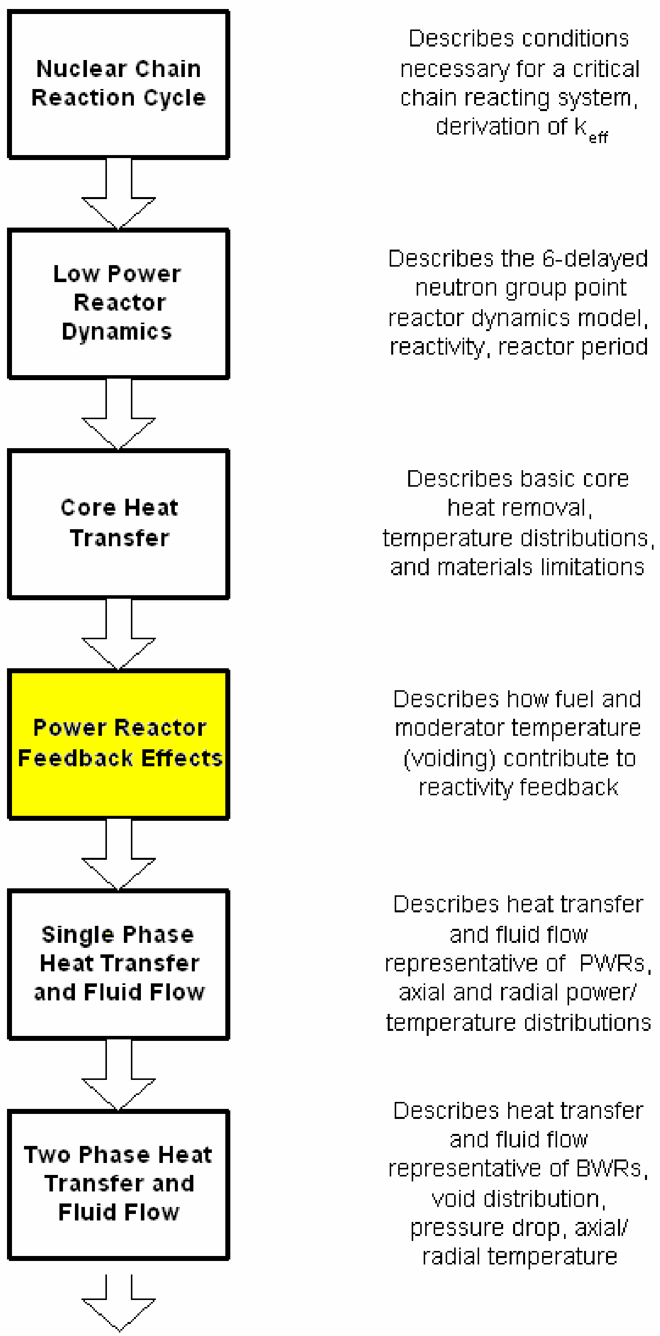
# Fundamentals of Nuclear Engineering

## Module 10: *Power Reactor Feedback Effects*

Dr. John H. Bickel



| | |
|---|---|
| **Nuclear Chain Reaction Cycle** | Describes conditions necessary for a critical chain reacting system, derivation of $k_{eff}$ |
| **Low Power Reactor Dynamics** | Describes the 6-delayed neutron group point reactor dynamics model, reactivity, reactor period |
| **Core Heat Transfer** | Describes basic core heat removal, temperature distributions, and materials limitations |
| **Power Reactor Feedback Effects** | Describes how fuel and moderator temperature (voiding) contribute to reactivity feedback |
| **Single Phase Heat Transfer and Fluid Flow** | Describes heat transfer and fluid flow representative of PWRs, axial and radial power/ temperature distributions |
| **Two Phase Heat Transfer and Fluid Flow** | Describes heat transfer and fluid flow representative of BWRs, void distribution, pressure drop, axial/ radial temperature |

# *Objectives*

Previous lectures described reactor criticality and effects of heat generation. This lecture will:

1. Describe effect of power generation on: $k_{eff}$, $\rho$
2. Describe net reactivity feedback model
3. Describe origin and magnitude of fuel (Doppler) temperature coefficient of Reactivity
4. Describe origin and magnitude of moderator temperature coefficients of reactivity
5. Describe origin and magnitude of void coefficient of reactivity
6. Demonstrate concept of power defect in reactivity and how reactor power is regulated

# Power Generation Impact on $k_{eff}$, $\rho$

- Reactor criticality is a delicate balance between:
- Neutron *birth rate* vs. *loss rate* from absorption and leakage
- To remain at constant neutron population level:

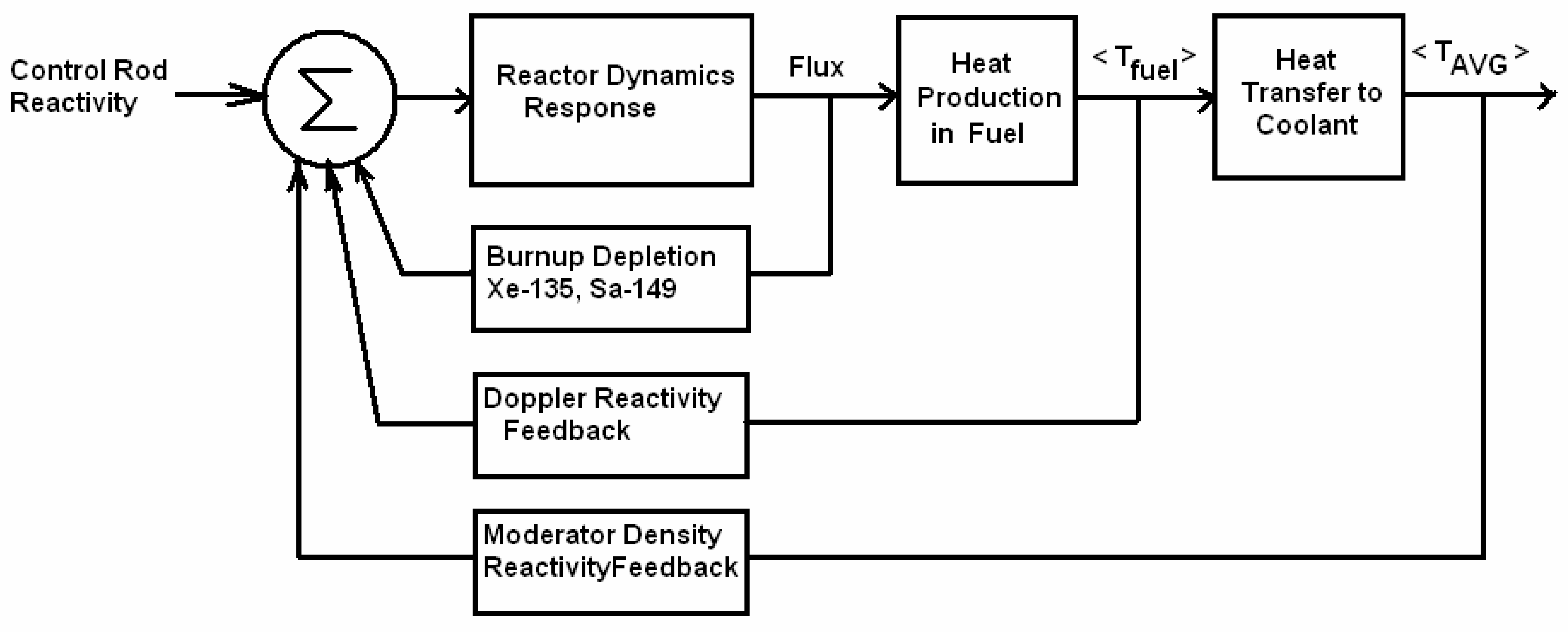
$$k_{eff} = \frac{k_{\infty}}{(1+L_f{}^2 B_f{}^2)(1+L_{th}{}^2 B_{th}{}^2)} = 1$$

- where: $k_{\infty} = \eta \varepsilon p f$

- Heat - makes materials expand (lowers assumed density)
- Heat - increases void content in water (boiling)
- Heat - increases neutron resonance absorption reactions
- Flux - consumes $U^{235}$ while converting $U^{238}$ to other fissionable isotopes (e.g.: $Pu^{239}$, $Pu^{241}$, etc.)
- Flux – produces strong neutron absorbing fission products such as: $Xe^{135}$, $Sm^{149}$

# *Net Reactivity Feedback*

# *Overall Reactivity Feedback Paths:*



# *Fuel Temperature Feedback*

- Previous lectures described relationship between effective fuel temperature: $< T_f >$, linear power density: $q$:

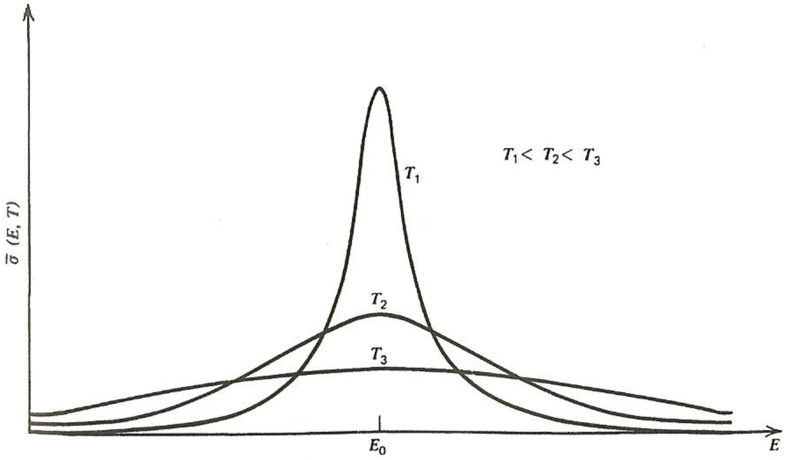
$$< T_f > = T_{coolant} + \frac{q}{2\pi}\left( \frac{1}{8k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} + \frac{1}{R_c h_{film}} \right)$$

- Lumped parameter model indicated that fuel temperature lagged behind linear power density by: $\tau_f \sim 3 - 5$ *seconds*

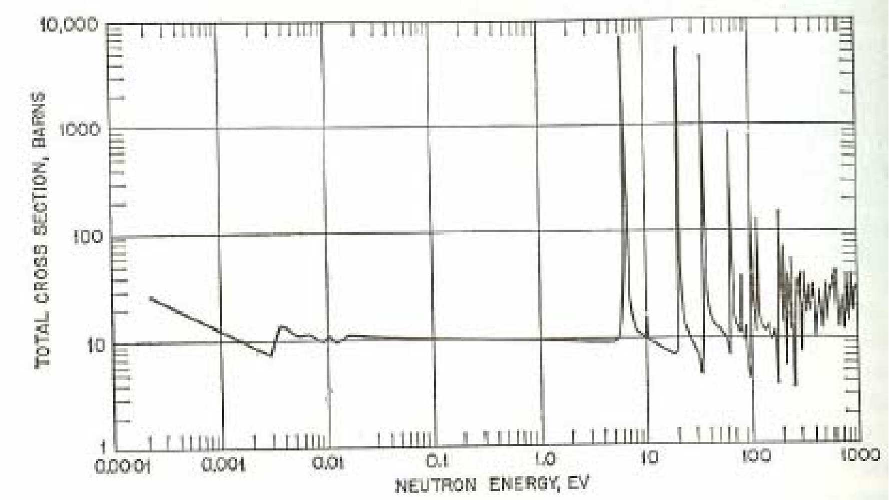
- Lumped parameter model indicated that fuel to coolant heat flux lags behind linear power density even more

