Chapter 3

- Radioactive Decay
- Specific Activity

RADIOACTIVE DECAY

Objectives

- Define the terms activity, radioactive decay constant, half-life, and specify the correct units
- State the equation for radioactive decay and explain each term
- Calculate activity (remaining or decayed away), decay constant, half-life, etc. given various terms in the radioactive decay equation

Activity

$$A = \lambda N$$

- Activity, A, is the term used to measure the decay rate of a radionuclide.
- The activity of a sample is based on the total number of radioactive atoms, N, and the probability of each atom undergoing radioactive decay.
- Activity has units of disintegrations per second or dps

Decay Constant, λ

$$\lambda = \frac{0.693}{T_{1/2}}$$

- The decay constant, λ, represents the probability that a radioactive atom will decay and is dependent on the half-life of the nuclide.
- Units of λ are 1/time (1/sec, sec-1 or per second)

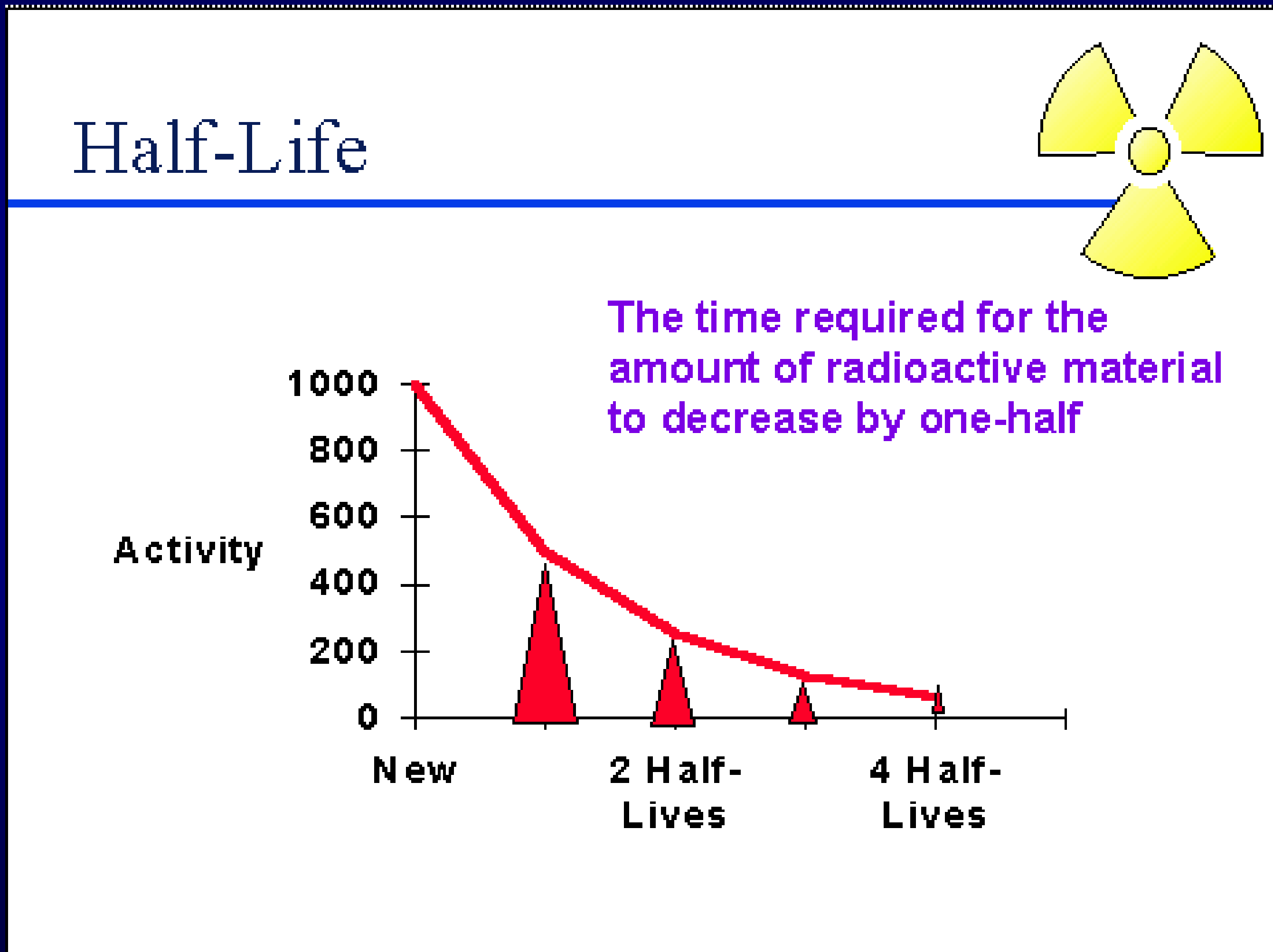
Activity Units

Curie (Ci) = 3.7×10^{10} dps

Becquerel (Bq) = 1 dps

1 Ci = 3.7×10^{10} Bq

Half-Life


Half-Life
The time required for the
amount of radioactive material
to decrease by one-half
Activity
1000
800
600
400
200
0
New
2 Half-
Lives
4 Half-
Lives

