Understanding diagnostic plots for well-test interpretation

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Keywords Hydraulic testing · Conceptual models · Hydraulic properties · Analytical solutions

Abstract In well-test analysis, a diagnostic plot is a scatter plot of both drawdown and its logarithmic derivative versus time. It is usually plotted in log–log scale. The main advantages and limitations of the method are reviewed with the help of three hydrogeological field examples. Guidelines are provided for the selection of an appropriate conceptual model from a qualitative analysis of the log-derivative. It is shown how the noise on the drawdown measurements is amplified by the calculation of the derivative and it is proposed to sample the signal in order to minimize this effect. When the discharge rates are varying, or when recovery data have to be interpreted, the diagnostic plot can be used, provided that the data are pre-processed by a deconvolution technique. The effect of time shift errors is also discussed. All these examples show that diagnostic plots have some limitations but they are extremely helpful because they provide a unified approach for well-test interpretation and are applicable in a wide range of situations.

Introduction

Among the techniques used to characterize the hydraulic properties of aquifers, well testing is one of the most commonly applied. It involves imposing a perturbation such as pumping in a well and measuring the response of the aquifer, for example in terms of head variations. Those data are then interpreted with the help of analytical or numerical models in order to infer the hydraulic properties of the aquifer.

In that framework, a diagnostic plot (Bourdet et al. 1983) is a simultaneous plot of the drawdown and the logarithmic derivative $(\partial s/\partial \ln t = t\partial s/\partial t)$ of the drawdown as a function of time in log–log scale. This plot is used to facilitate the identification of an appropriate conceptual model best suited to interpret the data.

The idea of using the logarithmic derivative in well-test interpretation is attributed to Chow (1952). He demonstrated that the transmissivity of an ideal confined aquifer is proportional to the ratio of the pumping rate by the logarithmic derivative of the drawdown at late time. He then developed a graphical technique to apply this principle, but this finding had a limited impact until the work of Bourdet and his colleagues (Bourdet et al. 1989; Bourdet et al. 1983). They generalized the idea and analyzed the behaviour of the log-derivative for a large number of classical models of flow around a pumping well. Doing so they showed that the joint use of the drawdown and its log-derivative within a unique plot had many advantages:

- The logarithmic derivative is highly sensitive to subtle variations in the shape of the drawdown curve. It allows detecting behaviours that are difficult to observe on the drawdown curve alone.
- The analysis of the diagnostic plot of a data set facilitates the selection of a conceptual model.
- For certain models, the values of the derivative can directly be used to estimate rapidly the parameters of the model.

Overall, one of the main advantages of the diagnostic plots is that they offer a unified methodology to interpret pumping test data. Indeed, between the work of Theis in 1935 and the work of Bourdet in 1983 (Bourdet et al. 1983), a wide range of models has been developed (bounded aquifers, double porosity, horizontal fracture, vertical fracture, unconfined aquifer, etc.). Many of these models required specific plots and interpretation techniques (see for example Kruseman and de Ridder 1994, for an excellent synthesis).

Using the diagnostic plot allows for the replacement of all these specialized tools with a simple and unique approach that can be applied to any new solutions, as it has been done

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NRC075
July 31, 2012

for example by Hamm and Bideaux (1996), Delay et al. (2004), or Beauheim et al. (2004). Today most of the theoretical works related to well testing include diagnostic plots both in hydrogeology and the petroleum industry.

However, the situation is different in these two fields of application (Renard 2005a). The technique has become a standard in petroleum engineering over the last 20 years. It has been popularized by a series of papers and books (Bourdet 2002; Bourdet et al. 1989; Bourdet et al. 1983; Ehlig-Economides et al. 1994b; Horne 1995; Horne 1994; Ramey 1992). In hydrogeology, the technique is used routinely only in some specific or highly technical projects such as the safety analysis of nuclear waste repositories. Therefore, even if the concept has already been introduced and described in hydrogeological journals (e.g. Spane and Wurstner 1993), only a minority of hydrogeologists are using diagnostic plots and the technique is not taught in specialized hydrogeology textbooks.

This is why there is still a need to promote the use of this technique in hydrogeology to reach a large part of the profession by explaining how the technique works and how it can be used in practice. The technique presents some difficulties and limitations that have been highlighted by the detractors of the approach. These difficulties are often real and must be discussed as well in order to understand what can be done and what cannot be done. In addition, most commercial and open-source pumping-test interpretation software are now providing the option to compute the logarithmic derivative and therefore many users would benefit from a better understanding of this tool.