# *Increased Fuel Temperature Causes Doppler Broadening*

- Recall definition of multiplication factor: $k_\infty = \eta\varepsilon pf$
- Resonance escape probability: $p$ is directly impacted by thermal broadening of $U^{238}$ absorption resonances
- Higher power → higher fuel temperature → higher absorption
- Doppler coefficient of reactivity: $(\partial\rho/\partial T_f)$ is obtained by performing many core physics calculations for spectrum of $T_f$

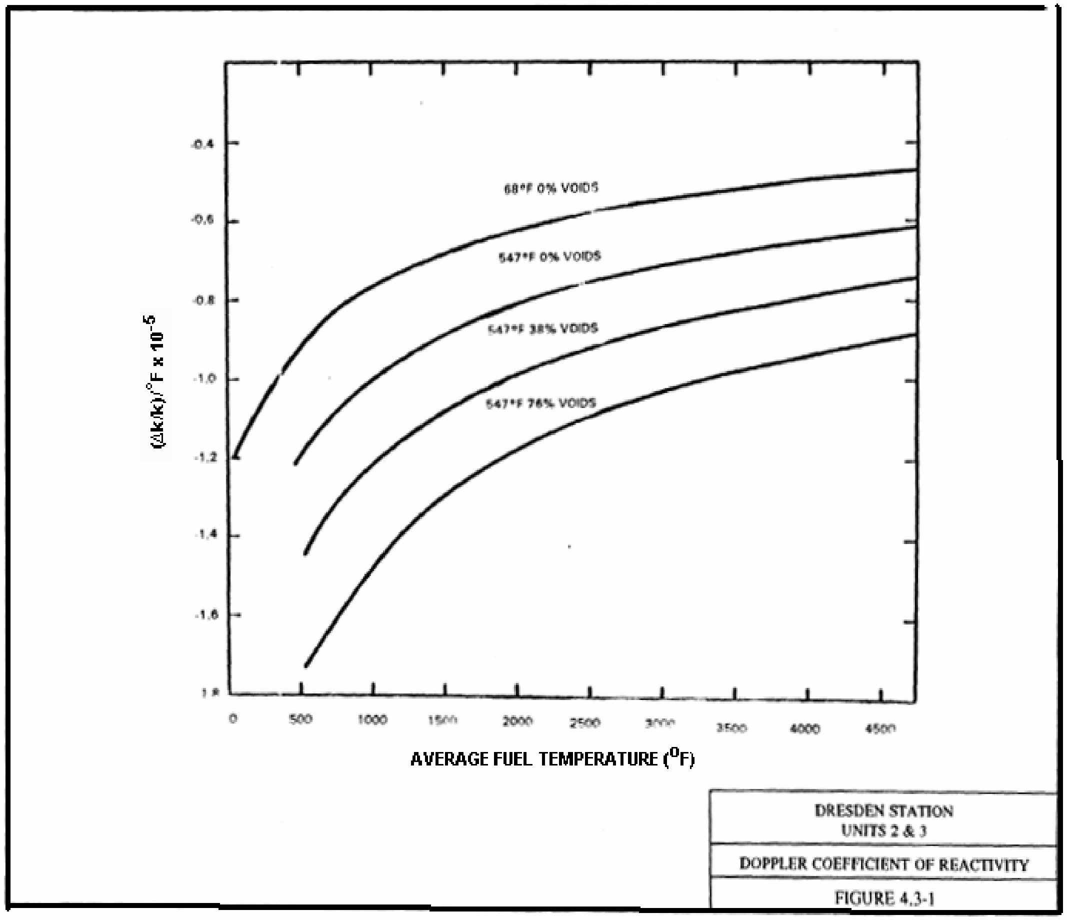


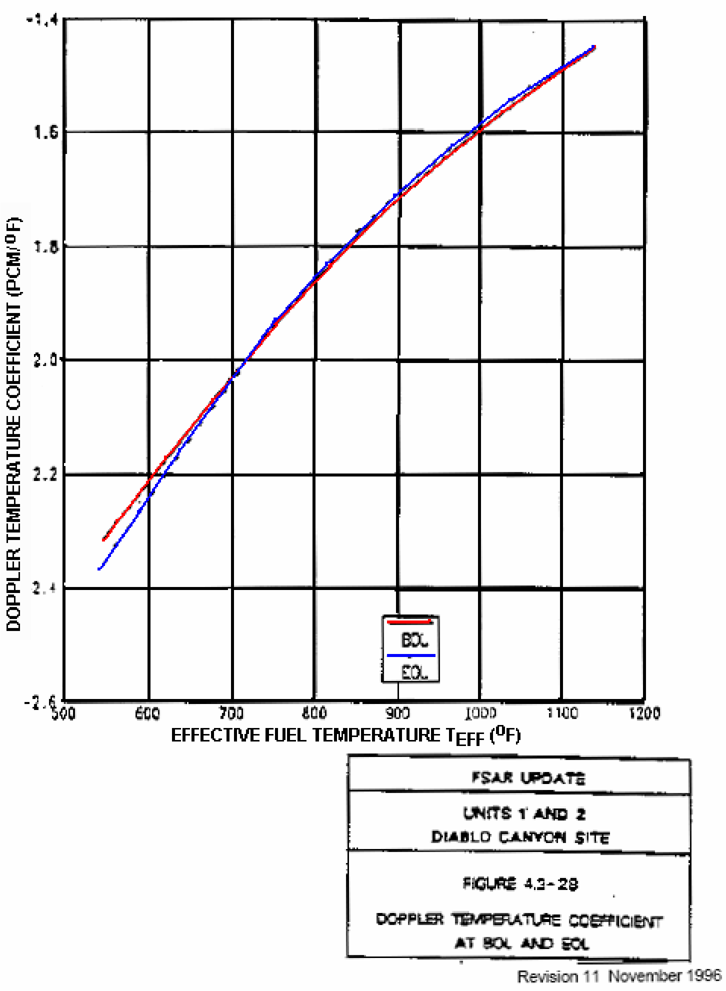
**Doppler-broadening of a resonance with increasing temperature.**



Total cross section of uranium-238 as function of neutron energy

# *Example Doppler Coefficients of Reactivity*





- Doppler coefficient of reactivity $(\partial\rho/\partial T_f)$ is *always negative* (-)

# Moderator Density Feedback

- Increase power:
  - → increase moderator temperature
  - → lower density
- Lower density (voiding in BWRs) impacts $k_{eff} = \eta \varepsilon p f P_f P_{th}$ as:
- Fast, thermal neutron leakage increase →$P_f P_{th}$ decrease
- Decrease in moderator density:
  - → decrease rate of neutron thermalization to below resonance region
  - → relative increase in resonance absorption
  - → lowers $p$
- Moderator temperature coefficient (or MTC): $\partial\rho/\partial T_{AVG}$ is obtained by performing core physics calculations at spectrum of coolant densities and soluble Boron concentrations
- In PWR with all fresh fuel compensated adding soluble Boron *MTC can be positive* due to temperature reducing [$B^{10}$]
- In specific case of BWR: core physics calculations performed at spectrum of void fractions ($\alpha$) to yield: $\partial\rho/\partial\alpha$

# *Example Moderator Coefficients of Reactivity*

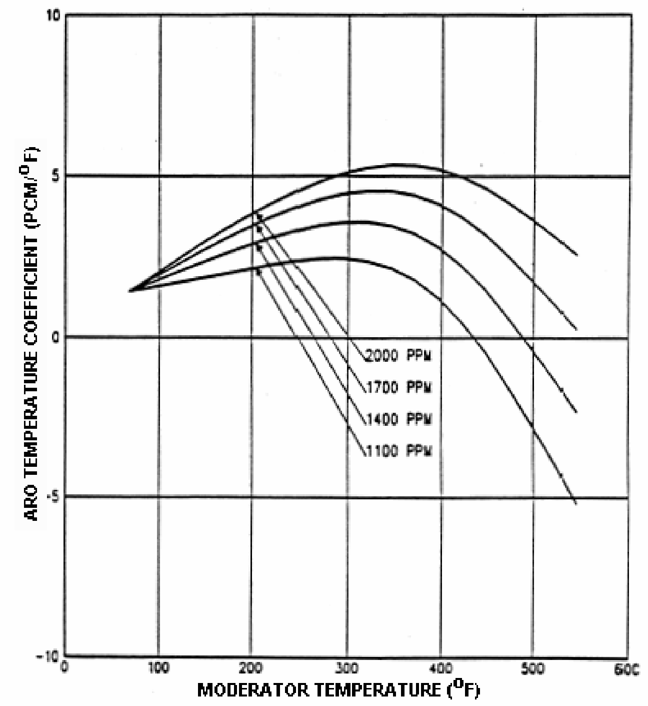


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FIGURE 4.3-31

MODERATOR TEMPERATURE COEFFICIENT
AT BOL, NO RODS

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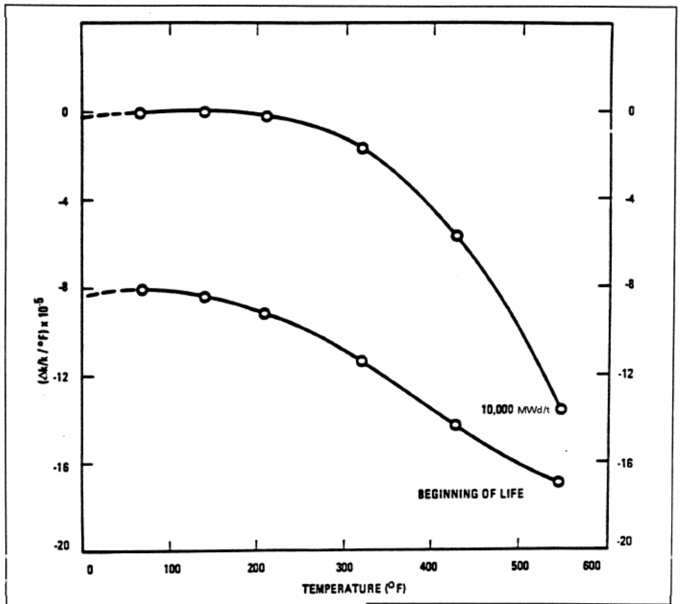

