Fundamentals of Nuclear Engineering

Module 3: *Neutron Induced Reactions*

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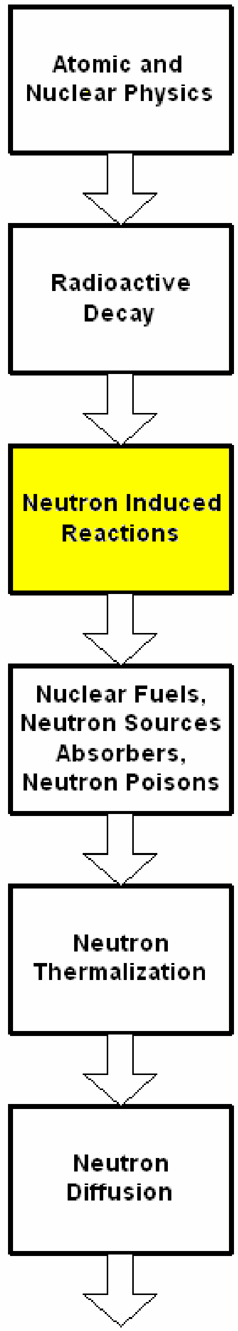


Basics of Atomic vs. Nuclear structure, definitions of terms used in future modules

Describes radioactive decay processes, half-life vs. decay rate used in future modules

Describes concept of cross sections, neutron induced reaction rates

Describes fissionable fuels, neutron sources, absorber materials, and fission product poisons impacting chain reactions

Describes process of neutron moderation used in thermal reactors, identifies impact of resonance capture which impacts chain reactions

Describes neutron diffusion model used to analyze chain reactions and other future reactor problems

Objectives:

1. Explain physical rate laws for nuclear reactions involving: *scattering, absorption, fission*

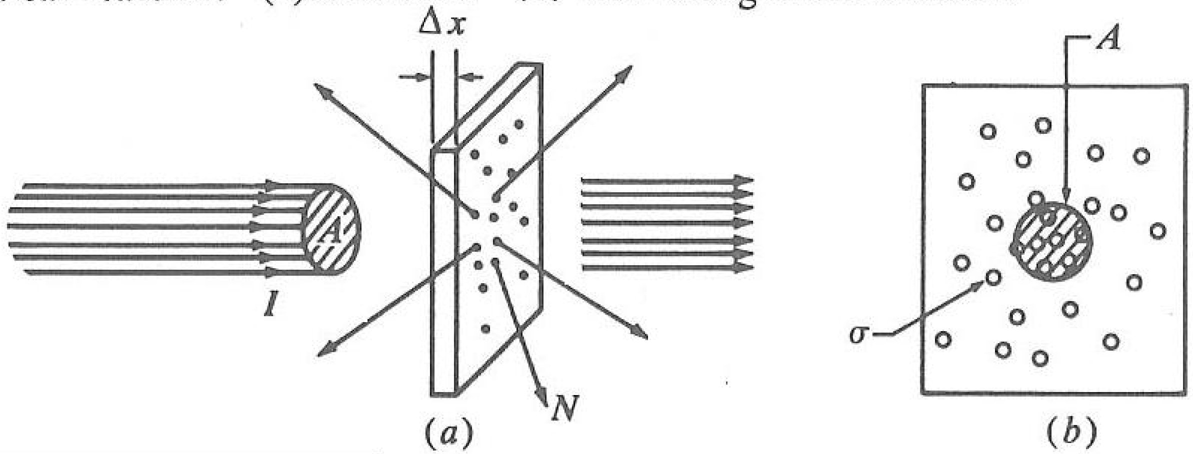
2. Explain concept of cross section: $\sigma(E)$, macroscopic cross section: $\Sigma(E)$, mean free path

3. Explain rate laws for *bulk power production* from fission

4. Explain how to get information from on-line *ENDF* data bases.

Neutron Reaction Rate Proportional to Neutron Flux and Target Area

Basic experimental arrangement to determine the cross section of a nuclear reaction. (*a*) Side view. (*b*) View along beam direction.



from "Elements of Nuclear Physics" by W.E. Meyerhof, Copyright 1967 McGraw-Hill

- Assume foil density "n" atoms/cm^3, width "Δx", bombarded with beam (area "A") of neutrons "I" (neutrons/sec.) with velocity "v_n".
- Each nucleus in foil represents possible target area: $\sigma = \pi R_o^2$ - where R_o is *nuclear radius*. Total target area ~ $A\ \Delta x\ n\ \sigma$
- Rate of removing neutrons from *I* is proportional to: #neutrons crossing through A and total area presented by all targets:

$$dN/dt = (I/A)(A\ \Delta x\ n\ \sigma)$$

Neutron reaction cross sections

- Total *microscopic neutron cross section* is expressed as:

  $$\sigma = dN/dt \;/\; [(I/A)\; n\; A\; \Delta x]$$

- Defining *neutron flux* as: $\varphi = I/A$ (neutrons/sec.cm^2)
- Then: $dN/dt = \varphi\;(A\;\Delta x\; n\; \sigma)$
- Neutron flux can also be defined: $\varphi = n_n v_n$ where:

  n_n is neutron density per cm^3 in beam,

  v_n relative velocity (cm/sec.) of neutrons in beam
- Cross section σ can be experimentally measured as function of energy: $\sigma(E)$
- For nucleus, $\sigma(E)$ units expressed in “barns” (b)
- 1b = 10^{-24} cm^2