The objective of this report is therefore to provide an introduction to the diagnostic plots for the practitioners. The main advantages and limitations of this tool are discussed and illustrated through the study of a few field examples. Along the presentation, some theoretical points are discussed to explain how the method works and to facilitate its understanding.

Before discussing the examples and the method, it is important to emphasize that diagnostic plots are calculated on drawdowns. Therefore, the original data must have been preprocessed in order to remove the head variations independent of pumping (both natural or induced by nearby pumping operations). For example, tools are available to remove barometric or earth tide effects (Toll and Rasmussen 2007). This is a standard procedure (Dawson and Istok 1991) that needs to be applied whether one uses diagnostic plots or not, therefore this aspect will not be discussed further here.

An introductory example

A pumping test has been conducted in a confined aquifer for 8 h and 20 min at a steady pumping rate of 50 m^3/h. The drawdown data, measured in an observation well located 251 m away from the pumping well, are reported by Fetter (2001, p. 172).

To analyse this data set with a diagnostic plot, the procedure starts by calculating the log-derivative of the data, and plotting simultaneously the drawdown and the log derivative. The corresponding diagnostic plot is illustrated in Fig. 1. The circles represent the original drawdown data and the crosses the log-derivative. The plot shows that the derivative is larger than the drawdown at early time and then it becomes smaller than the drawdown and tends to stabilize. At late time, the derivative slightly oscillates.

In order to identify which model can be used to interpret these data, one needs to compare the diagnostic plot with a set of typical diagnostic plots such as those shown in Fig. 2; note that Fig. 2 will be presented in detail in a later section. For the moment, it will be used as a catalogue of typical behaviours.

Neglecting the oscillations of the derivative at late time, the comparison between Figs. 1 and 2 shows that the overall behaviour resembles very much what is depicted in Fig. 2a in log–log scale. There is no major increase or decrease of the derivative at late time, as can be seen for example in Fig. 2h,d, or e. At early time, the derivative either does not follow the drawdown curve as it does in Fig. 2f, or it does not remain systematically smaller than the drawdown curve as in Fig. 2g. Additionally, there is not any major hole in the derivative like in Fig. 2b; therefore, one can conclude that the model that best represents the data is the Theis model corresponding to Fig. 2a.

Once the model (or the set of possible models) has been identified, the procedure consists of estimating the parameters of the model that allow for the best reproduction of the data. This is usually done with least-squares procedures and there are many examples of these both in the petroleum and hydrogeology literature (Bardsley et al. 1985; Horne 1994). What is important to highlight is that when diagnostic plots are used, it is interesting to display

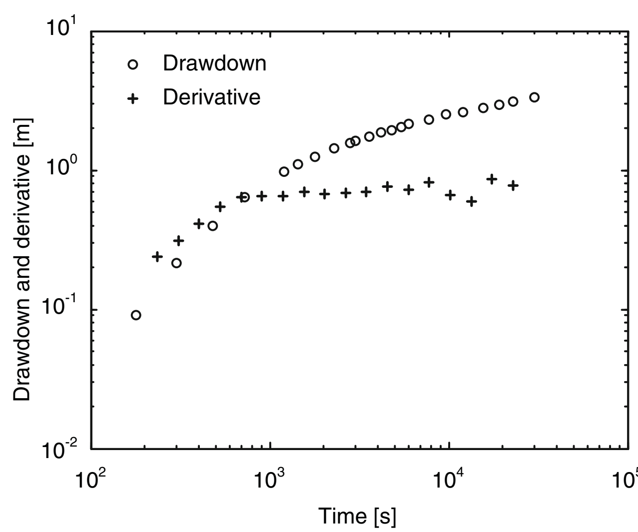

Fig. 1 Diagnostic plot of the Fetter (2001) drawdown data. The *vertical axis* represents both the drawdown and the logarithmic derivative

