REGULATORY GUIDE

OFFICE OF NUCLEAR REGULATORY RESEARCH

REGULATORY GUIDE 1.92
(This is an administratively changed version of Revision 2, issued July 2006)

COMBINING MODAL RESPONSES AND SPATIAL COMPONENTS IN SEISMIC RESPONSE ANALYSIS

A.   INTRODUCTION

This regulatory guide provides licensees and applicants with guidance concerning methods that the staff of the U.S. Nuclear Regulatory Commission (NRC) considers acceptable for combining modal responses and spatial components in seismic response analysis of nuclear power plant structures, systems, and components (SSCs) that are important to safety.

Appendix A, "General Design Criteria for Nuclear Power Plants," to Title 10, Part 50, "Domestic Licensing of Production and Utilization Facilities," to the *Code of Federal Regulations* (10 CFR Part 50), Criterion 2, "Design Bases for Protection Against Natural Phenomena," (Ref. 1) requires, in part, that nuclear power plant SSCs important to safety must be designed to withstand the effects of natural phenomena such as earthquakes without loss of capability to perform their safety functions. Such SSCs must also be designed to accommodate the effects of, and be compatible with the environmental conditions associated with normal operation and postulated accidents. Appendix S, "Earthquake Engineering Criteria for Nuclear Power Plants," to 10 CFR Part 50 (Ref. 2) specifies, in part, requirements for implementing General Design Criterion 2 with respect to earthquakes. Appendix S applies to applicants for a design certification or combined license pursuant to 10 CFR Part 52, "Licenses, Certifications, and Approvals for Nuclear Power Plants" (Ref. 3) or a construction permit or operating license pursuant to 10 CFR Part 50 after January 10, 1997. However, the earthquake engineering criteria in Section VI of Appendix A to 10 CFR Part 100 (Ref. 4) continue to apply for either an operating license applicant or an operating license holder whose construction permit was issued before January 10, 1997.

This guide describes methods that the NRC staff considers acceptable for complying with the agency's regulations regarding the following aspects of seismic response analysis:

(1)   Combining the responses of individual modes (in the case of the response spectrum method) to a component of the three orthogonal spatial components of earthquake motion (two horizontal and one vertical), to find the representative maximum response of interest (such as displacement, acceleration, shear, moment, stress, or strain) for a given element of a nuclear power plant SSC.

(2)   Combining the maximum responses (in the case of the time history method) or the representative maximum responses (in the case of the response spectrum method) of an SSC, when such

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responses are calculated either separately (for the response spectrum method or the time history method) or simultaneously (for the time history method) for each of the three orthogonal spatial components (two horizontal and one vertical) of an earthquake

The NRC issues regulatory guides to describe to the public methods that the staff considers acceptable for use in implementing specific parts of the agency's regulations, to explain techniques that the staff uses in evaluating specific problems or postulated accidents, and to provide guidance to applicants. Regulatory guides are not substitutes for regulations, and compliance with regulatory guides is not required.

This regulatory guide contains information collection requirements covered by 10 CFR Part 50 and 10 CFR Part 52 that the Office of Management and Budget (OMB) approved under OMB control number 3150 0011 and 3150-0151, respectively. The NRC may neither conduct nor sponsor, and a person is not required to respond to, an information collection request or requirement unless the requesting document displays a currently valid OMB control number. This regulatory guide is a rule as designated in the Congressional Review Act (5 U.S.C. 801–808). However, OMB has not found it to be a major rule as designated in the Congressional Review Act.

B.   DISCUSSION

Reason for Change

Revision 3 to this guide was issued as an administratively changed guide. The changes from the previous version were editorial with no substantive change in the Staff Regulatory Guidance. Among the changes in text were the footnote on the first page, the Congressional Review Act language at the end of the Introduction on page two, and the Implementation Section (pages 17 and 18).

Background

For several decades, the nuclear industry has used the response spectrum method and the time history method (described below) for the seismic analysis and design of nuclear power plant structures, systems, and components (SSCs) that are important to safety. In 1976, the NRC issued Revision 1 of this guide, which described then up-to-date guidance for using the response spectrum and time history methods for estimating SSC seismic response. Since that time, research in the United States has resulted in improved methods for combining modal responses and spatial components. This guide describes methods that the NRC staff considers acceptable in view of those improvements. The methods of combining modal responses, described in Revision 1, remain acceptable. If however, applicants for new licenses choose to use Revision 1 methods for combining modal responses, their analyses should address the residual rigid response of the missing mass modes as discussed in Regulatory Positions C.1.4.1 and C.1.5.1 of this guide. Licensees of existing operating plants should address the residual rigid response of the missing mass modes in their seismic analyses of SSCs when they choose to use Revision 1.

Systems or components (e.g., piping) may be supported at several locations either within a single structure or in two separate structures (multi-supported systems or components), and the motions of the primary structure(s) at each support location may be quite different. An acceptable approach for analyzing systems or components supported at multiple locations within a single structure is to define a uniform response spectrum (URS) that envelops all of the individual response spectra at the various support locations. The URS should be applied at all support locations to calculate the maximum inertial responses of the system or component. This is commonly referred to as the uniform support motion (USM) method. The modal and spatial combination methods described in this regulatory guide apply

only when using the USM method for response spectrum analysis of multi-supported systems or components (such as piping).

Designers should consider the relative displacements at the support points. Conventional static analysis procedures are acceptable for this purpose. In considering design, they should impose the maximum support displacements on the supported item in the most unfavorable combination and combine the responses attributable to the inertia effect and relative displacements using the absolute sum method.

The USM method can result in considerable overestimation of seismic responses. In the case of multiple supports located in a single structure, an alternative method is to use the independent support motion (ISM) approach, as described in Section 2 of Reference 5.

In lieu of the response spectrum approach, time histories of support motions may be used as excitations to the system or component. Because of the increased analytical effort compared to the response spectrum techniques, usually only a major equipment system would warrant a time history approach. However, compared to the response spectrum envelope method for multi-supported systems or components, the time history approach provides more realistic results in some cases.

Combination of Individual Modal Responses

For the purpose of seismic design of a nuclear power plant structure, system, or component (SSC), the representative maximum response of interest for design (e.g., displacement, acceleration, shear, moment, stress, strain) can be obtained by combining the corresponding maximum individual modal responses derived from the response spectrum method (e.g., see Refs 2 or 3). In general, it is unlikely that the maximum individual modal responses would all occur at the same time during an earthquake. Thus, it is necessary to identify appropriate combination methods to obtain the representative maximum response of interest from the maximum individual modal responses.

