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# ACOUSTIC RESONANCE IN A RESERVOIR - DOUBLE PIPE - ORIFICE SYSTEM

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## ABSTRACT

Acoustic resonance in a two-pipe system is simulated with four different models for the periodic excitation. Analytical solutions are provided in full for the three linear excitations. Exact numerical results are presented for the nonlinear excitation. The influence of a large-diameter supply pipe (instead of a constant-head reservoir) on the system's fundamental frequencies and mode shapes is studied. The peculiar behaviour of wave reflection at an orifice is fully explained.

**Key words**
Water hammer; Hydraulic transients; Acoustic resonance; Impedance; Orifice; Rotating valve; Analytical solution.

## INTRODUCTION

This study is a preliminary analysis of acoustic resonance tests carried out at Deltares, Delft, The Netherlands, within the framework of the European Hydralab III programme. The (idealised) test system is a 50 m long pipeline of 200 mm diameter (Pipe 2 herein) that is discharging water from a 25 m high reservoir through an 800 mm$^2$ orifice to the open atmosphere. See Fig. 1. The outflow is partly interrupted by a rotating disc which generates flow disturbances at a fixed frequency in the range 3 Hz to 100 Hz.

In a previous study [1] this system was analysed without taking into account the 30 m long pipe that connects the test pipeline to the elevated reservoir. This additional steel pipe has a diameter of 1 m, a wall thickness of 10 mm, and is lined with 1.5 mm thick synthetic material. It also contains one bend that is fitted with a flow straightener. See Fig. 2.

The influence of the additional pipe (Pipe 1 herein) on the resonance behaviour of the reservoir-pipeline-orifice system is investigated. Four different types of system excitation are considered: forcing velocity, forcing pressure, linear oscillating resistance and nonlinear oscillating resistance. Analytical solutions for the two-pipe system are given for the periodic responses to the first three excitations. The fundamental frequencies and mode shapes of the two-pipe system are calculated and compared with single-pipe solutions. Exact numerical solutions provide insight in the nonlinear behaviour of the rotating valve.

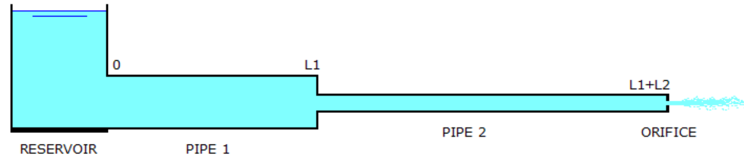


**Figure 1.** Sketch of reservoir - double pipe - orifice system; pipe lengths $L_1 = 30$ m and $L_2 = 50$ m; inner diameters $D_1 = 1$ m and $D_2 = 0.2$ m [1].

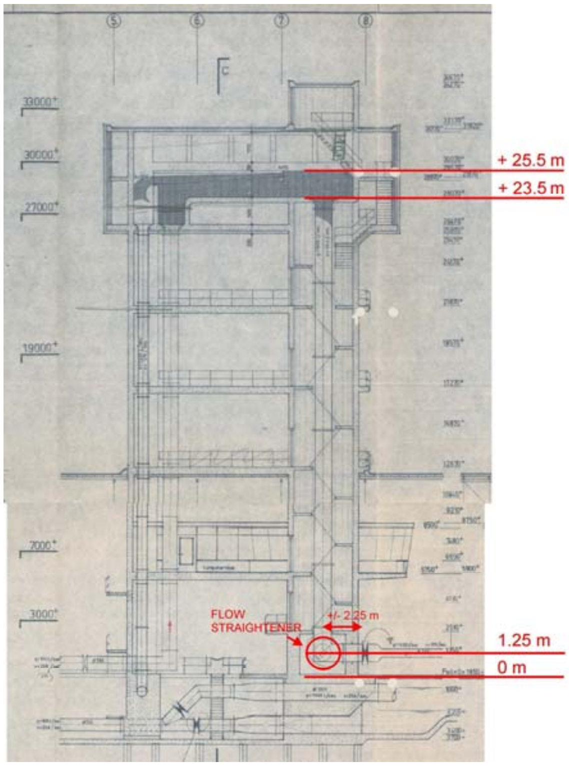


**Figure 2.** Technical drawing of water tower at Deltares, Delft.

## WATERHAMMER EQUATIONS

Classical waterhammer theory [2-4] adequately describes the low-frequency vibration of elastic liquid columns in fully-filled pipes. The two equations, governing velocity, $V$, and pressure, $P$, are

$$\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{\lambda_f}{2D} V\left|V\right| \ , \qquad (1)$$

$$\frac{\partial V}{\partial x} + \frac{1}{K^*}\frac{\partial P}{\partial t} = 0 \ , \qquad (2)$$

with

$$\frac{1}{K^*} := \frac{1}{K} + \frac{D}{Ee} \ . \qquad (3)$$

*Notation:* $D$ = inner pipe diameter, $E$ = Young modulus of pipe material, $e$ = wall thickness, $K$ = bulk modulus of liquid, $K^*$ = effective bulk modulus including wall elasticity, $x$ = distance along pipe, $t$ = time, $\lambda_f$ = Darcy-Weisbach friction factor, and $\rho$ = mass density of liquid. The friction term is ignored herein, i.e. $\lambda_f = 0$, to concentrate on the orifice as the sole cause of damping. Equations (1) and (2) can be combined to the standard wave equations

$$\frac{\partial^2 V}{\partial t^2} - c^2 \frac{\partial^2 V}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2 P}{\partial t^2} - c^2 \frac{\partial^2 P}{\partial x^2} = 0 \ , \qquad (4a, 4b)$$

where the acoustic wave speed is

$$c := \sqrt{\frac{K^*}{\rho}} \ . \qquad (5)$$

## SINUSOIDAL EXCITATION

The pressure of the reservoir at $x = 0$ is taken constant, i.e.