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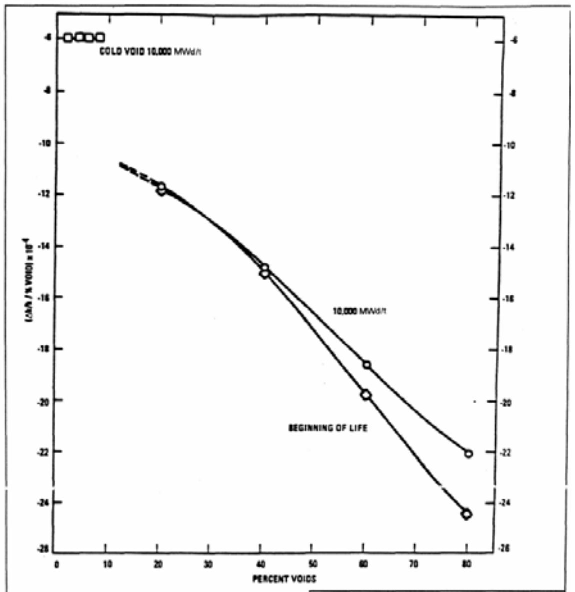
MODERATOR TEMPERATURE COEFFICIENT
OF REACTIVITY

FIGURE 4.3-6



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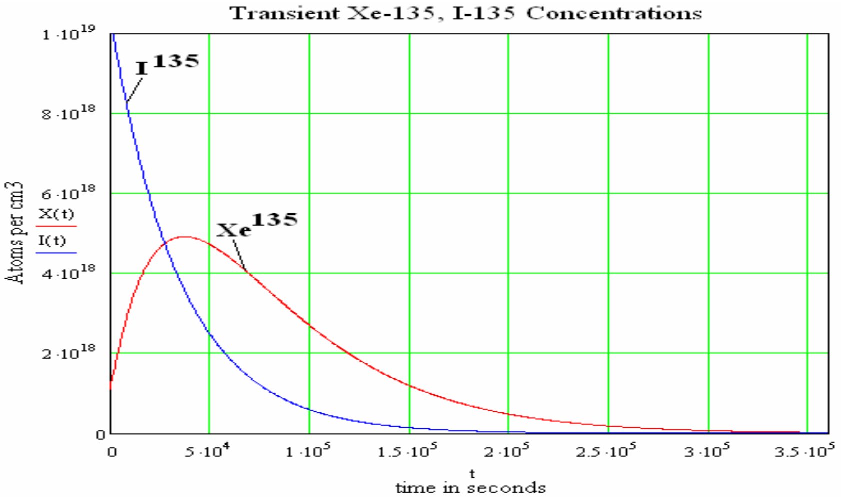
MODERATOR VOID COEFFICIENT OF
REACTIVITY

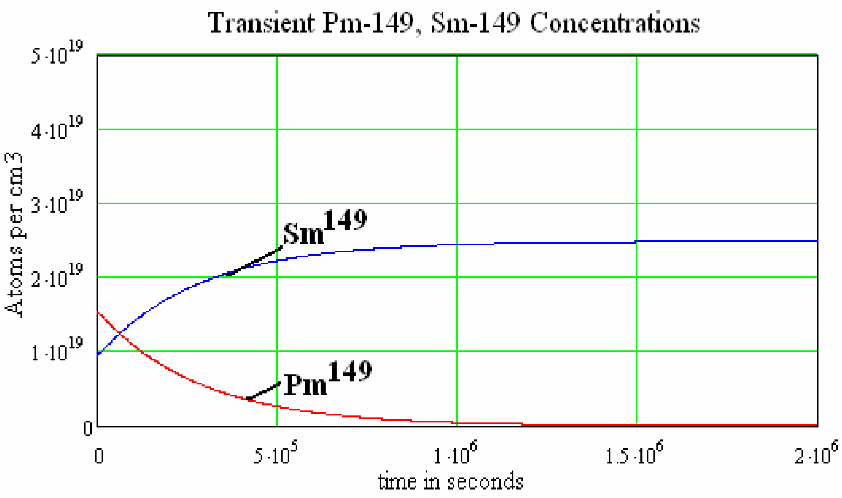
FIGURE 4.3-7

- Moderator coefficient of reactivity in PWR can be positive.

# $Xe^{135}$, $Sm^{149}$ Related Feedback

- Recall previously:
- If $\varphi_{th} \geq 10^{14}/cm^2\ sec$ - which is typical of power reactors
- $Xe^{135}$: peaks ~11.6 hr, then decays.
- $Xe^{135}$ capture competes with fission for neutrons
- $Sm^{149}$ maximizes at ~75hrs capture exceeds fission
- $Sm^{149}$ competes with fission for neutrons





# $Xe^{135}$, $Sm^{149}$ *Related Feedback:*

- Reactors are designed with capability to start-up and *override* certain level of transient $Xe^{135}$, $Sm^{149}$ poisoning

- Referred to as *Xenon Override* capability

- *Xenon Override* capability does not imply ability to override at time of peak Xenon concentration

- Under certain load following maneuvers using control rods it is possible to induce Xenon oscillations

- Xenon oscillations are an operational concern (not safety) that cause oscillating flux tilts with 15-30 hour period.

# *Reactivity Feedback Impact on Reactor Dynamics*

# *Revisiting Point Reactor Dynamics*

- Point reactor dynamics model previously described for zero-power case:

$$\frac{dN(t)}{dt} = \frac{(\rho(t)-\beta)}{\Lambda} N(t) + \sum_{i=1}^{6} \lambda_i c_i(t)$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i c_i(t)$$

- Model assumed an external reactivity change: $\Delta\rho(t)$
- Model can be solved for certain simplified cases
- To address reactivity feedback effects it is necessary to incorporate *additional simultaneous equations*

# *Additional Equations to Dynamics Model:*

$$\delta\rho_{NET}(t) = \rho(t) + \frac{\partial\rho}{\partial T_f}[T_f(t) - T_f(0)] + \frac{\partial\rho}{\partial T_{TAV}}[T_{AVG}(t) - T_{AVG}(0)]$$

$$\frac{dP(t)}{dt} = \frac{(\delta\rho_{NET}(t) - \beta)}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t)$$

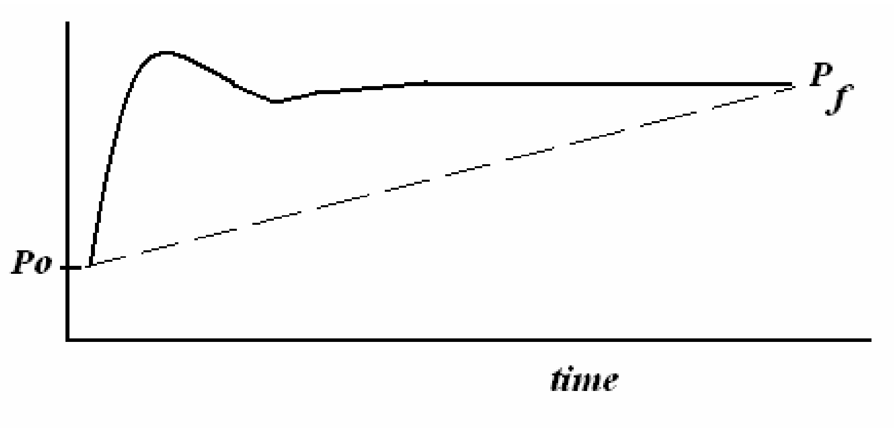
$$\tau_f \frac{dT_f(t)}{dt} = P(t) - \frac{T_f(t) - T_{AVG}(t)}{R_{f-c}}$$

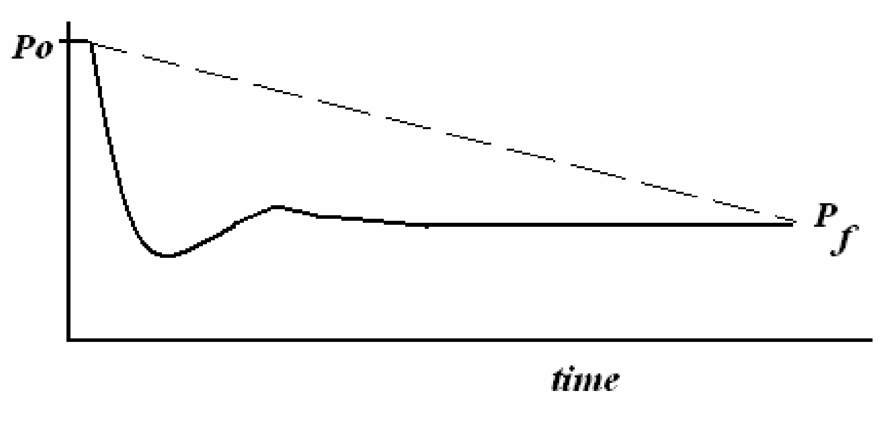
..................

- One quickly sees need for *reactor systems code* ?
- Good understanding of *feedback coefficients* allows good picture of actual reactor performance

# *Insights from Nonlinear Systems Theory:*

- Power reactor: *highly damped system* with negative feedback that seeks relaxation of $\rho$ back to steady state.
- Small increase in reactivity while at $Po$ → power initially rises $\Delta P$ → fuel, moderator temperature rise → negative Doppler, moderator density feedback counteract change
- New power level $P_f$ reached where: $\rho = 0$
- Reverse of process (decrease in $\rho$) behaves similarly





# *Systems Considerations:*

- To really understand reactor dynamics:
- Focus on system changes needed to restore: $\rho \rightarrow 0$

Doppler reactivity change: $\Delta\rho_D = (\partial\rho/\partial T_f)\Delta T_f$

Moderator reactivity change: $\Delta\rho_M = (\partial\rho/\partial T_{AVG})\Delta T_{AVG}$

Void (BWR) reactivity change: $\Delta\rho_V = (\partial\rho/\partial\alpha)\Delta\alpha$

Initial state: $P_o(T_f, T_{AVG}, \alpha),\ \rho = 0$

Perturbation: $\Delta\rho$

Final state: $P_F(T_f + \Delta T_f, T_{AVG} + \Delta T_{AVG}, \alpha + \Delta\alpha),\quad \rho = 0$

Final reactivity balance:

$$\rho = 0 = \Delta\rho + (\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG} + (\partial\rho/\partial\alpha)\Delta\alpha$$

- This walks us into concept of *Power Defect in Reactivity*

# *Power Defect in Reactivity*

# *Power Defect in Reactivity*

Power defect in reactivity is:

Net difference in reactivity between two power operating states characterized by core temperature and moderator density (or void fraction)

Power defect in reactivity is a static reactivity balance between two operating states which integrates impacts of Doppler and moderator density reactivity feedback

Start with simplified reactivity balance:

PWR: $\rho = 0 = \Delta\rho + (\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG}$

BWR: $\rho = 0 = \Delta\rho + (\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG} + (\partial\rho/\partial\alpha)\Delta\alpha$

# Doppler Power Defect in Reactivity

- Doppler reactivity defect is independent of reactor type
- Recall our simple relation between $T_f$ and $q$:

$$<T_f> = T_{coolant} + \frac{q}{2\pi}\left(\frac{1}{8k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} + \frac{1}{R_c h_{film}}\right)$$

$$<T_f> = T_{coolant} + q\kappa_{fc}$$

$$where : \kappa_{fc} = \frac{1}{2\pi}\left(\frac{1}{8k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} + \frac{1}{R_c h_{film}}\right)$$

- *NOTE:* Vendor's $T_f$ vs. $q$ models not typically linear in $q$. Example CE: $T_f = 2793[1 - 0.7995\ exp(-0.1062\ q)]$

# *Doppler Power Defect in Reactivity*

- First do simple conversion between $P$ and $q$:
- Define: $q = C_o P$ -where $C_o$ converts %Power to average kW/ft, or (Watts/cm)
- Doppler reactivity defect can be calculated:

$$\Delta\rho_D = \int_{P_0}^{P_F} \frac{\partial\rho}{\partial T_f}\frac{dT_f}{dP}\,dP \approx \frac{\partial\rho}{\partial T_f} C_o \kappa_{fc} (P_F - P_0)$$

- Values of Doppler reactivity defect can be found in many FSARs
- *NOTE:* Make sure *(+/-) reference starting point is known*. Typically it is reported from hot, zero power.

