MODULE C

Overview of the PRA Process and Basic PRA Techniques

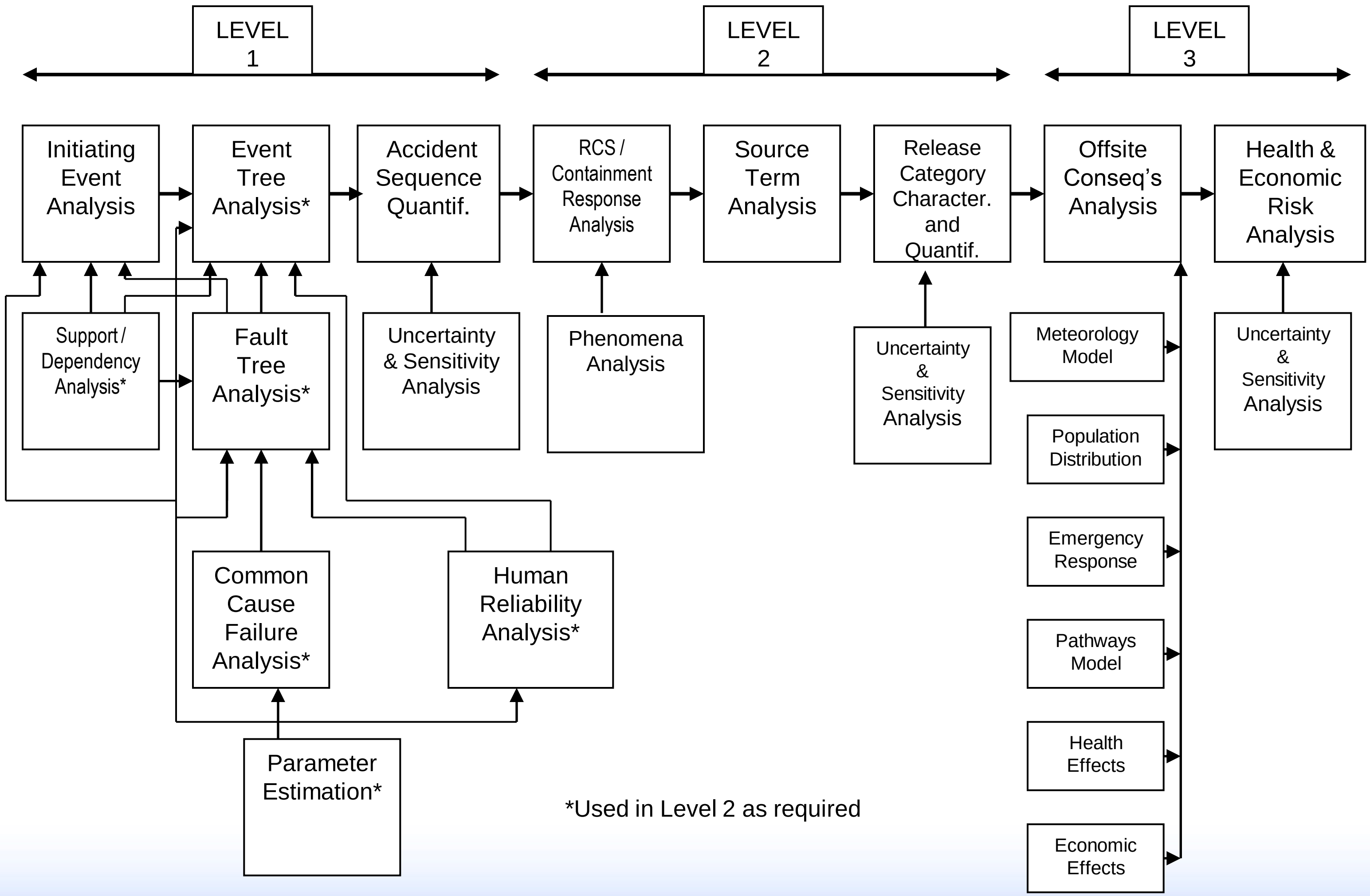
Purposes & Objectives

- **Purpose: Provide an overview of the PRA process and describe why probabilistic models are used.**
- **Objectives: Upon completion of this module, students should be able to**
  - **Describe the major steps in the PRA process**
  - **Describe the outputs of each of the "Levels" of PRA**
  - **Describe why probabilistic models are used**
  - **Give examples of disciplines required to perform a PRA**
  - **Give examples of where traditional engineering inputs are used in the PRA process**
  - **List basic probability operations**
  - **Describe the difference between frequency and probability**
  - **Calculate probabilities**
  - **Define cut sets**

Probabilistic Risk Assessment (PRA)

- PRA is a rigorous technical analysis that systematically attempts to answer
  - three questions commonly referred to as the risk triplet:
    - What can go wrong?
      - Identify accident scenarios
    - How likely is it?
      - Estimate likelihood (frequency, probability) of each accident scenario
    - What will be the outcome?
      - Estimate consequences of each accident scenario
  - A fourth question, reflecting the importance of uncertainty, has also been addressed in recent PRAs
    - How confident are we in our answers to these three questions?

