A METHOD FOR CONSTRUCTING REDUCED ORDER TRANSFORMER MODELS FOR SYSTEM STUDIES FROM DETAILED LUMPED PARAMETER MODELS

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Abstract - All power transformer manufacturers maintain computer programs that compute the internal transient voltage distribution when the transformer is subjected to transient voltages. This information is used to design the insulation structure for the transformer. Utility engineers also need to represent the power transformer in some detail for their system studies since significantly higher failure rates on large EHV units suggest the transient voltage that the system places on the terminals of the transformer are in some measure a function of the impedance characteristic of the transformer. Currently, no method exists to conveniently link the detailed lumped parameter model of the transformer designer to that required by the utility engineer. This paper presents a reduction technique which uses the detailed lumped parameter transformer model as a starting point and allows its reductions to any size specified by the user. The method is straightforward mathematically and while retaining the physical configuration of the transformer does not require any proprietary information from the transformer suppliers. The paper presents the necessary mathematics and illustrates the method by example using a 500 MVA autotransformer constructed by ABB for AEP.

INTRODUCTION

To provide a reliable and efficient insulation structure the transformer design engineer must have a knowledge of the transformer voltage distribution under impulse and switching waveforms. Transformer manufacturing companies generally have some form of computer program which allows the design engineer to calculate the required distribution for various types of waveform. These programs usually construct an inductance, capacitance, and resistance model from the transformer geometry and then solve for the voltage distribution using either time or frequency domain techniques. The validity of both these techniques have been verified with laboratory and field data from manufacturing and utility companies.

To perform system studies, utility engineers also need to represent the power transformer in some detail. It is impractical to use the detailed model in system studies because of its size and the resultant computational burden, therefore, reduced order models are used in system studies. Reduced order models for utility system studies are either constructed from terminal characteristics or by greatly simplifying the detailed model obtained from the transformer design study [2,3,9]. Both these models have impedance vs frequency characteristics which are in considerable error above the first resonance. If the transient waveform the system places on the transformer terminals is a function of both the system and the transformer impedance characteristic, then a model which more faithfully represents the impedance characteristic of the transformer is required to perform realistic system studies.

Failure rates of higher level EHV transformers (2.3% per year/phase for 765 Kv GSU's) compared to lower level units (0.7% per year for 345 Kv GSU's) is currently a significant concern to the utility industry and considerable effort has been spent investigating the cause of these failures [1]. Preliminary results suggest that the transient voltages at the terminals of a transformer in the field are, in some measure, a function of the impedance characteristic of the transformer itself. As indicated above, however, the transformer model used in utility system studies is not sufficiently detailed to accurately capture the transformer's required frequency characteristic.

A method which starts with the manufacturer's detailed model and systematically reduces it to a size suitable for use in utility system studies is presented. The method requires only that the retained nodes and desired size of the model be specified. The result obtained is a group of inductive and capacitive lumped elements suitable for use in utility transient programs such as EMTP.

DETAILED LINEAR TRANSFORMER MODEL

The following paragraphs describe the basic modules required in a typical program used in computing the internal transient voltage distribution in transformers. As indicated previously, all transformer manufacturer's possess similar programs. The detailed lumped parameter inductance and capacitance models constructed will provide the starting point for the reduction process developed in this paper. Only a detailed linear transformer model will be considered in this work.

An input initialization module will provide a link between the transformer designers concepts in inches of copper, paper, oil, and steel to the formulas and conventions necessary to compute the needed R, L, and C's.

The Topology module will convert users instructions specifying desired model detail and descritization into an orderly system of node numbers associated with the winding geometry. In addition, this module would order the node numbering sequence to make the node number sequence as straight-forward as possible. For example, the ground node would be assigned the largest node number, followed by impulsed nodes, switched nodes, nodes connected to non-linear resistors (ZnO), and finally the detailed linear model. The resulting network is shown pictorially in References 5 and 7.

It is worth noting that the ability of the detailed model to faithfully reproduce the transient characteristic of the transformer depends upon two independent model characteristics. First, the model must accurately provide R, L, and C's which are appropriate for the transformer geometry. (This fact is well appreciated and has been the topic of numerous papers in the literature.) The second requirement is that the model must possess sufficient detail to adequately represent the frequency response for the applied waveshape of interest. This often overlooked point can produce results which appear to be mathematically valid, but which have little physical basis [4]. The second condition is also required to meet the necessary assumption for a lumped parameter model - the system must be quasi-static. In other words, the greatest frequency of interest must have a period at least ten times larger than the travel time of the largest element in the model. Hence, for a model to accurately predict the

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switching surge response it must be valid to at least 10 kHz, 50 kHz for a full wave, and 250 kHz for a chopped wave [5], since 95% of the energy in the applied wave is contained below the frequencies listed.