# *Example Doppler Power Defect*

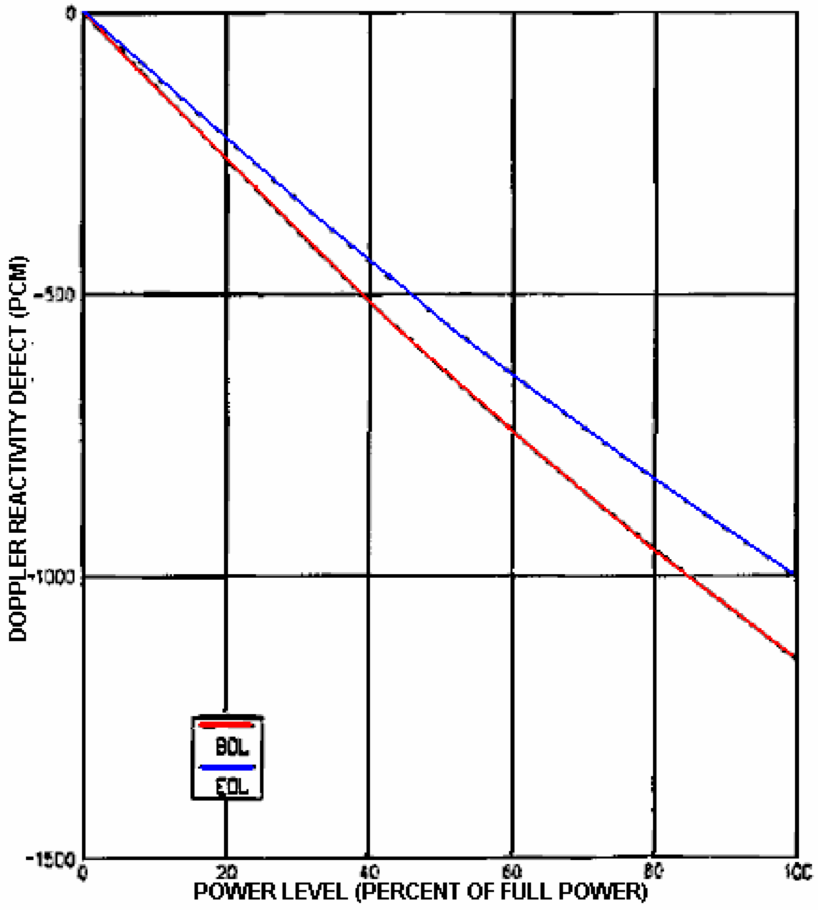


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FIGURE 4.3-30

DOPPLER ONLY POWER DEFECT
AT BOL AND EOL

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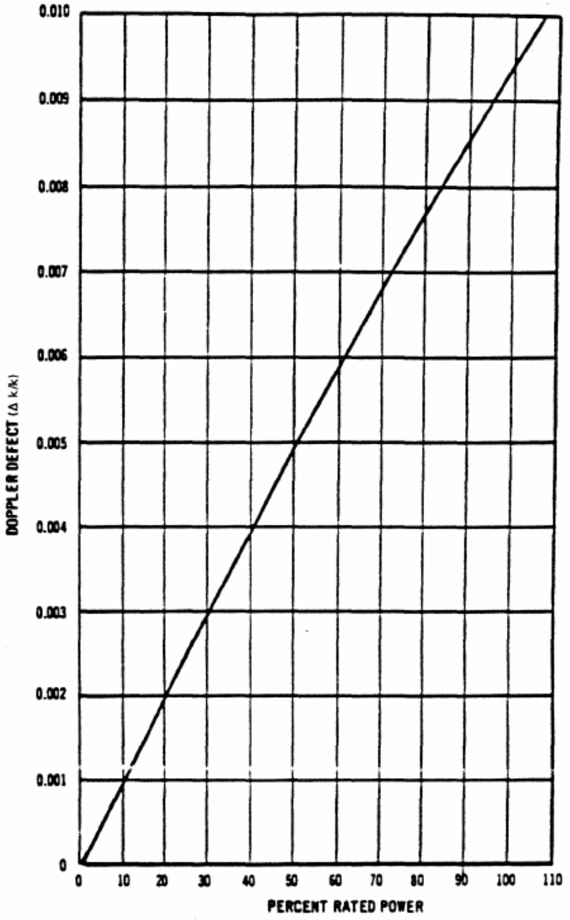

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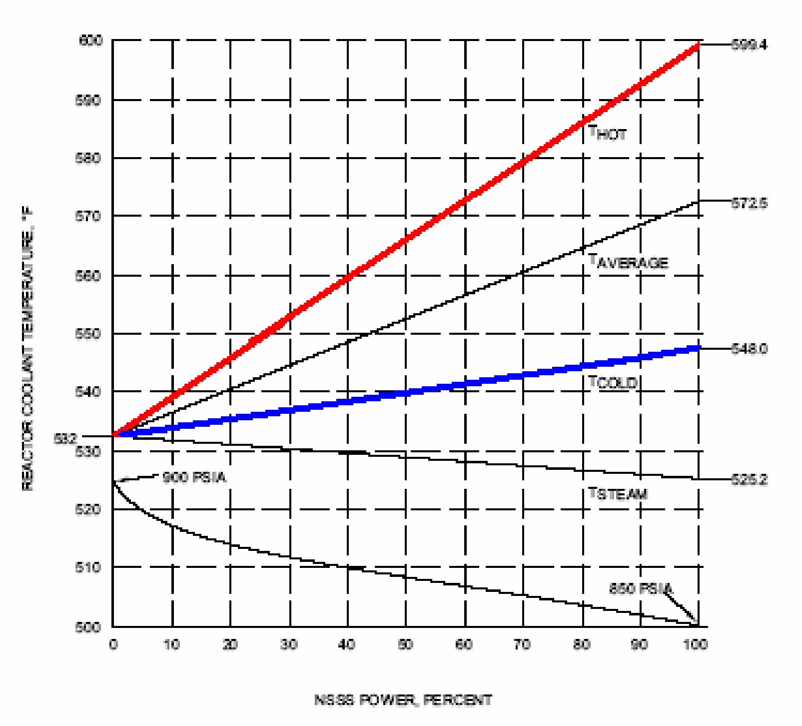
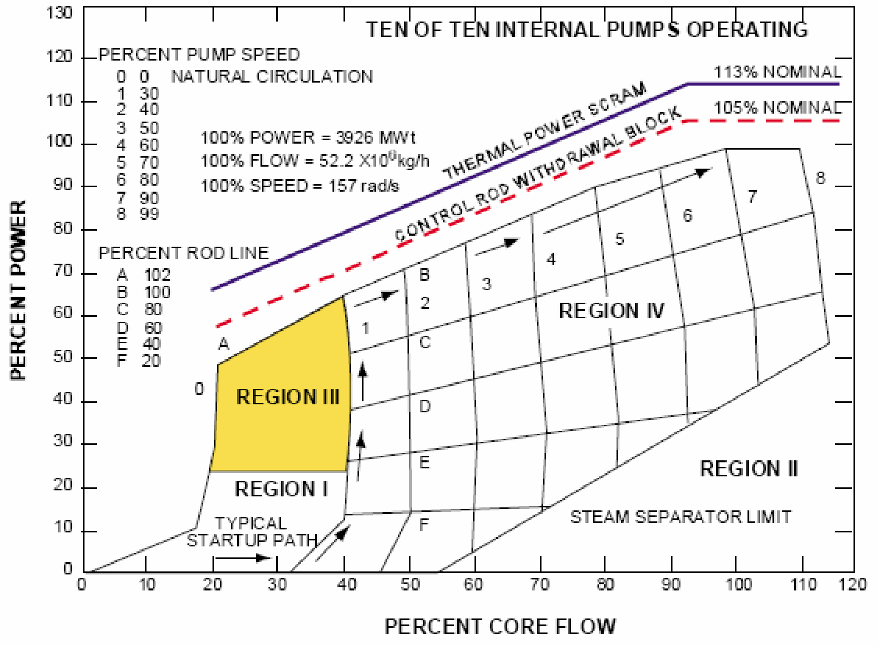
CORE AVERAGE DOPPLER DEFECT
VS. POWER LEVEL

FIGURE 4.3-3

# *PWR vs. BWR Power Defect in Reactivity*

- Essential control scheme for reactivity vs. power is different between PWR and BWR designs
- PWR regulates power based upon adjustment of control rods (or soluble Boron) and temperature (SG heat removal)
- BWR regulates power based upon adjustment of control rods and recirculation flow control (adjusts void content)





# *PWR Total Power Defect in Reactivity*

- Assuming *normal control system regulation* of coolant temperature:

$$\Delta\rho(P_o,P_F)=\int_{P_o}^{P_F}\left(\frac{\partial\rho}{\partial T_f}\frac{dT_f}{dP}+\frac{\partial\rho}{\partial T_{AVG}}\frac{dT_{AVG}}{dP}\right)dP$$

-where:

$P_0$ is initial power level, $P_F$ is final power level

$\partial\rho/\partial T_f$ is Doppler or fuel temperature coefficient (FTC) of reactivity

$\partial\rho/\partial T_{AVG}$ is moderator temperature coefficient (MTC) of reactivity

$dT_f/dP$ is local derivative of $T_f$ relative to power

$dT_{AVG}/dP$ slope of $T_{AVG}$ vs power program (typically dependent on steam generator design)

# *Example From Diablo Canyon FSAR:*

- Total power defect in reactivity is calculated:

$$\Delta\rho(0\%,100\%) = \int_{0\%}^{100\%} \left( \frac{\partial\rho}{\partial T_f}\frac{dT_f}{dP} + \frac{\partial\rho}{\partial T_{AVG}}\frac{dT_{AVG}}{dP} \right) dP$$

- At EOL conditions:
- Increasing power 0% → 100% requires: $\Delta\rho = +\,2250\ PCM$ or $(+0.0225\ \Delta k/k)$ to compensate for power defect in reactivity