Principal Steps in PRA



*Used in Level 2 as required

Overview of PRA Process

- Probabilistic Risk Assessments (PRAs) are performed to find severe accident weaknesses and provide quantitative results to support decision-making. Three levels of PRA have evolved:

Level	Type of Analysis	Assessment of:	Results
1	Systems analysis	Plant accident initiators and response of systems and operators	Core damage frequency & contributors
2	Containment analysis	Frequency and modes of containment failure	Categorization & frequencies of containment releases
3	Radiological consequences	Public health consequences	Estimation of public & economic risks

Level 1 PRA

- Level 1 PRA assesses frequency of core damage
  - Level 1 PRA consists of six major steps:
    - Identification and grouping of initiating events including initiators of traditional DBAs [operations experience]
    - Establishment of success criteria based on traditional engineering analyses [mechanical engineers/computer specialists]
    - Accident sequence modeling (event tree and fault tree development) [system engineers, operations & maintenance input, PRA modelers]
    - Parameter estimation (e.g., component failure rates) [statistical experts, human performance specialists]
    - Accident sequence quantification [PRA specialists]
    - Documentation and evaluation of results

Level 2 PRA

- Level 2 PRA assesses probability of containment failure and characteristics of releases from containment
  - Progression of severe core damage accidents evaluated by:
    - Investigating phenomenology of the core-melt process [experimentalists, physicists]
    - Analyzing response of containment to structural challenges based on structural analyses [structural engineers]
  - Level 2 analysis used to identify, order, and quantify physical phenomena that could affect progression of severe accidents (largely based on deterministic computer codes but with probabilistic input where outcome is random or uncertain)
  - Final product of Level 2 analysis includes:
    - Probabilities of particular containment failure modes
    - Timing of containment failure
    - Fraction of radionuclides released to atmosphere (source term)

Level 3 PRA

- **Level 3 PRA assesses public health and economic consequences of radiological releases**
  - **Comprises four major modeling processes (PRA specialists, meteorologists, health effects modelers...):**
    - **Atmospheric transport and deposition model to estimate**
      - **Direction and quantity of source-term plume release from containment**
      - **Area expected to be contaminated**
      - **Timing processes relative to emergency response**
    - **Pathways model considers:**
      - **Routes by which radiation enters body**
      - **Accumulated dose to various organs**
    - **Health effects model estimates:**
      - **Fatalities and injuries expected to occur within one year of accident (acute health effects)**
      - **Cancer deaths expected to occur over lifetime of exposed population (latent health effects)**
    - **Models relating to other consequence factors such as:**
      - **Population distribution**
      - **Emergency response**
      - **Economic effects**

Level 3 PRA (cont.)

- Integrated risk result is frequency with which a consequence of a particular magnitude will be exceeded
- NRC Quantitative Health Objectives (see Module A) constitute risk guidelines for commercial nuclear power plants

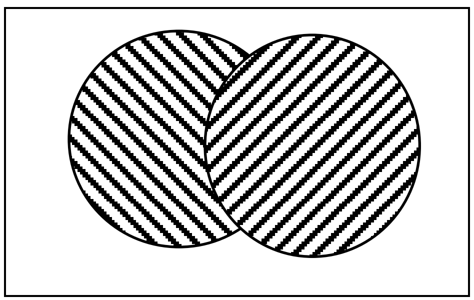
Why Probabilistic Modeling?

- **Some problems/issues are too complex to treat deterministically; for example**
  - **Want to determine if emergency diesel generator (EDG) will start on next demand**
  - **Would require complete knowledge of initial and boundary conditions (e.g., how wearing of piece parts affects start capability)**
  - **Our lack of knowledge forces us to treat EDG performance as a random process (i.e., probabilistically)**

Basic Probability Concepts Used in PRAs

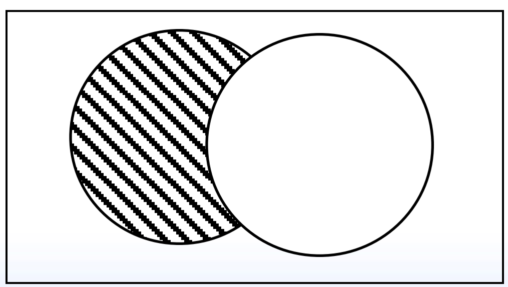
A or B

A + B



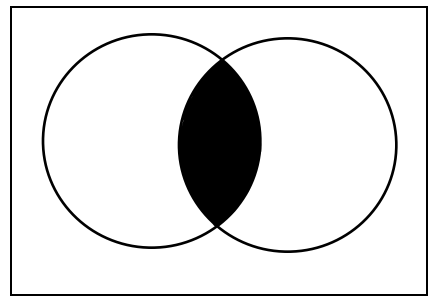
A and /B

*A * /B*



A and B

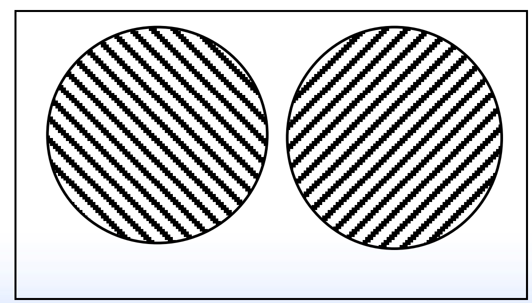
*A * B*



A or B

A + B

with the two events mutually exclusive



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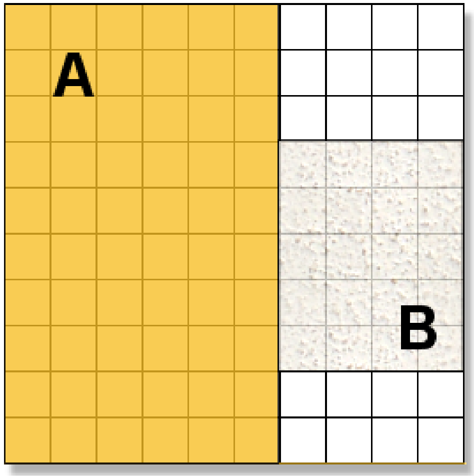
Basic Probability Concepts