$$P(0,t) = P_{\text{res}} \ . \qquad (6)$$

Sinusoidal excitation at the downstream end (at $x = L = L_1 + L_2$) is simply imposed by

$$V(L,t) = V(L,0) + \hat{V}\ \sin\left(2\pi \frac{t}{T}\right) \quad \text{or} \qquad (7a)$$

$$P(L,t) = P(L,0) + \hat{P}\ \sin\left(2\pi \frac{t}{T}\right) \ , \qquad (7b)$$

where $T$ is the period of oscillation, the circumflex (^) indicates the amplitude of oscillation, $V(L, 0) > \hat{V} > 0$ to prevent back-flow at $x = L$ and $P(L, t) = P_{\text{res}}$ to guarantee equilibrium when $\hat{V} = \hat{P} = 0$ or $T = \infty$.

<u>*Nonlinear orifice equation*</u>

In steady turbulent pipe flow the pressure loss, $\Delta P_0$, across an orifice is

$$\Delta P_0 = \xi_0 \tfrac{1}{2} \rho V_0 \left| V_0 \right| \ , \quad \text{where} \quad \xi_0 := \left( \frac{A}{C_d\ A_{\text{or},0}} \right)^2 \ , \qquad (8)$$

and $V_0$ is the steady flow velocity in the pipe, $A$ is the cross-sectional area of the pipe, $A_{\text{or},0}$ is the steady outflow area of the orifice and $C_d$ is the coefficient of discharge [3, Section 3-3]. In a quasi-steady manner, the same relation is assumed to hold for an orifice with an area that varies in time,

$$\Delta P = \xi(t)\ \tfrac{1}{2} \rho V \left| V \right| \ , \quad \text{where} \quad \xi(t) := \left( \frac{A}{C_d\ A_{\text{or}}(t)} \right)^2 \ , \qquad (9)$$

and $\xi(0) = \xi_0$ when starting the area variation from steady state at $t = 0$. Dividing (9) by (8) and introducing the dimensionless valve closure coefficient $\tau(t) := \sqrt{\xi_0 / \xi(t)} = A_{\mathrm{or}}(t) / A_{\mathrm{or},0}$ gives the nonlinear boundary condition

$$\Delta P_0 \, V|V| = \tau^2(t) \, V_0 |V_0| \, \Delta P \, , \quad V_0 \neq 0 \ . \qquad (10)$$

The specific function $\tau(t)$ used herein to generate oscillating flow is

$$\tau(t) := \frac{2-\alpha}{2} + \frac{\alpha}{2} \cos\left(2\pi \frac{t}{T}\right), \quad 0 \le \alpha \le 1 \ , \qquad (11)$$

in which $T$ is the period of the sinusoidal excitation and $\tau(0) = 1$. See Fig. 3. Equation (11) describes an orifice with constant flow area $A_{\mathrm{or},0}$ that is partly covered by a varying area $A_{\mathrm{disc}}(t)$ (rotating disc herein) according to the following relationships:

$$A_{\mathrm{or}}(t) := A_{\mathrm{or}}(0) - A_{\mathrm{disc}}(t) \qquad \text{with}$$

$$A_{\mathrm{disc}}(t) := A_{\mathrm{disc,max}} \left(\frac{1}{2} - \frac{1}{2}\cos\left(2\pi \frac{t}{T}\right)\right), \ 0 \le A_{\mathrm{disc,max}} \le A_{\mathrm{or}}(0) \qquad (12a)$$

or

$$A_{\mathrm{or}}(t) := A_{\mathrm{or}}(0) \left(\frac{2-\alpha}{2} + \frac{\alpha}{2}\cos\left(2\pi \frac{t}{T}\right)\right) \qquad \text{with}$$

$$\alpha := \frac{A_{\mathrm{disc,max}}}{A_{\mathrm{or}}(0)}, \ 0 \le \alpha \le 1 \ . \qquad (12b)$$

Here, $A_{\mathrm{disc}}(0) = 0$ so that the initial orifice area at $t$ = 0 is at largest, $A_{\mathrm{or}}(0) = A_{\mathrm{or},0}$ and $1 - \alpha \le \tau(t) \le 1$. This is in accordance with the experimental procedure, but of no importance for the final oscillatory steady state. However, it affects the initial transient.