A capacitance module computes the series capacitance along the winding and the shunt capacitance to different winding segments, other windings and ground. The module therefore provides a nodal capacitance matrix for the entire winding structure. In the past, the inductance module has been constructed by either building a self and mutual inductance matrix, inverting the matrix, and then using a loop to nodal transformation [6,7], or by constructing the inverse inductance matrix directly [8].

The influence of losses on the reduced model will not be considered in this paper but will be addressed in subsequent work. Historically, the influence of losses on the transient response has been ignored or handled empirically. Only recently have methods been suggested that represent losses based upon the geometry of the windings [6]. Ignoring losses will not change the frequency characteristics of the model appreciably and the transient response will, in general, be more pessimistic.

A solution routine applies preselected voltage and/or current waveforms to the lumped parameter model and computes the transient voltages. The solution module is normally one of two general types: time domain and frequency domain.

Time domain method solve the problem using some form of numerical integration. The most common form of time domain solution is that proposed Dommel [10]. Dommel's approach is based on the trapezoidal rule of integration and provides a very robust solution method with great flexibility. Nonlinear core characteristics and time dependent switches (used in failure analysis) can both be addressed with this technique. Its principal disadvantage is that it is difficult to model frequency dependent parameters such as eddy current losses.

A frequency domain approach to solving the problem has gained wide acceptance. In this method, a solution is obtained in the frequency domain and convolution used to obtain a time domain solution [11]. With this approach, frequency dependent parameters can be modeled with relative ease, whereas, nonlinear characteristics of the transformer (core and ZnO) tend to be much more difficult. For a detailed linear transformer model, however, both methods provide the same solution.

REDUCTION GOAL

The steady state and transient behavior of a circuit, for any applied voltage, is established by the poles and zeros of the circuit impedance in the complex frequency plane. By definition, the zeros of the terminal impedance function coincide with the natural frequencies of the circuit. McNutt [5] has defined terminal resonance as a terminal current maximum and a terminal impedance minimum. (This corresponds to what is also referred to as series resonance.) Terminal anti-resonance is defined as a terminal current minimum and impedance maximum. (This is also referred to as parallel resonance.) McNutt also defines internal resonance as an internal voltage maximum and internal anti-resonance as an internal voltage minimum. These relationships are all grouped under the heading of forced oscillations and form the basis of the transient response for a detailed lumped parameter transformer model. If a reduced order model of a transformer is to reproduce the transient voltage characteristics of the detailed model, the reduced model must essentially contain the same poles and zeros in its terminal impedance and transfer functions as the complete model over the frequency range of interest. In the past, this relationship has not been emphasized and, if not appreciated, will lead to a conclusion that reduction techniques apply over a much wider frequency range than is, in fact, the case.

The following reduction technique is based on an extension of Kron's work in the Laplace transform domain. The results will be compared with the transient voltage response and frequency characteristics of the detailed model which is used as a reference.

KRON's REDUCTION METHOD FOR LINEAR TIME-INVARIANT LOSSLESS TRANSFORMER MODELS

The mathematics behind Kron's reduction method is straightforward [12]. Equation (1) shows the relationship, in frequency domain, between nodal currents and voltages for an RLC network representing a linear-time invariant model of a power transformer.

$$[I(s)] = \left[\frac{I_1}{I_2}\right] = \left[\frac{Y_{11}}{Y_{21}}\Big|\frac{Y_{12}}{Y_{22}}\right]\left[\frac{E_1}{E_2}\right] = [Y(s)]\,[E(s)] \quad (1)$$

where $Y(s) = 1/s\,[\Gamma] + [G] + s[C]$

with
$[\Gamma]$ = inverse nodal inductance matrix
$[G]$ = nodal conductance matrix
$[C]$ = nodal capacitance matrix
s = Laplace operator

Matrix partitioning in equation (1) groups all input, output and nodes of interest under subscript 1. All remaining nodes in the detailed model are grouped under subscript 2. Therefore, $I_2 = 0$ and E_2 can be expressed as a function of E_1 yielding the reduced matrix equation (2).

