may produce a negative charge on a balloon by rubbing it against our hair. The balloon will then stick to a wall or the ceiling, which are uncharged. Relative to the negatively charged balloon the neutral wall and ceiling are oppositely charged.

We now define the *coulomb* (C), discussed in the previous section, by stating that the charge of an electron is a negative one of 1.6021×10^{-19} coulombs. Putting it another way, a coulomb is the charge of about 6.24×10^{18} electrons. These are, of course, mind-boggling numbers, but their sizes enable us to use more manageable numbers, such as 2 C, in the circuit theory to follow.

The symbol for charge will be taken as Q or q, the capital letter usually denoting constant charges such as $Q = 4$ C, and the lowercase letter indicating a time-varying charge. In the latter case we may emphasize the time dependency by writing $q(t)$. This practice involving capital and lowercase letters will be carried over to the other electrical quantities as well.

The primary purpose of an electric circuit is to move or transfer charges along specified paths. This motion of charges constitutes an *electric current,* denoted by the letters i or I, taken from the French word "intensité." Formally, current is the time rate of change of charge, given by

$$i = \frac{dq}{dt} \tag{1.1}$$

The basic unit of current is the *ampere* (A), named for André Marie Ampère (1775–1836), a French mathematician and physicist who formulated laws of electromagnetics in the 1820s. An ampere is 1 coulomb per second.

In circuit theory current is generally thought of as the movement of positve charges. This convention stems from Benjamin Franklin (1706–1790), who guessed that electricity traveled from positive to negative. We now know that in metal conductors the current is the movement of electrons that have been pulled loose from the orbits of the atoms of the metal. Thus we should distinguish *conventional* current (the movement of positive charges), which is used in electric network theory, and *electron* current. Unless otherwise stated, our concern will be with conventional current.

As an example, suppose the current in the wire of Fig. 1.3(a) is $I = 3$ A. That is, 3 C/s pass some specific point in the wire. This is symbolized by the arrow labeled 3 A, whose direction indicates that the motion is from left to right. This situation is equivalent to that depicted by Fig. 1.3(b), which indicates -3 C/s or -3 A in the direction from right to left.

FIGURE 1.3 Two representations of the same current

Figure 1.4 represents a general circuit element with a current i flowing from the left toward the right terminal. The total charge entering the element between time t_0 and t is found by integrating (1.1). The result is

$$q_T = q(t) - q(t_0) = \int_{t_0}^{t} i \, dt \tag{1.2}$$

We should note at this point that we are considering the network elements to be *electrically neutral*. That is, no net positive or negative charge can accumulate in the

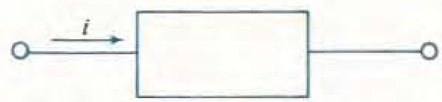

FIGURE 1.4 Current flowing in a general element

element. A positive charge entering must be accompanied by an equal positive charge leaving (or, equivalently, an equal negative charge entering). Thus the current shown entering the left terminal in Fig. 1.4 must leave the right terminal.

There are several types of current in common use, some of which are shown in Fig. 1.5. A constant current, as shown in Fig. 1.5(a), will be termed a *direct current,* or dc. An *alternating current,* or ac, is a sinusoidal current, such as that of Fig. 1.5(b). Figures 1.5(c) and (d) illustrate, respectively, an *exponential* current and a *sawtooth* current.