Neutron reaction cross sections

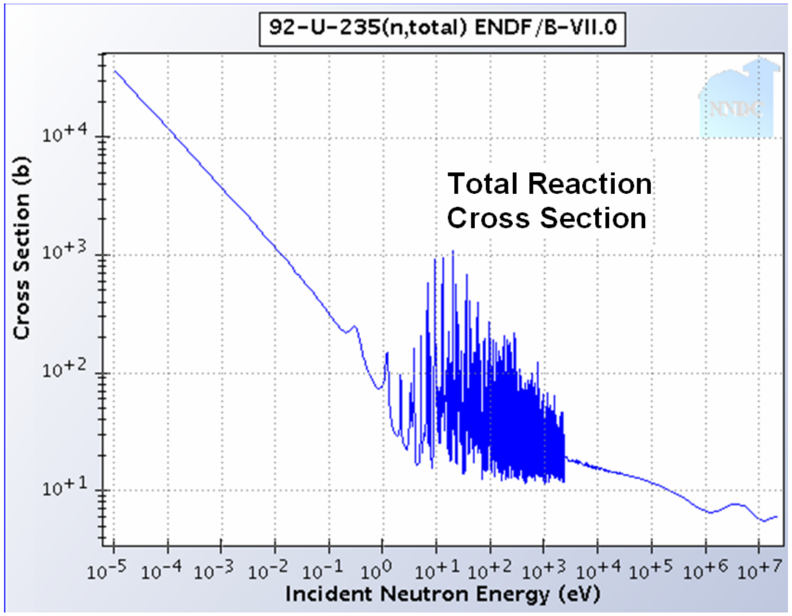
- Cross sections $\sigma(E)$ can be separated into different types of reactions – scattering, absorption, fission:

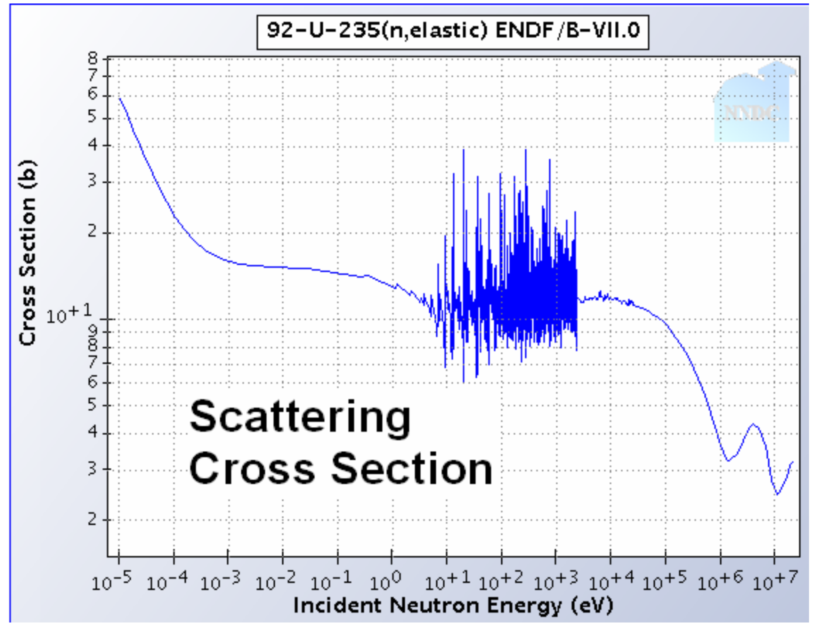
$$\sigma(E) = \sigma_s(E) + \sigma_c(E) + \sigma_f(E)$$

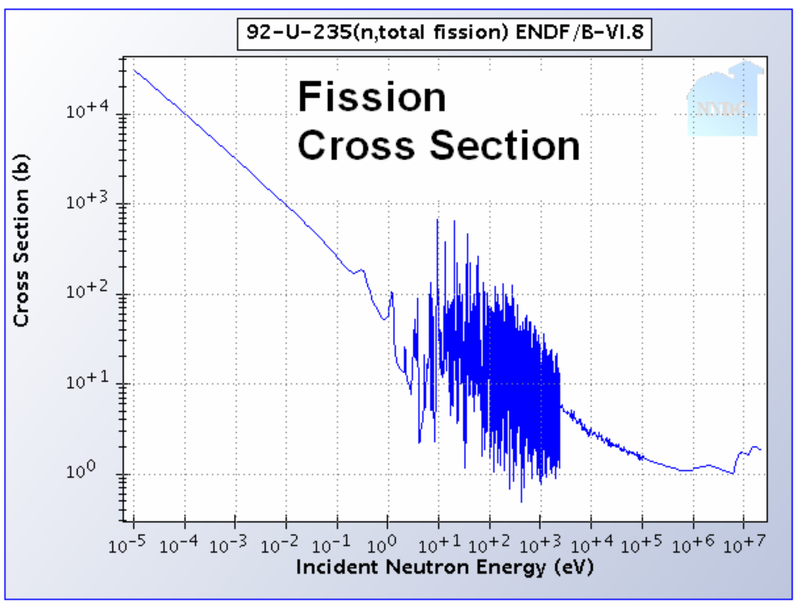
- Neutron cross section data is available from NNDC at:

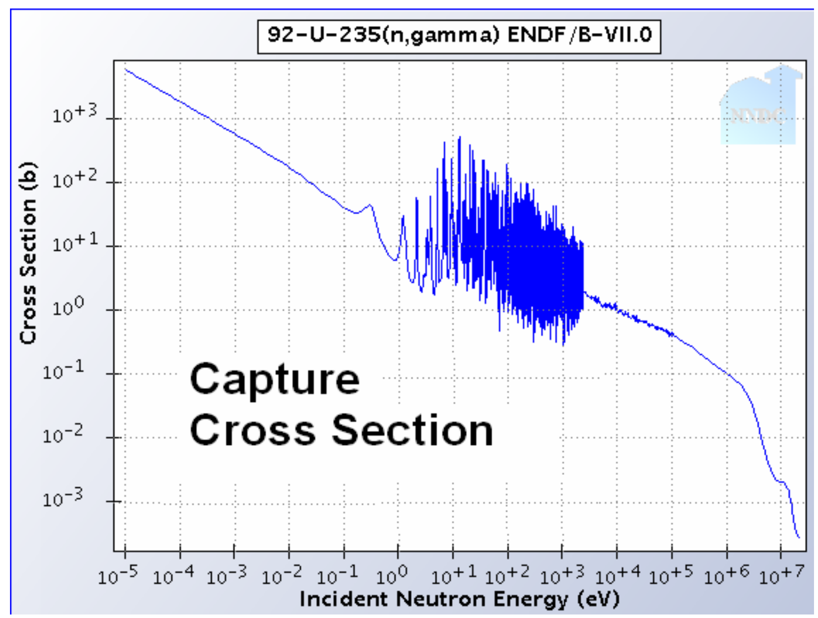
  http://www.nndc.bnl.gov/sigma/index.jsp

Example: published $_{92}U^{235}$ cross sections



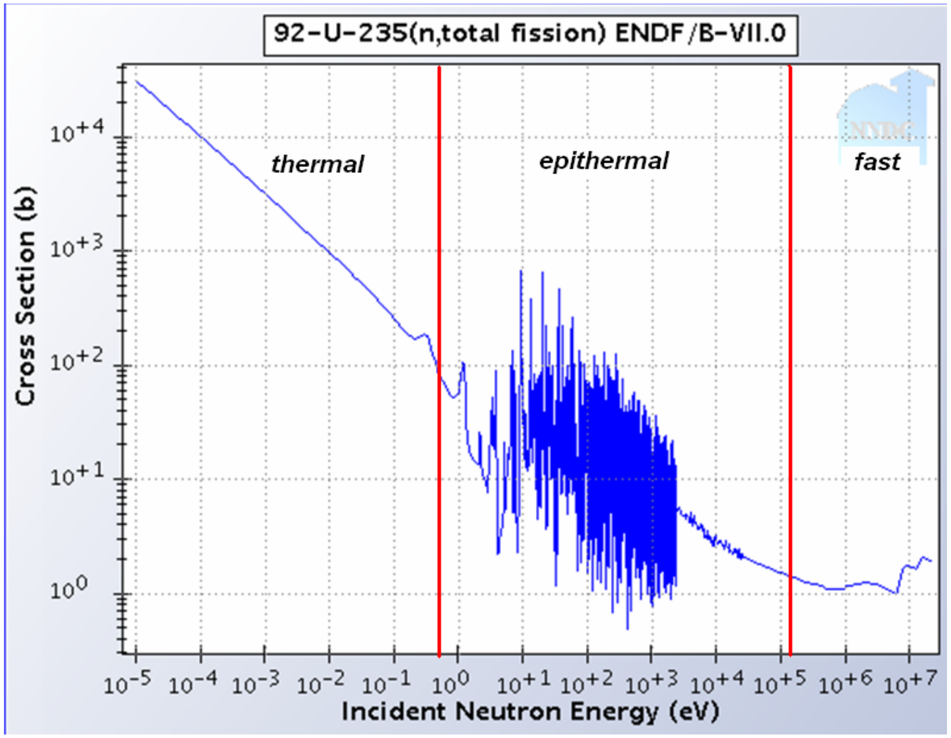






Some observations of neutron cross sections

- Three distinct regions exist: ***thermal***, ***epithermal***, ***fast***
- Thermal (<<1eV) neutron cross section varies: ~1 / $v_{neutron}$
- Epithermal (roughly:1 – 10^5eV) neutron cross section is ***highly structured***
- Structure is caused by resonance capture and scattering - more about this later
- Fast neutron (roughly > 10^5eV) cross section not as structured as epithermal region



Some observations of ${}_{92}U^{235}$ *cross sections*

- Fission: ${}_{92}U^{235}(n,f)$, inelastic neutron capture via: ${}_{92}U^{235}(\mathrm{n},\gamma)$ ${}_{92}U^{236}$ are predominant modes of neutron reaction for this isotope