All methods utilized to combine seismic responses of individual modes obtained from the response spectrum method can provide only approximate representative maximum values, which are not exact in the sense of a time history method. The goal is to develop methods that enable one to estimate the maximum responses of interest as accurately as possible for the design of nuclear SSCs. Researchers have used the time history method, applying either modal superposition or direct integration, as a benchmark for gauging the degree of accuracy of these combination methods.

Since the issuance of Revision 1 of Regulatory Guide 1.92 in 1976, research in the United States has resulted in improved methods for combining modal responses that provide better estimates of SSC seismic response, while reducing unnecessary conservatism. NUREG/CR-6645, "Reevaluation of Regulatory Guidance on Modal Response Combination Methods for Seismic Response Spectrum Analysis" (Ref. 8), documents the results of an NRC evaluation of these developments for modal response combination, which includes a literature review and extensive analytical efforts, and provides the technical bases for the regulatory positions on combination of individual modal responses delineated in Section C.1 of this guide.

For the purpose of discussion, the broad-banded spectrum in Figure 1 will be used. However, this guide and the following discussion are applicable to all types of response spectra. This includes broad-banded spectra, such as a design ground spectrum, as well as single-peaked, narrow-banded spectra (Figure 2) and multiple-peaked, narrow-banded spectra (Figure 3), typical of in-structure spectra. Regulatory Position C.1.3 of this guide defines f_l, f_2, and f_{zpa} as used in Figures 1, 2, and 3.
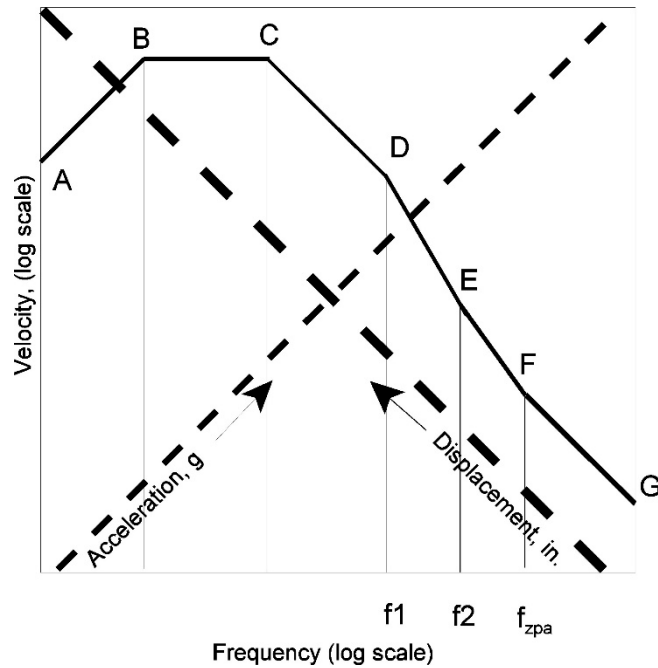
Figure 1. Regions of a broad-banded response spectrum

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Key Regions in Figure 1

AB - amplified periodic spectral displacement
BC - amplified periodic spectral velocity
CD - amplified periodic spectral acceleration
DE - transition from amplified periodic spectral acceleration to rigid spectral acceleration
EF - transition from rigid spectral acceleration to maximum base acceleration
FG - maximum base acceleration

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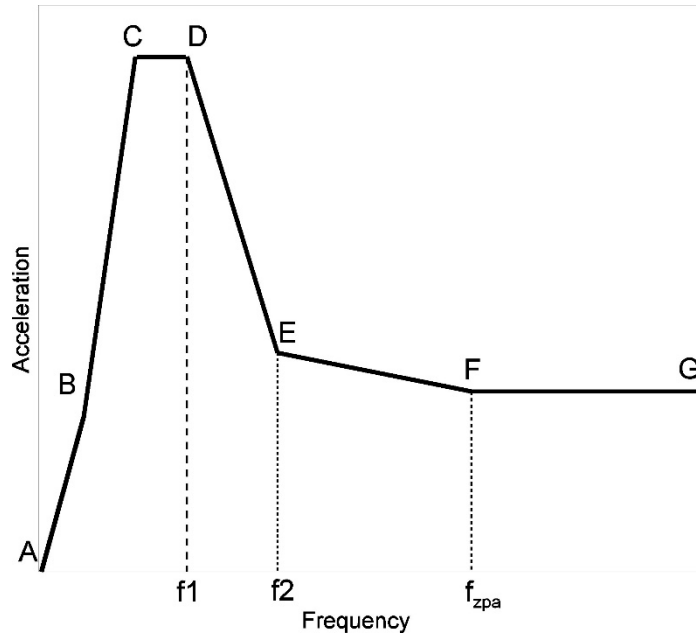
Figure 2. A narrow-banded response spectrum
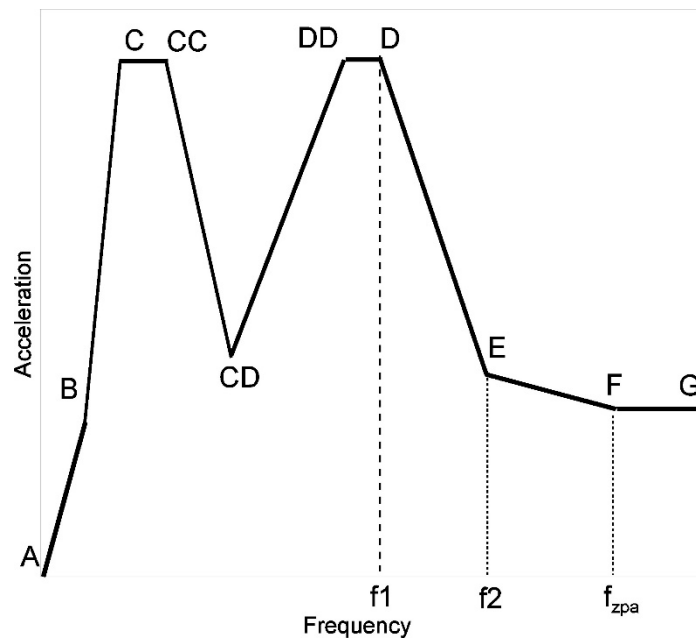
Figure 3. A multiple narrow-banded response spectrum

The seismic response of interest with regard to an SSC consists of two parts, which are referred to (in structural dynamics) as the damped-periodic (or simply "periodic") response and the "rigid" response. (In the theory of vibrations, these two parts are referred to as "transient" and "steady-state," respectively.) These two parts of the seismic response correspond respectively to the homogeneous and particular solutions of the differential equation of motion of an SSC. The periodic responses have the frequencies of

the oscillators (or individual modes), and the rigid responses have the frequencies of the input motion. For a more detailed discussion of periodic and rigid responses, see Chapter 3 of Reference 9.