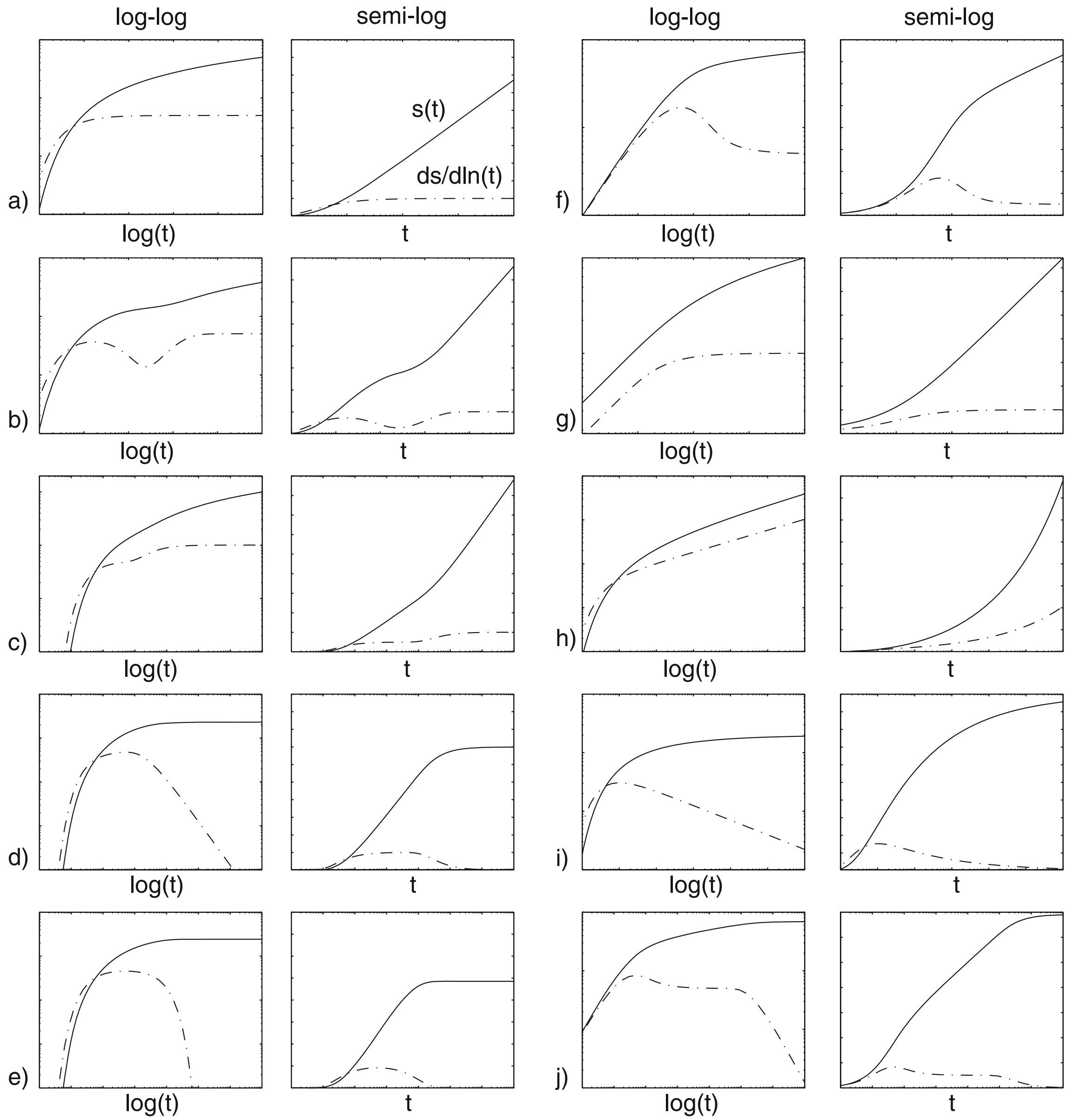

Fig. 2 Most typical diagnostic plots encountered in hydrogeology: **a** Theis model: infinite two-dimensional confined aquifer; **b** double porosity or unconfined aquifer; **c** infinite linear no-flow boundary; **d** infinite linear constant head boundary; **e** leaky aquifer; **f** well-bore storage and skin effect; **g** infinite conductivity vertical fracture.; **h** general radial flow—non-integer flow dimension smaller than 2; **i** general radial flow model—non-integer flow dimension larger than 2; **j** combined effect of well bore storage and infinite linear constant head boundary (modified from Renard 2005b)

(e.g. Fig. 3) the diagnostic plot of the data with the diagnostic plot of the fitted model in the same graph. One can then very rapidly check visually if the fit is acceptable and if the model derivative reproduces the observed data.

Infinite acting radial flow (IARF)

Figure 3 shows an important characteristic of the Theis model: the logarithmic derivative of the model stabilizes at late time. This is due to the fact that the Theis (1935) solutions tend asymptotically toward the Cooper and Jacob (1946) asymptote:

$$\lim_{t\to\infty} s(t) = \frac{Q}{4\pi T} \ln\left(\frac{2.25tT}{r^2 S}\right) \tag{1}$$

where s [m] represents the drawdown, t [s] the time since the pumping started, Q [m^3/s] the pumping rate, T [m^2/s] the transmissivity, r [m] the distance between the pumping well and the observation well, and S [–] the storativity. Note that ln represents the natural logarithm. Computing