- Probability of neutrons *<u>bouncing off</u>* ${}_{92}U^{235}$ nucleus is less than 1/1000

- Below 1eV: probability of neutron capture or fission increases dramatically as incident energy decreases

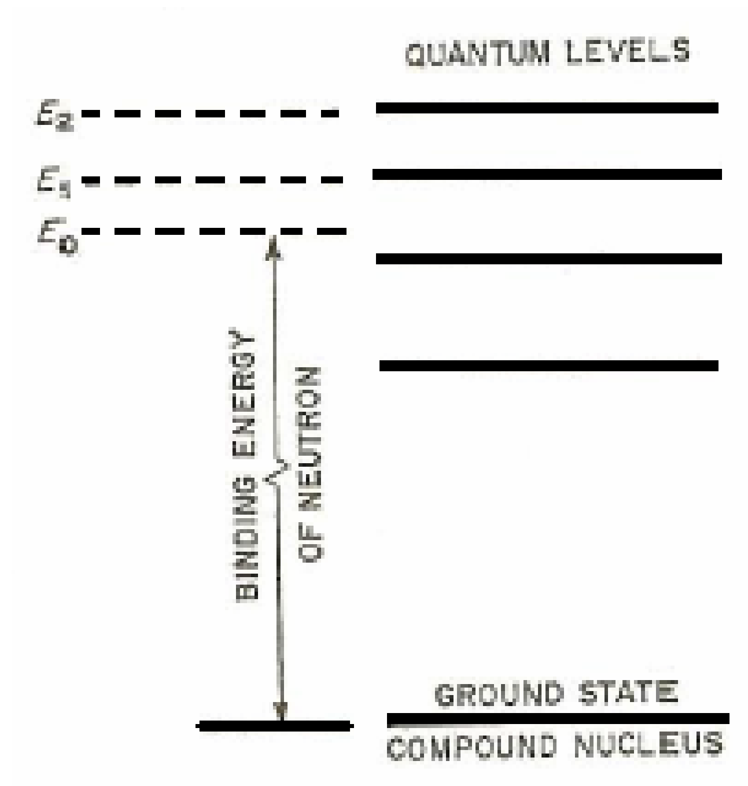
- 1MeV (fission emitted) neutrons have less 1/10,000 chance of being captured, or causing fission

Example Neutron Cross Sections evaluated in 0.01eV (Thermal) Region

Element	σ_s *(barns)*	σ_a *(barns)*	Element	σ_s *(barns)*	σ_a *(barns)*
Aluminum	1.4	0.24	Hydrogen	38 – 100	0.33
Berylium	7.0	0.10	Iron	11.0	2.62
Bismuth	9.0	0.034	Lead	11.0	11.0
Boron	4.0	755	Nitrogen	10.0	1.88
Cadmium	7.0	2450	Oxygen	4.2	0.0002
Carbon	4.8	0.0034	Sodium	4.0	0.53
Deuterium	7.0	0.0005	Uranium	8.3	7.68
Helium	0.8	0.007	Zirconium	8.0	0.185

Resonance Neutron Capture:

- Resonance capture occurs when incoming neutron's energy matches ΔE level between two discrete states
- Energy region where this occurs is "Epithermal" region
- Neutron is preferentially absorbed
- One or more Gamma rays (or *α*,*β*) emitted as compound nucleus begins decay back to lower energy state.



Resonance Neutron Capture:

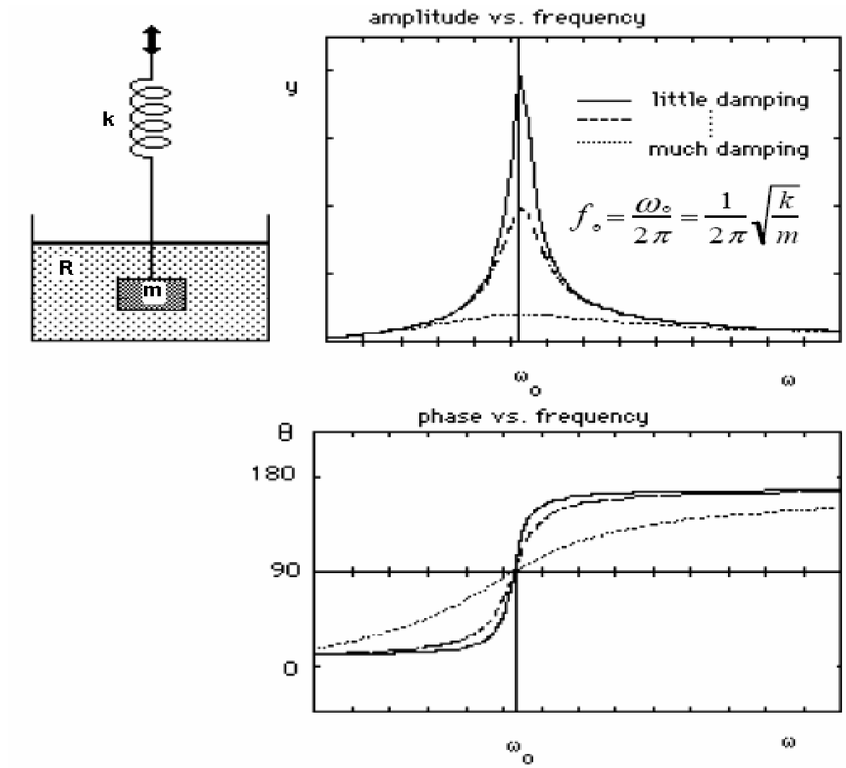
- Concept of *resonance excitation* is found in many classical physical systems