For periodic modal responses with sufficiently separated frequencies, as indicated in Revision 1 of this guide, Goodman, Rosenblueth, and Newmark (Ref. 10) showed that the Square-Root-of-the-Sum-of-the-Squares (SRSS) method is the appropriate method to combine these modal responses. When modes with closely spaced frequencies are present, several conservative methods presented in Revision 1 of this guide can be used to combine these modal responses. Research since the 1970s (e.g., Refs. 7 and 8) has shown that for periodic modal responses, the double sum equation with appropriate formulas for calculating modal correlation coefficients will more accurately combine modal responses for modes with closely spaced frequencies. For modes with sufficiently separated frequencies, this double sum equation reduces to the SRSS method.

When using the response spectrum method, in most cases, it is not practical to calculate all mode shapes and frequencies. Research since the 1980s has shown that in the regions of rigid modal responses, the appropriate method to combine rigid responses is the algebraic sum method (Ref. 13). Some nuclear power plant SSCs may have a number of important modes beyond the zero period acceleration (ZPA) frequency (f_{zpa}). As discussed in Regulatory Position C.1.4, the residual rigid response of the missing mass modes should be addressed (Refs. 10 and 11); otherwise, it may result in underestimation of some SSC element forces and moments in the vicinity of supports, as well as underestimation of some support forces and moments.

Research since the 1980s (e.g., Refs. 11, 12, and 13) has shown that between the end of the region of amplified spectral acceleration, D, and the beginning of the rigid region, E, in Figure 1, the modal response consists of both the periodic and rigid components. Appropriate methods, as discussed in Regulatory Position C.1.3, should be used to separate the two components in this transition region. The periodic components of modal responses are combined with the other periodic modal responses in accordance with Regulatory Position C.1.1; the rigid components of modal responses are combined with the other rigid responses in accordance with Regulatory Position C.1.2.

Finally, after calculating the total periodic response, total rigid response, and residual rigid response, an appropriate combination method, as discussed in Regulatory Position C.1.5, should be used to obtain the total response.

Combination of Spatial Components

Regulatory Guide 1.60, "Design Response Spectra for Seismic Design of Nuclear Power Plants" (Ref. 18), specifies that the design of all Seismic Category 1 SSCs should be based on three orthogonal components (two horizontal and one vertical) of a prescribed design earthquake motion.

Regulatory Position C.2 of this guide, for the combination of spatial components, is the same as in Revision 1 of this guide, with one notable addition. When using the response spectrum method, use of the 100-40-40 percent combination rule proposed by Newmark (Ref. 19), as described in Regulatory Position C.2.1 of this guide, is acceptable as an alternative to the SRSS method.

<u>Response Spectrum Method</u>

For response spectrum analysis, in which each of the three spatial components are calculated separately, Chu, Amin, and Singh (Ref. 20) concluded that for an SSC subjected to the action of the three components of an earthquake motion, the representative maximum response of interest of the SSC can be

satisfactorily obtained by taking the SRSS of the corresponding representative maximum response for each of the three components calculated separately.

The SRSS procedure for combining the responses to the three components of an earthquake motion is based on the consideration that it is very unlikely that the maximum response for each of the three spatial components would occur at the same time during an earthquake.

The 100-40-40 percent rule was originally proposed as a simple way to estimate the maximum expected response of a structure subject to three-directional seismic loading for response spectrum analysis, and is the only alternative method for spatial combination that has received any significant attention in the nuclear power industry. The results of the 100-40-40 spatial combination have been compared with the SRSS spatial combination. Generally, they indicate that the 100-40-40 combination method produces higher estimates of maximum response than the SRSS combination method by as much as 16 percent, while the maximum under-prediction is 1 percent.

Time History Method

When using the time history method, the representative maximum response of interest of the SSC can be obtained either by performing separate analyses for each of the three components of earthquake motion, or by performing a single analysis with all three components of earthquake motion applied simultaneously. In the latter case, the three components of earthquake must be statistically independent (Ref. 21).

C.   STAFF REGULATORY GUIDANCE

This guide describes methods that the NRC staff considers acceptable based on knowledge gained by research conducted in the United States since Revision 1 of this guide was issued in 1976. The methods of combining modal responses described in Revision 1 remain acceptable. If, however, applicants for new licenses choose to use Revision 1 methods for combining modal responses, their analyses should address the residual rigid response of the missing mass modes discussed in Regulatory Positions C.1.4.1 and C.1.5.1 of this guide. Licensees of existing operating plants should consider the residual rigid response of the missing mass modes in their seismic analyses of SSCs, as discussed in Regulatory Positions C.1.4.1 and C.1.5.1 when they choose to use Revision 1 methods for combining modal responses.

1.   Combination of Individual Modal Responses

1.1.   Combination of Periodic Modal Responses

Research has shown that the periodic responses are dominant in the regions of amplified spectral displacement, amplified spectral velocity, and amplified spectral acceleration of a spectrum (regions AB, BC, and CD in Figure 1.). Beyond amplified spectral acceleration region CD and up to E, the modal responses consist of both the periodic and rigid components. (Refer to Chapter 3 of Reference 9 for a discussion of periodic and rigid responses, as well as periodic and rigid components of responses.) The periodic modal responses and the periodic components of modal responses are combined using the following double sum ["complete quadratic combination" (CQC)] equation:

$$R_{pI} = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{ij} R_{p_i} R_{p_j} \right]^{1/2} \tag{1}$$

where: R_{pI} = combined periodic response for the Ith component of seismic input motion (I = 1, 2, 3, for one vertical and two horizontal components),

ε_{ij} = the modal correlation coefficient for modes i and j,

R_{p_i} = periodic response or periodic component of a response of mode i,

R_{p_j} = periodic response or periodic component of a response of mode j, and

n = number of modes considered in the combination of modal responses.

For completely correlated modes i and j, ε_{ij} = 1; for partially correlated modes i and j, $0 < \varepsilon_{ij} < 1$; for uncorrelated modes i and j, ε_{ij} = 0.

The modal correlation coefficients are uniquely defined, depending on the method chosen for evaluating the correlation coefficient, as follows.

