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 DENTON, H.R. Office of Nuclear Reactor Regulation, Director

SUBJECT: Forwards addl info re Mark I torus mods to torus attached piping, as part of containment plant-unique analysis rept, per D Vassallo 840322 request.

SEE REPTS.

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Extra  
 Cys To: B. Siegel

Iowa Electric Light and Power Company

April 23, 1984  
NG-84-1615

Mr. Harold Denton, Director  
Office of Nuclear Reactor Regulation  
U.S. Nuclear Regulatory Commission  
Washington, DC 20555

Subject: Duane Arnold Energy Center  
Docket No: 50-331  
Op. License No: DPR-49  
Mark I Torus Modifications-Torus Attached Piping  
Reference: NRC Letter, D. Vassallo to L. Liu, dated  
March 22, 1984

Dear Mr. Denton:

The attachment to this letter provides the information requested by the referenced letter regarding our Plant Unique Analysis Report (PUAR) for our Mark I torus modifications.

If you have any additional questions, please contact us.

Very truly yours,

*R. W. McGaughy*  
Richard W. McGaughy  
Manager, Nuclear Division

RWM/SS/pt\*

Attachment: A. Response to NRC Request for Additional Information  
Regarding Mark I Containment Plant Unique Analysis Report  
B. User Manual for Computer Program CMDOF  
C. Frequency Response Analysis

cc: S. Swails  
L. Liu  
S. Tuthill  
M. Thadani  
NRC Resident Office  
Commitment Control No: 84-0077

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50-331

MARK I CONTAINMENT PLANT UNIQUE ANALYSIS REPORT  
USER MANUAL FOR COMPUTER PROGRAM CMDOF  
FREQUENCY RESPONSE ANALYSIS

Docket # 50-331

Control # 8404260205

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DUANE ARNOLD ENERGY CENTER

DOCKET NO. 50-331

RESPONSE TO NRC REQUEST FOR

ADDITIONAL INFORMATION REGARDING MARK I

CONTAINMENT PLANT UNIQUE ANALYSIS REPORT

TORUS ATTACHED PIPING

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Item 1: With respect to the Section 6-2.4.1 (1), provide justifications for truncating piping models (in some cases) at locations beyond the point where combined stress levels due to Mark I Program Loadings are less than 10% of the appropriate ASME code allowables. Indicate the effect of model truncation on the overall piping analysis.

Response: The majority of the torus attached piping systems analyzed as a requirement of NUREG-0661 were small enough in extent so as to allow complete modeling. This means that the piping models extended out from the torus to an anchor at either a piece of equipment (pump, turbine) or an attachment to a larger main piping run. The RHR pump discharge systems, however, were of such extent due to the length of piping and the number of interconnections that it was necessary to truncate these models.

Two separate truncated models were developed for each of the RHR pump discharge piping systems. The first one was somewhat shorter than the second model and was utilized for Mark I loads. The second, longer model was utilized for non-Mark I loads (thermal, seismic, etc.).

The use of shortened models for Mark I loads is possible since the overall piping response attenuates with distance from the torus (load source). The validity of the short model was confirmed by analyzing the piping system for CO torus motions (1 case) with both the short and long model and comparing the results.

In the case of non-Mark I loads, the model used was longer as it was not required to perform coupling analysis on this model. The model extent was such that sufficient piping and supports were modeled beyond the point of interest (10% of allowable).

Item 2: Tables 6-2.5-1, 6-5.4-2, and 6-6.5-1 of Reference 1 indicate that the calculated values of certain stresses are close to or equal to the respective allowables. Indicate conservatism sources (if any) in the analysis to justify high stress levels.

Response: The stresses shown in the referenced tables are calculated utilizing loads developed in accordance with the LDR and analytical methods, both defined in the PUAR and in accordance with the PUAAG. As such, the stress results, as they are less than code allowables, meet the requirements of NUREG-0661.

Although the specific details of the load and analytical conservatisms are not quantified here, Section 1-1.4 of the PUAR contains a description of some of the conservatisms inherent in the plant unique analysis. It is felt that the small margins to code allowables do not require additional justification as there is conservatism in the calculated stress results and the code allowables themselves are conservatively determined.

Item 3(a): Provide the theory used to develop the computer program CMDOF. Also, show how this program has been been verified. Indicate whether any simplified dynamic model including both the torus and piping (direct coupled analysis) has been used to verify the proposed approach. It should be noted that the torus attached piping system exhibits a complex dynamic behavior; it is not advisable to verify the proposed approach by using a simple model (if this is the case).

Response: CMDOF is a NUTECH Proprietary Computer Program which was developed to perform coupling analysis in association with the NUTECH Piping Computer Program PISTAR. CMDOF is based upon, and has as a central building block, the CMDOF program developed by Structural Mechanics Associates (SMA) for the Mark I owners. The theory behind the SMA computer program CMDOF is presented in detail in the SMA report (Attachment B), SMA 12101.03.R001, by R. P. Kennedy and R. H. Kincaid of SMA. This SMA report also includes a verification of the computer program. NUTECH's computer program CMDOF was verified through demonstration that NUTECH's program gave the same coupled response (displacement time histories and acceleration time histories) as the SMA program.

As indicated in the SMA report, the program CMDOF is intended primarily for MARK I torus attached piping (TAP) coupling analysis. The coupling analysis results were verified using both coupled structure-equipment models and an independent computer program HDYNE.

Four sample problems were specifically selected by the authors and are included in the SMA report. These problems provide a complete and thorough test of the CMDOF program. The test models are designed such that the uncoupled equipment modeled has similar mass, stiffness, and frequency characteristics as the corresponding uncoupled structure. This approach provides an excellent test since the similarity in dynamic characteristics makes the dynamic coupling effect both significant and complex. This verification demonstrates that similar results are obtained using both the CMDOF program and a full explicitly coupled model.

Item 3(b): Indicate whether the transfer function approach has been adequately verified in view of the fact that the final solution obtained via this approach must rely on a significant number of intermediate calculation steps. Also, provide step-by-step procedures (mathematical expressions) of the transfer function approach beginning with the equations of motion of the torus (subject to a white noise motion).

Response: The transfer function approach has been verified by comparing explicit time history results with those results obtained using transfer functions. A brief description of the verification is provided below to show the adequacy of the verification. The transfer function method is performed through use of the TFUNCTION computer program.

The test model contains a typical uncoupled torus and a complex uncoupled torus attached piping system. Two parallel coupling analyses are performed to calculate two different sets of coupled piping responses due to two different dynamic loadings. The first loading is an actual time history, while the other loading is an artificial white-noise time history. The verification involves the comparison of the piping response time history due to the actual loading with the response obtained using the transfer function approach with both the knowledge of the actual loading time history and the calculated piping response time history due to the artificial white-noise loading.

Attachment C contains a detailed theoretical description of the transfer function approach.



Item 3(c): Indicate the characteristics of the white noise time-history used as input to the mathematical model of the torus to obtain a desired transfer function. Explain the relationship between the white noise function and the specified hydrodynamic loads.

Response: Comparison of the characteristics of a white-noise function and a specified hydrodynamic loading can be made based on their spatial and temporal distributions.

The specified hydrodynamic loading has a spatial distribution assumed to be independent of time. This time-invariant spatial distribution of loading is also assumed for the white-noise function used in the transfer function approach. The only adjustment is that the white-noise spatial distribution is normalized.

The time history or temporal distribution of the white-noise function was initially defined analytically by the following equation during the development of the piping approach:

$$WN(t) = \frac{\sin \alpha t}{\alpha t}$$

where C is a constant and  $\alpha$  is a cut-off frequency parameter of the time history. Later, it was determined that the time history should be mirrored in order to give a flatter fourier transform. Thus, the time history used in the final torus attached piping analyses is defined by:

$$WN(t) = C \frac{\sin(2\pi\alpha - T)}{2\pi\alpha - T}$$

where T is the time of maximum amplitude. In the time history shown in Figure 3, the cut-off frequency is 83 Hz. For the DAEC torus attached piping analyses, a cut-off frequency of 62.5 Hz was used.