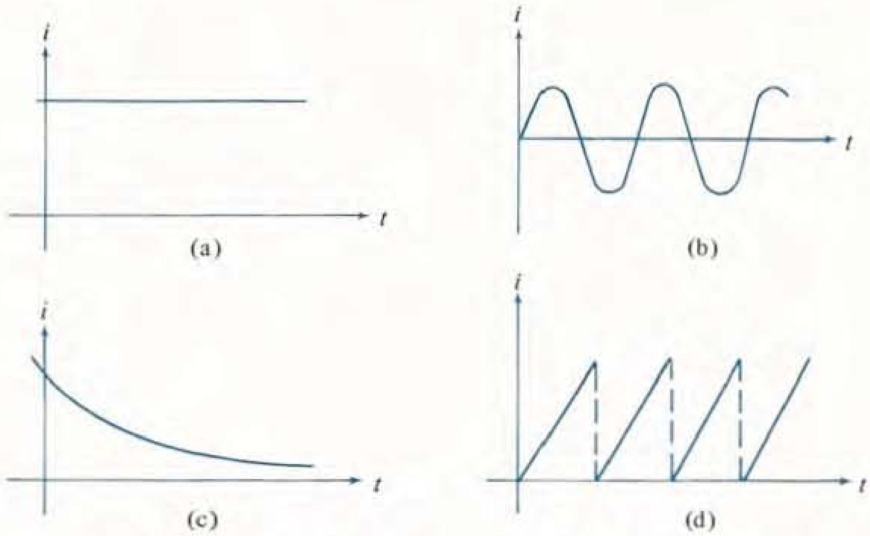

FIGURE 1.5 (a) Dc; (b) ac; (c) exponential current; (d) sawtooth current

There are many commercial uses for dc, such as in flashlights and power supplies for electronic circuits, and, of course, ac is the common household current found all over the world. Exponential currents appear quite often (whether we want them or not!) when a switch is actuated to close a path in an energized circuit. Sawtooth waves are useful in equipment, such as oscilloscopes, used for displaying electrical characteristics on a screen.

EXERCISES

1.2.1 Find the number of electrons equivalent to a charge of -0.016021 pC.
ANS. 100,000

1.2.2 The total charge entering a terminal of an element is given by

$$q = 5t^2 - 8t \text{ mC}$$

Find the current i at $t = 0$ and at $t = 2$ s. ANS. -8, 12 mA

1.2.3 The current entering a terminal is given by

$$i = 20t - 5 \text{ mA}$$

Find the total charge entering the terminal between $t = 1$ s and $t = 4$ s.
ANS. 135 mC

1.3 VOLTAGE, ENERGY, AND POWER

Charges in a conductor, exemplified by free electrons, may move in a random manner. However, if we want some concerted motion on their part, such as is the case with an electric current, we must apply an external or so-called *electromotive force* (EMF). Thus work is done on the charges. We shall define *voltage* "across" an element as the work done in moving a unit charge (+1 C) through the element from one terminal to the other. The unit of voltage, or *potential difference,* as it is sometimes called, is the *volt* (V), named in honor of the Italian physicist Alessandro Guiseppe Antonio Anastasio Volta (1745–1827), who invented the voltaic battery.

Since voltage is the number of joules of work performed on 1 coulomb, we may say that 1 V is 1 J/C. Thus the volt is a derived SI unit, expressible in terms of other units.

We shall represent a voltage by v or V and use the +, − polarity convention shown in Fig. 1.6. That is, terminal A is v volts positive with respect to terminal B. Putting it another way in terms of potential difference, terminal A is at a *potential* of v volts higher than terminal B. In terms of work, it is clear that moving a unit charge from B to A requires v joules of work.

Some authors prefer to describe the voltage across an element in terms of voltage *drops* and *rises*. Referring to Fig. 1.6, a voltage drop of v volts occurs in moving from A to B. In contrast, a voltage rise of v volts occurs in moving from B to A.

FIGURE 1.6 Voltage polarity convention

As examples, Figs. 1.7(a) and (b) are two versions of exactly the same voltage. In (a), terminal A is +5 V above terminal B, and in (b), terminal B is −5 V above A (or +5 V below A).

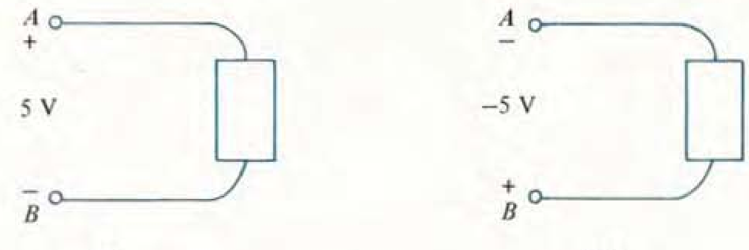

FIGURE 1.7 Two equivalent voltage representations

We may also use a *double-subscript* notation v_{ab} to denote the potential of point a with respect to point b. In this case we have in general, $v_{ab} = -v_{ba}$. Thus in Fig. 1.7 (a), $v_{AB} = 5$ V and $v_{BA} = -5$ V.

In transferring charge through an element work is being done, as we have said. Or, putting it another way, energy is being supplied. To know whether energy is being supplied *to* the element or *by* the element to the rest of the circuit, we must know not only the polarity of the voltage across the element but also the direction of the current through the element. If a positive current enters the positive terminal, then an external force must be driving the current and is thus supplying or *delivering* energy to the element. The element is *absorbing* energy in this case. If, on the other hand, a positive current leaves the positive terminal (enters the negative terminal), then the element is delivering energy to the external circuit.

As examples, in Fig. 1.8(a) the element is absorbing energy. A positive current enters the positive terminal. This is also the case in Fig. 1.8(b). In Figs. 1.8(c) and (d) a positive current enters the negative terminal, and therefore the element is delivering energy in both cases.