*Linearised orifice equation*

The orifice equation (10) is linearised around the initial steady state $V_0$, $P_0$ and $\tau_0$ defined by relation (8) [3, Section 13-1]. This linearisation is needed for analysis in the frequency domain and for finding symbolic solutions. Assuming flow discharging to the atmosphere, the gauge pressure downstream of the orifice is taken zero, so that $\Delta P_0 = P_0$ and $\Delta P = P$. The small fluctuations $v$, $p$ and $\tau'$ (*not* the derivative of $\tau$) around the steady state are defined by

$$V := V_0 + v \ > 0 \, , \quad v \ll V_0 \ ; \qquad P := P_0 + p \ > 0 \, , \quad p \ll P_0 \ ;$$
$$\tau := \tau_0 + \tau' \ > 0 \, , \quad \tau' \ll \tau_0 \ . \qquad (13)$$

Substituting (13) into Eq. (10) and neglecting the small quadratic terms gives

$$\frac{v}{V_0} - \frac{1}{2}\frac{p}{P_0} = \frac{\tau'(t)}{\tau_0} \ . \qquad (14)$$

The last term in Eq. (14) is zero for a fixed orifice. For Eq. (11) (with $\tau' = \tau - \tau_0$ and $\tau_0 = \tau(0) = 1$) it is

$$\tau'(t) = \frac{\alpha}{2}\left[\cos\left(2\pi \frac{t}{T}\right) - 1\right], \quad 0 \le \alpha \le 1 \ . \qquad (15)$$

The linearised excitation function $\tau'$ has a non-zero average value of $-\alpha/2$, because $V_0$ has been chosen to be the steady flow velocity for the orifice in its most open position.

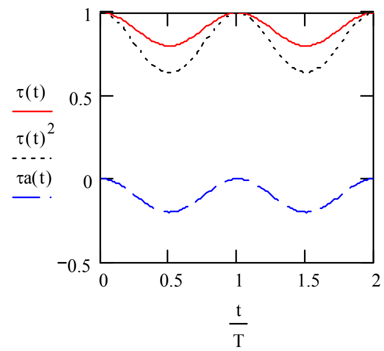


**Figure 3.** Functions $\tau(t)$, $\tau^2(t)$ and $\tau a(t) = \tau'(t)$ for $\alpha = 0.2$.

## ANALYTICAL DOUBLE-PIPE SOLUTIONS

Analytical steady-oscillatory solutions are given for two pipes in series that are excited harmonically at the downstream end. Constant pressure prevails at the upstream end. At the junction of the two pipes, the pressure ($P$) and discharge ($AV$) are continuous; that is, the pressure upstream and downstream of the junction is the same, but the velocity may jump. The solutions were derived by separation of variables. Equivalent linear solutions can be found in Refs [4-14], but here they are – for easy use – written out in full. In Ref. [11] the current Eq. (4) was formulated in terms of displacements. The single-pipe solutions in Ref. [1] are obtained by taking $L_1$ = 0. The lengths, cross-sectional areas, initial velocities, wave speeds and wave numbers are denoted by $L$, $A$, $V_0$, $c$ and $\kappa$, where the subscripts 1 and 2 indicate the upstream and downstream pipe, respectively. The total length is $L = L_1 + L_2$ and the angular frequency of excitation is $\omega$.

The symbolic steady-oscillatory solution of Eqs (4a), (6) with $P_{\mathrm{res}} = 0$ and (7a) for velocity excitation with $V(L,0) = 0$, is:

$V_v(x,t) :=$

$$\frac{\hat{V}\sin(\omega t)\cos(\kappa_1 x_1)}{\frac{A_1}{A_2}\cos(\kappa_1 L_1)\cos(\kappa_2 L_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 \le x_1 = x < L_1$ ;

$$\frac{\hat{V}\sin(\omega t)\left[\frac{A_1}{A_2}\cos(\kappa_1 L_1)\cos(\kappa_2 x_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\sin(\kappa_2 x_2)\right]}{\frac{A_1}{A_2}\cos(\kappa_1 L_1)\cos(\kappa_2 L_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 < x_2 = x - L_1 \le L_2$ ; and

$P_v(x,t) :=$

$$\frac{-\rho c_1 \hat{V}\cos(\omega t)\sin(\kappa_1 x_1)}{\frac{A_1}{A_2}\cos(\kappa_1 L_1)\cos(\kappa_2 L_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 \le x_1 = x < L_1$ ;

$$\frac{\rho c_2 \hat{V}\cos(\omega t)\left[-\frac{A_1}{A_2}\cos(\kappa_1 L_1)\sin(\kappa_2 x_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\cos(\kappa_2 x_2)\right]}{\frac{A_1}{A_2}\cos(\kappa_1 L_1)\cos(\kappa_2 L_2) - \frac{c_1}{c_2}\sin(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 < x_2 = x - L_1 \le L_2$ . (16a)

Resonance occurs when the denominator vanishes.