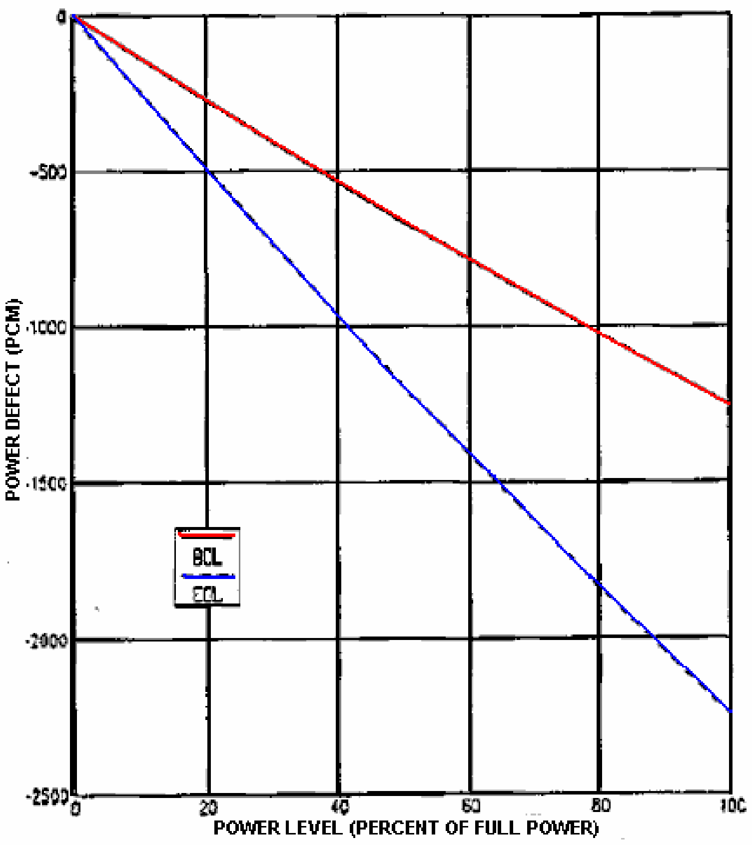


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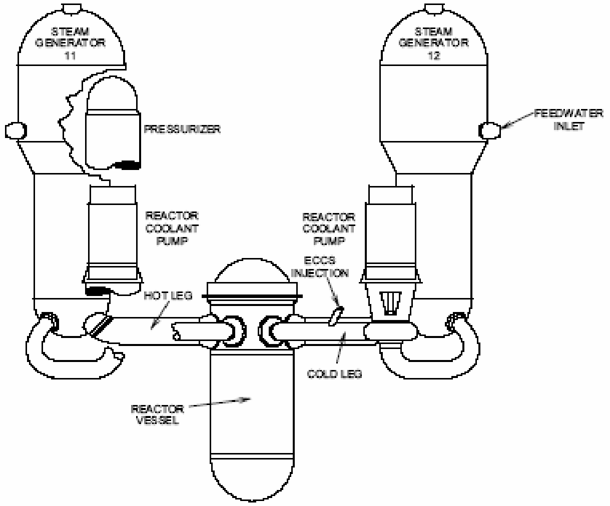
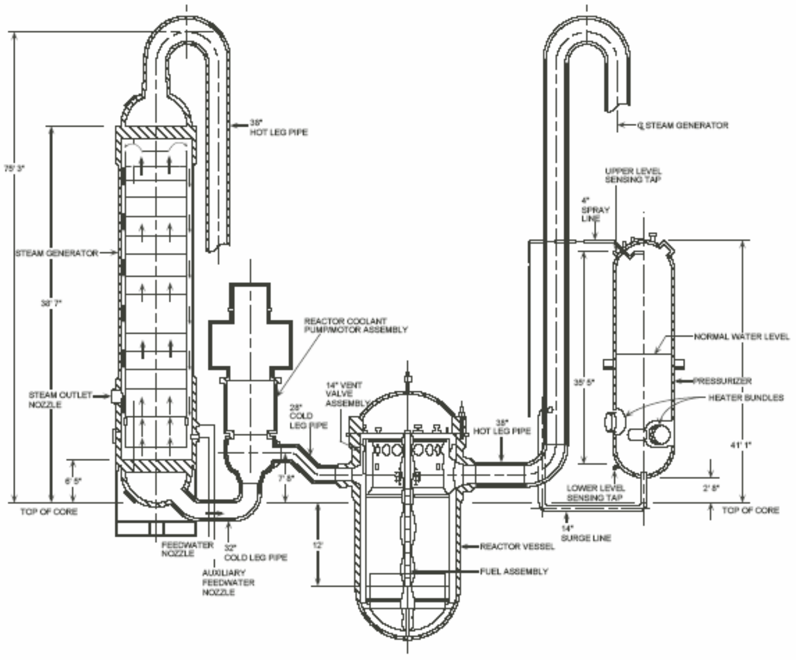
FIGURE 4.3-36

TOTAL POWER DEFECT AT
AT BOL AND EOL

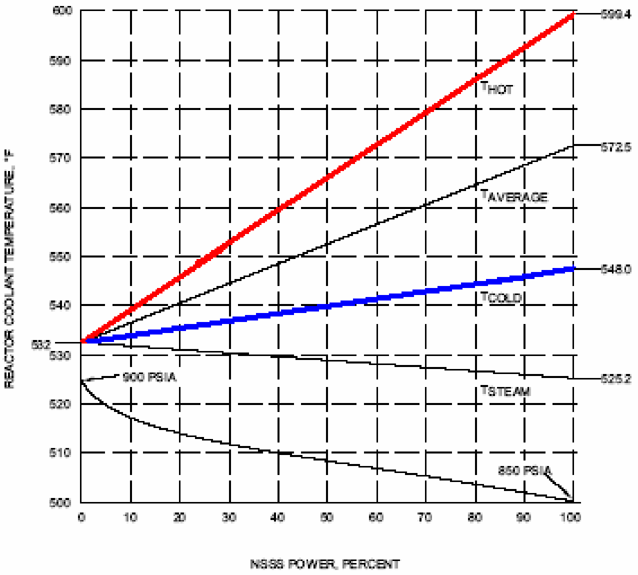
# Two Types of PWRs:

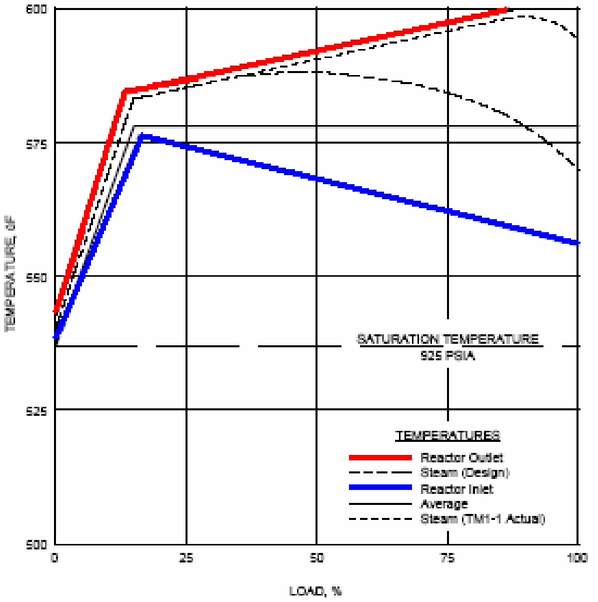


**CE 2-Loop Nuclear Steam Supply System**

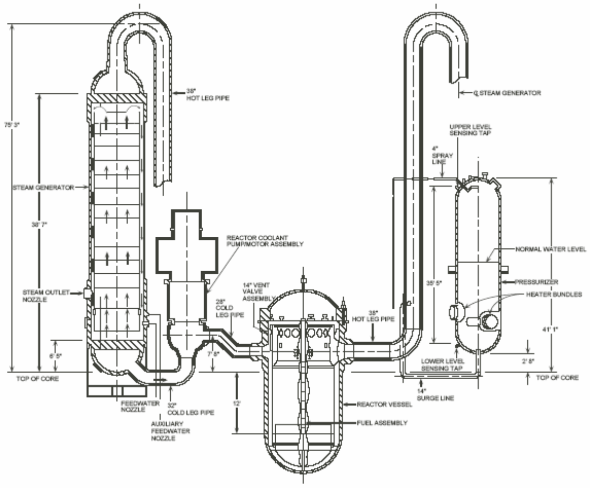


**B&W 2-Loop Nuclear Steam Supply System**

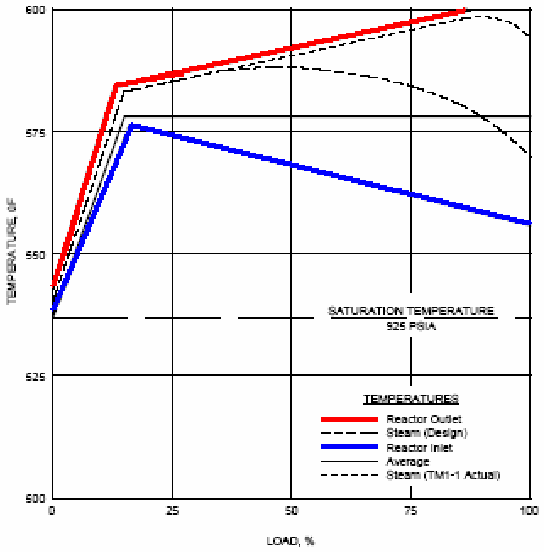




# *Power Defect in Reactivity for B&W NSSS:*



**B&W 2-Loop Nuclear Steam Supply System**



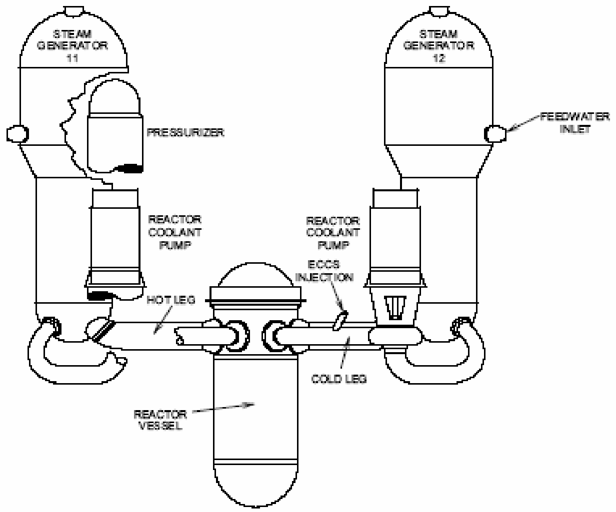
- Reactivity needed to raise power from 0% →100% ?
- $\Delta\rho_D \sim -0.01\ \Delta k/k$, and: $\partial\rho/\partial T_{AVG} = -2.5\text{x}10\text{-}4\ \Delta k/k{}^\circ F$
- Note: flat temperature *above 15% power*

$$\Delta\rho_D = \frac{\partial\rho}{\partial T_f} C_o \kappa_{fc}(100\%) + \int_{0\%}^{15\%} \frac{\partial\rho}{\partial T_{AVG}} \frac{dT_{AVG}}{dP} dP$$