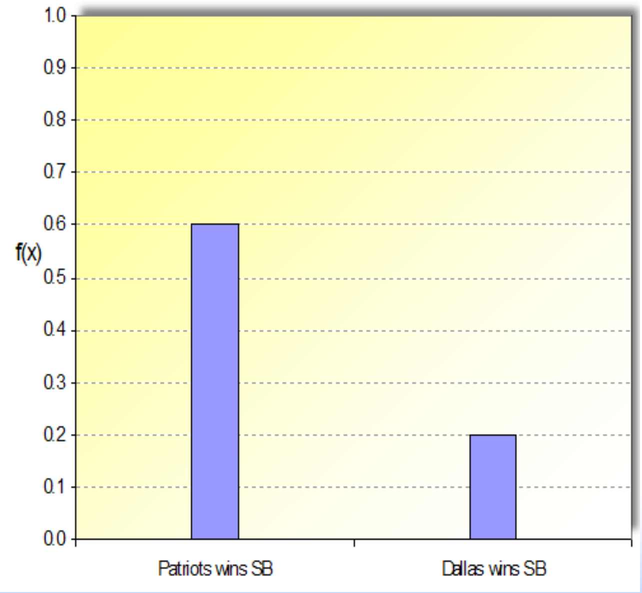
- Mutually Independent - Means that the occurrence (or non-occurrence) of an event (such as A) has no influence on the subsequent occurrence (or non-occurrence) of another event (such as B) and vice versa
  - If a coin is tossed randomly, the occurrence of Heads on the first toss should not influence the probability of Tails on the second toss.
  - If A and B are two mutually independent events, then P(A and B) = P(A) * P(B)
- Mutually Interdependent (or also called dependent) - Means that the occurrence (or non-occurrence) of an event (such as A) has an influence on the subsequent occurrence (or non-occurrence) of another event (such as B) and vice versa
  - If a resistor overheats in an electronic circuit, it may very well change the failure probability of a nearby transistor or of related circuitry.
  - If A and B are two mutually interdependent events, then P(A and B) = P(A) * P(B|A) = P(B) * P(A|B)
- Mutually Exclusive - Means that events (such as A and B) cannot both happen on a single trial of an experiment
  - Either a Head or a Tail is the expected outcome on a single toss of a coin, cannot possibly get both a Head and a Tail as an outcome on a single toss.
  - If A and B are two mutually exclusive events, then P(A or B) = P(A) + P(B)
  - Note: If Mutually Exclusive, P(A and B) = P(A)*P(B|A) = P(B)*P(A|B) = 0

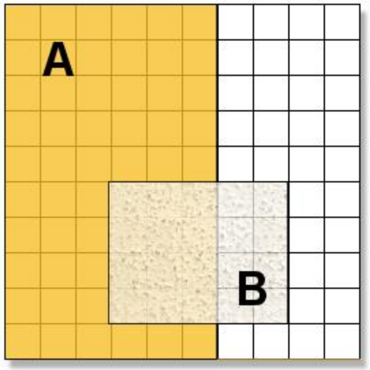
Independent versus Disjoint

- **An example using disjoint events**
  - **If two events A and B are disjoint (mutually exclusive)**
    - **Pr(A AND B) = 0**
    - **If Pr(A) = 0.6 while Pr(B) = 0.2 then the “Venn” diagram is**