The symbolic steady-oscillatory solution of Eqs (4b), (6) with $P_{\text{res}} = 0$ and (7b) for pressure excitation with $P(L,0) = 0$ , is:

$P_p(x,t) :=$

$$\frac{\hat{P}\sin(\omega t)\sin(\kappa_1 x_1)}{\frac{A_1}{A_2}\sin(\kappa_1 L_1)\cos(\kappa_2 L_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 \le x_1 = x < L_1$ ;

$$\frac{\hat{P}\sin(\omega t)\left[\frac{A_1}{A_2}\sin(\kappa_1 L_1)\cos(\kappa_2 x_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\sin(\kappa_2 x_2)\right]}{\frac{A_1}{A_2}\sin(\kappa_1 L_1)\cos(\kappa_2 L_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 < x_2 = x - L_1 \le L_2$ ; and

$V_p(x,t) :=$

$$\frac{\frac{\hat{P}}{\rho c_1}\cos(\omega t)\cos(\kappa_1 x_1)}{\frac{A_1}{A_2}\sin(\kappa_1 L_1)\cos(\kappa_2 L_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 \le x_1 = x < L_1$ ;

$$\frac{\frac{\hat{P}\cos(\omega t)}{\rho c_2}\left[-\frac{A_1}{A_2}\sin(\kappa_1 L_1)\sin(\kappa_2 x_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\cos(\kappa_2 x_2)\right]}{\frac{A_1}{A_2}\sin(\kappa_1 L_1)\cos(\kappa_2 L_2) + \frac{c_1}{c_2}\cos(\kappa_1 L_1)\sin(\kappa_2 L_2)}$$

for $0 < x_2 = x - L_1 \le L_2$ . (16b)

The analytical solutions (16a) and (16b) can be combined to satisfy any linear boundary condition at $x = L$. If one defines the amplification factors related to (16a) and (16b) as

$$\mathcal{A}_{P_v} := P_v(L,t) \big/ \rho c_2 \hat{V}\cos(\omega t) \quad \text{and}$$
$$\mathcal{A}_{V_p} := \rho c_2 V_p(L,t) \big/ \hat{P}\cos(\omega t),$$

and one chooses (subscript 2 indicates Pipe 2)

$$\hat{\hat{V}} := \frac{\alpha}{2}\beta_0 V_{0,2} \quad \text{and} \quad \hat{\hat{P}} := \alpha\,\beta_0\,P_0 \quad \text{with}$$
$$\beta_0 := \frac{\gamma}{\gamma^2 \mathcal{A}_{V_p} - \mathcal{A}_{P_v}} \quad \text{and} \quad \gamma := \frac{2P_{0,2}}{\rho c_2 V_{0,2}} \qquad (16c)$$

as the new amplitudes in the formulas (16a) and (16b), respectively, then the symbolic steady-oscillatory solution

$$V(x,t) := V_v(x,t) + V_p(x,t) - \hat{\hat{V}} \quad \text{and}$$
$$P(x,t) := P_v(x,t) + P_p(x,t) \ , \qquad (16d)$$

satisfies the linearised orifice equation (14) with excitation function (15). Equations (16) hold for zero initial conditions, but the steady states $V_0(x)$ and $P_0(x)$ may simply be added. Because $\tau'$ in Eq. (15) has a non-zero average, the velocity amplitude $\hat{\hat{V}}$ has been subtracted in Eq. (16d). The ratio of pressure to velocity excitation at $x = L$ (or $x_2 = L_2$) is $\left|\hat{\hat{P}}/\hat{\hat{V}}\right| = 2P_{0,2}/V_{0,2}$, from Eq. (16c). For constant impedance at $x = L$, that is $\tau' = 0$ in boundary condition (14), $|p/v| = 2P_{0,2}/V_{0,2}$ too.

## EXACT MOC SOLUTION

In addition to the analytical solutions (16), this solution is needed to find the transient leading to the steady-oscillatory state and to determine the exact response to the nonlinear excitation (10-11). The waterhammer equations (1) and (2), in combination with the boundary conditions (6), and (7) or (10), are solved exactly with the MOC-based method described in [15]. The quadratic equation (10) is solved simultaneously with one linear compatibility equation. The initial condition is the undisturbed steady flow in the pipes. The reservoir pressure, $P_{res}$, and the resistance of the valve, $\xi_0$, determine the initial flow velocity in Pipe 2 according to relation (8). The calculation has to be repeated for each excitation frequency of interest.

## TEST PROBLEM

The reservoir - two-pipe - orifice system is simulated with the following input data: pipe length $L_2$ = 50 m, wave speed $c_2$ = 1250 m/s, pipe flow area $A_2$ = 31416 mm$^2$, orifice area $A_{or,0}$ = 800 mm$^2$, discharge coefficient $C_d$ = 0.6, orifice cover fraction $\alpha$ = 0.2, mass density $\rho$ = 1000 kg/m$^3$ and friction factor $\lambda_{f,2}$ = 0. These values correspond to an idealised laboratory system [1] with pipe inner diameter $D_2$ = 200 mm and wall thickness $e_2$ = 6 mm. The pipeline leading to the elevated reservoir is simulated with the following input data: pipe length $L_1$ = 30 m, wave speed $c_1$ = 1000 m/s, pipe flow area $A_1$ = 785398 mm$^2$, reservoir pressure $P_{res}$ = 2.5 bar and friction factor $\lambda_{f,1}$= 0. These values correspond to an idealised water tower with pipe inner diameter $D_1$ = 1000 mm, wall thickness $e_1$ = 10 mm and the reservoir water-level $H_{res}$ = 25 m above the elevation of the pipe's central axis. The synthetic lining and flow straightener (Fig. 2) are ignored.