1.1.1.   Square Root of the Sum of the Squares (SRSS) Method

At the foundation of all methods for combining uncorrelated modal responses is the SRSS method. All methods for combination of periodic modal response components are equivalent to the SRSS method if the frequencies of the modes are all sufficiently separated. In this case,

ε_{ij} = 1.0   for   i = j

and

ε_{ij} = 0.0   for   i ≠ j

and Equation 1 reduces to the following:

$$R_{pI} = \left[\sum_{i=1}^{n} R_{p_i}^{2} \right]^{1/2} \tag{2}$$

If modes with closely spaced frequencies exist, the SRSS method is not applicable, and one of the two methods in Regulatory Positions C.1.1.2 and C.1.1.3 (below) should be used instead. The definition of modes with closely spaced frequencies is a function of the critical damping ratio (Ref. 8, page 66):

(1)   For critical damping ratios ≤2%, modes are considered closely spaced if the frequencies are within 10% of each other (i.e., for $f_i < f_j$, $f_j \leq 1.1 f_i$).

(2)   For critical damping ratios >2%, modes are considered closely spaced if the frequencies are within five times the critical damping ratio of each other (i.e., for $f_i < f_j$ and 5% damping, $f_j \leq 1.25 f_i$; for $f_i < f_j$ and 10% damping, $f_j \leq 1.5 f_i$).

1.1.2. Rosenblueth Correlation Coefficient

Rosenblueth (Ref. 11) provided the first significant mathematical approach to the evaluation of modal correlation for seismic response spectrum analysis. It is based on the application of random vibration theory, utilizing a finite duration of white noise to represent seismic loading. A formula for calculation of the coefficient ε_{ij} as a function of modal frequencies (f_i, f_j), modal damping ratios (λ_i, λ_j), and the time duration of strong earthquake motion (t_D) was derived as follows:

$$\varepsilon_{ij} = \left[1 + \left(\frac{f_i' - f_j'}{\lambda_i' f_i + \lambda_j' f_j}\right)^2\right]^{-1} \tag{3}$$

where

$$f_i' = f_i\left[1 - \lambda_i^2\right]^{1/2}$$

$$\lambda_i' = \lambda_i + \frac{1}{\pi\, t_D\, f_i}$$

and f_j', λ_j' are similarly defined.

Appendix D to Reference 8 tabulates numerical values of ε_{ij} for the Rosenblueth formula as a function of frequency, frequency ratio, and strong motion duration time for constant modal damping of 1%, 2%, 5% and 10%. The effect of t_D is most significant at 1% damping and low frequency. For 5% and 10% damping, $t_D = 10$ sec. and 1,000 sec. produced similar values for ε_{ij} regardless of frequency. The most significant result is that ε_{ij} is highly dependent on the damping ratio for: 2%, 5%, and 10% damping, $\varepsilon_{ij} = 0.2$, 0.5, and 0.8, respectively, at a frequency ratio of 0.9 (modal frequencies within 10%).

1.1.3. Der Kiureghian Correlation Coefficient

Der Kiureghian (Ref. 12) presents an expression for ε_{ij} similar to Rosenblueth's. It is also based on the application of random vibration theory, but utilizes an infinite duration of white noise to represent seismic loading. A formula for calculation of the coefficient ε_{ij} as a function of modal frequencies (f_i, f_j) and modal damping ratios (λ_i, λ_j) was derived as follows:

$$\varepsilon_{ij} = \frac{8\left(\lambda_i\lambda_j f_i f_j\right)^{1/2}\left(\lambda_i f_i + \lambda_j f_j\right)f_i f_j}{\left(f_i^2 - f_j^2\right)^2 + 4\lambda_i\lambda_j f_i f_j\left(f_i^2 + f_j^2\right) + 4\left(\lambda_i^2 + \lambda_j^2\right)f_i^2 f_j^2} \tag{4}$$

While the form of Equation 4 differs significantly from that of Equation 3, the two equations produce equivalent results if t_D is assumed to be very large in Equation 3. This is shown in Appendix D to Reference 8, where ε_{ij} is tabulated for the Rosenblueth formula (with $t_D = 1,000$ sec.) and the Der Kiureghian formula.

1.2. __Combination of Rigid Modal Responses__

In the high-frequency regions (regions EF and FG in Figure 1), the rigid responses predominate. Also, beyond the amplified acceleration region of CD and up to E in Figure 1, the modal responses consist of both periodic and rigid components.

The rigid responses and rigid components of responses are combined algebraically, as follows:

$$R_{rI} = \sum_{i-1}^{n} R_{r_i} \qquad (5)$$

where R_{rI} = combined rigid response for the Ith component of seismic input motion (I =1, 2, 3, for one vertical and two horizontal components), R_{r_i} = rigid response or rigid component of a response of mode i, and n = number of modes considered in the combination of modal responses.

1.3. Modes with Both Periodic and Rigid Response Components

Beyond the amplified acceleration region of CD and up to E in Figure 1, the modal responses consist of both the periodic and rigid components. Several methods were examined for the separation of periodic and rigid response components (Ref. 8). The Gupta method (Refs. 15, 16, and 22) and Lindley-Yow method (Ref. 17) are considered acceptable by the NRC staff, subject to the limitations discussed below. For the Ith direction of seismic input motion, the periodic components of modal responses obtained in this section should be combined with the other periodic modal responses (or periodic components of modal responses) using Equation 1. Similarly, for the Ith direction of seismic input motion, the rigid components of modal responses obtained in this section should be combined with the other rigid modal responses (or rigid components of modal responses) using Equation 5.

1.3.1. Gupta Method

Gupta separated the periodic and rigid components of a response by a rigid response coefficient α_i. Using the notations in Regulatory Positions C.1.1 and C.1.2 above, the rigid response component of a modal response, R_i, is defined as follows:

$$R_{r_i} = \alpha_i R_i \qquad (6.1)$$

The periodic response component of R_i can then be expressed as follows:

$$R_{p_i} = \left[1 - \alpha_i^2\right]^{1/2} R_i, \qquad \text{where} \qquad R_i^2 = R_{p_i}^2 + R_{r_i}^2 \qquad (6.2)$$

With proper selection of key frequencies $f1$ and $f2$, Gupta determined that the rigid response coefficient, α_i, can be idealized as follows:

$$\alpha_i = \frac{\ln(f_i / f1)}{\ln(f2 / f1)}, \qquad\qquad f1 \le f_i \le f2 \qquad (7.1)$$

and

$$\alpha_i = 0 \text{ for } f_i \le f1, \qquad \alpha_i = 1 \text{ for } f_i \ge f2$$

Gupta expressed the key frequencies $f1$ and $f2$ as follows:

$$f1 = \frac{S_{a_{\max}}}{2\pi S_{v_{\max}}}, \text{ and } f2 = fr \tag{7.2}$$

where $S_{a_{max}}$ = the maximum spectral acceleration, $S_{v_{max}}$ = the maximum spectral velocity, fr = the rigid frequency. fr is the lowest frequency at which the responses of single degree of freedom (SDOF) oscillators become completely correlated with the input motion (i.e., α_i = 1 for all $f_i \geq f_r$).