Disjoint

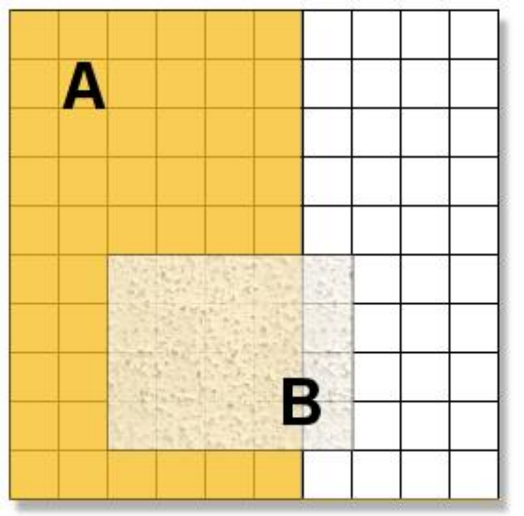




Pr(A AND B) = 0.12 if A, B were independent…

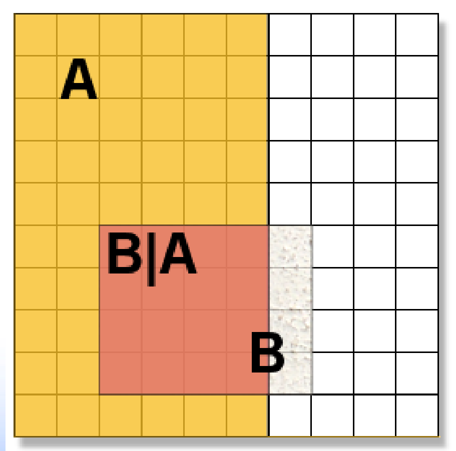
Independent versus Dependent

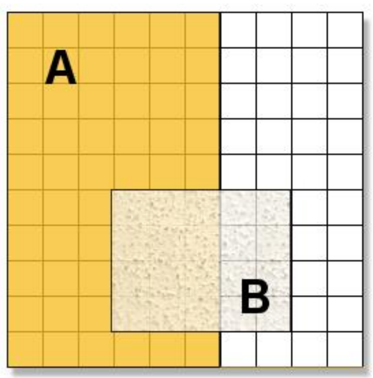
- **An example using dependent events**
  - If Pr(A) = 0.6, Pr(B) = 0.2, and Pr(A AND B) = 0.16, then
    - Pr(B|A) = Pr(A AND B)/Pr(A) = 0.16/0.6 = 0.2667
      - since Pr(A AND B) = Pr(A) • Pr(B|A)
    - Pr(A|B) = Pr(A AND B)/Pr(B) = 0.16/0.2 = 0.80
      - since Pr(A AND B) = Pr(B) • Pr(A|B)



A and B are dependent

Where is Pr(B|A) on the Venn diagram?
16 blocks/60 blocks = 0.2667





Pr(A AND B) = 0.12
if A, B were
independent…

Some Events have an Associated Frequency which is used to Calculate a Probability

- **Frequency**
  - Parameter used in model for stochastic (aleatory) uncertainty
  - Events per unit of time
  - Frequency can be any positive value (i.e., can be greater than one)
  - Typically used for initiating events and failure rates
- **Probability**
  - Internal measure of certainty about the truth of a proposition
  - Always conditional
  - Unitless
  - Value between 0 and 1
  - Used for all events in a PRA except the initiating event
- **Different concepts; sometimes numerically equal**

Common Probability Models

- **Binomial (used for failures on demand)**
  - **P{r failures in N trials |p } =** $\binom{N}{r} p^r (1-p)^{N-r}$
    - **Recall:** $\binom{N}{r} = \frac{N!}{(N-r)!r!}$
    - **Probability of failure for a single demand**

$$\mathrm{P}\{1 \text{ failure in } 1 \text{ trial} \mid \mathrm{p}\} = \frac{N!}{(N-r)!r!} p^r (1-p)^{N-r} = \frac{1!}{(1-1)!1!} p^1 (1-p)^{1-1} = (1) p^1 (1) = p$$

- **Binomial Example:**
  - **Pump data failing to start on demand p = 0.001**
  - **Probability of 1 failure to start in 1 demand?**

$$\mathrm{P}\{1 \text{ failure in } 1 \text{ trial} \mid 0.001\} = \frac{1!}{(1-1)!1!} 0.001^1 (1-0.001)^{1-1} = \frac{1!}{(0)!1!} 0.001^1 (0.999)^0 = (1)(0.001)(1) = 0.001$$

Common Probability Models (cont.)

- Poisson (used for failures/events in time)
  - P{r failures in (0,t) | λ } = $\dfrac{(\lambda t)^r e^{-\lambda t}}{r!}$
  - Probability of one or more failures (Poisson simplifies to exponential)
    - $P\{T_f < t \mid \lambda\} = 1 - e^{-\lambda t} \approx \lambda t$ (for small λt; when $\lambda t < 0.1$)
      - Example of exact $1 - e^{-\lambda t}$ versus estimate λt

0.39	vs	0.5
0.095	vs	0.1
0.04877	vs	0.05
0.00995	vs	0.01
0.0049875	vs	0.005

- Poisson Example:
  - Pump data failing to run λ = 1E-4 failures per operating hour
  - Probability of failure to run for 24 hours?
    - $P\{T_f <$ 24 hours | 1E-4 failures/hour}

      = 1 - $e^{-(\text{1E-4failures/hour})(\text{24 hours})}$ = 1 - $e^{-(\text{2.4E-3})}$ = 1 – (0.997602878) = 0.002397122

      ≈ 2.4E-3 [i.e., product of λt = (1E-4)(24) = 2.4E-3 as the $\lambda t < 0.1$]

Probability of Core Damage

- **Assume 100 plants, each with CDF = 1E-4/yr**
- **Assume operation over 40 years**
- **What is probability of at least one core damage accident during that time?**

P(≥ 1 core damage | CDF = 1E-4/yr)
= 1 – exp[-(1E-4/plant-yr)(40 yr)(100 plants)]
= 0.33