*Orifice*

The orifice is a horizontal slit of 100 mm width and 8 mm height. The atmospheric pressure downstream of the orifice has been set equal to zero, so that $P_0 = \Delta P_0$ and $P = \Delta P$ in Eqs (8-10) are gauge pressures. The resistance coefficient of the orifice is $\xi_0$ = 4284 from Eq. (8). With a frictionless pipe at constant initial pressure $P_{0,2} = P_{res}$ = 250000 Pa, this gives a constant initial flow velocity $V_{0,2}$ = 0.342 m/s and a steady Reynolds number $\mathrm{Re}_{0,2}$ = 68329. The neglected steady pressure loss due to skin friction along the second pipe is $\lambda_{f,2}(L_2/D_2)(\rho V_{0,2}^2/2)$ = 292 Pa, assuming that $\lambda_{f,2}$ = 0.02. This is small compared to the steady pressure loss $P_{0,2}$ at the orifice. The skin friction loss in Pipe 1 is negligible.

Initially the orifice is fully open. At $t$ = 0 the outflow is interrupted by a frequency-controlled rotating disc [16] that has three 10 mm sinusoidal variations in its 263 mm radius as drawn in Fig. 4. The specific function $\tau(t)$ in Eq. (10) used to describe the orifice with rotating disc is:

$$\tau(t) = 0.9 + 0.1\cos\left(2\pi\frac{t}{T}\right), \quad \text{so that}$$

$$\tau'(t) = -0.1 + 0.1\cos\left(2\pi\frac{t}{T}\right), \tag{17}$$

in which $T$ is the period of the induced oscillation (see Fig. 3). The frequency ($f$) range studied herein is from 1 Hz to 25 Hz. In its most closed position at $t/T$ = 1/2 (mod 1) (Fig. 3) the orifice is a horizontal slit of 80 mm width and 8 mm height (Fig. 4) with resistance coefficient $\xi_0$ = 6693 from Eq. (8) and flow velocity $V_{0,2}$ = 0.273 m/s. If $\tau(t)$ = 0.9 (average $\tau$-value) $\xi_0$ = 5288 and $V_{0,2}$ = 0.307 m/s. Thus, the velocity amplitude for slow ( $f < 1$ Hz ) variations is $\hat{V}$ = 0.0342 m/s. The corresponding acoustic pressure variation (Joukowsky) is $\rho c_2 \hat{V}$ = 42750 Pa.

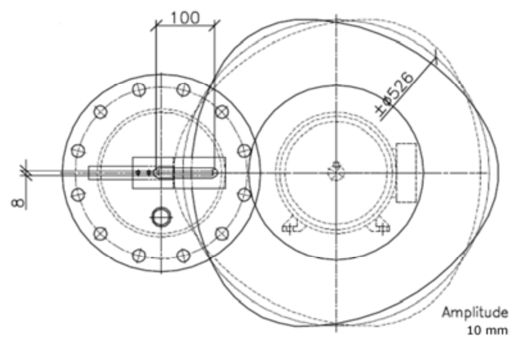


**Figure 4.** Front view of downstream pipe end with orifice and Svingen-type rotating disc.

*Fundamental frequencies and mode shapes*

The frequency range 1 Hz – 25 Hz covers the fundamental natural frequencies. If the forcing function is a velocity (Eq. 7a) the natural frequencies in the single pipe are: $c_2/(4L_2)$ = 6.25 Hz plus odd multiples of $c_2/(4L_2)$. For the double pipe the fundamental frequencies are found from a vanishing denominator in Eq. (16a), that is ($\kappa = \omega / c$)

$$\tan(\kappa_1 L_1)\tan(\kappa_2 L_2) = \frac{A_1/c_1}{A_2/c_2} \ . \tag{18}$$

This is exactly Eq. (5) in Ref. [2, Ch. 20] and Eq. (59) in Ref. [5]. It reduces to Eq. (29) in Ref. [11] if $L_1 = L_2$ and $c_1 = c_2$. The double-pipe results are shown in Table 1. The additional pipe (Pipe 1) reduces the fundamental frequency by 4%. The effect on the higher harmonics is negligible. However, a second fundamental frequency of 8.59 Hz is introduced. The question is whether this leads to resonance when it is excited. The corresponding mode shapes are shown in Fig. 5. These are formulas (16a) – without time dependence – for the double-pipe fundamental frequencies $f = 1/T$ given in Table 1. Note that the downstream end is a velocity node, because natural modes correspond to self-sustained free vibrations that do not need excitation.

**Table 1.** Fundamental frequencies in Hz.

| | $c_2 / 4L_2$ | $c_1 / 4L_1$ | $3c_2 / 4L_2$ | $3c_1 / 4L_1$ |
|---|---|---|---|---|
| single pipes | 6.25 | 8.33 | 18.75 | 25 |
| double pipe | 5.98 | 8.59 | 18.70 | 25.00 |