Gupta has postulated that fr can be identified as the frequency where response spectral curves for different damping values converge, and that above this frequency, the periodic component of the modal response is essentially zero. It was found (Ref. 8) that when using Gupta's method, the results of combining modal responses are somewhat sensitive to the value of $f2$ used, and there are situations that $f2$ may not be uniquely determined by postulating convergence of spectral curves of different damping values. In such cases, Appendix B to this guide recommends a more systematic method to determine $f2$, as first proposed in Appendix F to Reference 8.

The definition of $f2$ in Equation 7.2 (above) is applicable to all types of response spectra (broad-banded, narrow-banded, or multiple narrow-banded).

The definition of $f1$ in Equation 7.2 (above) is applicable to single-peaked, unbroadened spectra. When the spectral peak has been broadened ±15% to account for uncertainty, as shown in Figure 2, it is acceptable to select $f1$ at point D in Figure 2. The staff will review alternative $f1$ selections on a case-by-case basis.

For multiple-peaked, narrow-banded spectra, as shown in Figure 3, the selection of $f1$ is not straightforward, because there is more than one frequency region of the spectrum that exhibits significant amplification above the ZPA, due to predominantly periodic response. To avoid possible overestimation of the rigid response component in this higher frequency, amplified response region, an acceptable approach is to select the highest frequency of all significant peaks as $f1$, such as point D in Figure 3. Alternative selection of $f1$ will be reviewed by the staff on a case-by-case basis.

For broad-banded spectra, as shown in Figure 1, typical of a design ground spectrum or a design spectral envelope of multiple in-structure response spectra, an acceptable approach is to select $f1$ at point D in Figure 1. The staff will review alternative $f1$ selections on a case-by-case basis.

1.3.2. Lindley-Yow Method

In the Lindley-Yow method, separate analyses are performed for periodic and rigid response components. The periodic response component is calculated as follows:

$$R_{p_i} = R_i \left[\overline{S_{a_i}} \middle/ S_{a_i} \right] \tag{8.1}$$

where a modified spectral acceleration is used and defined as follows:

$$\overline{S_{a_i}} = \left[S_{a_i}^2 - ZPA^2 \right]^{1/2}, \qquad \overline{S_{a_i}} \geq 0 \tag{8.2}$$

where S_{a_i} = spectral acceleration of mode i, and ZPA = zero period acceleration, which is the maximum acceleration of the base input time history record.

Employing Gupta's notations, the rigid response component is calculated using the following definition for the rigid response coefficient:

$$\alpha_i = ZPA / S_{a_i}, \qquad 0 \le \alpha_i \le 1 \tag{9}$$

The rigid response component is calculated in accordance with Equation 6.1, while the periodic response component is calculated in accordance with either Equation 6.2 or Equation 8.1.

There is one limitation on the use of Lindley-Yow's method. Specifically, Equation 9 gives $\alpha_i = 1$ when $S_{a_i} = ZPA$ at f_{zpa} (the ZPA frequency) and higher frequencies. It has its minimum value at the spectral acceleration peaks (C–D in Figure 2, C–CC and DD–D in Figure 3), where the modified spectral acceleration from Equation 8.2 is essentially equal to the spectral acceleration. However, at frequencies below point C in Figures 2 and 3, α_i begins to increase and would exceed 1.0 for $S_{a_i} < ZPA$. Therefore, Lindley-Yow's method should not be used for SSCs that have natural frequencies less than the frequency of the lowest-frequency spectral acceleration peak (point C in Figures 2 and 3), unless it is modified to set $\alpha_i = 0$ for frequencies below point C.

1.4.   Residual Rigid Response

Unlike tall buildings and other relatively flexible systems, nuclear power plant SSCs may have important natural vibration modes at frequencies higher than the ZPA frequency, f_{zpa}. In most cases, it is not practical to calculate these high-frequency modes, which are not excited by the seismic ground or in-structure motion, with sufficient accuracy to warrant the effort. If only modes with frequencies below f_{zpa} are included in the dynamic analysis, the mass associated with the modes with frequencies higher than f_{zpa} has not been included in (i.e., is "missing" from) the dynamic analysis. It is important to account for the residual rigid response if a nuclear power plant SSC has significant natural vibration modes at frequencies higher than f_{zpa}. Ignoring the residual rigid response in these cases may result in underestimation of some SSC element forces and moments in the vicinity of supports, as well as underestimation of some support forces and moments (e.g., Ref. 13). The residual rigid response of the missing mass modes (or the "missing mass response") can be calculated using the Missing Mass method of Kennedy (Ref. 13) or the Static ZPA method. These two methods, which were examined in Reference 8, have been selected as providing acceptable results as noted below.

Use of the Missing Mass method for calculating the contribution of high frequency modes is acceptable for both response spectrum analysis and modal superposition time history analysis. In modal superposition time history analysis, a procedure analogous to the approach used in Combination Method A (see Regulatory Position 1.4.1) for response spectrum analysis is acceptable. Only modes with $f < f_{zpa}$ participate in the modal solution; the missing mass contribution, scaled to the instantaneous input acceleration, is treated as an additional mode in the algebraic summation of modal responses at each time step. The missing mass contribution is considered for all degrees of freedom.

1.4.1.   Missing Mass Method

The Missing Mass method (Ref. 13) is a convenient, computationally efficient and accurate method for the following uses:

(1)   Account for the contribution of all modes with frequencies above the frequency (f_{zpa}) at which the response spectrum returns to the zero period acceleration (ZPA).

(2)   Account for the contribution to support reactions of mass that is apportioned to system support points.

The Missing Mass method constitutes the total effect of all system mass that is not included in the modes with frequencies below f_{zpa}. The system response to the missing mass is calculated by performing a static analysis for an applied load that equals the missing mass multiplied by the spectrum ZPA. This method is considered the only acceptable method to account for high-frequency modal contributions ($f \geq f_{zpa}$) and the effects of mass apportioned to system support points.

Kennedy (Ref. 13) documented this method and recommended including it in regulatory guidance. The mathematical details are presented in References 9 and 22, while the mathematical formulation is included as Appendix I to Reference 9 and is reproduced as Appendix A to this guide.