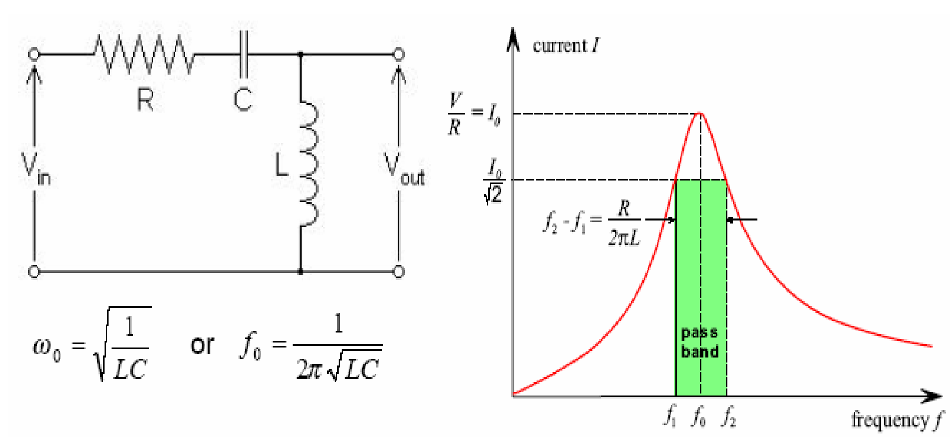
- Idealized mass, spring, damper model of mechanical system

- $d^2y/dt^2 - 2\gamma\, dy/dt + k/m\, y(t) = F(t)/m$

- Tuning radio to match specific radio frequency signal vs. damping out other frequencies

- Physics is *analogous* in neutron – nucleus interaction

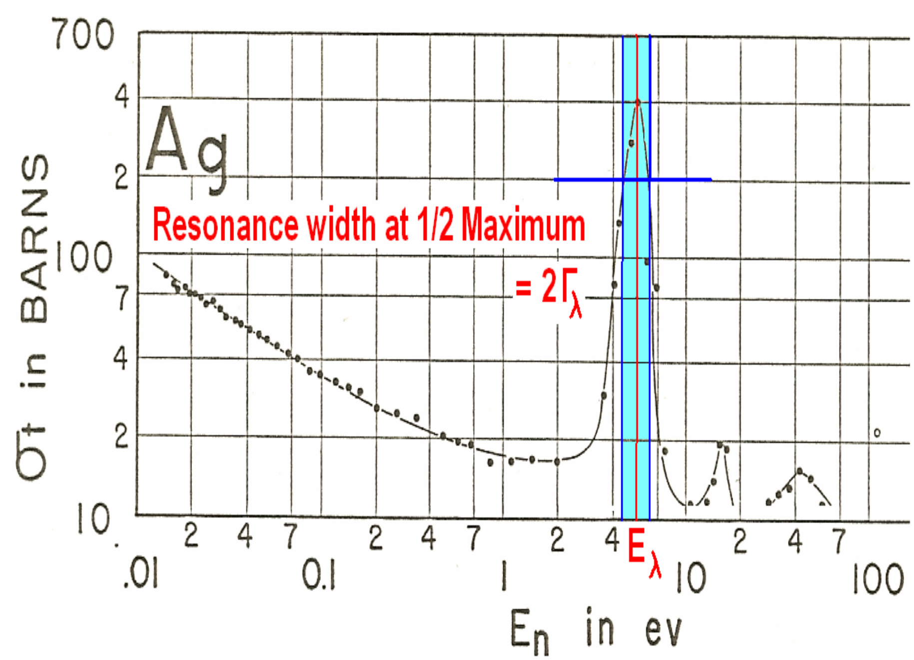




Resonance Neutron Capture:

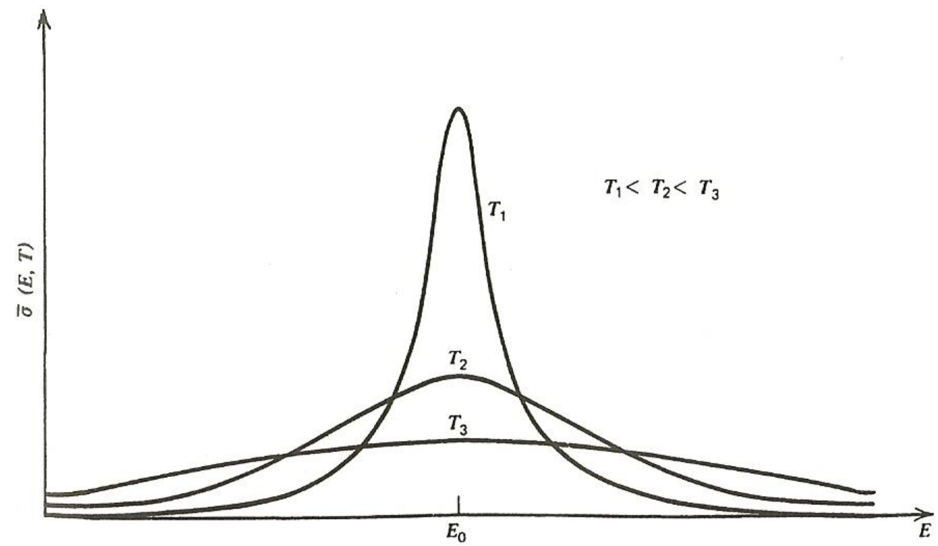
- G. Breit and E. Wigner first described physics of neutron capture reactions in 1936
- $\sigma(E)$ can be derived almost exactly via *"Quantum Mechanics"….*
- Nuclear Engineers interested in specific location of resonances "E_λ" and widths of resonances "Γ_λ"
- $\sigma(E)$ shape is identical to mechanical, electrical analogs

$$\sigma_{ab} = \frac{\pi g_a}{k_a^2} \sum_\lambda \frac{\Gamma_{\lambda a}\Gamma_{\lambda b}}{(E - E_\lambda)^2 + (\Gamma_\lambda / 2)^2}$$



Temperature Affects Resonance Shape

- Target atom temperature increases causes molecular vibrations
- Adds uncertainties to resonance energy E_o
- $E_o \rightarrow E_o +/- \Delta E$
- Effect is: *Doppler Broadening*
- Peak value decreases
- Resonance width increases
- Overall effect on *φ(E)σ(E,T)* is *increased interaction rate*
- Doppler effect causes fuel temperature reactor feedback

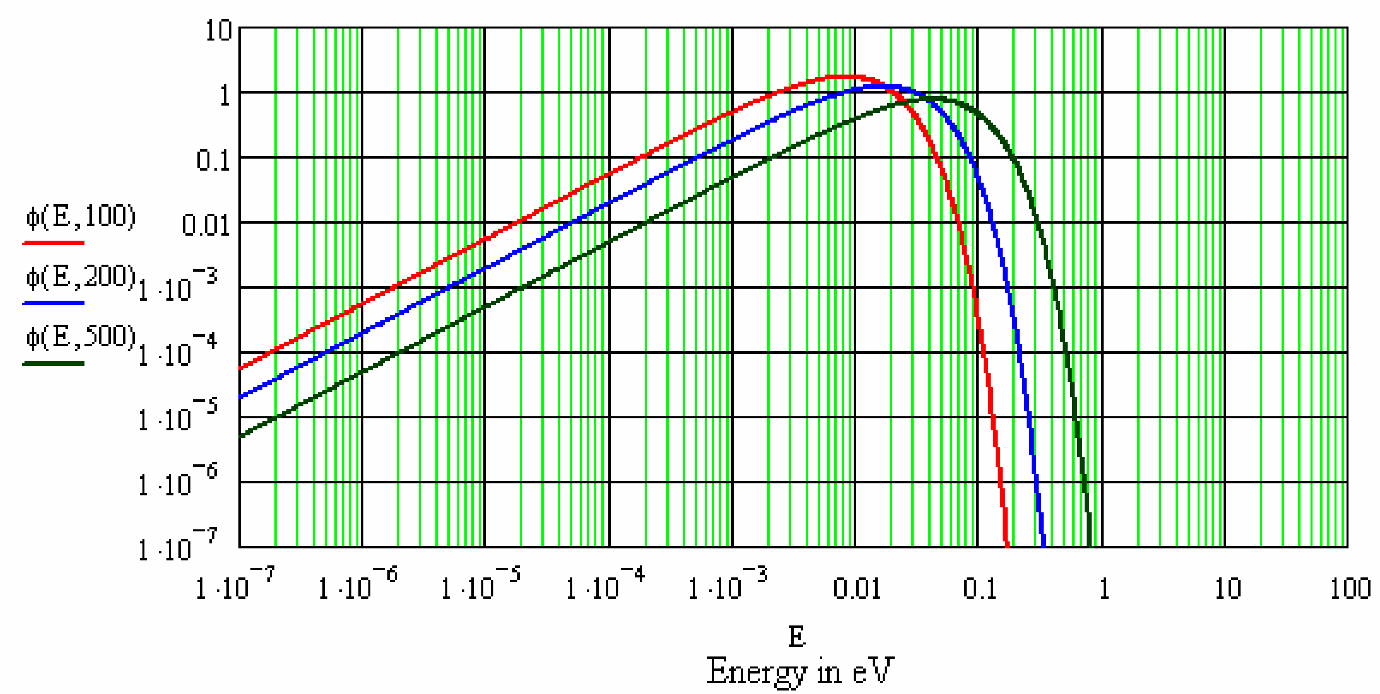


Doppler-broadening of a resonance with increasing temperature.