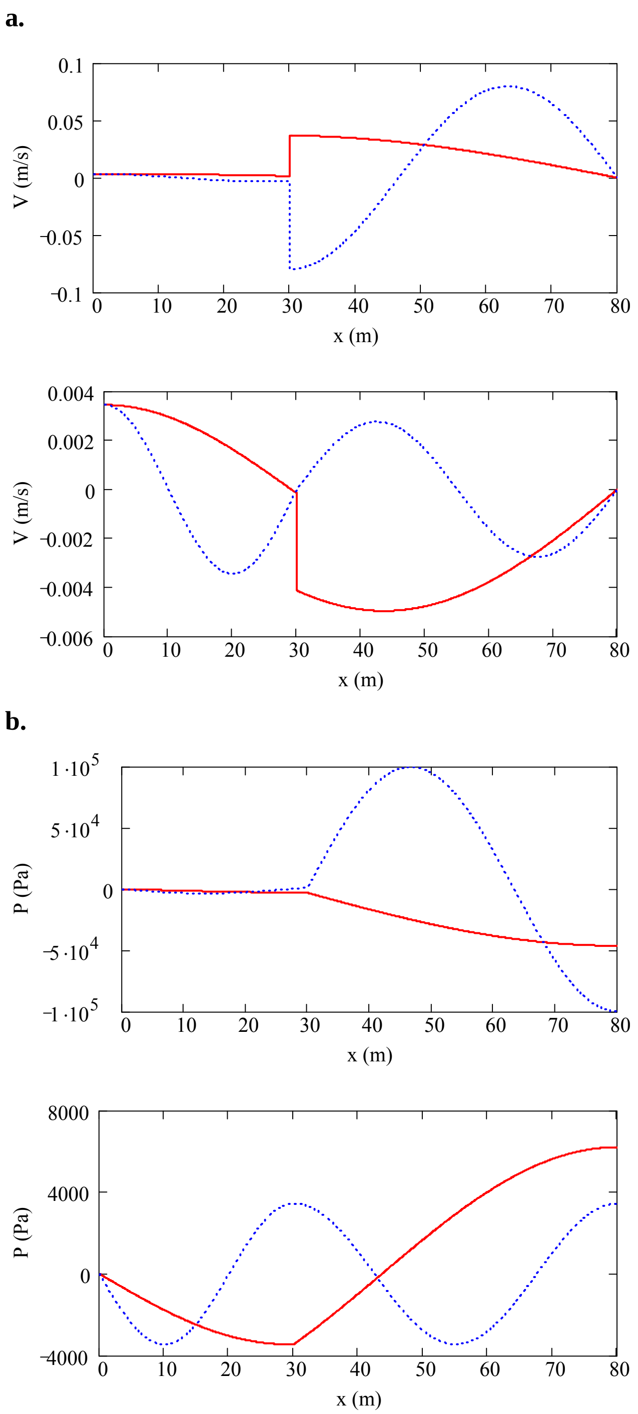


**Fig. 5.** Double-pipe natural mode shapes for velocity excitation for **a**) velocity and **b**) pressure. Pipe 2 modes (5.98 and 18.70 Hz) at top and Pipe 1 modes (8.59 and 25.00 Hz) at bottom. $L_1$ = 30 m and $L_2$ = 50 m.

## EXACT MOC SOLUTIONS

### *Nonlinear orifice excitation*

This (Eqs 10 and 17) is the most realistic excitation. Figure 6 shows system responses for excitation frequencies of 1, 6 and 12 Hz. The dashed blue line is the single-pipe solution [1] and the continuous red line is the MOC transient double-pipe solution. Both solutions start from the constant steady state $V_0(x) = V_0$ and $P_0(x) = P_{res}$. The main observation is that the 200 mm diameter test pipe (Pipe 2) has not a constant pressure at its upstream end (middle three graphs in Fig. 6). The upstream end is not a pressure node anymore. Acoustic inflow pressure fluctuations occur. The inflow velocities themselves are not much influenced by the additional pipe (left three graphs in Fig. 6). The outflow velocity fluctuations near resonance are significantly influenced by the presence of the 1000 mm diameter supply pipe (Fig. 6b, right). As already seen in Ref. [1], there is no resonance – i.e. pressure variations at least twice "Joukowsky" – for the current valve setting, because it introduces a large amount of damping. This fact is fully explained in the Appendix.

## CONCLUSIONS

A frequency-controlled rotating valve that generates oscillating pipe flow in the acoustic range has been modelled in the time domain in four different ways: as a forcing velocity, as a forcing pressure, as a linear oscillating resistance and as a nonlinear oscillating resistance. Analytical steady-oscillatory solutions are presented for the first three cases for two pipes in series. The influence of a large diameter supply pipe (instead of a constant-head reservoir) is not entirely negligible if the considered system is excited near resonance. The fundamental frequency dropped by 4%. The analytical steady-oscillatory solutions for a two-pipe system provide insight in the complex resonance and beat behaviour observed in the results of numerical simulations.