The guideline provided in References 13 and 22 that the missing mass contribution needs to be considered only if the fraction of missing mass at any degree of freedom exceeds 0.1, is non-conservative and should not be used. This guideline does not consider the total mass that is missing, which, in the limit, could be 10%. In a static analysis, this represents a 10% reduction in the applied load. The missing mass contribution should be calculated in all response spectrum analyses because its potential effect on support reactions is difficult to judge based on the fraction of missing mass. This calculation has been automated in a number of piping analysis codes and does not represent a significant computational effort.

The missing mass contribution to the response spectrum analysis solution represents response that is completely in-phase with the time-varying acceleration input and can be scaled to the instantaneous acceleration to obtain its contribution at any specific point in time. This characteristic is not important in response spectrum analysis because only peak response is predicted. In this case, the ZPA is used to calculate the missing mass contribution. However, the importance of the missing mass contribution is not limited to response spectrum analyses alone. Mode superposition time-history analysis is most accurately and efficiently performed by a procedure similar to that employed in response spectrum analysis (Ref. 9). Only modes that vibrate at frequencies below f_{zpa} need to be included in the transient mode superposition solution. The missing mass contribution, scaled to the instantaneous acceleration, is then algebraically summed with the transient solution at the corresponding time to obtain the total solution. This method is more rigorous and accurate than including additional modes in the transient mode superposition solution. Even if additional modes are included, it is still necessary to calculate the missing mass for the excluded, higher frequency modes and system support points.

1.4.2. Static ZPA Method

The Lindley-Yow method (Ref. 17) defines the acceleration of the rigid response component of all modes to be the ZPA of the response spectrum. The algebraic summation of the rigid response components for all modes (R_{rI}) is equivalent to the static response for a load equal to the total mass times the ZPA. When using the Lindley-Yow method, an alternative approach to including the contribution of high-frequency ($f \geq f_{zpa}$) modes is to calculate R_{rI} directly by the Static ZPA method. This eliminates the need for calculation of the missing mass, since it is automatically included in the static analysis of total mass times ZPA. The periodic response component (R_{pI}) is calculated in accordance with the Lindley-Yow method.

1.5.   Complete Solution for Response Spectrum Analysis

Two methods are acceptable for obtaining the complete (periodic plus rigid) response spectrum analysis solution for each of the three orthogonal component motions (two horizontal and one vertical) of

a prescribed design earthquake. The coefficients ε_{ij} in each method are determined by one of the combination methods for periodic modal responses (see Regulatory Position C.1.1).

The Lindley-Yow method is not suitable for analysis of systems with significant low-frequency response $(f < f_{spectral\ peak})$, unless the low-frequency correction is implemented (see Regulatory Position C.1.3.2).

The contribution of high-frequency modes $(f \geq f_{zpa})$ should be included in all response spectrum and modal superposition time history analyses. (See Regulatory Position C.1.4.)

1.5.1. Combination Method A

Combination Method A introduces the concept of periodic and rigid modal response components for the amplified modes $(f < f_{zpa})$. Mathematically, the complete solution is represented as follows:

$$R_{p_i} = \left[1 - \alpha_i^2\right]^{1/2} R_i$$
$$R_{r_i} = \alpha_i R_i$$
$$R_{pI} = \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon_{ij}R_{p_i}R_{p_j}\right]^{1/2} \text{, where n = number of modes below } f_{zpa} \tag{10}$$
$$R_{rI} = \sum_{i=1}^{n}R_{r_i} + R_{MissingMassI}$$
$$R_I = \left[R_{rI}^2 + R_{pI}^2\right]^{1/2}$$

where $R_{MissingMassI}$ is the residual rigid response of the missing mass modes for the Ith component of seismic input motion ($I = 1, 2, 3$, for one vertical and two horizontal components), calculated by using the missing mass method described in Regulatory Position C.1.4.1.

Combination Method A is equally applicable to both the Lindley-Yow and Gupta methods (Regulatory Position C.1.3, "Modes with Both Periodic and Rigid Response Components"). Only the definition of α_i changes. When using Revision 1 of this guide for combining modal responses, R_{r_i} terms are all identically zero.

1.5.2. Combination Method B

Combination Method B is to be used only when implementing Regulatory Positions C.1.3.2 and C.1.4.2. This method utilizes the Static ZPA method to calculate R_{rI}. Combination Method B is completely compatible with the Lindley-Yow method only when the low frequency correction (see Regulatory Position C.1.3.2) is **not** necessary, because calculation of R_{rI} by the Static ZPA method is based on the Lindley-Yow definition for α_i using Equation 9.

However, use of Combination Method B is acceptable even when using the low-frequency correction, because the predicted response will always be more conservative than Combination Method A.

Mathematically, the complete solution is represented as follows:

$$R_{p_i} = \left[1 - \alpha_i^2\right]^{1/2} R_i$$
$$R_{pI} = \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon_{ij}R_{p_i}R_{p_j}\right]^{1/2} \text{, where n = number of modes below } f_{zpa} \tag{11}$$

$$R_{rI} = R_{StaticZPA\,I}$$

$$R_I = \left[R_{rI}^2 + R_{pI}^2\right]^{1/2}$$

where $R_{StaticZPA\,I}$ is the rigid response for the Ith component of seismic input motion ($I = 1, 2, 3$, for one vertical and two horizontal components), calculated by using the static ZPA method described in Regulatory Position C.1.4.2.

2. Combining Effects Caused by Three Spatial Components of an Earthquake

Depending on which basic method is used in the seismic analysis (i.e., response spectra or time history method); the following two approaches are considered acceptable for the combination of three-dimensional earthquake effects.

2.1. Response Spectra Method

When the response spectra method is used, the representative maximum earthquake-induced response of interest in an SSC should be obtained by the SRSS combination of the maximum representative responses from the three earthquake components calculated separately as follows:

$$R = \left[\sum_{I=1}^{3} R_I^2\right]^{1/2} \tag{12}$$

Where R = any response of interest of an SSC, R_I = combined response for the Ith component of seismic input motion ($I = 1, 2, 3$ for one vertical and two horizontal components), as obtained from Equations 10 or 11.