$$I_1 = (Y_{11} - Y_{12}\,Y_{22}^{-1}\,Y_{21})\,E_1 \quad (2)$$

Equation (1) represents the detailed linear-time invariant transformer model corresponding to a RLC network of n nodes plus ground. On the other hand, equation (2) represents the reduced linear-time invariant transformer model corresponding to a RLC network of m nodes, plus ground, where $m < n$. The reduction method outlined yields an exact reduced model valid for one frequency. Kron's reduction method allows a detailed (even turn-to-turn) transformer model to be reduced to any number of nodes, $0 < m < n$. This reduced model can be applied for steady-state studies or normal operation of power systems where one single frequency (60 Hz) is normally considered.

For electrical transient studies in power systems, where a spectrum of frequencies is involved, Kron's reduction method must be modified in order to obtain a reduced model which approximates the response of detailed model. This is a trade-off between simplicity and accuracy.

Equation (2) can be rewritten as:

$$I_1 = (Y_{11} - Y_r)E_1$$

where Y_r is the reducing nodal admittance and is shown in equation (3):

$$Y_r = Y_{12}\,Y_{22}^{-1}\,Y_{21} \quad (3)$$

where:

$$Y_{12} = sC_{12} + G_{12} + \frac{\Gamma_{12}}{s} \quad (4)$$

$$Y_{22}^{-1} = \left(sC_{22} + G_{22} + \frac{\Gamma_{22}}{s}\right)^{-1} \quad (5)$$

$$Y_{21} = sC_{21} + G_{21} + \frac{\Gamma_{21}}{s} \quad (6)$$

substituting (4) and (6) into (3):

$$Y_r = s^2 C_{12} Y_{22}^{-1} C_{21} + s(C_{12} Y_{22}^{-1} G_{21} + G_{12} Y_{22}^{-1} C_{21})$$
$$+ C_{12} Y_{22}^{-1} \Gamma_{21} + \Gamma_{12} Y_{22}^{-1} C_{21} + G_{12} Y_{22}^{-1} G_{21} \quad (7)$$
$$+ \frac{1}{s}(G_{12} Y_{22}^{-1} \Gamma_{21} + \Gamma_{12} Y_{22}^{-1} G_{21}) + \frac{1}{s^2} \Gamma_{12} Y_{22}^{-1} \Gamma_{21}$$

Obtaining the inverse of Y_{22} may be cumbersome, however, this inversion can be simplified if it is reduced one node at a time, by partitioning matrices in equation (1) such that Y_{21} is a row matrix, Y_{12} is a vector and Y_{22} is a scalar which represents the nodal admittance associated to the node which is neglected. By reducing one node at a time, the inverse of the nodal admittance of the neglected node (eq. 5) became a scalar fraction. Substituting equation (5) into equation (7) yields:

$$Y_r = Y_{r1} = \frac{s^4 A_4 + s^3 A_3 + s^2 A_2 + s A_1 + A_0}{s(s^2 C_{22} + s G_{22} + \Gamma_{22})} \quad (8)$$

where:

$$A_4 = C_{12} C_{12}^t$$
$$A_3 = C_{12} G_{12}^t + G_{12} C_{12}^t$$
$$A_2 = C_{12} \Gamma_{12}^t + \Gamma_{12} C_{12}^t + G_{12} G_{12}^t$$
$$A_1 = G_{12} \Gamma_{12}^t + \Gamma_{12} G_{12}^t$$
$$A_o = \Gamma_{12} \Gamma_{12}^t$$

The post subscript t means transpose and Y_{r1} represents the reducing admittance by reducing one node at a time. Additionally, Y_{r1}, A_4, A_3, A_2, A_1, A_o, are n-1 x n-1 matrices, with n equal to the number of nodes in the model not counting ground.