Item 3(d): Show (by some figures) how the critical piping frequencies were obtained from the transfer function based on the prescribed hydrodynamic load frequency range. Also, show how the hydrodynamic load was tuned to the critical piping frequencies.

Response: The transfer function of a piping response component is first calculated and plotted as a function of frequency. Due to the small damping values that are used in the analysis of the piping system, the transfer function plot shows typically sharp peaks and deep valleys. The locations of the peaks (or local maxima) within the prescribed hydrodynamic load frequency range then define the critical frequencies for the piping response component under consideration (see Figure 1). The selection of critical frequencies for the piping analysis or "tuning", is performed by determining those frequencies within the load range with the highest number of peaks.

Use of the above procedure for many piping component responses on the same piping system yields a collection of critical frequencies of the piping system. In general, the critical frequencies for different piping component responses on a given system are grouped at a few select frequencies. Therefore, there are not excessive critical frequencies of a piping system within its prescribed hydrodynamic load frequency range.

As given the Load Definition Report (LDR), the frequency of a hydrodynamic loading may assume any single value within its prescribed frequency range (Figure 2). This apparent frequency is tuned (or scaled) to one of the critical piping frequencies previously determined. The tuning defines a design hydrodynamic loading for which piping responses are calculated using the transfer function approach.

Iteration of the above tuning and analysis procedure over many critical frequencies yields a collection of maximum credible piping responses. Bounding values of these maximum credible piping responses are then used conservatively for design.

A-7

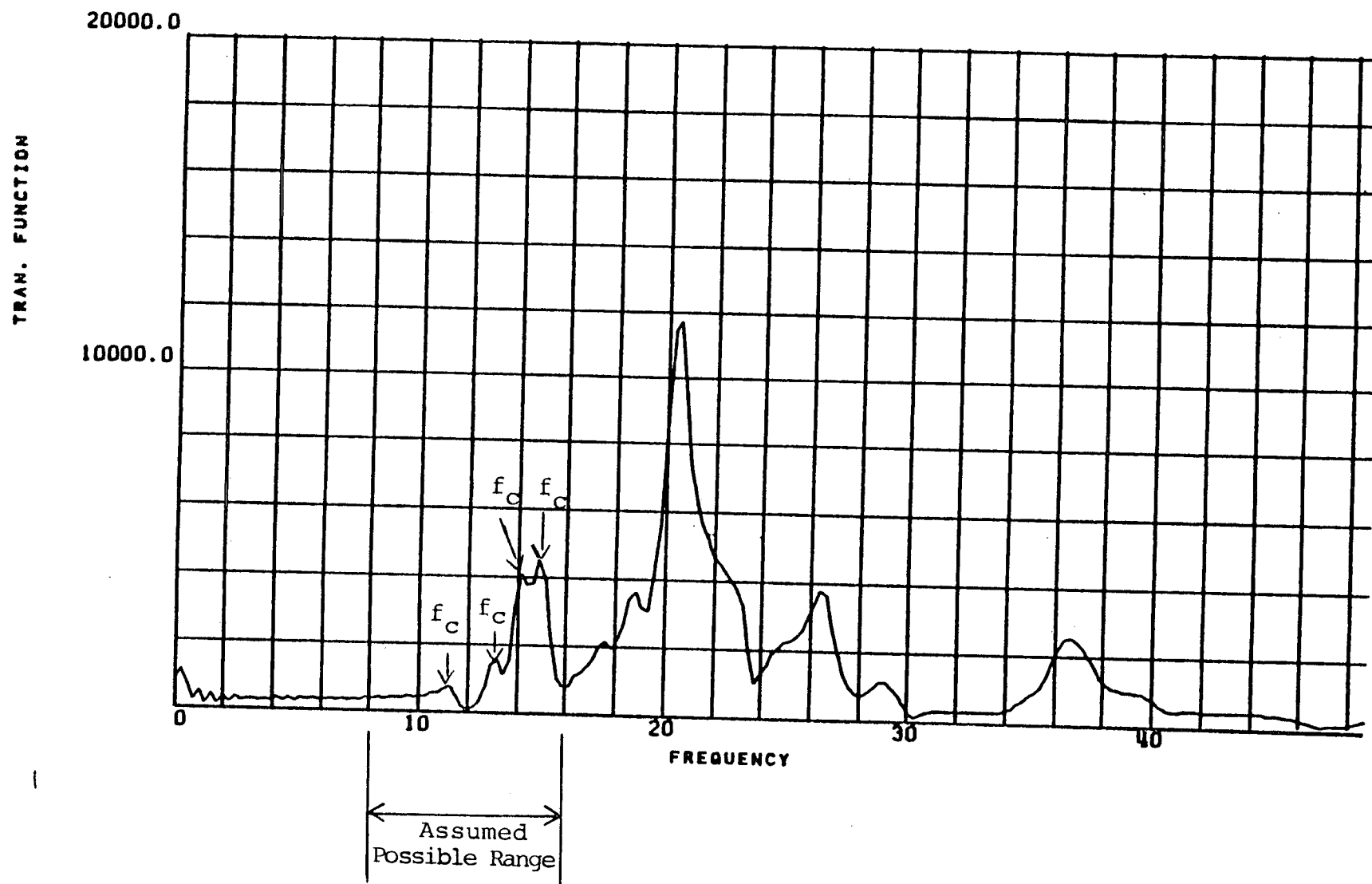


Figure 1 TRANSFER FUNCTION AND CRITICAL FREQUENCIES ( $f_c$ )