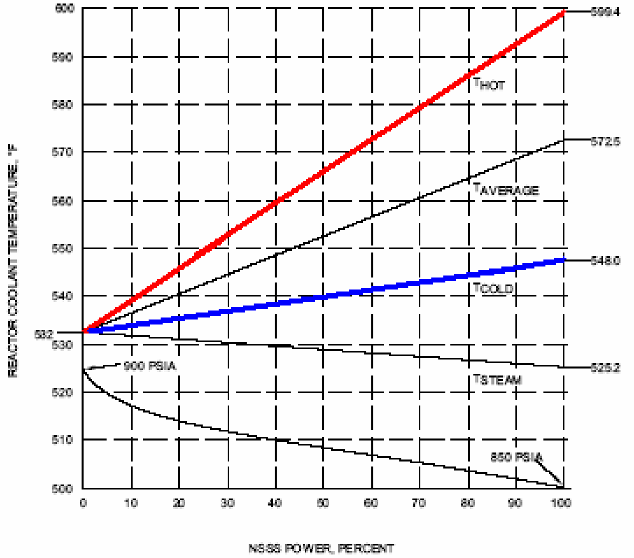
$$= \frac{\partial\rho}{\partial T_f} C_o \kappa_{fc}(100\%) + \frac{\partial\rho}{\partial T_{AVG}}(45^\circ F) = -0.01\Delta k/k - 0.0113\Delta k/k$$

$$= -0.0213\Delta k/k$$

# *Power Defect in Reactivity for W, CE:*



**CE 2-Loop Nuclear Steam Supply System**



- Reactivity needed to raise power from 0% →100% ?
- $\Delta\rho_D \sim -0.01\ \Delta k/k$, and: $\partial\rho/\partial T_{AVG} = -2.5\text{x}10\text{-}4\ \Delta k/k°F$
- $\Delta T_{AVG} = 40.5°F$

$$\rho_D = \frac{\partial\rho}{\partial T_f} C_o \kappa_{fc} (100\%) + \int_{0\%}^{100\%} \frac{\partial\rho}{\partial T_{AVG}} \frac{dT_{AVG}}{dP} dP$$

$$= \frac{\partial\rho}{\partial T_f} C_o \kappa_{fc} (100\%) + \frac{\partial\rho}{\partial T_{AVG}} (40.5°F) = -0.01\Delta k/k - 0.0101\Delta k/k$$

$$= -0.0201\Delta k/k$$

# *Criticality With All Control Rods Inserted?*

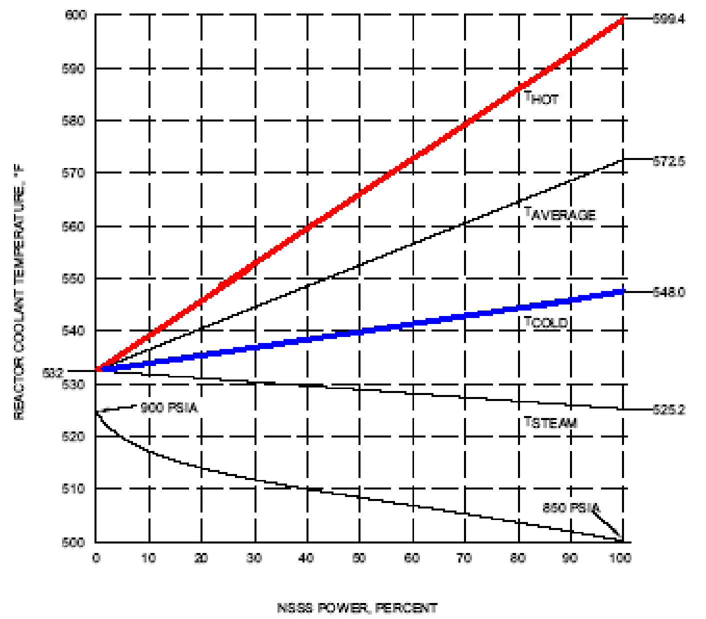
- From previous we found that at EOL conditions getting from 100% power to 0% power involved $\Delta\rho \sim -0.02\Delta k/k$
- This however results in *being critical* at ~0% power
- Many FSARs credit an additional 5% shutdown margin to assure getting reactor subcritical, $k_{eff} \sim 0.95$
- Are there any situations where this is insufficient?
- Power defect model presumes: $\Delta T_{AVG}$ vs. $\%Power$ relation
- Certain scenarios cause significant deviation in $\Delta T_{AVG}$
- Examples: steam line break, loss of feedwater heater
- Start with: $0 = \Delta\rho_{CR} + (\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG}$
- Assume constant reactivity coefficients:
- $\partial\rho/\partial T_f = \gamma_f$ and $\partial\rho/\partial T_{AVG} = \gamma_{AVG}$

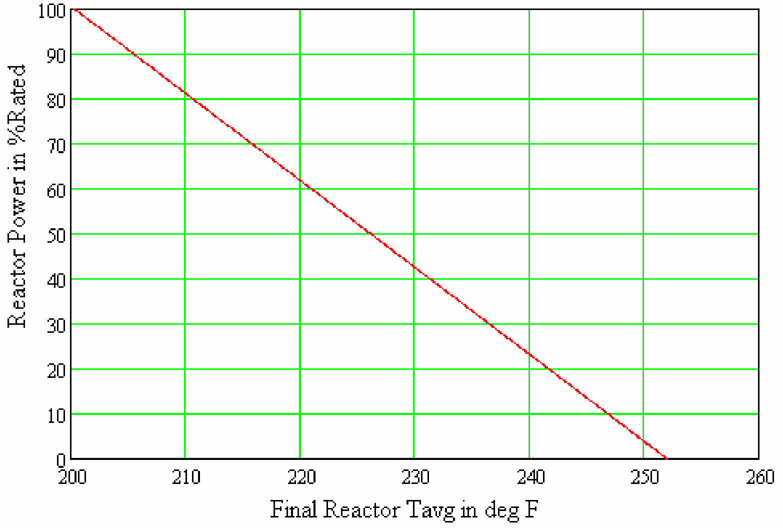
# *Criticality With All Control Rods Inserted?*

- Assume linear relationship between fuel temperature and power: $\Delta T_f = \kappa_{fc} \Delta P$
- $\kappa_{fc} = 12.92\ °F / \%Power$
- Rearranging power defect equation and solving for $\Delta P$ as a function of $T_{AVG}$ yields:

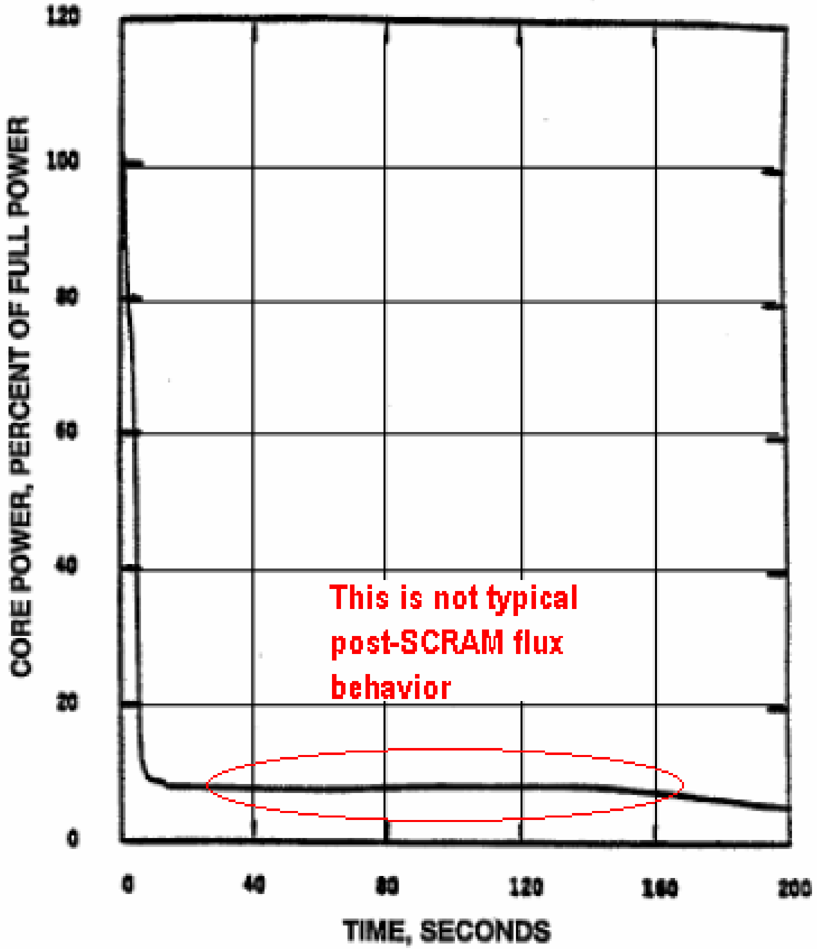
  $$\Delta P = [\Delta\rho_{CR} + \gamma_{AVG} \Delta T_{AVG}] / - \gamma_{AVG} \kappa_{fc}$$

- $\Delta T_{AVG} = T_{AVG} - 532°F$
- Results →





# *SONGS 2,3 Steam Line Break Re-criticality*



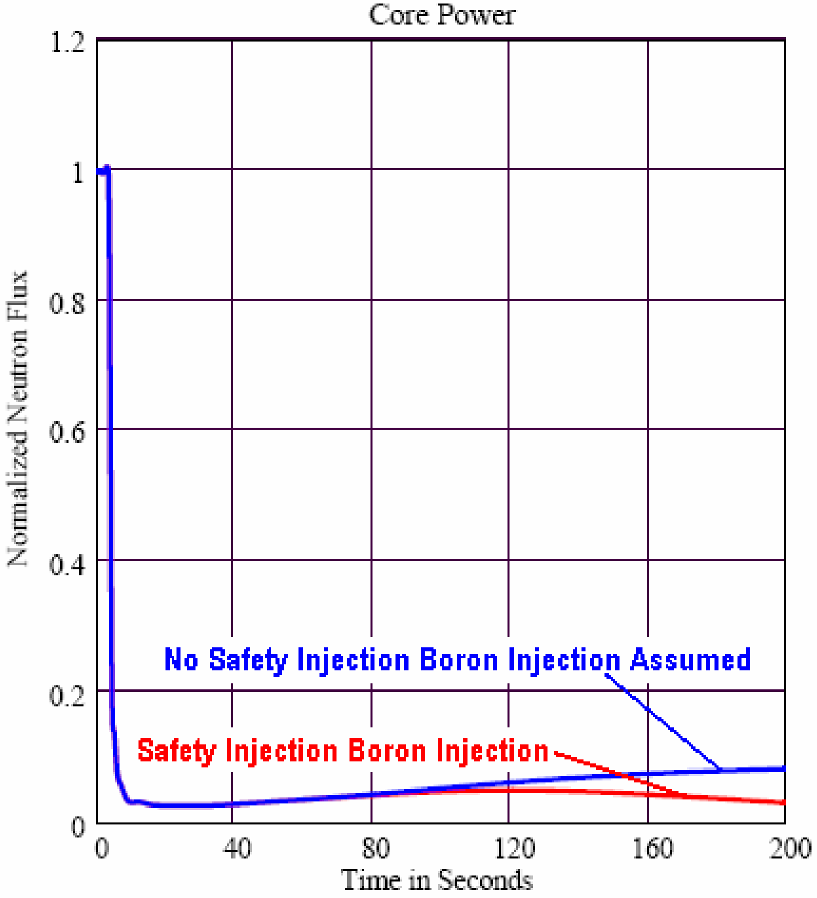
This is not typical post-SCRAM flux behavior