Examining equation (8), we see that the associated RLC parameters of the Y_r are frequency dependent, however, it is possible to approximate the reducing nodal admittance by an equivalent that is not frequency dependent. Using this approximation maintains the linearity and time invariant property of the reduced model as indicated in equation (9)

$$\tilde{Y}_{r1} = s C_{r1} + G_{r1} + \frac{\Gamma_{r1}}{s} \quad (9)$$

Equating equations (8) and (9) yields an overdetermined set of five equations where only three matrices are unknowns: C_{r1}, G_{r1}, and Γ_{r1}. They must satisfy as closely as possible the following equations:

$$A_4 = C_{22}\ C_{r1} \quad (10a)$$
$$A_3 = G_{22}\ C_{r1} + C_{22}\ G_{r1} \quad (10b)$$
$$A_2 = \Gamma_{22}\ C_{r1} + G_{22}\ G_{r1} + C_{22}\ \Gamma_{r1} \quad (10c)$$
$$A_1 = \Gamma_{22}\ G_{r1} + G_{22}\ \Gamma_{r1} \quad (10d)$$
$$A_o = \Gamma_{22}\ \Gamma_{r1} \quad (10e)$$

Since this analysis is restricted to the lossless case, we can set conductance matrices to zero, therefore equation (10) becomes

$$A_4 = C_{22}\ C_{r1} \quad (11a)$$
$$A_2 = \Gamma_{22}\ C_{r1} + C_{22}\ \Gamma_{r1} \quad (11b)$$
$$A_o = \Gamma_{22}\ \Gamma_{r1} \quad (11c)$$

where:

$$A_4 = C_{12}\ C_{12}^t$$
$$A_3 = A_1 = 0$$
$$A_2 = C_{12}\ \Gamma_{12}^t + \Gamma_{12}\ C_{12}^t$$
$$A_o = \Gamma_{12}\ \Gamma_{12}^t$$

In equation 11, we have two unknown matrices, C_{r1} and Γ_{r1} and three equations. Taking two equations only, there are three possibilities:

I - Equations (11b) and (11c) give the low frequency model

II - Equations (11a) and (11b) give the high frequency model

III - Equations (11a) and (11c) give the disintegrated Kron's reduction model.

The characterization of high or low frequency model is given in Appendix 1 for models I and II and the neglected equations for each type may be used for error evaluation.

It can be shown that model III, called disintegrated Kron's reduction model, can be obtained by applying Kron's reduction method to the nodal capacitance matrix and to the inverse nodal inductance matrix independently. This model can be obtained by neglecting one or more nodes at a time. It can in fact be constructed with the application of equation (2) to the L and C matrix independently.

In Table 1 the equivalent reducing nodal capacitance and inverse nodal inductance matrices are shown for Models I, II, and III. Those relationships have been obtained from equation 11. In Appendix 1 the range of application for Models I and II is determined by the nodal frequency of the node to be deleted. Model I is applicable for $\omega < \omega_n$. Model II is applicable for $\omega > \omega_n$ where the nodal frequency is defined by equation (A4). Model III is a hybrid model, it combines the high frequency reducing nodal capacitance with the low frequency reducing inverse nodal inductance.

The minimum retained nodes in the reduced model are input, output, and nodes of interest. Additionally, nodes may also be retained to improve the accuracy of the reduced model. These additional nodes can be selected automatically by the program, which optimizes the reduced model by neglecting the highest or lowest nodal frequency nodes at each reduction step for the low (model I) and high (model II) frequency model, respectively. For Model III, (disintegrated Kron's reduction) optimization is not necessary because the reduced model is independent of the order in which nodes are reduced.

The number of nodes contained in the reduced model is a trade-off between simplicity and accuracy. In general, increasing the number of retained nodes in the reduced model diminishes its simplicity but improves its accuracy.

ILLUSTRATIVE EXAMPLE

The following example will illustrate the computed transient voltage response for standard full and switching surge waveforms applied to the HV terminal of a single phase, core-form, 500 MVA, 765/345/34.5 kV YYD autotransformer. Voltages presented are either line-to-ground at the common terminal, or a voltage difference between two layers ends in the series winding.

Results from four different models of this transformer are presented. The first is considered to be the base case and is generated from a model of the transformer which divided each layer into 5 segment. This model was developed from the transient analysis program used to design the insulation structure for the transformer.

TABLE 1

Reducing Admittance

Reducing Parameters	Models I	II	III
C_{r1}	$\frac{A_2}{\Gamma_{22}} - \frac{C_{22}A_o}{\Gamma_{22}^2}$	$\frac{A_4}{C_{22}}$	$\frac{A_4}{C_{22}}$
Γ_{r1}	$\frac{A_o}{\Gamma_{22}}$	$\frac{A_2}{C_{22}} - \frac{\Gamma_{22}A_4}{C_{22}^2}$	$\frac{A_o}{\Gamma_{22}}$

The transformer contains 1294.5 total turns and 26 layers. The layer with the greatest number of turns (69.5) was divided into 5 segments which placed 14 turns in each segment. It is this segment which determines the highest frequency the detailed model is valid for. For an average mean turn length of 18 feet, and assuming a velocity of propagation in the winding of 500 feet per microsecond, the highest frequency this model is valid for is on the order of 200 kHz. The lumped parameter model for this condition contains 134 nodes.