Taken from "Nuclear Reactor Analysis", by J. J. Duderstadt, and L. J.Hamilton

Thermal Neutrons

- Fast neutrons slow down to point where energy distribution gets close to Maxwell-Boltzmann distribution
- $\varphi(E) = 2\pi n/(\pi kT)^{3/2}\,(2/m)^{1/2}\,E\,\exp(-E/kT)$
- Plot below shows $\varphi(E)$ for: *100°K, 200°K, 500°K*
- Or: *-279.6°F, -99°F, 440°F*



How Fast is a *Thermal Neutron?*

- Most probably speed is where: $d\varphi(E_T)/dE = 0$

$$E_T = kT$$

- Kinetic energy: $\frac{1}{2} m v_T^2 = kT$, thus: $v_T^2 = (2kT/m)$
- Assume room temperature ~70°F, or 294.26°K
- $k = 8.63x10^{-5} eV/°K$ (Boltzmann's constant)
- $E_T = kT = (8.63x10^{-5} eV/°K)(294.26°K) = 0.025\ eV$
- $m = 1.675x10^{-27} kg$

$$v_T^2 = \frac{2(8.63x10^{-5} eV/°K)(294.26°K)(1.602x10^{-19} Joule/eV)}{(1.675x10^{-27} kg)}$$

$$v_T = 2.204x10^3 \text{ meters/sec (4930 m.p.h.)}$$

Attenuation of Neutron Beam

- From conservation of neutrons in beam: number scattered, absorbed, reacted removed from beam: $dN = - dI$

- Since: $N/I = n\Delta x\sigma$ -> $N = In\sigma\,\Delta x$

  $- dI = In\sigma\, dx$

- Integrated, this yields attenuation formula in terms of total *reaction cross section* and foil density:

$$I(x) = I_o exp(-n\sigma x)$$

- $I(x)/I_o = exp(-n\sigma x)$ is probability of *non-interaction*

Macroscopic Cross Section: $\Sigma(E)$ Definition

- For nuclear engineering calculations macroscopic neutron cross section $\Sigma(E)$ becomes more useful

- $\Sigma(E)$ can be expressed as linear sum of cross sections of different materials: $\Sigma(E) = n_1\sigma_1(E) + n_2\sigma_2(E) + n_3\sigma_3(E)$

- Allows generating effective cross section of composite material: U_3O_8 fuel pellets, Ag-In-Cd control rods

- $\Sigma(E)$ is defined: $\Sigma(E) = n\sigma(E)$

- $\Sigma(E)$ effectively has units of: #/cm^3 x cm^2 = #/cm

Example Neutron Macroscopic Cross Sections evaluated in 0.01eV (Thermal) Region

Material	Σ_s (cm^{-1})	Σ_a (cm^{-1})
Water (H_2O)	3.45	0.022
Heavy Water (D_20)	0.449	3.3×10^{-5}
Graphite	0.385	2.73×10^{-4}
Natural Uranium (UO_2)	0.372	0.17
Boron	0.5124	97.23
Indium	0.08435	7.438
Cadmium	0.3245	113.6
Halfnium	0.5494	5.836
Gadolinium	0.1218	1401

Probability of Interaction

- Probability of neutron interaction event in dx is expressed:

$$p(x)\,dx = \Sigma_{tot}\,exp(-\,\Sigma_{tot}x)\,dx$$

- Probability of at least one interaction while traversing infinite media:

$$\int_0^{\infty} p(x)\,dx = \int_0^{\infty} \Sigma_{tot}\,exp(-\,\Sigma_{tot}x)\,dx = 1$$

- Average distance traveled without interaction, or *mean free path*:

$$\lambda = \int_0^{\infty} xp(x)\,dx = 1/\,\Sigma_{tot}$$

Fission mean free path fast vs. thermal neutron?

- Density of pure U^{235} is: $\rho_U \sim 19.0\ gram/cm^3$
- $n_U = \rho_U\ (Avogadro's\ Number)/\ 235\ grams/mole$
- $\sigma_f(1MeV) \sim 1\ barn$

$$\Sigma_f\,(1MeV) = n\ \sigma_f(1MeV)$$

$$= \frac{(18.95\ gram/cm^3)(6.02x10^{23} atoms/mole)(1\cdot 10^{-24} cm^2)}{235\ grams/mole}$$

$$= 0.0487\ cm^{-1}$$

- $\lambda_f = 1/\ \Sigma_f$
- $\lambda_f\,(1MeV) = 2.0546\ cm.$ ← *Fast Neutron*

Fission mean free path fast vs. thermal neutron?

- Density of pure U^{235} is: $\rho_U \sim 19.0\ gram/cm^3$
- $n_U = \rho_U$ *(Avogadro's Number)/ 235 grams/mole*
- $\sigma_f(0.01eV) \sim 1000\ barn$

$$\Sigma_f(0.01eV) = n\,\sigma_f(0.01eV)$$

$$= \frac{(18.95\ gram/cm^3)(6.02x10^{23} atoms/mole)(1\cdot 10^{-21} cm^2)}{235\ grams/mole}$$

$$= 48.7\ cm^{-1}$$

- $\Sigma_f(0.01eV) = 48.7\ cm^{-1}$
- $\lambda_f = 1/\Sigma_f$
- $\lambda_f(0.01eV) = 0.020546\ cm.$ ← *Thermal Neutron*