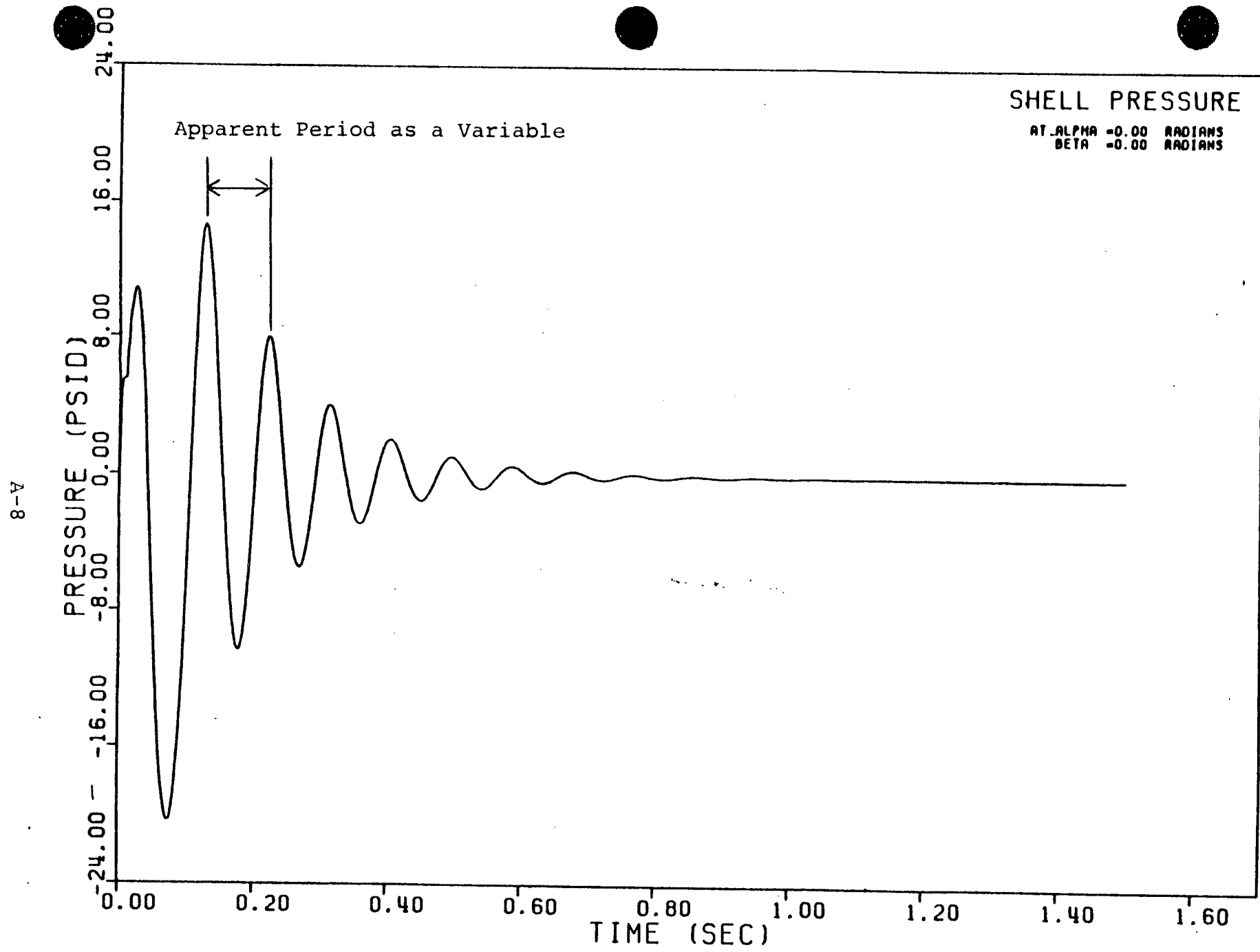


Figure 2 HYDRODYNAMIC BUBBLE LOAD

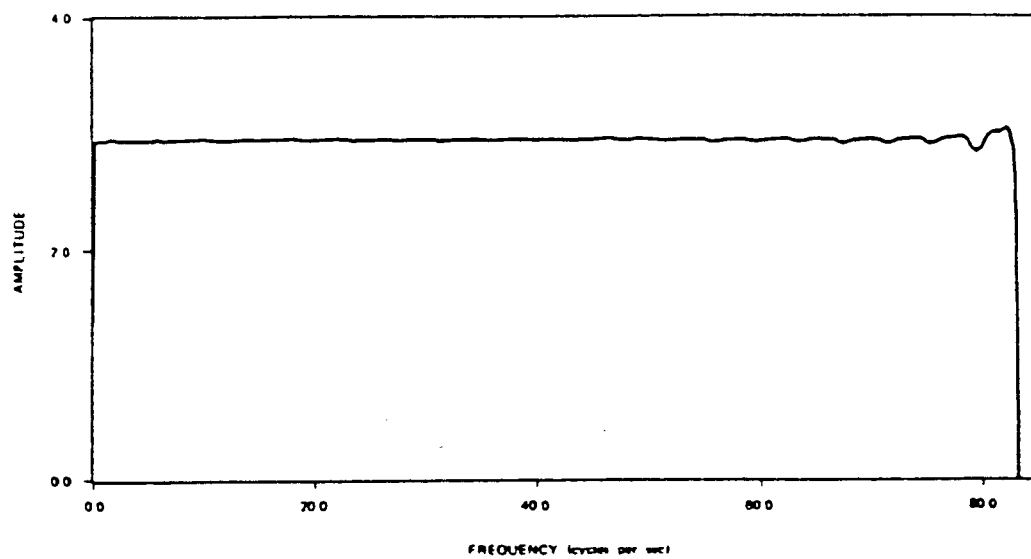
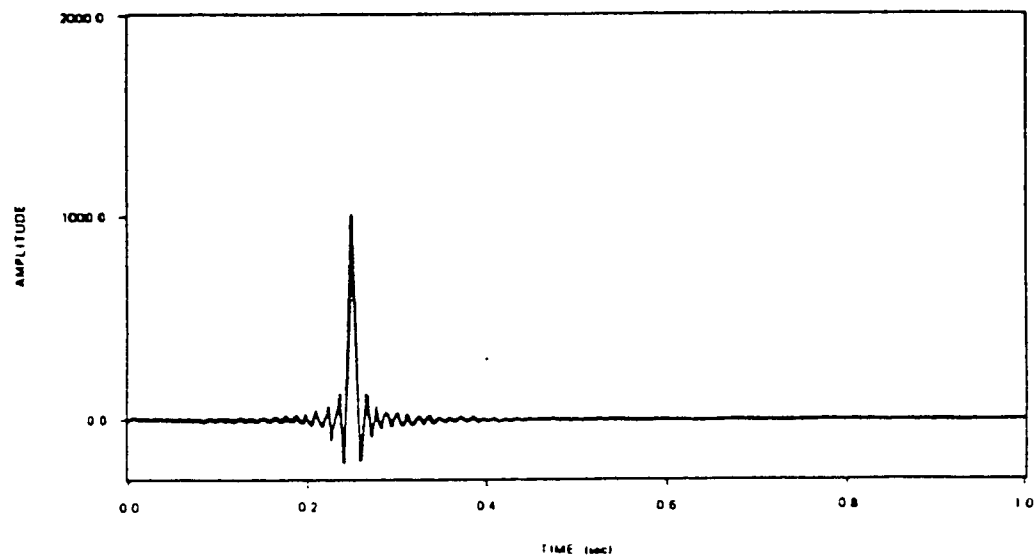


Figure 3 Time-History and Fourier transform of White Noise

Attachment B  
NG-84-1615  
April 23, 1984

Attachment B

USER MANUAL FOR COMPUTER PROGRAM CMDOF

by

Structural Mechanics Associates, Inc.

Attachment A

USER MANUAL FOR COMPUTER PROGRAM CMDOF

by

Structural Mechanics Associates, Inc.



SMA 12101.03.R001

CMDOF

(COUPLING OF MULTIPLE DEGREES OF FREEDOM)

A COMPUTER PROGRAM TO COUPLE THE  
RESPONSE OF STRUCTURES AND SUPPORTED EQUIPMENT  
FOR MULTIPLE DEGREES OF COUPLING USING  
THE RESULTS FROM UNCOUPLED STRUCTURE AND EQUIPMENT ANALYSIS

by

R. P. Kennedy  
R. H. Kincaid

prepared for

A-E TASK FORCE, MARK I OWNERS GROUP

November, 1980



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## 1. INTRODUCTION

### 1.1 DESCRIPTION OF PROBLEM

The most prevalent method for performing dynamic analyses of structures and light supported equipment when the structure is subjected to dynamic input is to perform independent uncoupled dynamic analyses of the structure and of the supported equipment. In this approach, a detailed structural model is developed (often this model includes the mass content of the supported equipment but does not incorporate the frequency or stiffness characteristics of the supported equipment) and a dynamic analysis of the structure subjected to the dynamic input is performed using this uncoupled model. The response time history at the attachment point of the supported equipment is obtained. Then, this response time history (or else the corresponding floor spectra obtained from this response time history) is used in conjunction with an uncoupled dynamic model of the equipment in order to calculate equipment response. This approach is known as an uncoupled analysis because the dynamic model of the structure and the equipment are never directly coupled.