The following analysis compares the response of the base case to reduced models developed using the technique presented. The three reduced models contain 80, 26, and 10 nodes, respectively. Eighty nodes corresponds to the number of nodes a lumped parameter model would have if each layer were divided into 3 segments. The 26 node model corresponds to a model constructed with only one segment or division per layer. The results presented correspond to the response of Model I (low-frequency reduced model) only. In general, the response of Model II agrees with the reference model for the first few microseconds, and the response of Model III (disintegrated Kron's reduction) is comparable to Model I. However, Model III has shown lower accuracy than Model I. The applied waves are a standard 1.2/50 μs full wave and a 75/800 μs switching surge. For this example the common winding is left floating and the tertiary is short circuited.

Table 2 contains a comparison of the eigenvalues of the base case and three reduced models below 100 kHz. Each model contains the same number of eigenvalues (poles) or natural frequencies (zeros) as their are nodes in the model. Table 2 contains only the lowest values. The base case would be expected to contain accurate natural frequencies through approximately 200 kHz with those above 200 kHz of questionable accuracy. It is against these lower frequency eigenvalues the reduced models are compared.

From Table 2 it is noted that the 80 node reduced model agrees well through 100 kHz or the 25th eigenvalue. The 26 node model agrees well through 34 kHz or the 8th eigenvalue and the 10 node model has good agreement up to 12 kHz or the 3rd eigenvalue. From this it would be expected that all the models would give very good agreement in the switching surge range. For a full wave the agreement would be good for the base case, the 80 and 26 node model but the 10 node model would be less accurate. Subsequent examination of the voltage wave forms verify this condition.

Figures 1a, 1b and 1c compare the base case full wave response of the common terminal to ground with the 80, 26, and 10 node reduced model respectively. Figures 2a, 2b, and 2c compare the full wave response between adjacent layers for the 80, 26, and 10 node reduced model, respectively. As was anticipated from the analysis, the 80 node reduced model follows the base case response very closely, the 26 node model reasonably well, and the 10 node model's response is almost a mean value of the base case.

Figure 3a, 3b, and 3c compare the base case switching surge response of the common terminal to ground with that of the 80, 26, and 10 node reduced model, respectively. As was anticipated from the natural frequency characteristics of the models, the agreement is quite good for all models. Figures 4a, 4b, and 4c present the comparison of the layer to layer voltage differences when the transformer is subjected to a switching surge. All reduced models produce good results.

Table 3 presents an error analysis for the wave forms presented in Figures 1 to 4. The base case detailed model response is used as reference and the error has been computed by the RMS norm and the infinity norm.

TABLE 2

EIGENVALUES COMPARISON (kHz)

Full Model 134 N	Reduced Models 80 N	26 N	10 N
4.53	4.53	4.53	4.53
5.82	5.82	5.83	5.94
12.41	12.41	12.42	12.46
17.78	17.78	17.98	
19.37	19.37	19.66	20.14
21.82	21.82	21.96	21.32
31.90	31.91	32.29	23.50
34.56	34.59	36.03	36.34
39.43	39.46		
39.53	39.56	40.27	
40.78	40.85	41.87	44.01
47.50	47.58	43.09	48.78
53.92	53.99	49.80	
59.93	60.03	55.47	
60.69	60.98		
62.49	62.69	62.06	
64.26	64.40	69.94	68.49
69.59	69.77	70.35	
72.95	73.32	74.31	
78.16	78.42	78.70	
83.20	83.37	86.12	
85.92	87.01		
87.96	88.37	90.04	
92.08	92.67		
96.49	96.96	95.29	
99.95	100.55		