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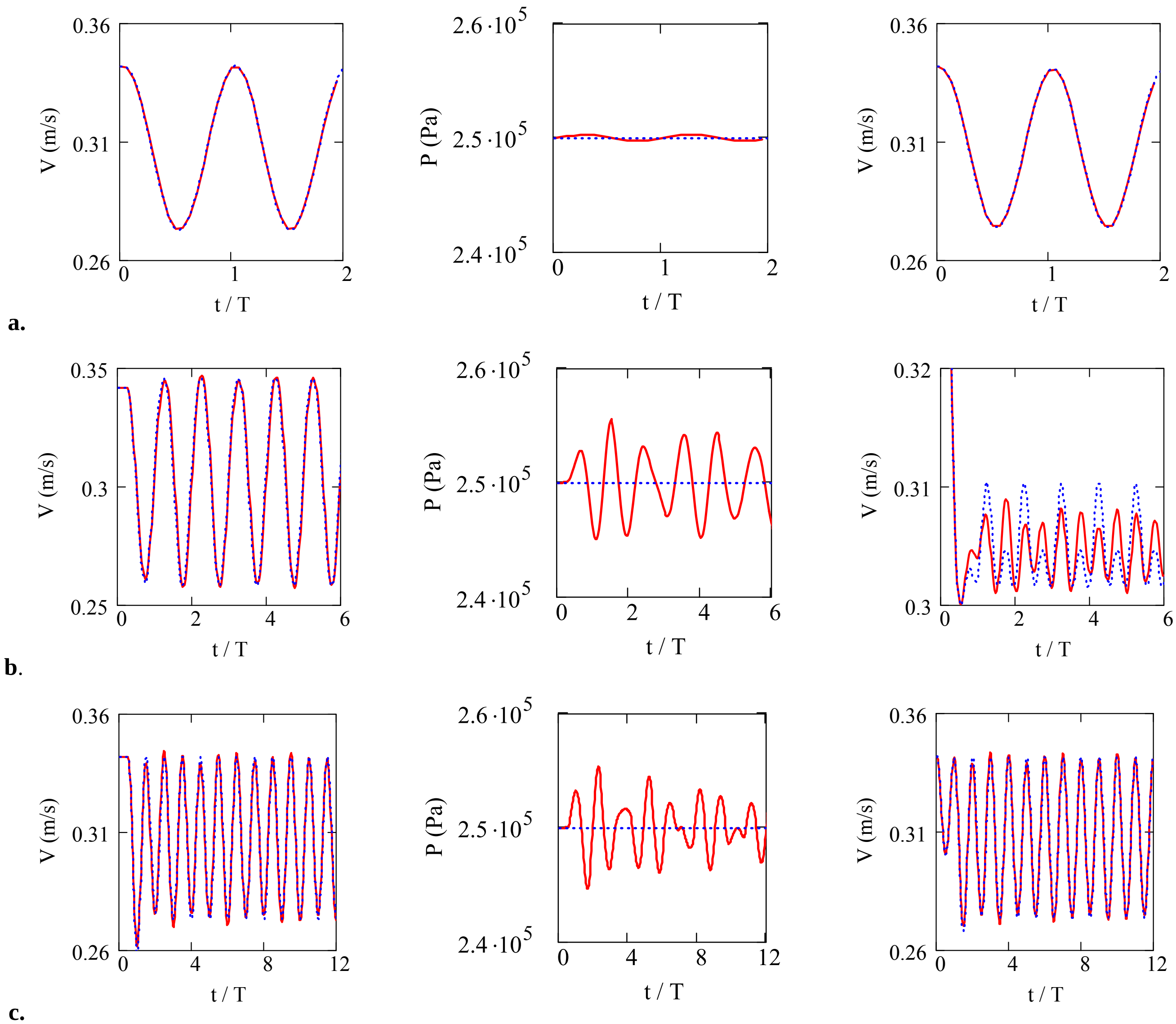


**Figure 6.** Velocity (at $x_2 = 0$), pressure (at $x_2 = 0$) and velocity (at $x_2 = L_2$) responses for nonlinear orifice excitation at fixed frequency $f = 1/T$. Continuous line = double pipe (with water tower); dashed line = single pipe (Fig. 8 in [1]);
**a.** $f = 1/T = 1$ Hz, **b.** $f = 1/T = 6$ Hz, **c.** $f = 1/T = 12$ Hz.

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## APPENDIX: ORIFICE REFLECTION CRITERION

The steady oscillatory state of liquid in a single pipe excited by a rotating valve (linear Eqs 14 and 15) has been derived in Ref. [1]. These are Eqs (16cd-ab) herein for $L_1 = 0$. Ignoring a phase shift, the solution for the pressure can be written as

$$\frac{P(x,t)}{P_0} = \frac{\alpha\ \cos(\omega t)\ \sin(\kappa x)}{\sqrt{\gamma^2 \cos^2(\kappa L) + \sin^2(\kappa L)}} \ , \quad \text{with}$$

$$\gamma := \frac{2P_0}{\rho c V_0} = \xi_0 \frac{V_0}{c} = \sqrt{2\xi_0 \frac{P_0}{K^*}} \quad \text{and} \quad \kappa = \frac{\omega}{c} . \tag{A1}$$

This formula tells the complete story. The pressure (relative to $P_0$) is proportional to the amount of valve closure $\alpha$ (Eq. 12b) and oscillates harmonically at angular frequency $\omega$. The factor $\sin(\kappa x)$ determines the locations of the nodes and anti-nodes. The denominator accounts for the pressure amplification – if any – and is studied below as a function of the important parameter $\gamma$, which depends on the orifice resistance $\xi_0$ (Eq. 8). One is therefore interested in the extreme values of

$$\gamma^2 \cos^2(\kappa L) + \sin^2(\kappa L) \tag{A2}$$

as a function of wave number $\kappa$ and hence frequency $f$. Setting the derivative of expression (A2) equal to zero gives ($L > 0$):

$$(1-\gamma^2)\cos(\kappa L)\sin(\kappa L) = 0 \ . \tag{A3}$$

Extreme values of (A2) occur when

$$\kappa L = 0 \ \bmod\left(\frac{\pi}{2}\right) \quad \text{or} \quad f = 0 \ \bmod\left(\frac{c}{4L}\right) . \tag{A4}$$