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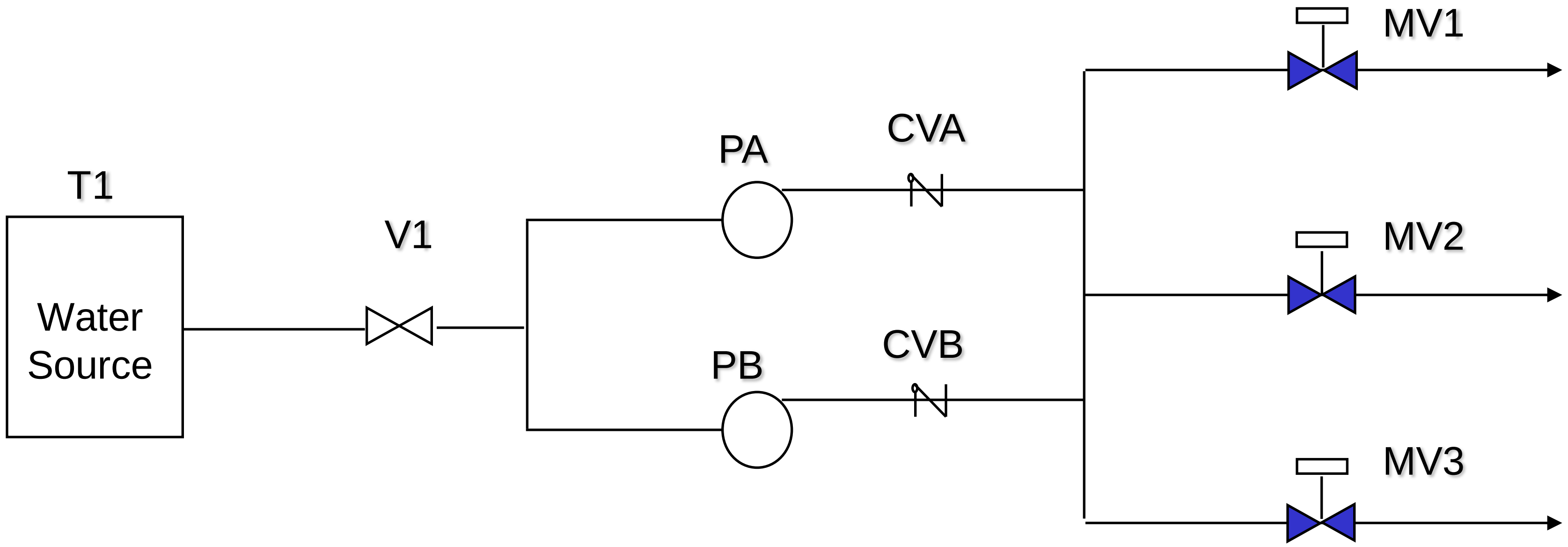
Cut Sets

- **Combinations of events that result in a particular outcome**
- ***Minimal Cut Sets* are those combinations that are both *necessary* and *sufficient* to produce the particular outcome**
  - **i.e., minimal combination**
- **Each cut set represents a failure scenario that must be "ORed" together with all other cut sets for the top event when calculating the total probability of the top event**
- **Boolean algebra (discussed later) used for processing cut sets**

Cut Set Example



Emergency Coolant Injection (ECI) System: success if there is flow from the tank through any one pump train through any one motor-operated valve. ECI components include;

T# - tank

V# - manual valve, normally open

P# - pump

CV# - check valve

MV# - motor-operated valve, normally closed

Cut Sets for ECI

By inspection of the ECI piping and instrumentation diagram (P&ID):

ECI-System-Failure =

T1 +
V1 +
*PA * PB +*
*PA * CVB +*
*PB * CVA +*
*CVA * CVB +*
*MV1 * MV2 * MV3*

Cut Sets Can Be Quantified Using Various Methods

- **Exact Solution for Cut Sets = A + B**
  - **P(Exact Solution for Cut Sets) = P(A + B) = P(A) + P(B) - P(AB)**
- **Cross terms become unwieldy for large lists of cut sets. For example, if Cut Sets = A + B + C, then:**
  - **P(Exact Solution for Cut Sets) = P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)**
- **Cut Sets typically quantified using one of two approximation methods:**
  - **Rare-Event Approximation**
  - **Minimal Cut Set Upper Bound Approximation**

Cut Sets Can Be Quantified Using Various Methods (cont.)

- **Rare Event Approximation for Cut Sets = A + B**
  - **P(Union of Cut Sets) = sum of the probabilities of each individual cut set**
  - **P(Union of Cut Sets) =** $\sum_{k=1}^{K} P(Cutset_k)$
  - **P(AB) judged to be sufficiently small (rare) and thus can be ignored (i.e., cross-terms are simply dropped)**
- **In general,**
  - **P{*Exact* Solution for Cut Sets} ≤** $\sum_{k=1}^{K} P(Cutset_k)$

Cut Sets Can Be Quantified Using Various Methods (cont.)