TABLE 3

ERROR EVALUATION
REFERENCE: FULL MODEL 134 NODES
BASE: FULL MODEL MAXIMUM

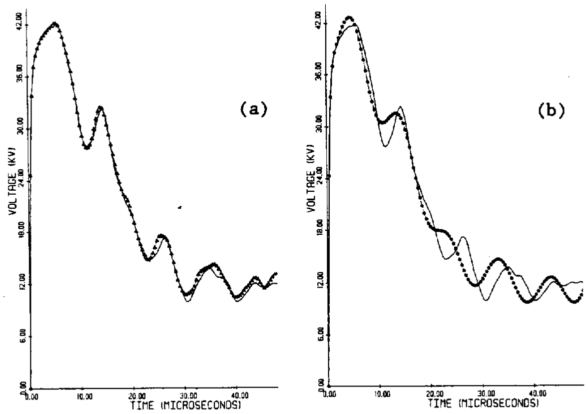
COMMON TERMINAL
ERROR (%)

Reduced Model	Full Wave RMS	Full Wave MAX	Switching RMS	Switching MAX
80 N	1.19	3.32	1.25	3.05
26 N	4.18	9.83	0.90	2.63
10 N	4.64	12.22	2.70	7.90

VOLTAGE DIFFERENCES BETWEEN LAYER ENDS

ERROR (%)

Reduced Model	Full Wave RMS	Full Wave MAX	Switching RMS	Switching MAX
80 N	4.36	12.74	0.77	2.28
26 N	8.34	22.35	2.19	6.36
10 N	16.20	42.20	3.23	8.89



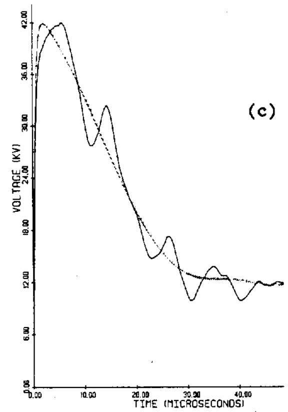
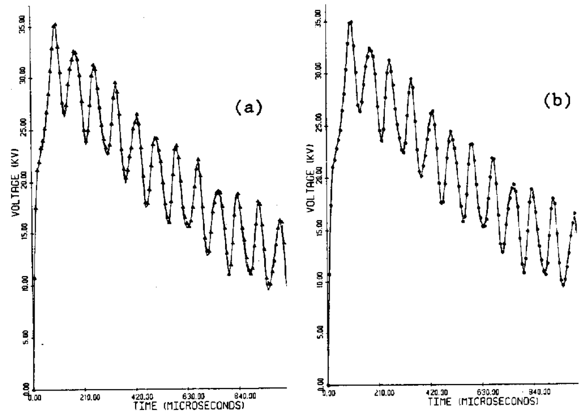


Fig. 1. Reduced Model vs Base Case - Common Terminal to Ground Voltage for Full Wave

a) Base vs 80 node
b) Base vs 26 node
c) Base vs 10 node

Base case is the solid line.



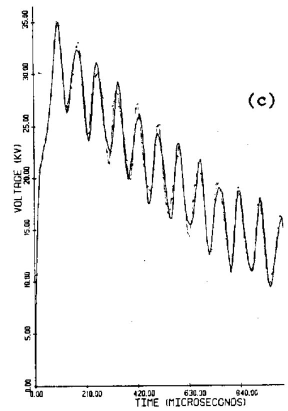
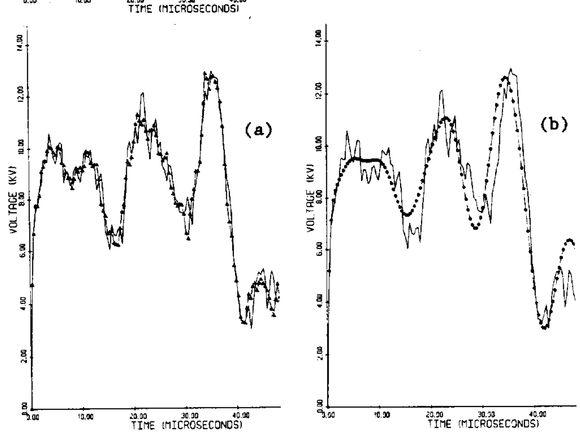


Fig. 3. Reduced Model vs Base Case - Common Terminal to Ground Voltage for Switching Surge

a) Base vs 80 node
b) Base vs 26 node
c) Base vs 10 node

Base case is the solid line.