As an alternative, the 100-40-40 percent combination rule proposed by Newmark (Ref. 20) may be used in lieu of the SRSS method. The 100-40-40 procedure is as follows:

(1)   Let R_1, R_2, R_3, be the maximum responses of an SSC caused by each of the three earthquake components calculated separately, such that

$$|R_1| \geq |R_2| \geq |R_3|$$

(2)   The maximum seismic response attributable to earthquake loading in three orthogonal directions is given by the following equation:

$$R = (1.0|R_1| + 0.4|R_2| + 0.4|R_3|) \tag{13}$$

2.2. Time History Method

When time history analysis method is employed for seismic analyses, two types of analyses are generally performed:

(1)   For time history analysis when each of the three spatial components are calculated separately, the representative maximum response of interest of an SSC can be satisfactorily obtained by taking the SRSS of the maximum responses from the time history analysis for each of the three earthquake components:

$$R = \left[\sum_{I=1}^{3} R_I^2 \right]^{1/2} \tag{14}$$

(2)   If the three components of earthquake motion are statistically independent (e.g., Ref. 21), the maximum response of interest of an SSC can be obtained from algebraic summation of the three component responses at each time step.

When the effect of all three components of earthquake motion is calculated simultaneously, in a single dynamic analysis, algebraic summation is automatically achieved.

When the effect of each component of earthquake motion is calculated in a separate dynamic analysis, algebraic summation is obtained as follows:

$$R(t) = \sum_{I=1}^{3} R_I(t) \tag{15}$$

After algebraic summation at each time step, it is necessary to search the entire response time history, in order to find the maximum response. Because the time of maximum response may vary from location to location within the SSC, and also may vary for different responses at the same location (e.g., stresses vs. deflections), this process is carried out for each response of interest.

When using algebraic summation, it is important that the response of interest be consistent with the structural acceptance criterion. For example, an acceptance criterion for a beam-type structural member may be the maximum allowable axial stress, which is composed of a component attributable to axial force and two components attributable to bending moments. In this case, the response of interest would be the maximum axial stress, not the maximum axial force and maximum moment responses.

3.   Methods Used

If the applicant has used the methods described in this guide, each applicable section of the safety analysis report (SAR) should state specifically which acceptable methods was used in analyzing the SSCs covered by that section.

D. IMPLEMENTATION

The purpose of this section is to provide information on how applicants and licensees[1] may use this guide and information regarding the NRC's plans for using this regulatory guide. In addition, it describes how the NRC staff complies with the Backfit Rule (10 CFR 50.109) and any applicable finality provisions in 10 CFR Part 52.

Use by Applicants and Licensees

Applicants and licensees may voluntarily[2] use the guidance in this document to demonstrate compliance with the underlying NRC regulations. Methods or solutions that differ from those described in this regulatory guide may be deemed acceptable if they provide sufficient basis and information for the NRC staff to verify that the proposed alternative demonstrates compliance with the appropriate NRC regulations. Current licensees may continue to use guidance that the NRC found acceptable for complying with the identified regulations so long as their current licensing basis remains unchanged. Licensees may use the information in this regulatory guide for actions which do not require NRC review and approval such as changes to a facility design under 10 CFR 50.59. Licensees may use the information in this regulatory guide or applicable parts to resolve regulatory or inspection issues.

Use by NRC Staff

The staff may discuss with licensees, various actions consistent with staff positions in this regulatory guide, as one acceptable means of meeting the underlying NRC regulatory requirement. Such discussions would not ordinarily be considered backfitting even if prior versions of this regulatory guide are part of the licensing basis of the facility. However, unless this regulatory guide is part of the licensing basis for a facility, the staff may not represent to the licensee that the licensee's failure to comply with the positions in this regulatory guide constitutes a violation.

If an existing licensee voluntarily seeks a license amendment or change and (1) the NRC staff's consideration of the request involves a regulatory issue directly relevant to this new or revised regulatory guide, and (2) the specific subject matter of this regulatory guide is an essential consideration in the staff's determination of the acceptability of the licensee's request, then the staff may request that the licensee either follow the guidance in this regulatory guide or provide an equivalent alternative process that demonstrates compliance with the underlying NRC regulatory requirements. This is not considered backfitting as defined in 10 CFR 50.109(a)(1) or a violation of any of the issue finality provisions in 10 CFR Part 52.

The NRC staff does not intend or approve any imposition or backfitting of the guidance in this regulatory guide. The NRC staff does not expect any existing licensee to use or commit to using the guidance in this regulatory guide, unless the licensee makes a change to its licensing basis. The NRC staff does not expect or plan to request licensees to voluntarily adopt this regulatory guide to resolve a generic regulatory issue. The NRC staff does not expect or plan to initiate NRC regulatory action which would require the use of this regulatory guide. Examples of such unplanned NRC regulatory actions include issuance of an order requiring the use of the regulatory guide, requests for information under

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[1]   In this section, "licensees" refers to licensees of nuclear power plants under 10 CFR Parts 50 and 52; and the term "applicants," refers to applicants for licenses and permits for (or relating to) nuclear power plants under 10 CFR Parts 50 and 52, and applicants for standard design approvals and standard design certifications under 10 CFR Part 52.

[2]   In this section, "voluntary" and "voluntarily" mean that the licensee is seeking the action of its own accord, without the force of a legally binding requirement or an NRC representation of further licensing or enforcement action.

10 CFR 50.54(f) as to whether a licensee intends to commit to use of this regulatory guide, generic communication, or promulgation of a rule requiring the use of this regulatory guide without further backfit consideration.

Additionally, an existing applicant may be required to adhere to new rules, orders, or guidance if 10 CFR 50.109(a)(3) applies.

If a licensee believes that the NRC is either using this regulatory guide or requesting or requiring the licensee to implement the methods or processes in this regulatory guide in a manner inconsistent with the discussion in this Implementation section, then the licensee may file a backfit appeal with the NRC in accordance with the guidance in NUREG-1409 and NRC Management Directive 8.4.

REFERENCES

(1)   *U.S. Code of Federal Regulations*, "Domestic Licensing of Production and Utilization Facilities," Appendix A, "General Design Criteria for Nuclear Power Plants," Part 50, Chapter I, Title 10, "Energy."

(2)   *U.S. Code of Federal Regulations*, "Domestic Licensing of Production and Utilization Facilities," Appendix S, "Earthquake Engineering Criteria for Nuclear Power Plants," Part 50, Chapter I, Title 10, "Energy."

(3)   *U.S. Code of Federal Regulations*, "Licenses, Certifications, and Approvals for Nuclear Power Plants," Part 52, Chapter I, Title 10, "Energy."

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(5)   U.S. Nuclear Regulatory Commission, "Report of the U.S. Nuclear Regulatory Commission Piping Review Committee; Volume 4: Evaluation of Other Loads and Load Combinations," NUREG-1061, Washington, DC, December 1984, available through ADAMS under Accession No. ML11343A034 [3].