Calculation of Fission Reaction Rates

- Rate of any general neutron induced reaction for thin target "Δx" was previously defined as:

$$dN/dt = \Phi\ (A\ \Delta x\ n\ \sigma) = \Phi\ (A\ \Delta x)\ \Sigma$$

- Assumption of thin target is necessary to avoid issue of reactions *depleting* incoming neutron flux of neutrons due to reactions.
- Strongly reacting target with Δx > *mean free path*: *λ,* will deplete neutron flux resulting in spatial distribution.
- Reaction cross section is function of energy: $\Sigma_f(E)$
- Fission reaction rate would most accurately be expressed:

$$\frac{dN}{dt} = \int_0^V \int_0^\infty \Phi(\vec{r}, E)\Sigma_f(\vec{r}, E)\,dE\,d^3r$$

Calculation of Local Power Generation Rate

- Exact expression: $$\frac{dN}{dt} = \int_0^V \int_0^\infty \Phi(\vec{r}, E)\Sigma_f(\vec{r}, E) dE d^3r$$
- Simplified based on averaging flux and macroscopic cross section for just thermal neutron spectrum:

$$\frac{dN}{dt} = \int_0^\infty \left\langle \Phi(\vec{r}) \right\rangle_{th} \left\langle \Sigma_f(\vec{r}) \right\rangle_{th} d^3r$$

- Simplified by neglecting local flux depression:

$$\frac{dN}{dt} \approx \left\langle \Phi \right\rangle_{th} \left\langle \Sigma_f \right\rangle_{th} V$$

- ***Power generation rate: proportional to fission reaction rate:*** $dP/dt = dN/dt$ ***·(Energy / fission)***

Power Generation Rate from Fission Rate

- Energy yield per individual fission depends on nuclear binding energy calculation of resultant fission products
- Net fission energy release is from:

  Fission fragment recoil kinetic energy *(165-170 MeV)*

  Prompt γ-ray emission *(~7 MeV)*

  Kinetic energy from prompt neutrons *(1 – 7 MeV)*

  Fission fragment *β-particle* emission *(~ 7 MeV)*

  γ-ray emission from fission fragments *(~ 6 MeV)*

  ν-emission (Neutrino emission) *(10 – 12 MeV)*
- Neutrino kinetic energy is lost because mean free path for neutrino interaction with any substance is very very large.
- Neglecting long term *α,β,γ* decay of fission products
- *Average prompt energy release per fission is ~ 200 MeV*

Power Generation Rate from Fission Rate

- To produce 1 Watt of thermal energy requires:

$(1W_t)/[(2.0x10^8 eV/fission)(1.6x10^{-19} W_t\text{-}sec/eV)] =$

$3.1x10^{10}$ *fissions/sec.*

- Thus:

$$\frac{dN}{dt} \approx \frac{\langle \Phi \rangle_{th} \langle \Sigma_f \rangle_{th} V}{3.1x10^{10}} Watts$$

Power Generation Rate in 3.5% Enriched U

- 4000 MWt reactor has core loading of 1.211×10^5kg 3.5% enriched UO_2.
- What would be *order of magnitude* of neutron flux?
- Volume: 1.211×10^5kg/*10.96 gr/cm*^{3}= $1.105 \times 10^7 cm^3$
- UO_2 density $\rho = 10.96\ gr/cm^3$, 3.5% U^{235} enriched, Molecular weight = 270.07gr/mole, $\sigma_f \sim 389\ barns$

$$n_{U235} = (0.035)(10.96\ gr/cm^3)(6.022 \times 10^{23})/(270.07 gr/mole)$$
$$= 8.53 \times 10^{20} cm^{-3}$$

$$\Sigma_f = n_{U235}\,\sigma_f = (8.53 \times 10^{20} cm^{-3})(389 \times 10^{-24}\ cm^2)$$
$$= 0.333 cm^{-1}$$

$$\Phi = \frac{(4 \times 10^9 W_t)\ \ (3.1 \times 10^{10}/sec)}{(0.333 cm^{-1})(1.105 \times 10^7 cm^3)}$$

$$\Phi \sim 3.4 \times 10^{13}\ neutrons/cm^2\text{-}sec.$$

Summary: Neutron Induced Reactions

- Interaction rate of neutrons with target nuclei is characterized by cross section: $dN/dt = \Phi\, n\, V\, \sigma = \Phi\, \Sigma\, V$
- Cross section: σ (in barns, or 10^{-24} cm^2) is probability of interaction as function of neutron energy: $\sigma(E)$
- Low energy thermal neutrons tend to interact more
- Epithermal neutrons tend to support resonance capture/scattering reactions
- Neutrons with energy levels corresponding to specific quantum shifts in compound nuclei will preferentially interact
- U^{235} fission rate is high for thermal neutrons, low for fast neutrons

Neutron Cross Section Data Online:

- *http://www.nndc.bnl.gov/sigma/index.jsp* - link to National Nuclear Data Center at Brookhaven National Lab.

- Site contains:

- $\sigma_{tot}(E)$, $\sigma_s(E)$, $\sigma_a(E)$, $\sigma_f(E)$ *raw* data, plots

- Java scripts for looking at specific localized resonances

- Downloadable data files suitable for use in large scale physics codes to generate various $\Sigma(E)$ data sets