It has been recognized that uncoupled analyses can significantly overestimate the response of the supported equipment in some cases. The response time history at the equipment attachment point obtained from an uncoupled structure analysis will contain harmonic oscillatory response at each of the uncoupled structure modes excited by the input time history. Thus, the floor spectra from this time history will show substantial amplified spectral acceleration at each of the important uncoupled structure modes. If the uncoupled equipment model has any important equipment modes with frequencies in the vicinity of any of the frequencies of these important uncoupled structure modes, then this uncoupled response time history (or corresponding floor spectra) when applied as input to the uncoupled equipment model will excite a substantial amplified response of the equipment. However, in reality, when any uncoupled structure natural frequencies and any uncoupled equipment natural

frequencies are nearly the same, the supported equipment will modify and generally suppress to some extent the response of the structure at that frequency in the vicinity of the equipment attachment point. Thus, in this case, the structure response time history input will not be the same as that obtained from an uncoupled structure analysis, but will be partially suppressed at each of the important equipment natural frequencies that correspond to a structure natural frequency. In this case, the equipment response will generally be reduced.\* This effect on reducing equipment response is often very important in those cases where uncoupled structure and uncoupled equipment natural frequencies are similar. In fact, uncoupled analysis will often result in substantial overestimation of equipment response whenever uncoupled frequencies are similar and the ratio of effective modal mass associated with the attachment point for the equipment and the structure is greater than about 0.1 percent. Thus, if the effective equipment modal weight is 1000 pounds, this effect is important if the effective modal weight of the structure at the equipment attachment point for a corresponding structure mode is less than about  $1 \times 10^6$  pounds (Reference 3). Actually, the effective modal weight of the structure is often much less than the total structure weight. This is particularly true for the higher modes. Thus, if the uncoupled response floor spectra shown significant amplified response at higher mode frequencies, equipment coupling is particularly effective in reducing these responses.

For major equipment items, this overestimation of equipment response is most easily corrected by performing a coupled analysis in which a single dynamic model of both the structure and equipment is used. In this way, the coupling effects between the structure and equipment are automatically included. However, a coupled analysis is not practical for every item of equipment or for the majority of the piping loops. For these cases, the only practical option is to perform uncoupled analyses, even where such analyses may be overconservative.

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\* Coupling effects can also shift a coupled natural frequency more into resonance with the input forcing function so that in some cases the coupled response will exceed the uncoupled response. However, an increase in response is substantially less likely than suppression.

The computer program CMDOF (Coupling of Multiple Degrees of Freedom) enables coupling effects to be incorporated into the results of uncoupled structures and equipment analyses. The procedure is as follows:

1. Perform uncoupled structure response analysis using standard uncoupled analysis techniques.
2. Determine modal response characteristics of an uncoupled structure model subjected to an applied unit force/moment at the equipment attachment points for each coupling degree-of freedom (dof) for which coupling effects are to be considered.
3. Determine modal response characteristics of each uncoupled equipment model subjected to an applied unit force/moment at each attachment dof.
4. Input into the coupling program (CMDOF) the uncoupled structure response time histories at the equipment attachment points for each dof to be considered in the coupling analysis (from Step #1), the uncoupled structure modal response characteristics (from Step #2), and the uncoupled equipment modal response characteristics (from Step #3). The output of program CMDOF is the coupled response time history at the equipment attachment points for all dof when coupling effects between the equipment and structure are considered.
5. Use the coupled response time histories at the equipment attachment point (or corresponding floor response spectra) as input to an uncoupled equipment analysis. The resultant equipment response includes the effect of coupling between the structure and equipment.

This procedure enables the effects of structure and equipment coupling to be incorporated for equipment or piping in those many cases that it is not practical to perform a single coupled analysis. Keeping in mind that uncoupled analyses are nearly always conservative, the following are suggested for practical use of Computer Program CMDOF:

- A. Perform uncoupled structure analysis (Step #1).
- B. Perform uncoupled equipment analysis using input from Step #1. If equipment responses are unacceptable, then proceed to the following steps.
- C. Estimate whether a coupled analysis might relieve the calculated unacceptable responses. If so, perform Steps #2 through #5 and use the coupled response results.

## 1.2 PRACTICAL APPLICATIONS

A Mark I torus is schematically illustrated in Figure 1-1. The torus at the Peach Bottom Power Plant has dimensions which are typical for the Mark I series. This torus has an overall center to center diameter of approximately 111 feet and a cross-sectional diameter of 30 feet. The torus shell wall thickness is 0.675 inches in regions below the top of the suppression pool and 0.60 inches above the top of the suppression pool. Since the shell has a relatively large radius to thickness ratio there are a number of significant structure modes which exist at frequencies less than 30 hertz. For this torus, approximately 60 modes have frequencies less than 30 hertz. Thus, there are many opportunities for an equipment or piping modal frequency to be close to a structure modal frequency. For the higher modes, the effective structure modal mass is very low compared to the total mass due to the localized nature of the response. Thus, a very small equipment modal weight can be effective in modifying the amplified structure response at the equipment attachment degrees-of-freedom at the corresponding uncoupled equipment frequencies.

The lower portion (about 14 feet) of this steel torus holds water in the suppression pool. Safety relief valves discharge into this pool (Figure 1-2). The resultant discharge subjects the steel shell to oscillatory input loads from the water with predominant frequency content up to the 18 hz range but also with significant input content at much higher frequencies. Typically, an uncoupled analysis of the shell shows substantial response amplification and high floor spectra at approximately 21 hz. When this uncoupled structure response data is used as input to attached equipment or piping (such as schematically shown in Figure 1-2),



attached pipe or equipment for which a coupled analysis were desired. Either option would be very costly. Furthermore, if piping supports were changed, it would be necessary to repeat the full coupled analysis.

Because of the 1) expected substantial modification of response of the piping due to coupling effects, and 2) difficulty in performing a coupled analysis, this problem represents a good case for the application of the response coupling program CMDOF. In fact, the program was specifically developed with this problem in mind. However, the program is also useful for many other similar problems in which coupled responses are designed using uncoupled models and analyses.

The uncoupled response time history ( $a_u$ ) for each attachment degree-of-freedom of the structure at attachment point A (Figure 1-2) is obtained for the applied loading using an uncoupled, structure model with the asymmetric SRV discharge oscillatory loading (Step #1). Actually, a single uncoupled analysis provides the uncoupled response time history at all attachment points for any piping or equipment attached along the structure. The next step is to determine the modal responses of the containment shell to an externally applied force/moment at all attachment degrees-of-freedom (Step #2). This will require a complete eigensystem analysis of the torus for all modes below an appropriate cutoff frequency. Again, a single eigensystem analysis can develop this information for all attachment nodes of interest on the shell. For the attached piping system, the modal responses of the piping to an externally applied force/moment at all attachment degrees-of-freedom must also be determined. This is done in a manner analogous to the eigensystem analysis of the uncoupled torus. With this information, the program CMDOF (Step #4) corrects the uncoupled response time history ( $a_u$ ) for each attachment point degree-of-freedom to account for coupling effects and outputs a coupled acceleration time history ( $a_c$ ) and coupling reaction time history (R) for all attachment degrees-of-freedom. These output time histories or their response spectra are used as input to an uncoupled piping model (Step #5) in order to calculate the true coupled response of the piping.

$d_{c_j}(t_i)$  = Coupled displacement of structure attachment point degree-of-freedom  $j$  at time  $t_i$ .

$d_{u_j}(t_i)$  = Uncoupled displacement of structure attachment point degree-of-freedom  $j$  at time  $t_i$ .

$f_{E_m}$  = Equipment mode frequency (Hz) for mode  $m$ .

$f_{S_m}$  = Equipment mode frequency (Hz) for mode  $m$ .

$j, k$  = Attachment point degree-of-freedom numbers.

$m$  = Mode number.

$M_E$  = Total number of equipment modes.

$M_S$  = Total number of structure modes.

$NC$  = Total number of degrees-of-freedom at which coupling is considered between structure and equipment.

$PF_{E_{j,m}}$  = Uncoupled equipment participation factor for mode  $m$  associated with an applied unit force/moment at attachment degree-of-freedom  $j$  for the equipment attachment node.

$PF_{S_{j,m}}$  = Uncoupled structure participation factor for mode  $m$  associated with applied unit force/moment at attachment degree-of-freedom  $j$  for the equipment attachment node on the structure.

$R_j(t_i)$  = Coupling reaction acceleration at time  $t_i$  for degree-of-freedom  $j$ .