SAN ONOFRE NUCLEAR GENERATING STATION
Units 2 & 3
Updated Final Safety Analysis Report

FULL POWER STEAM LINE BREAK
WITH LOSS OF AC POWER
CORE POWER vs. TIME

Figure 15.1-40

**Comparison of Power Projection With SI vs.Without SI**



No Safety Injection Boron Injection Assumed

Safety Injection Boron Injection

# *PWR Response is More Sluggish at EOL?*

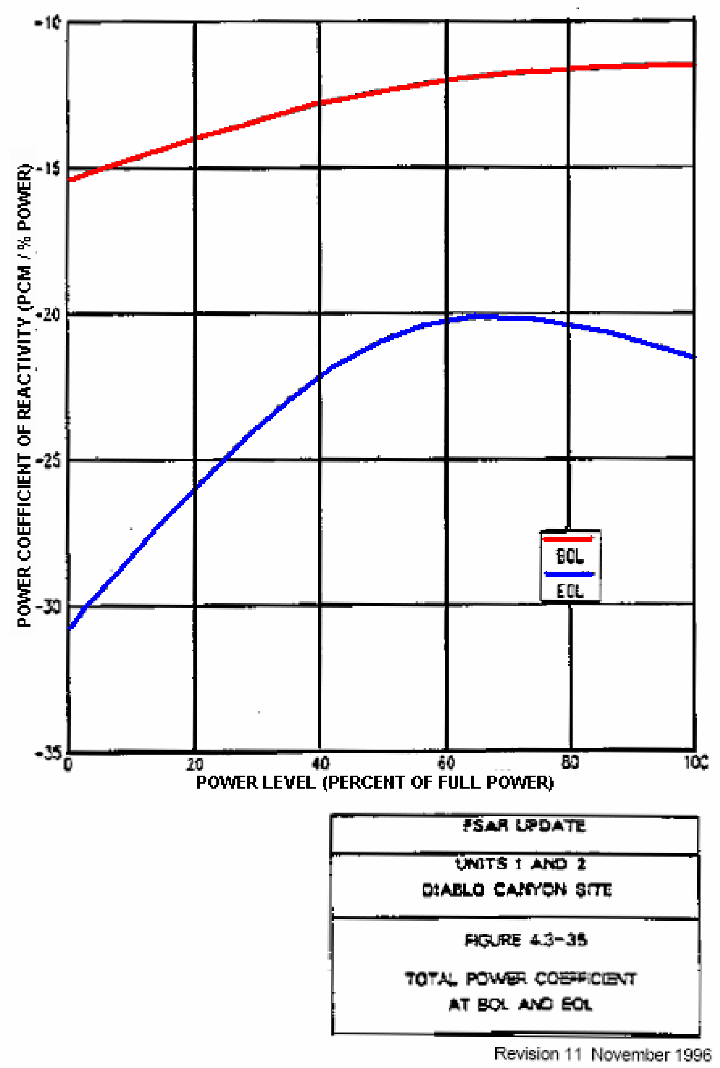
- Total power coefficient of reactivity is calculated:

$$\partial\rho/\partial P = (\partial\rho/\partial T_f)(dT_f/dP) + (\partial\rho/\partial T_{AVG})(dT_{AVG}/dP)$$

- Assume rate of reactivity insertion (control rods or Boron dilution) is fixed
- Rate of power change: $dP/dt$ is proportional to rate of reactivity change: $d\rho/dt$, as:

$$dP/dt = (d\rho/dt) / (\partial\rho/\partial P)$$

- Total power coefficient of reactivity is more negative at EOL, hence more sluggish power response !



FSAR UPDATE

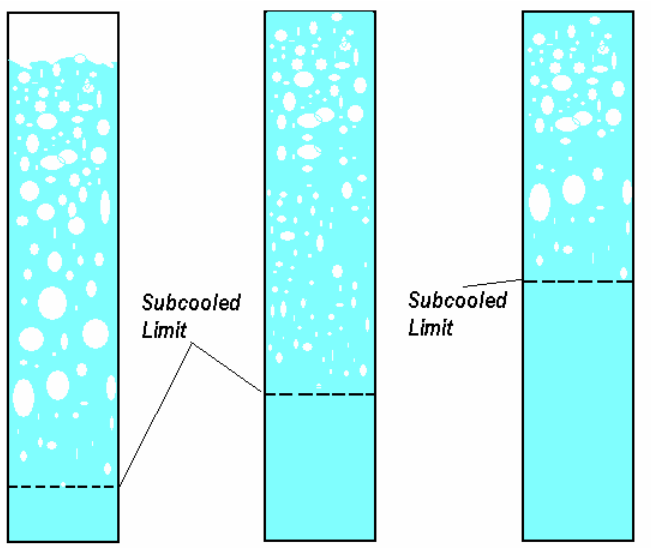
UNITS 1 AND 2
DIABLO CANYON SITE

FIGURE 4.3-35
TOTAL POWER COEFFICIENT
AT BOL AND EOL

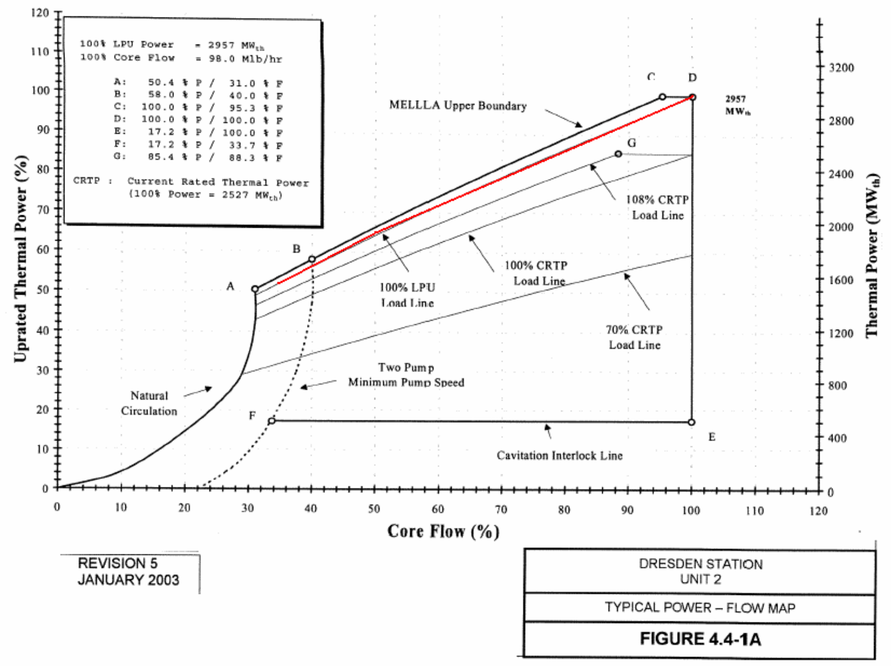
Revision 11 November 1996

# *BWR Reactivity Control*

- BWR regulates power based upon adjustment of control rods and *recirculation flow control* (adjusts void content)
- As power is changed, Doppler defect in reactivity is same as in PWR
- What differs: within certain operational limits BWR can adjust void content up/down by varying recirculation flow



*Effect of Increasing Flow on Subcooled Height and Void Fraction Assuming Same Channel Power*



# *Minimum BWR Shutdown Reactivity*

Cold shutdown requires getting from 550°F to 68°F using only control rods (*using: Sodium Pentaborate* *__not desirable__*)

Basic formula:

$$\rho = 0 = \Delta\rho + (\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG} + (\partial\rho/\partial\alpha)\Delta\alpha$$

Minimum control rod reactivity is thus:

$$\Delta\rho = -[(\partial\rho/\partial T_f)\Delta T_f + (\partial\rho/\partial T_{AVG})\Delta T_{AVG} + (\partial\rho/\partial\alpha)\Delta\alpha]$$

Use available EOL data from Dresden 2,3 FSAR

Doppler Defect *already calculated* – shown in Figures

Assume transition from power → Cold Shutdown involves void fraction transition from $\alpha = 0.8$ to $\alpha = 0.0$