The most interesting thing is that the parameter $\gamma$ determines the character of the extreme values of (A2). The fundamental frequency $c/(4L)$ minimises (A2) when $\gamma > 1$, but it maximises it when $\gamma < 1$. Defining the amplification factor

$$\frac{1}{\sqrt{\gamma^2 \cos^2\left(\frac{2\pi L f}{c}\right) + \sin^2\left(\frac{2\pi L f}{c}\right)}} \tag{A5}$$

and looking at the periodic solution (A1), "resonance" occurs at frequency $c/(4L)$ for $\gamma > 1$, but at frequency $c/(2L)$ for $\gamma < 1$. This confirms the *orifice reflection criterion* derived in [3] and discussed in [1]:

if $\gamma > 1$ the orifice behaves like a closed end (small pipe flow velocity means small orifice opening); there is much damping and some reduction, i.e. $1/\gamma <$ (A5) $< 1$, if $\gamma < 1$ the orifice behaves like a fully open end (large pipe flow velocity means large orifice opening); there is less damping and some amplification, i.e. $1 <$ (A5) $< 1/\gamma$,

if $\gamma = 1$ there is no reflection because the orifice impedance equals the pipe's characteristic impedance; it is a reflection-free boundary, where (A5) = 1.

(A6)

Wylie, Streeter and Suo [3, Section 12-5] derived this criterion employing the impedance approach to calculate free vibration in a reservoir-pipe-orifice system (using Eq. 14 with $\tau' \equiv 0$). Peng and Moody [13] applied a sinusoidal pressure variation at the reservoir and – using Laplace transforms – arrived at the same criterion via infinite series solutions. Here, solution (A1) holds for a linearised rotating valve such that the forced oscillation is caused by the valve itself.

For the test problem considered herein, $L = L_2 = 50$ m, $c = c_2 = 1250$ m/s and $\gamma = 1.17$. The amplification factor (A5) is displayed in Fig. 7 as a function of frequency. This factor is always smaller than or equal to 1 (for $\gamma = 1.17$) and in that sense there is a reduction of pressure [compare for example with the velocity-excitation solution (16a) for $L_1 = 0$, which has an amplification factor $1/\cos(\kappa L)$ allowing for resonance]. For illustration, if $\gamma = 0.83$ there is amplication of pressure. More important, the fundamental frequency has changed from that of an open-closed system to that of an open-open system. One might think that the transformation from a $c/(4L)$ system to a $c/(2L)$ system at $\gamma = 1$ is a discontinuity. But this is not so as is demonstrated in Fig. 7. The solutions for $\gamma = 1.02$ and $\gamma = 0.98$ are very close to each other and near the critical solution $\gamma = 1$. For $\gamma = 1$ the amplification is independent of the frequency of excitation, because there is no wave reflection at the orifice. It is noted that $P_0$ increases with $\gamma$ and hence that the amplification factor in Fig. 7 is not a direct measure of the pressure amplitude.

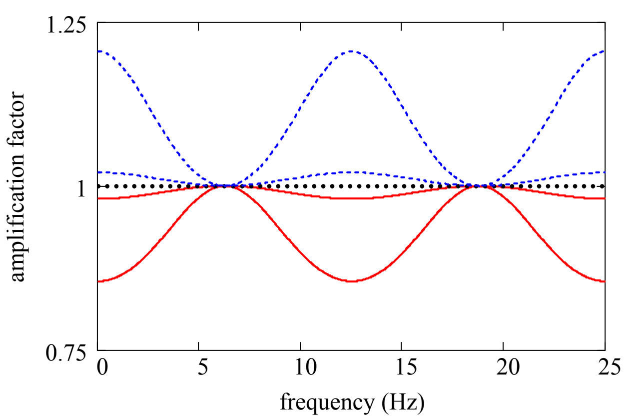


**Figure 7.** Illustration and confirmation of criterion (A6). Pressure amplification factor (A5) for $\gamma = 1.17$ and 1.02 (continuous red lines), $\gamma = 1$ (dotted black line) and $\gamma = 0.98$ and 0.83 (broken blue lines).

For $\gamma \to 0$, where the valve is fully open, solution (A1) tends to the pressure-excitation solution, that is Eq. (16b) for $L_1 = 0$, however with a damping factor $\alpha$.

For $\gamma \to \infty$, where the valve is fully closed, solution (A1) tends to the velocity-excitation solution, that is Eq. (16a) for $L_1 = 0$, however with a damping factor $\alpha / \gamma$.

An apparent paradox is that pressure amplification (A5) > 1 (for varying $\xi_0$ and fixed $P_0$) takes place when the valve is (more) open ($\gamma \to 0$), whereas in classical waterhammer pressure amplification (doubling) takes place when the valve is closed ($\gamma \to \infty$). See also Eq. (25) in Ref. [13]. Damping and valve characteristics play a prominent role here. Be aware that the choice of $\xi_0$ in the linearisation is to a certain extent arbitrary and that nonlinear behaviour becomes of importance near resonance [1].