$\Delta R_j(t_i)$  = Change in coupling reaction acceleration from time  $t_{i-1}$  to time  $t_i$  at degree-of-freedom  $j$

$t_i$  = Time at time step  $i$ .

$V_{c_j}(t_i)$  = Coupled velocity of structure attachment point degree-of-freedom  $j$  at time  $t_i$ .

$\ddot{Y}_{m,j}(t_i), \dot{Y}_{m,j}(t_i), Y_{m,j}(t_i)$  = Modal acceleration, velocity, and displacement for uncoupled structure mode  $m$  dof  $j$  at time  $t_i$ , respectively.

$\ddot{Z}_{m,j}(t_i), \dot{Z}_{m,j}(t_i), Z_{m,j}(t_i)$  = Modal relative acceleration, velocity, and displacement for uncoupled equipment mode  $m$ , dof  $j$  at time  $t_i$ , respectively.

$\Delta t_i$  = Time step size from time  $t_{i-1}$  to time  $t_i$ .

$\lambda_{E_m}$  = Ratio of m-th equipment mode damping to critical damping.

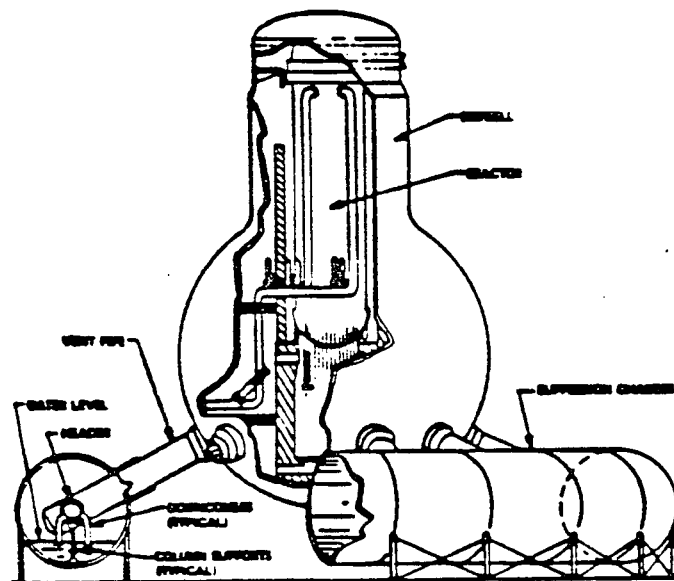
$\lambda_{S_m}$  = Ratio of m-th structure mode damping to critical damping.

$\phi_{E_{j,m}}$  = Uncoupled equipment eigenvector for degree-of-freedom j and mode m.

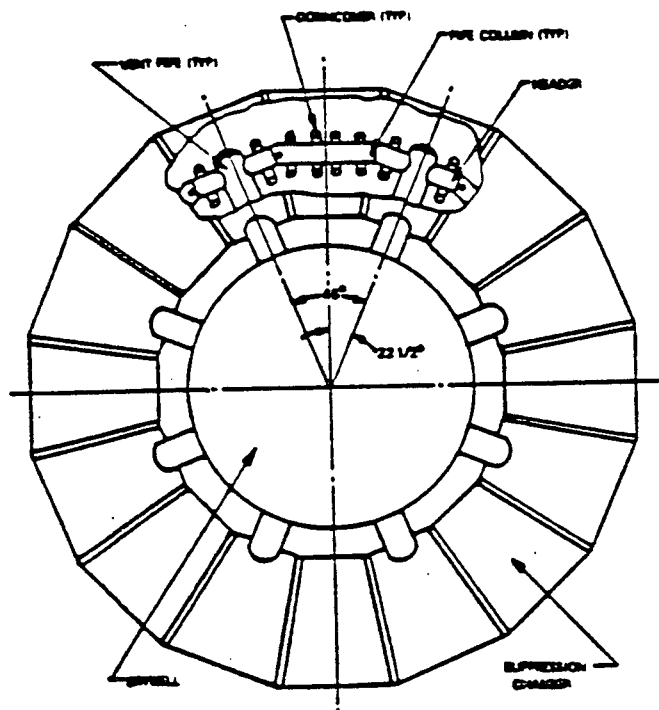
$\phi_{S_{j,m}}$  = Uncoupled structure eigenvector value for degree-of-freedom j and mode m.

$\omega_{E_m}$  = Circular natural frequency for equipment mode m.

$\omega_{S_m}$  = Circular natural frequency for structure mode m.