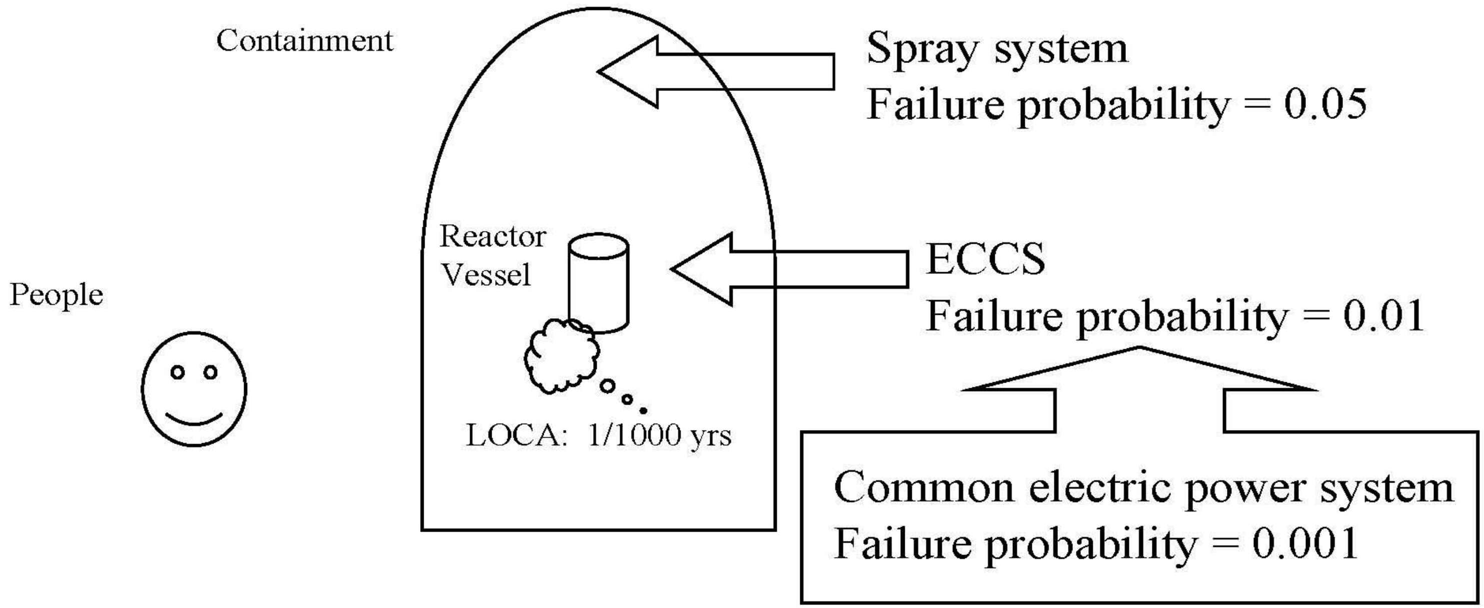
- **Minimal Cut Set Upper Bound Approximation for Cut Sets = A + B**
  - **P(Minimal Cut Set Upper Bound for Cut Sets) = 1.0 minus the product of each individual cut set NOT occurring**
  - **P(MCSUB for Cut Sets) =** $1 - \prod_{k=1}^{K} [1 - P(Cutset_k)]$
  - **P(MCSUB for Cut Sets) = 1 - [(1 - P(A)) * (1 - P(B))]**
- **Assumes cut sets are independent (i.e., no shared basic events in individual cut sets)**
- **In general,**
  - **P(*Exact* Solution for Cut Sets) ≤ P(MCSUB for Cut Sets) ≤ P(Rare Event for Cut Sets)**

Examples of Cut Set Quantification Methods for P(A+B)

	Cut Sets A & B independent; individual cut set values low	Cut Sets A & B independent; individual cut set values high	Cut Sets A & B are not independent (they have shared basic events); individual cut set values low
Cut-Sets = A + B	P(A) = 0.01 P(B) = 0.03	P(A) = 0.4 P(B) = 0.6	Cut Set A = BE1 * BE2 Cut Set B = BE2 * BE3 P(BE1) = 0.1 P(BE2) = 0.1 P(BE3) = 0.3
Exact	= 0.01 + 0.03 - (0.01 * 0.03) = 0.04 – 0.0003 = 0.0397	= 0.4 + 0.6 - (0.4 * 0.6) = 1.0 - (0.24) = 0.76	= (BE1*BE2) + (BE2*BE3) – (BE1*BE2)*(BE2*BE3) = (BE1*BE2) + (BE2*BE3) – (BE1*BE2*BE3) = 0.01 + 0.03 – 0.003 = 0.04 – 0.003 = 0.037
Rare Event	= 0.01 + 0.03 = 0.04	= 0.4 + 0.6 = 1.0	= 0.01 + 0.03 = 0.04
MinCut UB	= 1 - [(1-0.01) * (1-0.03)] = 1 - [(0.99) * (0.97)] = 1 - [0.9603] = 0.0397	= 1 - [(1-0.4) * (1-0.6)] = 1 - [(0.6) * (0.4)] = 1 - [0.24] = 0.76	= 1 - [(1-0.01) * (1-0.03)] = 1 - [(0.99) * (0.97)] = 1 - [0.9603] = 0.0397

Exercise Demonstrating PRA Process

- For the simple plant shown (next page)
  - What can go wrong?
    - Assume Loss-of-Coolant Accident (LOCA) is the initial challenge (initiating event) during normal plant operation
    - What else could go wrong in terms of the three systems shown?
      - Success or failure of electric power system
      - Success or failure of emergency core cooling system (ECCS)
      - Success or failure of containment spray system (CSS)
  - How likely is each combination of events identified above?
    - Use LOCA frequency and given probabilities to calculate scenario frequencies
  - What are the consequences?
    - What happens to core in each scenario?
    - What happens to containment?
    - Characterize expected release offsite
    - Which level of PRA would be involved in each of these questions?



Assumptions:

1 - Electric system powers ECCS and spray system
2 - If ECCS fails, core is damaged
3 - If spray system fails, containment is damaged
4 - If spray system successful, containment does NOT fail - even with core damage (i.e., ECCS failed)
5 - If spray system fails, containment is damaged and ECCS will subsequent fail if not already failed

****IPE Exercise****

- **Using your choice of a plant's IPE, determine the following:**
  - **Level of PRA detail that was analyzed**
  - **Estimated core damage frequency (CDF)**
  - **Compare estimated CDF with 1E-4 goal**
  - **Dominant (highest frequency) type of accident sequence**
  - **Estimated large, early release frequency (LERF)**
  - **Compare estimated LERF with 1E-5 goal**

WORKSHOP - Probability and Frequency Questions – (question 1 of 4)

- 1. An event occurs with a frequency of 0.02 per year.
  - 1.1. What is the probability that at least one event will occur within a given year?
  - 1.2. What is the probability that at least one event will occur within 50 years?