Half-Life

$$T_{1/2} = \frac{0.693}{\lambda}$$

Half-Life

Activity Problem

A criticality accident occurs in a Japanese uranium processing facility. 10^{19} fissions of U-235 occur over a 17-hour period. Given that the U-235 fission yield for I-131 is 0.03 and the half-life of I-131 is 8 days, calculate the I-131 activity at the end of the accident. Neglect I-131 decay during the accident.

$$A = \lambda N$$

Calculating N

A fission yield of 0.03 means that for every 100 fissions of U-235, three I-131 atoms are created.

$$N = 10^{19} \times 0.03$$
$$= 3 \times 10^{17} \text{ I-131 atoms}$$

Solution

Activity = λN

= (0.693/8 days) x (1/86,400 sec/day) x (3 x 10^{17} atoms)

= 3 x 10^{11} atoms/sec I-131

= 3 x 10^{11} dps I-131

Converting to traditional units:

3 x 10^{11} / (3.7 x 10^{10} dps/Ci) = 8.1 Ci I-131

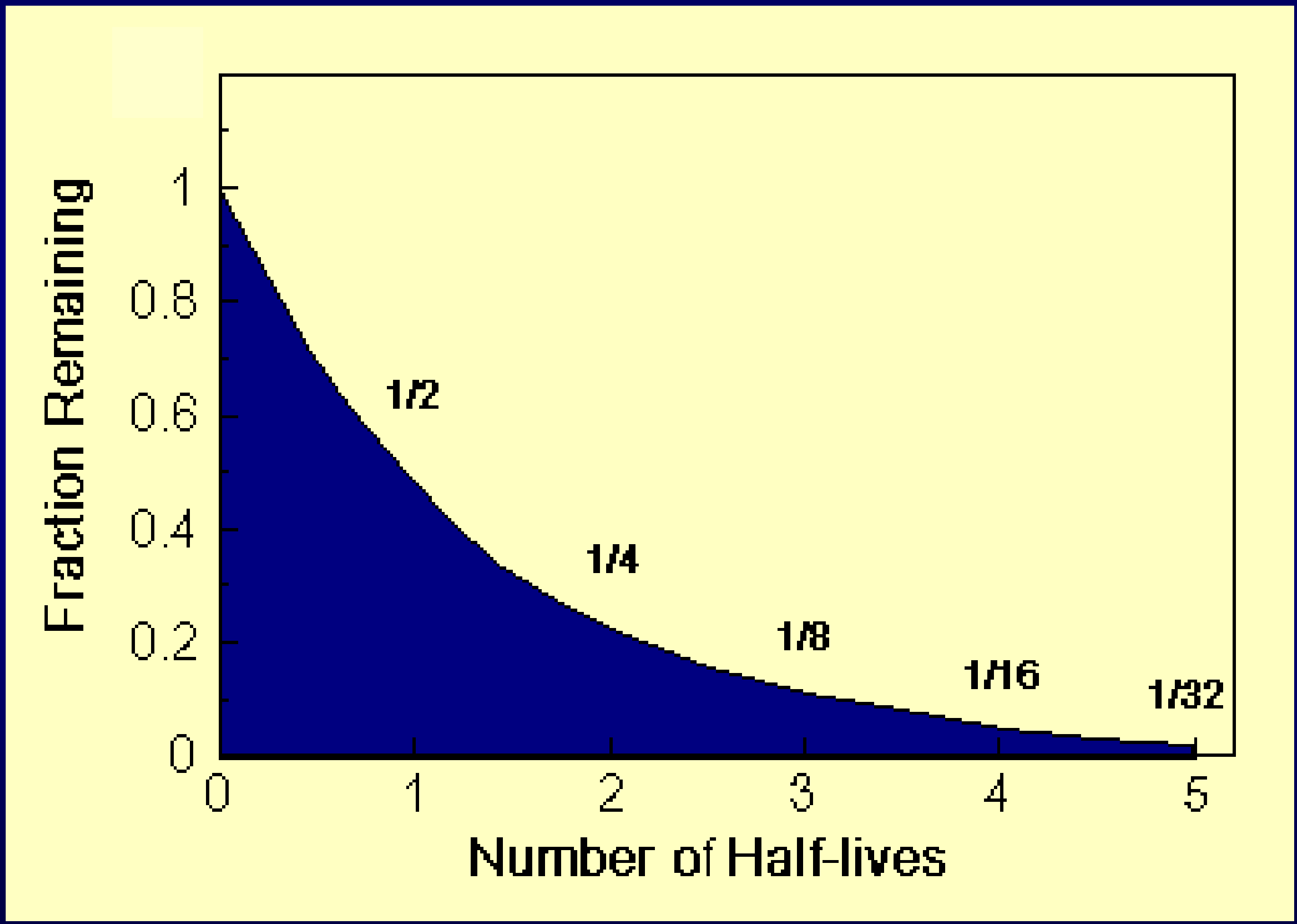
Decay Equation

$$\frac{dN}{dt} = -\lambda N$$

Decay Equation

$$N(t) = N_0 \, e^{-\lambda t}$$

Radioactive Decay



Activity Equation

Multiply both sides by λ,

$$\lambda N(t) = \lambda N_0 e^{-\lambda t}$$

Activity Equation

Recall $A = \lambda N$

$$A(t) = A_0 e^{-\lambda t}$$

Radioactive Decay

The fraction of activity A remaining after n half-lives is given by:

$$\frac{A}{A_0} = \frac{1}{2^n}$$

$$A = A_o e^{(-\lambda t)} \quad \text{or} \quad A = A_o (½)^n$$

These two equations are identical! Here's how:

Example

$A = A_o e^{(-\lambda t)}$ but $\lambda = \ln(2)/T_{1/2}$ so that

$A = A_o e^{\{-\ln(2)/T_{1/2} * t\}}$

but -ln(2) = ln(1/2) and t can be measured in the number n, of half-lives that have passed ($t = nT_{1/2}$) Putting these values in our equation, we get:

$= A_o e^{\{\ln(1/2)/T_{1/2} * nT1/2\}}$

$= A_o e^{\{n\ln(½)\}} = A_o e^{\{\ln[(½)^n]\}} = A_o (½)^n$

Since the exponential of a logarithm $e^{\ln(A)}$ is just the value "A"

Radioactive Decay

The fraction of activity decayed away after n half-lives is given by:

$$1 - (A/A_0)$$

Problem

Suppose you have 10^6 atoms of F-18 that were created in a water target at a cyclotron facility. How many F-18 atoms remain after the target sits and decays for 220 minutes?

Recall that $A(t) = A_0 e^{-\lambda t}$ and in this case, the half-life of F-18 is ~ 110 minutes, so

$$A(t) = A_0 e^{-\lambda t} = 10^6 \text{ atoms} * e^{-.693/110 \text{ min} * 220 \text{ min}} = 2.5E5 \text{ atoms}$$

Solution

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Solution

Another way of solving this would be to use the relationship:

$$A = \frac{A_0}{2^n}$$

Since two half-lives have passed (220 min), n = 2 and:

$$A = \frac{A_0}{2^n} = \frac{10^6 \text{ atoms}}{2^2} = \frac{10^6 \text{ atoms}}{4} = 2.5\text{E}5 \text{ atoms}$$

END OF RADIOACTIVE DECAY

SPECIFIC ACTIVITY

Objectives

- Define the term specific activity
- Explain each term given the equation for specific activity
- Calculate the specific activity of various radioisotopes

Specific Activity

Specific Activity is the activity per unit mass

Typical units: Ci/kg or Bq/g

Atoms per Gram

The number of atoms of a radionuclide in one gram is given by

$$N = \frac{6.02 \times 10^{23} \dfrac{\text{atoms}}{\text{mole}}}{A_w \dfrac{\text{grams}}{\text{mole}}}$$

This gives us the number of atoms per gram of the radionuclide

Grams per Mole

Examples of calculating number of grams in one mole of a radionuclide:

- **In one mole of Co-60, there are 60 grams**
- **In one mole of U-235, there are 235 grams**
- **In one mole of Na-24, there are 24 grams**
- **In one mole of P-32, there are 32 grams**

Specific Activity

The activity in one gram is then given by:

$$SA = \lambda N$$

$$= \lambda \times 6.02 \times 10^{23} / A_w \ (\text{dps} \cdot \text{gram}^{-1})$$

$$= \text{Bqs/gram}$$

Specific Activity

Specific Activity (S.A.) in curies/gram =

$$= \lambda \times 6.02 \times 10^{23} / A_w \ (\text{dps} \cdot \text{gram}^{-1})$$

$$\frac{0.693}{T_{1/2}\ (\text{secs})} \cdot \frac{6.02 \times 10^{23}\ \text{atoms}}{\text{mole}} \cdot \frac{\text{mole}}{A_w\ \text{grams}} \cdot \frac{\text{curie}}{3.7 \times 10^{10}\ \text{atoms/sec}}$$

Specific Activity

0.693	6.02×10^{23} ~~atoms~~	~~mole~~	curie
$T_{1/2}$ ~~(secs)~~	~~mole~~	A_w grams	3.7×10^{10} ~~atoms/sec~~

S.A.

$$= (1.13 \times 10^{13}) / A_w T_{1/2} \text{ (curies/gram)}$$

where A_w = atomic weight in grams

and $T_{1/2}$ = half-life in seconds*

*Recall that units on λ are 1/s

Problem

Calculate the specific activity of Pu-239, given that the half-life is 24,400 years

Problem

Given that the specific activity of natural U is 7 x 10^{-7} Ci per g, calculate the ratio of the specific activities of Pu-239 and natural U.

Mass vs Activity

0.001 g

$^{60}_{27}Co$

1 g

$^{226}_{88}Ra$

1,428,571 g

^{Nat}U

Amount in grams of each isotope equaling one curie of activity

END OF SPECIFIC ACTIVITY

END OF CHAPTER 3