OVERALL SCHEMATIC OF MARK I VENT HEADER SYSTEM



PLAN VIEW OF VENT HEADER SYSTEM

FIGURE 1-1: TYPICAL MARK I TORUS AND REACTOR

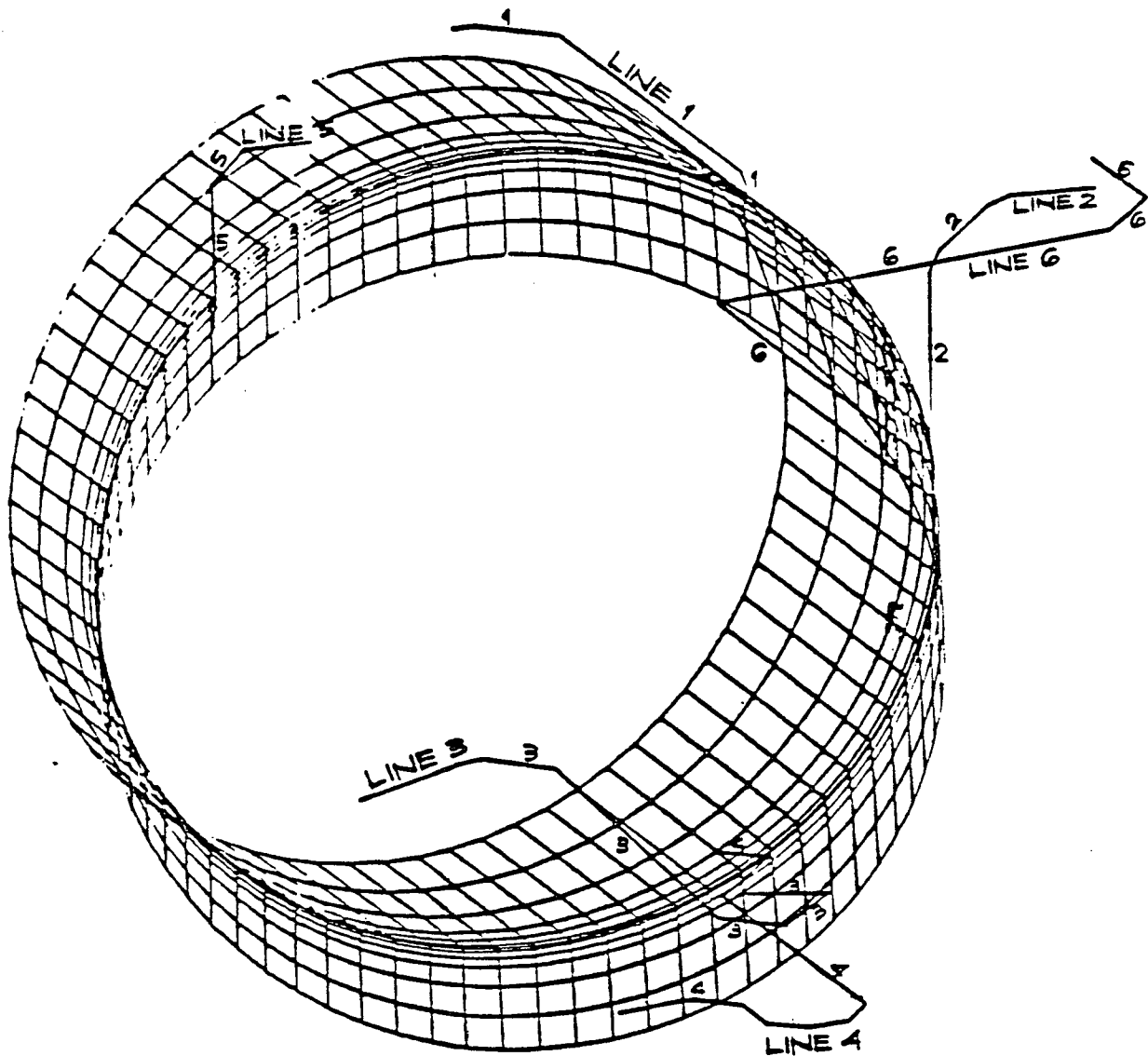


FIGURE 1-3: TYPICAL COUPLED MARK I TORUS AND PIPING SYSTEMS

## 2. TECHNICAL BASIS

### 2.1 ASSUMPTIONS

1. At the attachment point, the structure uncoupled and coupled acceleration time histories and the equipment reaction time history for all degrees-of-freedom are assumed to vary linearly between time steps. This assumption is common to many dynamic analysis programs and the time step size for this coupling program does not have to be any smaller than for most other dynamic analysis programs. A time step equal to  $(0.1/f)$  where  $f$  is the highest natural frequency of interest is generally considered adequate.
2. Both the structure and the attached equipment are considered to behave linearly elastic. Superposition is used extensively in this program.
3. Sufficient structural and equipment modes are included to accurately model the structure and equipment at the degree-of-freedom (dof) of interest.

### 2.2 COMPATIBILITY AND SUPERPOSITION

Figure 2-1 illustrates an uncoupled structure model and uncoupled equipment models which are attached to the structure at a series of different nodes with one dof (dof) of coupling per node. Another model which would be just as valid for use in conjunction with the theory which follows is one in which the equipment is attached to the structure at only a couple of nodes but there are multiple degrees of coupling at each attachment node. In the following discussion, there are assumed to be  $NC$  coupled dof between the structure and equipment.

As shown in Figure 2-1, the uncoupled structure is subjected to an input force loading  $F(t_i)$  which in the absence of the attached equipment results in a series of uncoupled accelerations  $a_{u_j}(t_i)$  for each attachment mode at time  $t_i$ . By knowing the modal characteristics of the uncoupled structure and uncoupled equipment models (frequency, eigenvector, participation factor for each of the NC attachment dof) the coupling reaction  $R_j(t_i)$  for each dof applied by the equipment on the structure at time  $t_i$  can be determined from the coupled accelerations  $a_{c_j}$  defined at all previous time steps  $t_0$  through  $t_{i-1}$ . This reaction  $R_j(t_i)$  on the uncoupled structure model results in the structural response acceleration  $a_{R_j}(t_i)$  at dof  $j$  where  $a_{R_j}(t_i)$  is the acceleration at time  $t_i$  due to the equipment reaction  $R_j$  for all previous times considering the contributions of all NC attachment dof between the equipment and structure. By using superposition, the coupled acceleration of the attachment point for dof  $j$  is given by:

$$a_{c_j}(t_i) = a_{u_j}(t_i) + a_{R_j}(t_i) \quad (2.1)$$

Note that  $a_{R_j}(t_i)$  is due to the equipment reaction time histories  $R_k$  for all coupling DOF through time  $t_i$  which, in turn, are due to the coupled acceleration time histories  $a_{c_j}$  for all coupled DOF through time  $t_i$ .

Compatibility requires that the coupled accelerations  $a_{c_j}(t_i)$  and the reactions  $R_j(t_i)$  at the attachment points be identical for the structure and attached equipment for every time point  $t_i$ . Similarly, superposition requires that Equation 2.1 be satisfied



at every point  $t_i$ . At the outset, only the uncoupled response time histories defined by  $a_{U_j}(t_i)$  are known at each time point  $t_i$ . The essence of this program is to determine the reaction  $R_j(t_i)$  and reaction acceleration  $a_{R_j}(t_i)$  for all NC attachment dof time point by time point so that compatibility and superposition are satisfied at every time point  $t_i$ .

### 2.3 STRUCTURE RESPONSE DUE TO EQUIPMENT REACTIONS

The reaction acceleration  $a_{R_k}(t_i)$  for the structure at the equipment attachment node for dof k can be defined in terms of the reaction forces  $R_j(t_i)$  by:

$$a_{R_k}(t_i) = \sum_{j=1}^{NC} \sum_{m=1}^{M_S} PF_{S_{j,m}} \cdot \phi_{S_{k,m}} \cdot \ddot{y}_{j,m}(t_i) \quad (2.2)$$

where NC represents the total number of coupling degrees-of-freedom considered,  $M_S$  represents the total number of important structure modes,  $PF_{S_{j,m}}$  is the uncoupled structure participation factor for mode m associated with an applied unit force/moment at dof j, and  $\phi_{S_{k,m}}$  is the structural eigenvector for attachment dof k, for uncoupled structure mode m. The uncoupled structure m-th mode acceleration due to reaction  $R_j(t_i)$  at dof j and time  $t_i$  is given by:

$$\ddot{y}_{j,m}(t_i) + 2\lambda_{S_m} \omega_{S_m} \dot{y}_{j,m}(t_i) + \omega_{S_m}^2 y_{j,m}(t_i) = -R_j(t_i) \quad (2.3)$$

where  $\lambda_{S_m}$  is the modal structural damping ratio and  $\omega_{S_m}$  is the modal angular natural frequency for uncoupled structure mode m.

Given the response at time  $t_{i-1}$ , the modal response at time  $t_i$  for the attachment node, dof  $j$ , can be determined using the Nigam-Jennings technique (Reference 1) as follows:

$$\left. \begin{aligned} Y_{j,m}(t_i) &= Y_{j,m}^*(t_i) + B_{12S_m} \cdot \Delta R_j(t_i) \\ Y_{j,m}^*(t_i) &= A_{11S_m} \cdot Y_{j,m}(t_{i-1}) + A_{12S_m} \cdot \dot{Y}_{j,m}(t_{i-1}) + B_{11S_m} \cdot R_j(t_{i-1}) \\ \dot{Y}_{j,m}(t_i) &= \dot{Y}_{j,m}^*(t_i) + B_{22S_m} \cdot \Delta R_j(t_i) \\ \dot{Y}_{j,m}^*(t_i) &= A_{21S_m} \cdot Y_{j,m}(t_{i-1}) + A_{22S_m} \cdot \dot{Y}_{j,m}(t_{i-1}) + B_{21S_m} \cdot R_j(t_{i-1}) \end{aligned} \right\} (2.4)$$

By substituting Equation 2.4 into Equation 2.3,

$$\left. \begin{aligned} \ddot{Y}_{j,m}(t_i) &= \ddot{Y}_{j,m}^*(t_i) - \Delta R_j(t_i) \cdot \left[ 1 + 2 \lambda_{S_m} \omega_{S_m} B_{22S_m} + \omega_{S_m}^2 B_{12S_m} \right] \\ \ddot{Y}_{j,m}^*(t_i) &= -R_j(t_{i-1}) - 2 \lambda_{S_m} \omega_{S_m} \dot{Y}_{j,m}^*(t_i) - \omega_{S_m}^2 Y_{j,m}^*(t_i) \end{aligned} \right\} (2.5)$$

where  $R_j(t_{i-1})$  represents the equipment reaction for attachment dof  $j$  at the previous time point  $t_{i-1}$ , and  $\Delta R_j(t_i)$  represents the change in reaction for attachment dof  $j$  during the time interval from  $t_{i-1}$  to  $t_i$ . The change in reaction  $\Delta R_j(t_i)$  is unknown because the equipment solution has not been completed through time  $t_i$ . However, the reaction  $R_j(t_{i-1})$  for the previous time point is known so that the quantities  $Y_{j,m}^*(t_i)$ ,  $\dot{Y}_{j,m}^*(t_i)$ ,  $\ddot{Y}_{j,m}^*(t_i)$  can be evaluated for all the attachment dof for each structure mode  $m$  at time  $t_i$ . Using  $\ddot{Y}_{j,m}^*(t_i)$  the reaction acceleration  $a_{R_k}(t_i)$  at dof  $k$  for all NC dof can be determined using Equation 2.2 as follows:

$$\ddot{a}_{R_k}(t_i) = \sum_{j=1}^{NC} \sum_{m=1}^{M_S} PF_{S_{j,m}} \cdot \phi_{S_{k,m}} \cdot \ddot{Y}_{j,m}^*(t_i) \quad (2.6)$$

The reaction acceleration  $a_{R_k}^*(t_i)$  represents the reaction acceleration for the attachment node, dof k, that would result if no change in reactions occurred between time  $t_{i-1}$  and  $t_i$  (i.e.,  $\Delta R_j(t_i) = 0$ ).