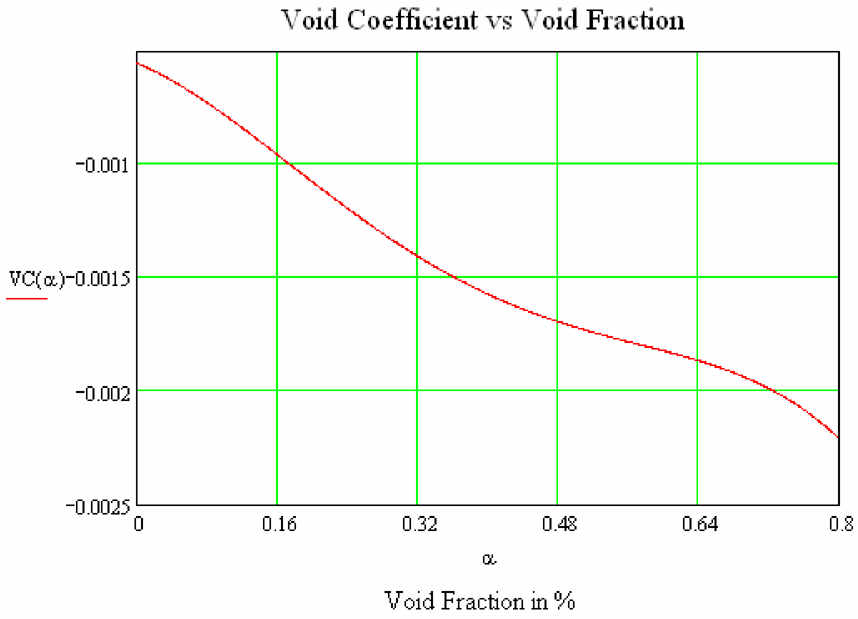
# *BWR Void Reactivity Defect*

**Dresden Shutdown Reactivity Requirements**

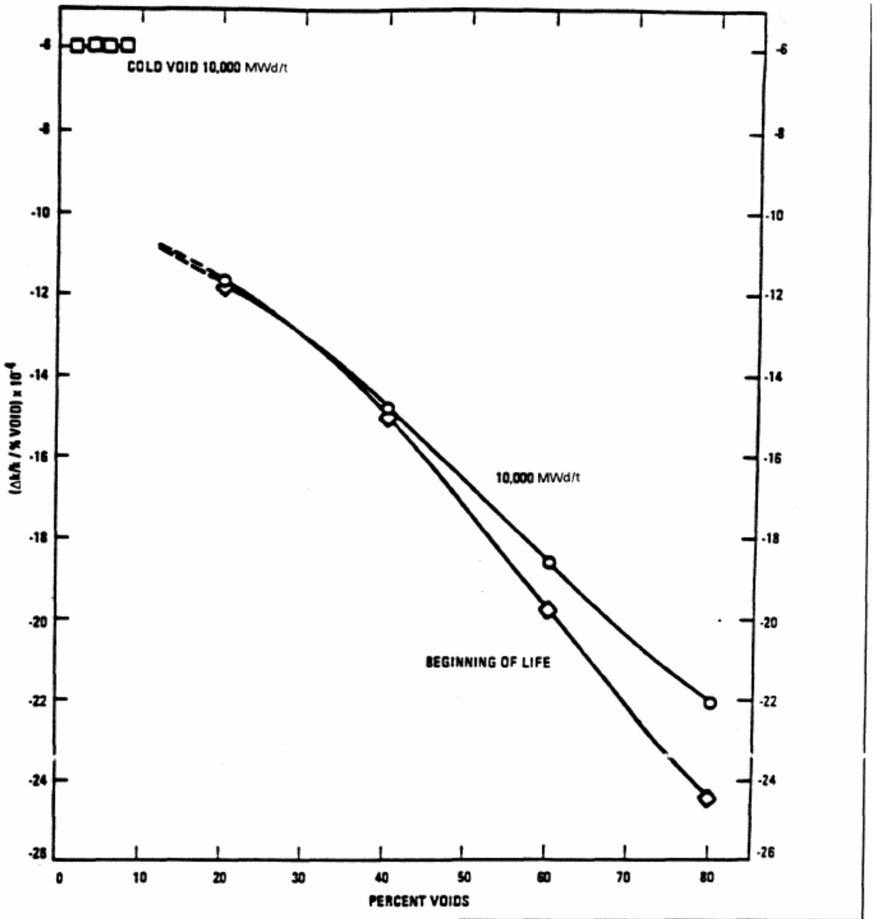
Doppler Power Defect: $\Delta\rho D := 9.5 \cdot 10^{-3}$ Δk/k No calculation needed

**Void Reactivity Coefficient vs. Void Fraction α fit using DATAFIT 6.0:**

$$VC(\alpha) := \left(-156.043\cdot\alpha^{4} + 233.079\cdot\alpha^{3} - 92.9591\cdot\alpha^{2} - 15.56257\cdot\alpha - 5.52955\right)\cdot 10^{-4}$$



$$\Delta\rho V := \int_{0.8}^{0} VC(\alpha)\, d\alpha$$ $\Delta\rho V = 1.163 \times 10^{-3}$ Δk/k



DRESDEN STATION
UNITS 2 & 3

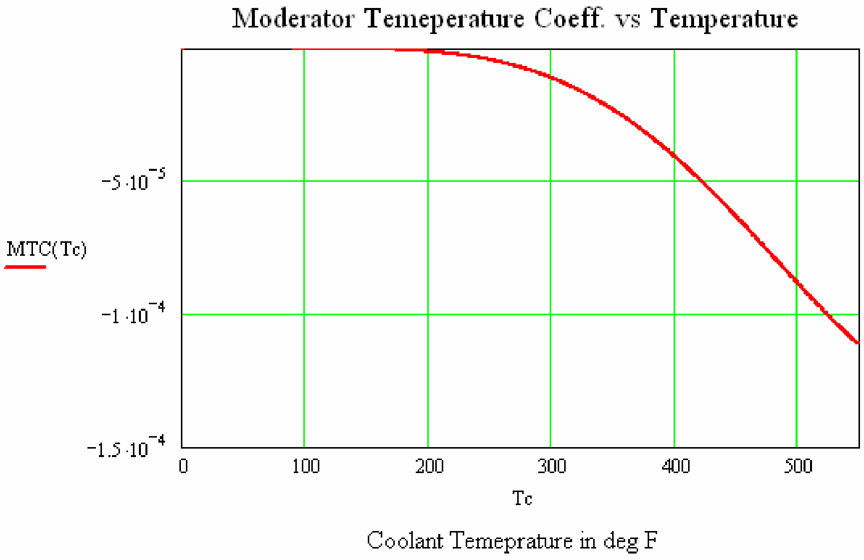
MODERATOR VOID COEFFICIENT OF REACTIVITY

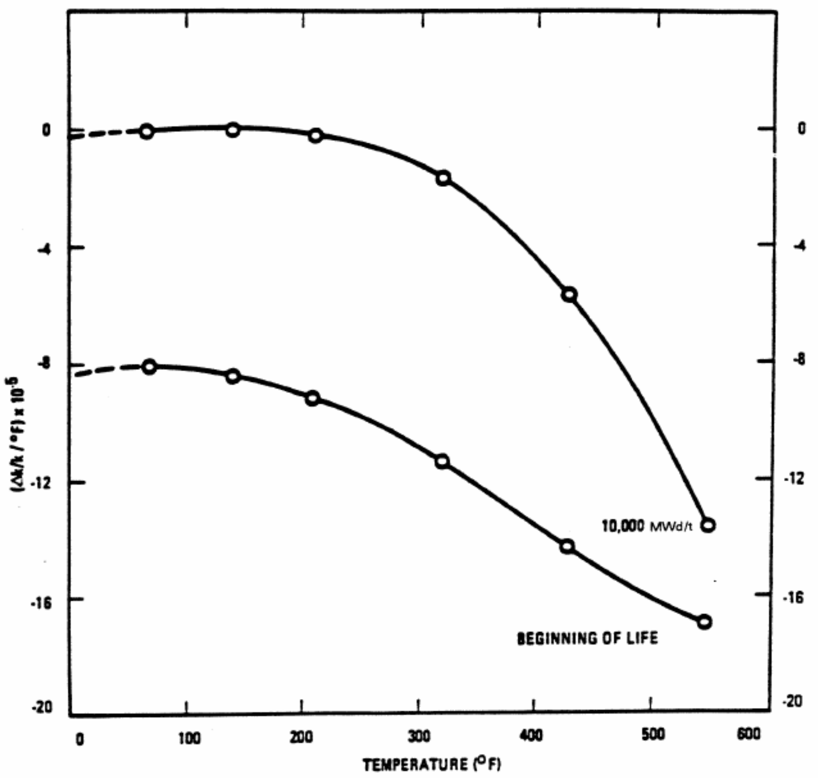
FIGURE 4.3-7

# BWR Moderator Defect Reactivity

**Moderator Reactivity Coefficient vs. Coolant Temperature fit using DATAFIT 6.0:**

$$\mathrm{MTC(Tc)} := \left(1.28341\cdot10^{-12}\cdot\mathrm{Tc}^5 - 1.42716\cdot10^{-9}\cdot\mathrm{Tc}^4 + 4.119771\cdot10^{-7}\cdot\mathrm{Tc}^3 - 4.89005\cdot10^{-5}\cdot\mathrm{Tc}^2 + 2.05643\cdot10^{-3}\cdot\mathrm{Tc} - 2.60031\cdot10^{-3}\right)\cdot10^{-5}$$





DRESDEN STATION
UNITS 2 & 3

MODERATOR TEMPERATURE COEFFICIENT
OF REACTIVITY

FIGURE 4.3-6

Integrating the MTC yields: $\Delta\rho\mathrm{M} := \int_{547}^{68} \mathrm{MTC(Tc)}\,d\mathrm{Tc}$

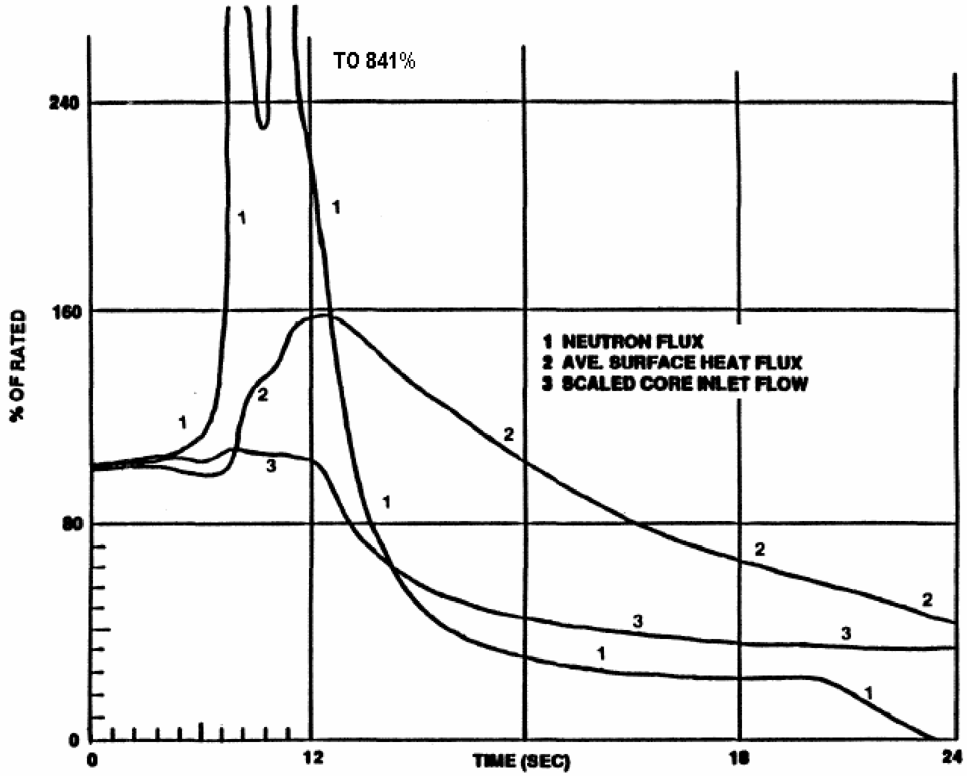
Net Moderator Defect: $\Delta\rho\mathrm{M} = 0.014$ Δk/k

Net Shutdown Reactivity: $\Delta\rho := \Delta\rho\mathrm{D} + \Delta\rho\mathrm{V} + \Delta\rho\mathrm{M}$

$\Delta\rho = 0.025$ Δk/k

# *Sudden Collapse of Voids:*

- Unique issue for BWRs is sudden collapse of voids due to MSIV closure
- MSIV closure ATWS from full power at Dresden 2,3 results in momentary neutron flux to 841%
- Heat flux as we learned lags behind flux and does not get this high.



DRESDEN STATION
UNITS 2 & 3

ATWS-MSIV CLOSURE TRANSIENT:
NEUTRON FLUX RESPONSE

FIGURE 15.8-2

# *Summary:*

- Reactor dynamics while at power can be represented by point reactor dynamics model
- Feedback reactivity effects would need to be incorporated
- Yes ! Full blown integrated systems model (RELAP5, TRAC) would always be nice
- Understanding static reactivity balances explains significant amount of PWR reactor behavior because $T_{avg}$ and Power are *inter-related*
- BWRs are more flexible: power control is via both control rods and recirculation flow control which alters void fraction
- Net reactivities are comparable for both reactor types.