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[3]   Publicly available documents from the U.S. Nuclear Regulatory Commission (NRC) are available electronically through the NRC Library on the NRC's public Web site at http://www.nrc.gov/reading-rm/doc-collections/. The documents can also be viewed on-line for free or printed for a fee in the NRC's Public Document Room (PDR) at 11555 Rockville Pike, Rockville, MD; the mailing address is USNRC PDR, Washington, DC 20555; telephone (301) 415-4737 or (800) 397-4209; fax (301) 415 3548; and e-mail pdr.resource@nrc.gov.

[4]   Copies may be purchased from Computers and Structures, Inc., 1995 University Ave., Suite 540, Berkeley, CA 94704 (telephone: 510-845-2177). Purchase information is available through the Computers and Structures Web site at http://www.csiberkeley.com/dynamics.html.

[5]   Copies may be purchased from the publisher, Prentice Hall, One Lake Street, Upper Saddle River, NJ 07458 (telephone: 201-236-7000). Purchase information is available through the publisher's Web site at http://vig.prenhall.com/catalog/academic/product/0,1144,0130869732,00.html.

[6]   Copies may be purchased from the publisher, CRC Press, 6000 Broken Sound Parkway, NW, Suite 300, Boca Raton, FL 33487 (telephone: 800-272-7737). Purchase information is available through the publisher's Web site at http://www.crcpress.com/shopping_cart/products/product_detail.asp?sku=8628&parent_id=&pc=.

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(18)   U.S. Nuclear Regulatory Commission, "Design Response Spectra for Seismic Design of Nuclear Power Plants," Rev. 1, Regulatory Guide 1.60, Washington, DC.

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[10]   Copies may be purchased from the American Society for Civil Engineers (ASCE), 1801 Alexander Bell Drive, Reston, VA 20190 [phone: 800-548-ASCE (2723)]. Purchase information is available through the ASCE Web site at http://www.pubs.asce.org/WWWdisplay.cgi?0002398.

APPENDIX A

CALCULATION OF MISSING MASS CONTRIBUTION
TO TOTAL RESPONSE
(From Appendix I to Reference 8)

Mathematical descriptions of the "missing mass" contribution to total response are contained in References 9, 10, and 14, while Reference 14 presents a step-by-step, mechanistic approach. Reference 10 presents a more complete mathematical description, which provides additional insight, and Reference 9 essentially incorporates that mathematical description. It is recommended that Section 3.4 of Reference 10 be reviewed to attain an understanding of the procedure.

The following steps can be utilized to calculate the response contribution of all system modes of vibration with frequencies equal to or greater than f_{zpa}. (Note that each direction of earthquake input motion must be considered separately.)

Step 1. Determine the modal responses only for those modes with natural frequencies less than that at which the spectral acceleration approximately returns to the ZPA (f_{zpa}).

Step 2. For each degree-of-freedom (DOF) included in the dynamic analysis, determine the fraction of DOF mass included in the summation of all modes included in Step 1. This fraction d_i for each DOF i is given by the following equation:

$$d_i = \sum_{n=1}^{N}\left[(c_{n,j})(\phi_{n,i})\right] \tag{A.1}$$

where

n = mode number ($1, 2, ..., N$)
N = the number of modes included in Step 1
$\phi_{n,i}$ = eigenvector value for mode n and DOF i
j = direction of input motion
$C_{n,j}$ = participation factor for mode n in the jth direction:

$$c_{n,j} = \frac{\{\phi_n\}^T[m]\{\delta_{ij}\}}{\{\phi_n\}^T[m][\phi_n]} \tag{A.2}$$

where δ_{ij} is the Kronecker delta, which is 1 if DOF i is in the direction of the earthquake input motion j and 0 if DOF i is a rotation or not in the direction of the earthquake input motion j. This assumes that the three orthogonal directions of earthquake input motion are coincident with the DOF directions. Also, $[m]$ is the mass matrix.

Next, determine the fraction of DOF mass not included in the summation of these modes:

$$e_i = d_i - \delta_{ij} \tag{A.3}$$

Step 3 Higher modes can be assumed to respond in phase with the ZPA and, thus, with each other; hence, these modes are combined algebraically, which is equivalent to pseudostatic response to the inertial forces from these higher modes excited at the ZPA. The pseudostatic inertial forces associated with the summation of all higher modes for each DOF *I* are given by the following:

$$P_i = (ZPA)(M_i)(e_i) \qquad\qquad (A.4)$$

where P_i is the force or moment to be applied at DOF *i*, M_i is the mass or mass moment of inertia associated with DOF *i*.

The structure is then statically analyzed for this set of pseudostatic inertial forces applied to all degrees of freedom to determine the maximum responses associated with high-frequency modes not included in Step 1.

This procedure requires the computation of individual modal responses only for lower-frequency modes. Thus, the more difficult higher-frequency modes need not be determined. The procedure ensures inclusion of all modes of the structural model and proper representation of DOF masses.

APPENDIX B

ALTERNATIVE METHOD FOR DETERMINATION OF THRESHOLD FREQUENCY FOR RIGID MODAL RESPONSE
(From Appendix F to Reference 8)

This appendix presents an alternative method for determining the threshold frequency for rigid modal response ($f2$ in Gupta's method, Regulatory Position C.1.3.1 of this guide), which was first proposed in Appendix F to Reference 5.

During the generation of a response spectrum from a ground or in-structure time history record, the complete time history of each single degree of freedom (SDOF) oscillator response is calculated and processed to identify the peak response. This peak response becomes a single point on the response spectrum plot. Each SDOF oscillator peak response has an associated time of occurrence and direction of the peak response, although this information is typically not retained because it is not needed in the generation of response spectrum. Nonetheless, valuable conclusions can be derived by comparing this information to the time and direction of the peak acceleration from the input time history record.

The lowest SDOF oscillator frequency ($f2$ in Gupta's method) for which the time and direction of peak response coincide with the time and direction of the peak of the input time history represents the onset of rigid modal response that is in-phase with the input, provided that all higher-frequency SDOF oscillators exhibit the same behavior (i.e., for $f \geq f2$, all SDOF oscillator peak responses occur at the same time and in the same direction as the peak of the input time history). To further verify that rigid modal response exists, a comparison of the crossings of the acceleration equal to zero datum between the input time history and SDOF oscillator time history response should be performed for SDOF oscillator frequencies in the vicinity of $f2$.

The calculation of $f2$, for each critical damping ratio of interest, can be fully automated and made a part of the response spectrum generation algorithm.