The change in reaction acceleration for dof k,  $a_{\Delta R_k}(t_i)$  due to the change in reaction  $\Delta R_j(t_i)$  as obtained from Equations 2.2 and 2.5 is:

$$a_{\Delta R_k}(t_i) = \sum_{j=1}^{NC} A_{\Delta S_{k,j}} \cdot \Delta R_j(t_i) \quad (2.7)$$

where:

$$A_{\Delta S_{k,j}} = - \sum_{m=1}^{M_S} PF_{S_{j,m}} \cdot \phi_{S_{k,m}} \left[ 1 + 2\lambda_{S_m} \omega_{S_m} B_{22S_m} + \omega_{S_m}^2 B_{12S_m} \right] \quad (2.8)$$

The quantity  $A_{\Delta S_{k,j}}$  represents the change in the structure reaction acceleration at dof k, due to a unit change in the reaction/moment at attachment dof j during the time step and is independent of the structural response or time step. This quantity simply depends upon the uncoupled structure modal properties and the time step size  $\Delta t$ .

The total reaction acceleration for attachment dof k at time  $t_i$  is given by:

$$a_{R_k}(t_i) = a_{R_k}^*(t_i) + a_{\Delta R_k}(t_i) \quad (2.9)$$

The coupled acceleration for attachment dof K as obtained from Equation 2.1 is thus:

$$a_{c_k}(t_i) = a_{cs_k}^*(t_i) + a_{\Delta R_k}(t_i) \quad (2.10)$$

where:

$$a_{cs_k}^*(t_i) = a_{u_k}(t_i) + a_{R_k}^*(t_i) \quad (2.11)$$

The Nigam-Jennings coefficients used in Equations 2.4 and 2.5 for calculating the modal structural response are derived from the coefficients in Reference 1 and are as follows:

$$\begin{aligned}
A_{11} &= \omega^2 B_{11} + 1 \\
A_{12} &= -B_{21} \\
A_{21} &= \omega^2 B_{21} \\
A_{22} &= A_{11} + 2\lambda\omega B_{21} \\
B_{11} &= \Delta t_i B_{22} \\
B_{12} &= -e^{-\lambda\omega\Delta t_i} \left[ \left( \frac{2\lambda^2-1}{\omega^2\Delta t_i} \right) \frac{\sin \omega_D \Delta t_i}{\omega_D} + \frac{2\lambda}{\omega^3\Delta t_i} \cos \omega_D \Delta t_i \right] - \frac{1}{\omega^2} + \frac{2\lambda}{\omega^3\Delta t_i} \\
B_{21} &= -e^{-\lambda\omega\Delta t_i} \left[ \frac{\sin \omega_D \Delta t_i}{\omega_D} \right] \\
B_{22} &= e^{-\lambda\omega\Delta t_i} \left[ \frac{\cos \omega_D \Delta t_i}{\omega^2 \Delta t_i} + \left( \frac{\lambda}{\omega\Delta t_i} \right) \frac{\sin \omega_D \Delta t_i}{\omega_D} \right] - \frac{1}{\omega^2\Delta t_i}
\end{aligned} \tag{2.12}$$

where the damped modal natural frequency  $\omega_D$  is given by

$$\omega_D = \omega \sqrt{1 - \lambda^2} \tag{2.13}$$

Note that all of the coefficients are defined in terms of  $B_{12}$ ,  $B_{21}$ , and  $B_{22}$ . Further note that when the product  $\omega\Delta t_i$  is small, both  $B_{12}$  and  $B_{22}$  are obtained by the subtraction of two nearly equal numbers. On a computer with seven to eight significant figure accuracy it has been found that the coefficients  $B_{12}$ , and  $B_{22}$  can be evaluated both faster and more accurately using a series expansion in lieu of Equation 2.12 whenever

$$\omega\Delta t_i < 0.3$$

The series expansion used is

$$\begin{aligned}
B_{12} &= -e^{-\lambda\omega\Delta t_i} \left( \frac{\Delta t_i^2}{6} \right) \left[ 1 + \frac{\lambda\omega\Delta t_i}{2} + \frac{(4\lambda^2-1)(\omega\Delta t_i)^2}{20} + \frac{\lambda(3\lambda^2-1)(\omega\Delta t_i)^3}{60} + \frac{(1-5\lambda^2+9\lambda^4)(\omega\Delta t_i)^4}{840} \right] \\
B_{22} &= -e^{-\lambda\omega\Delta t_i} \left( \frac{\Delta t_i}{2} \right) \left[ 1 + \frac{\lambda\omega\Delta t_i}{3} + \frac{(2\lambda^2-1)(\omega\Delta t_i)^2}{12} + \frac{\lambda(2\lambda^2-1)(\omega\Delta t_i)^3}{60} + \frac{(1-3\lambda^2+3\lambda^4)(\omega\Delta t_i)^4}{360} \right]
\end{aligned} \tag{2.14}$$

## 2.4 UNCOUPLED EQUIPMENT RESPONSE TO ATTACHMENT POINT REACTION

Using a development similar to that shown for the structure in Section 2.3, the following definitions are possible. The reaction acceleration  $a_{ce_k}(t_i)$  for dof k at the corresponding equipment attachment node is identical to the coupled acceleration  $a_{c_k}(t_i)$ . Thus, the coupled acceleration at dof k can be defined in terms of the coupling reaction forces  $R_j(t_i)$  by:

$$a_{c_k}(t_i) = - \sum_{j=1}^{NC} \sum_{m=1}^{M_E} PF_{E_{j,m}} \cdot \phi_{E_{k,m}} \cdot \ddot{z}_{j,m}(t_i) \quad (2.15)$$

where NC represents the total number of degrees-of-freedom considered at the equipment attachment nodes,  $M_E$  represents the total number of important equipment modes,  $PF_{E_{j,m}}$  is the uncoupled equipment participation factor for mode m associated with an applied unit force/moment at attachment dof j, and  $\phi_{E_{k,m}}$  is the equipment eigenvector for attachment dof k for uncoupled equipment mode m. The uncoupled equipment m-th mode acceleration for an applied reaction  $R_j(t_i)$  at dof j and time  $t_i$  is given by:

$$\ddot{z}_{j,m}(t_i) + 2\lambda_{E_m} \omega_{E_m} \dot{z}_{j,m}(t_i) + \omega_{E_m}^2 z_{j,m}(t_i) = -R_j(t_i) \quad (2.16)$$

where  $\lambda_{E_m}$  is the modal equipment damping ratio and  $\omega_{E_m}$  is the modal angular natural frequency for uncoupled equipment mode m.