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Useful Values for Workshops

- $e^{-0.02} = 0.9802$
- $e^{-0.10} = 0.9048$
- $e^{-0.30} = 0.7408$
- $e^{-0.50} = 0.6065$
- $e^{-1.00} = 0.3679$
- $e^{-1.50} = 0.2231$

WORKSHOP - Probability and Frequency Questions – (question 2 of 4)

- **2. Event A occurs with a frequency of 0.1 per year. Event B occurs with a frequency of 0.3 per year.**
  - **2.1. What is the probability that at least one event (either A or B) will occur within a given year?**
  - **2.2. What is the probability that at least one event (either A or B) will occur within 5 years?**

WORKSHOP - Probability and Frequency Questions – (question 3 of 4)

- 3. An experiment has a probability of 0.2 of producing a failure.
  - 3.1. What is the probability of observing exactly one failure if the experiment is repeated 4 times?
  - 3.2. What is the probability of observing at least one failure if the experiment is repeated 4 times?
  - 3.3. If the experiment is repeated 4 times, what is the probability of observing the following number of failures;
    - 0
    - 0 or 1
    - 0 or 1 or 2
    - 0 or 1 or 2 or 3
    - 0 or 1 or 2 or 3 or 4

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WORKSHOP - Probability and Frequency Questions – (question 4 of 4)

- 4. An experiment has a probability of 0.1 of producing a failure.
  - 4.1. What is the probability of observing exactly one failure if the experiment is repeated 4 times?
  - 4.2. What is the probability of observing at least one failure if the experiment is repeated 4 times?
  - 4.3. If the experiment is repeated 4 times, what is the probability of observing the following number of failures;
    - 0
    - 0 or 1
    - 0 or 1 or 2
    - 0 or 1 or 2 or 3
    - 0 or 1 or 2 or 3 or 4

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WORKSHOP - workspace

Probability and Frequency Questions

- 1. An event occurs with a frequency of 0.02 per year.
  - 1.1. What is the probability that at least one event will occur within a given year?
    - P{event <1 year} = $1\text{-}e^{-(2E\text{-}2)(1)}$ = 1-0.9802 = 0.0198 = 1.98E-2
    - Or P{event <1 year} ≈ λt ≈ (2E-2)(1) ≈ 2E-2
  - 1.2. What is the probability that at least one event will occur within 50 years?
    - P{event <50 years} = $1\text{-}e^{-(2E\text{-}2)(50)}$ = $1\text{-}e^{-1}$ = 1-0.3679 = 0.6321 = 6.321E-1
- 2. Event A occurs with a frequency of 0.1 per year. Event B occurs with a frequency of 0.3 per year.
  - 2.1. What is the probability that at least one event (either A or B) will occur within a given year?
    - P(A) = $1 - e^{-(\lambda A)t} = 1 - e^{-(0.1)1}$ = 1 – 0.9048 = 0.0952
    - P(B) = $1 - e^{-(\lambda B)t} = 1 - e^{-(0.3)1}$ = 1 – 0.7408 = 0.2592
    - P(A + B)=P(A)+P(B)-P(AB)=0.0952+0.2592–[(0.0952)(0.2592)]=0.3543–0.0247=0.3297
    - Or P(A + B) = P(A) + P(B) - P(AB) = $1 - e^{-(\lambda A + \lambda B)\,t} = 1 - e^{-(0.1 + 0.3)\,1}$ = 1 – 0.6703 = 0.3297
  - 2.2. What is the probability that at least one event (either A or B) will occur within 5 years?
    - P(A) = $1 - e^{-(\lambda A)t} = 1 - e^{-(0.1)5}$ = 1 – 0.6065 = 0.3935
    - P(B) = $1 - e^{-(\lambda B)t} = 1 - e^{-(0.3)5}$ = 1 – 0.2231 = 0.7769
    - P(A + B)=P(A)+P(B)-P(AB)=0.3935+0.7769–[(0.3935)(0.7769)]=1.1703–0.3057=0.8647
    - Or P(A + B) = P(A) + P(B) - P(AB) = $1 - e^{-(\lambda A + \lambda B)\,t} = 1 - e^{-(0.1 + 0.3)\,5}$ = 1 – 0.1353 = 8.647E-1

Probability and Frequency Questions

- **3. An experiment has a probability of 0.2 of producing a failure.**
  - **3.1. What is the probability of observing exactly one failure if the experiment is repeated 4 times?**
    - **P[exactly 1 failure in 4 trials | 0.2] =**

$$= \frac{4!}{1!(4-1)!}0.2^1(1-0.2)^{4-1} = \frac{4!}{1!3!}0.2^1 0.8^3 = (4)(0.2)(0.512) = 0.4096$$

  - **3.2. What is the probability of observing at least one failure if the experiment is repeated 4 times?**
    - **P[at least 1 failure in 4 trials | 0.2] =**
    - **P[1] + P[2] + P[3] + P[4] = 0.4096 + 0.1536 + 0.0256 + 0.0016 = 0.5904**
    - **Or**
    - **1 – P[0 failures in 4 trials | 0.2] = 1 – 0.4096 = 0.5904**

$$= 1 - \frac{4!}{0!(4-0)!}0.2^0(1-0.2)^{4-0} = 1 - \frac{4!}{0!4!}0.2^0 0.8^4 = 1 - (1)(1)(0.4096) = 0.5904$$