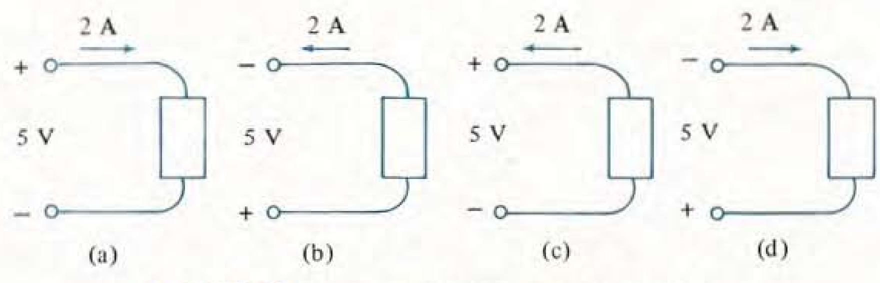

FIGURE 1.8 Various voltage-current relationships

Let us consider now the *rate* at which energy is being delivered to or by a circuit element. If the voltage across the element is v and a small charge Δq is moved through the element from the positive to the negative terminal, then the energy absorbed by the element, say Δw, is given by

$$\Delta w = v\,\Delta q$$

If the time involved is Δt, then the rate at which the work is being done, or the energy w is being expended, is given by

$$\lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \lim_{\Delta t \to 0} v \frac{\Delta q}{\Delta t}$$

or

$$\frac{dw}{dt} = v \frac{dq}{dt} = vi \tag{1.3}$$

Since by definition the rate at which energy is expended is power, denoted by p, we have

$$p = \frac{dw}{dt} = vi \tag{1.4}$$

We might observe that (1.4) is dimensionally correct since the units of vi are (J/C)(C/s) or J/s, which is watts (W), defined earlier.

The quantities v and i are generally functions of time, which we may also denote by $v(t)$ and $i(t)$. Therefore p given by (1.4) is a time-varying quantity. It is sometimes called the *instantaneous* power because its value is the power at the instant of time at which v and i are measured.

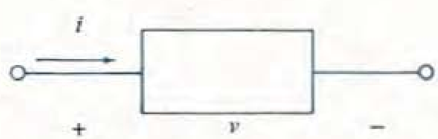

FIGURE 1.9 Typical element with voltage and current

Summarizing, the typical element of Fig. 1.9 is absorbing power, given by $p = vi$. If either the polarity of v or i (but not both) is reversed, then the element is delivering power, $p = vi$, to the external circuit. Of course, to say that an element delivers a negative power, say -10 W, is equivalent to saying that it absorbs a positive power, in this case $+10$ W.

As examples, in Fig. 1.8(a) and (b) the element is absorbing power of $p = (5)(2) = 10$ W. [In Fig. 1.8(b) the 2 A leave the negative terminal, and thus 2 A enter the positive terminal.] In Figs. 1.8(c) and (d) it is delivering 10 W to the external circuit, since the 2 A leave the positive terminal, or, equivalently, -2 A enter the positive terminal.

Before ending our discussion of power and energy, let us solve (1.4) for the energy w delivered to an element between time t_0 and t. We have, upon integrating both sides between t_0 and t,

$$w(t) - w(t_0) = \int_{t_0}^{t} vi \, dt \tag{1.5}$$

For example, the energy delivered to the element of Fig. 1.8(a) between $t = 0$ and $t = 2$ s is given by

$$w(2) - w(0) = \int_0^2 (5)(2) \, dt = 20 \text{ J}$$

Since the left member of (1.5) represents the energy delivered to the element between t_0 and t, we may interpret $w(t)$ as the energy delivered to the element between the beginning of time and t and $w(t_0)$ as the energy between the beginning of time and t_0. In the beginning of time, which let us say is $t = -\infty$, the energy delivered to the element was zero; that is,

$$w(-\infty) = 0$$

If $t_0 = -\infty$ in (1.5), then we shall have the energy delivered to the element from the beginning up to t, given by

$$w(t) = \int_{-\infty}^{t} vi \, dt \tag{1.6}$$

This is consistent with (1.5) since

$$\begin{aligned} w(t) &= \int_{-\infty}^{t} vi \, dt \\ &= \int_{-\infty}^{t_0} vi \, dt + \int_{t_0}^{t} vi \, dt \end{aligned}$$

By (1.6) this may be written

$$w(t) = w(t_0) + \int_{t_0}^{t} vi \, dt$$

which is (1.5).