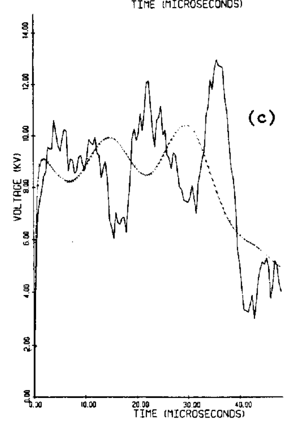
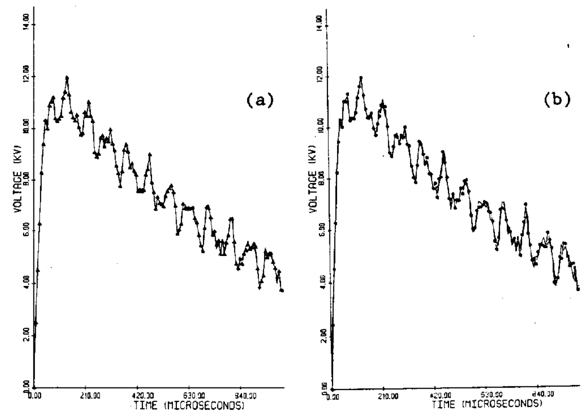


Fig. 2. Reduced Model vs Base Case - Layer-to-Layer Voltage Difference for Full Wave

a) Base vs 80 node
b) Base vs 26 node
c) Base vs 10 node

Base case is the solid line.



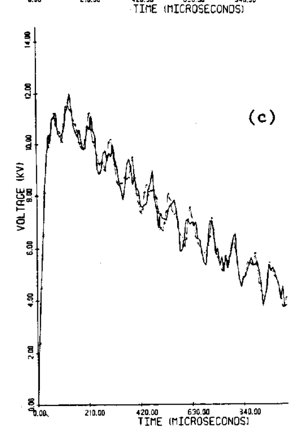


Fig. 4. Reduced Model vs Base Case - Layer-to-Layer Voltage Differences for Switching Surge

a) Base vs 80 node
b) Base vs 26 node
c) Base vs 10 node

Base case is the solid line.

DISCUSSION

The examples have shown that the reduction method presented is valid over a predicted frequency range. Agreement between the reduced and detailed model's transient response improves as the number of eigenvalues that match in each model increase. Agreement between the voltage versus time profile for a reduced model compared to the base should not be construed as validity of the reduced model for all frequencies. The number of nodes to be retained in the model for the reduction method presented is a user decision and a function of the task at hand. If the analysis requires the investigation of the switching surge response for the example given, a 10 node model would be adequate. However, if the analysis were for a chopped wave the model would have to retain at least the detail of the base model. This method does not require the transformer manufacturer to provide any proprietary information about transformers dimensions. Finally, it should be pointed out that the accuracy of the reduced model, at best, will be that of the detailed model so the detailed model must be constructed so that it is accurate in the frequency range of interest. This seems to be a point that is overlooked in recent efforts to create reduced order models.

Results presented in this paper have assumed zero initial conditions. However, since this model is completely linear, if the solution routine used to solve the reduced network will allow initial conditions, the model will reproduce situations with non-zero initial conditions. Clearly, for the non-linear case this model is not adequate and a subsequent paper will address this concern.

The application of this method requires that a detailed R,L, and C model of the transformer be available. Most suppliers of power transformers have this capability since the design of the transformers insulation structure depends very greatly on the ability to predict the transient voltages prior to construction and this requires the R, L, and C data. There are other techniques available to construct a reduced model if for any reason the R, L, and C matrices are not available [2,3].

The presented method is limited in detail only to the model created by the transient program. In the case presented this is a full multi-winding, three-phase linear iron core representation.

CONCLUSION

This paper has presented a method of reducing the size of the detailed lumped parameter model normally used for transformer insulation design to a size acceptable to a utility engineer performing systems studies. A significant advantage of this is that it can retain the frequency response characteristics of the transformer in a range of interest. The reduction technique retains the physical characteristics of the transformer model and the model is suitable to be used in transient analysis programs such as EMTP. The reduction technique is mathematically straight-forward and requires only the detailed model and user guidance on which nodes to retain. The method does not require that proprietary information about the transformer construction be used in the reduction process.