Probability and Frequency Questions

- 3.3. If the experiment is repeated 4 times, what is the probability of observing the following number of failures;
  - 0
    - P[0] = 0.4096
  - 0 or 1
    - P[0] + P[1] = 0.4096 + 0.4096 = 0.8192
  - 0 or 1 or 2
    - P[0] + P[1] + P[2] = 0.4096 + 0.4096 + 0.1536 = 0.9728
  - 0 or 1 or 2 or 3
    - P[0] + P[1] + P[2] + P[3] = 0.4096 + 0.4096 + 0.1536 + 0.0256 = 0.9984
  - 0 or 1 or 2 or 3 or 4
    - P[0] + P[1] + P[2] + P[3] + P[4] = 0.4096 + 0.4096 + 0.1536 + 0.0256 + 0.0016 = 1.0000

Probability and Frequency Questions

- **P{exactly 0 failures in 4 trials | 0.2 } =**
- **=** $\frac{4!}{0!(4-0)!}$ **$0.2^{0}(1-0.2)^{4}$ = (1)(1)(0.4096) = 0.4096**
- **P{exactly 1 failure in 4 trials | 0.2 } =**
- **=** $\frac{4!}{1!(4-1)!}$ **$0.2^{1}(1-0.2)^{3}$ = (4)(0.2)(0.512) = 0.4096**
- **P{exactly 2 failures in 4 trials | 0.2 } =**
- **=** $\frac{4!}{2!(4-2)!}$ **$0.2^{2}(1-0.2)^{2}$ = (6)(0.04)(0.64) = 0.1536**
- **P{exactly 3 failure in 4 trials | 0.2 } =**
- **=** $\frac{4!}{3!(4-3)!}$ **$0.2^{3}(1-0.2)^{1}$ = (4)(0.008)(0.8) = 0.0256**
- **P{exactly 4 failures in 4 trials | 0.2 } =**
- **=** $\frac{4!}{4!(4-4)!}$ **$0.2^{4}(1-0.2)^{0}$ = (1)(0.0016)(1) = 0.0016**

Probability and Frequency Questions

- **4. An experiment has a probability of 0.1 of producing a failure.**
  - **4.1. What is the probability of observing exactly one failure if the experiment is repeated 4 times?**
    - **P[exactly 1 failure in 4 trials | 0.1] =**

$$= \frac{4!}{1!(4-1)!} 0.1^1 (1-0.1)^{4-1} = \frac{4!}{1!3!} 0.1^1 0.9^3 = (4)(0.1)(0.7290) = 0.2916$$

  - **4.2. What is the probability of observing at least one failure if the experiment is repeated 4 times?**
    - **P[at least 1 failure in 4 trials | 0.1] =**
    - **P[1] + P[2] + P[3] + P[4] = 0.2916 + 0.0486 + 0.0036 + 0.0001 = 0.3439**
    - **Or**
    - **1 – P[0 failures in 4 trials | 0.1] = 1 – 0.6561 = 0.3439**

$$= 1 - \frac{4!}{0!(4-0)!} 0.1^0 (1-0.1)^{4-0} = 1 - \frac{4!}{0!4!} 0.1^0 0.9^4 = 1 - (1)(1)(0.6561) = 0.3439$$

Probability and Frequency Questions

- **4.3. If the experiment is repeated 4 times, what is the probability of observing the following number of failures;**
  - **0**
    - **P[0] = 0.6561 = 0.6561**
  - **0 or 1**
    - **P[0] + P[1] = 0.6561 + 0.2916 = 0.9477**
  - **0 or 1 or 2**
    - **P[0] + P[1] + P[2] = 0.6561 + 0.2916 + 0.0486 = 0.9963**
  - **0 or 1 or 2 or 3**
    - **P[0] + P[1] + P[2] + P[3] = 0.6561 + 0.2916 + 0.0486 + 0.0036 = 0.9999**
  - **0 or 1 or 2 or 3 or 4**
    - **P[0] + P[1] + P[2] + P[3] + P[4] = 0.6561 + 0.2916 + 0.0486 + 0.0036 + 0.0001 = 1.0000**

Probability and Frequency Questions

- P{exactly 0 failures in 4 trials | 0.1 } =
- $= \frac{4!}{0!(4-0)!} \; 0.1^{0}(1-0.1)^{4} = (1)(1)(0.6561) = 0.6561$
- P{exactly 1 failure in 4 trials | 0.1 } =
- $= \frac{4!}{1!(4-1)!} \; 0.1^{1}(1-0.1)^{3} = (4)(0.1)(0.729) = 0.2916$
- P{exactly 2 failures in 4 trials | 0.1 } =
- $= \frac{4!}{2!(4-2)!} \; 0.1^{2}(1-0.1)^{2} = (6)(0.01)(0.81) = 0.0486$
- P{exactly 3 failure in 4 trials | 0.1 } =
- $= \frac{4!}{3!(4-3)!} \; 0.1^{3}(1-0.1)^{1} = (4)(0.001)(0.9) = 0.0036$
- P{exactly 4 failures in 4 trials | 0.1 } =
- $= \frac{4!}{4!(4-4)!} \; 0.1^{4}(1-0.1)^{0} = (1)(0.0001)(1) = 0.0001$