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APPENDIX 1

The reducing admittance matrix generated by eliminating one node at a time is shown in equation (8), and is rewritten as follows:

$$Y_{r1} = \frac{A_4}{C_{22}} s + \left(\frac{A_3}{C_{22}} - \frac{G_{22} A_4}{C_{22}^2} \right) + \left(\frac{A_2}{C_{22}} - \frac{\Gamma_{22} A_4 + G_{22} A_3}{C_{22}^2} + \frac{G_{22}^2 A_4}{C_{22}^3} \right) \frac{1}{s} + R_m \tag{A1}$$

Equation (A1) is obtained from equation (8), by long division where R_m represents the remainder which is given in equation (A2):

$$R_m = \frac{R_{m1} s - R_{m2}}{s(s^2 C_{22} + s G_{22} + \Gamma_{22})} \tag{A2}$$

where:

$$R_{m1} = A_1 - \frac{G_{22}A_2 + \Gamma_{22}A_3}{C_{22}} + \frac{2\Gamma_{22}G_{22}A_4 + G_{22}^2 A_3}{C_{22}^2} - \frac{G_{22}^3 A_4}{C_{22}^3}$$

$$R_{m2} = \frac{\Gamma_{22}A_2}{C_{22}} - \frac{\Gamma_{22}^2 A_4 + \Gamma_{22}G_{22}A_3}{C_{22}^2} + \frac{\Gamma_{22}G_{22}^2 A_4}{C_{22}^3} - A_o$$

Again, for the lossless case, the conductances in equation (A1) and (A2) can be set to zero which yields:

$$Y_{r1} = \frac{A_4}{C_{22}} s + \left(\frac{A_2}{C_{22}} - \frac{\Gamma_{22}A_4}{C_{22}^2} \right) \frac{1}{s} + R_m \qquad \text{(A3)}$$

where:

$$R_m = \frac{-R_{m2}}{sC_{22}\left(s^2 + \dfrac{\Gamma_{22}}{C_{22}}\right)}$$

$$R_{m2} = \frac{\Gamma_{22}A_2}{C_{22}} - \frac{\Gamma_{22}^2 A_4}{C_{22}^2} - A_o$$

Defining the nodal frequency associated with the node to be neglected as:

$$\omega_n = \sqrt{\frac{\Gamma_{22}}{C_{22}}} \qquad \text{(A4)}$$

Then, the remainder is given by:

$$R_m = \frac{-R_{m2}}{sC_{22}(s^2 + \omega_n^2)} \qquad \text{(A5)}$$

The remainder can be expanded by partial fractions as follows:

$$R_m = \frac{-R_{m2}}{C_{22}\omega_n^2} \frac{1}{s} + \frac{R_{m2}}{C_{22}} \omega_n^4 \frac{s}{\left[\left(\dfrac{s}{\omega_n}\right)^2 + 1 \right]} \qquad \text{(A6)}$$

setting $s = j\omega$ and substituting(A4) in (A6), the remainder can be approximated as:

$$R_m = \frac{-R_{m2}}{\Gamma_{22}} \frac{1}{s} + \frac{C_{22}R_{m2}}{\Gamma_{22}^2} s \quad \text{for } \omega < \omega_n ; \qquad \text{(A7)}$$

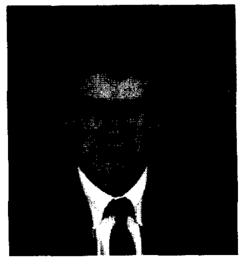
$$R_m = 0 \qquad \text{for } \omega > \omega$$

Therefore, for the lower frequencies ($\omega < \omega_n$) the remainder can be approximated by an equivalent inverse nodal inductance plus an equivalent nodal capacitance matrices. For higher frequencies the remainder can be neglected.

Substituting (A7) in (A3):

$$Y_{r1} = s\left(\frac{A_2}{\Gamma_{22}} - \frac{C_{22}A_o}{\Gamma_{22}^2} \right) + \frac{1}{s} \frac{A_o}{\Gamma_{22}} \ \ for\ \omega < \omega_n$$

$$Y_{r1} = s\frac{A_4}{C_{22}} + \frac{1}{s}\left(\frac{A_2}{C_{22}} - \frac{\Gamma_{22}A_4}{C_{22}^2} \right) \ \ for\ \omega > \omega_n \qquad \text{(A8)}$$



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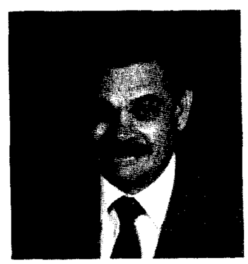

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