A set of equations identical to Equations 2.4 and 2.5 may be written for the equipment by making the following substitutions. All letters  $Y, \dot{Y}, \ddot{Y}, Y^*, \dot{Y}^*, \ddot{Y}^*$  must be replaced by  $Z, \dot{Z}, \ddot{Z}, Z^*, \dot{Z}^*, \ddot{Z}^*$  respectively while keeping all subscripts the same. All Nigam-Jennings coefficients for the structure  $A_{11S}, A_{12S}, \dots, B_{22S}$  must be replaced by the analogous equipment Nigam-Jennings response coefficients  $A_{11E}, A_{12E}, \dots, B_{22E}$ . Interpretation of the resulting equations for the equipment modal accelerations, velocities, and displacements, are similar to those given in Section 2.3 for the uncoupled structure.

Since the quantity  $R_j(t_{i-1})$  for the previous time point is known, the quantities  $Z_{j,m}^*(t_i), \dot{Z}_{j,m}^*(t_i), \ddot{Z}_{j,m}^*(t_i)$  can be evaluated for all NC attachment dof for each equipment mode  $m$  at time  $t_i$ . Using  $\ddot{Z}_{j,m}^*(t_i)$ , the reaction acceleration  $a_{ce_k}^*(t_i)$  can be determined from Equation 2.15 as follows:

$$a_{ce_k}^*(t_i) = - \sum_{j=1}^{NC} \sum_{m=1}^{M_E} PF_{E_{j,m}} \cdot \phi_{E_{k,m}} \cdot \ddot{Z}_{j,m}^*(t_i) \quad (2.17)$$

The reaction acceleration  $a_{ce_k}^*(t_i)$  represents the reaction acceleration for the equipment attachment node, dof  $k$ , that would result if no change in reactions occurred between time  $t_{i-1}$  and  $t_i$ .

The change in the reaction acceleration for dof  $k$ ,  $a_{\Delta c_k}(t_i)$  due to the change in reaction  $\Delta R_j(t_i)$  is:

$$a_{\Delta c_k}(t_i) = \sum_{j=1}^{NC} A_{\Delta E_{k,j}} \cdot \Delta R_j(t_i) \quad (2.18)$$

where

$$A_{\Delta E_{k,j}} = \sum_{m=1}^{M_E} PF_{E_{j,m}} \cdot \phi_{E_{k,m}} \left[ 1 + 2\lambda_{E_m} \omega_{E_m} B_{22E_m} + \omega_{E_m}^2 B_{12E_m} \right] \quad (2.19)$$

The quantity  $A_{\Delta E_{k,j}}$  represents the change in the equipment reaction acceleration at dof k due to a unit change in reaction/moment at attachment dof j during the time step.

The coupled acceleration for dof k is equal to:

$$a_{c_k}(t_i) = a_{ce_k}^*(t_i) + a_{\Delta c_k}(t_i) \quad (2.20)$$

In order to find the quantities  $a_{\Delta R_k}(t_i)$  for the structure (Equation 2.7) and  $a_{\Delta c_k}(t_i)$  for the equipment (Equation 2.18) for all NC attachment dof, the change in the coupling reactions  $\Delta R_j(t_i)$  must be found for all dof at time step  $t_i$ . This may be done by setting Equation 2.10 equal to Equation 2.20 and solving for  $\Delta R_j(t_i)$ . Thus, for time step  $t_i$ :

$$[B_{\Delta}] \{ \Delta R \} = \{ C \} \quad (2.21)$$

where:

$$\{ \Delta R \} = [B_{\Delta}]^{-1} \{ C \} \quad (2.22)$$

$$B_{\Delta_{k,j}} = A_{\Delta S_{k,j}} - A_{\Delta E_{k,j}} \quad (2.23)$$

$$C_k(t_i) = a_{ce_k}^*(t_i) - a_{cs_k}^*(t_i) \quad (2.24)$$



By Maxwell's Law the displacement of dof k due to a unit force at dof j must equal the displacement at dof j due to a unit force of dof k, therefore the  $[B_{\Delta}]$  matrix must be symmetric. Thus:

$$B_{\Delta_{k,j}} = B_{\Delta_{j,k}} \quad (2.24a)$$

Lack of symmetry is an indication that improper eigenvector values or improper participation factors have been input to the coupling program.

Thus, the change in reaction for each dof j,  $\Delta R_j(t_i)$  during the time step are determined from all of the dof accelerations  $C_k(t_i)$  corresponding to the difference in the structure and equipment accelerations resulting from no change in the reactions during the time step  $t_i$  and the matrix  $[B_{\Delta}]^{-1}$  which is a constant and is independent of the time history analysis.

Once the change in the coupling reactions  $\Delta R_j(t_i)$  are known for all NC attachment dof, the coupled acceleration for each attachment dof may be calculated for time step  $t_i$  by Equations 2.7 and 2.10. The total coupling reactions for time step  $t_i$ , dof j, are then given by:

$$R_j(t_i) = R_j(t_{i-1}) + \Delta R_j(t_i) \quad (2.25)$$

Thus, it is possible to calculate the coupled structural response at time step  $t_i$  using the results of the previous time step  $t_{i-1}$ . Once the coupled response is known for all NC attachment dof for time step  $t_i$ , the analysis can proceed to the next time point  $t_{i+1}$  and continue the correction of the uncoupled acceleration time history by marching time point by time point.

Step #1

Input the acceleration time history  $a_{u_k}(t_i)$  for all NC attachment dof and for all time points. For each of the  $M_S$  uncoupled structure modes, input the modal frequency  $\omega_{S_m}$ , the modal damping  $\lambda_{S_m}$ , the structural eigenvector  $\phi_{S_{k,m}}$  and the structural participation factor  $PF_{S_{k,m}}$  for all NC attachment dof due to a unit force/moment applied at each dof. Similarly, for the uncoupled equipment model for each of the  $M_E$  equipment modes input the modal frequency  $\omega_{E_m}$ , the modal damping  $\lambda_{E_m}$ , the equipment eigenvector  $\phi_{E_{k,m}}$  and the equipment participation factors  $PF_{E_{k,m}}$  for all NC attachment dof.

Step #2

Calculate the Nigam-Jennings coefficients  $A_{11S_m}$  through  $A_{22S_m}$  for each structure mode and  $A_{11E_m}$  through  $A_{22E_m}$  for each equipment mode using Equations 2.12 through 2.14.

Step #3

Calculate the factors  $A_{\Delta S_{k,j}}$  and  $A_{\Delta E_{k,j}}$  for all degrees-of-freedom from Equation 2.8 and 2.19, respectively. Once these factors are known, the matrices  $[B_{\Delta}]$  and  $[B_{\Delta}]^{-1}$  can be found from Equation 2.23.

As noted in Section 2.4, the  $[B_{\Delta}]$  matrix must be symmetric. An unsymmetric  $[B_{\Delta}]$  matrix indicates either an erroneous eigenvector or participation factor has been input for either the structure or equipment model. Generally, this error would result from an improper participation factor being computed and input or from a keypunch error. This error indicator is checked and the solution is terminated if an unsymmetric  $[B_{\Delta}]$  matrix is determined.

Each of the remaining Program Steps are performed time point by time point using the results from time  $t_{i-1}$  to calculate the results at time  $t_i$ .

#### Step #4

Calculate  $a_{R_k}^*(t_i)$  for all NC attachment dof of the structure using Equation 2.4, 2.5, and 2.6. Next, calculate  $a_{CS_k}^*(t_i)$  for all attachment dof using Equation 2.11.

#### Step #5

Similarly for the equipment, calculate  $a_{ce_k}^*(t_i)$  for all NC attachment dof due to no change in the coupling reaction during time step  $t_i$  using Equation 2.17.

Program CMDOF

Main program in which computer memory is dynamically allocated.

Subroutine CMDOFMN

Subroutine in which essentially all calculations described in the previous section are performed.

Subroutine PCN04

Develops Nigam-Jennings coefficients  $A_{11}$  through  $B_{22}$  for specified frequency  $\omega$ , damping  $\lambda$ , and time step size  $\Delta t$ .

Subroutine WTTB

Writes uncoupled acceleration, coupled acceleration, and reaction time histories off to either a RMS device or magnetic tape for all NC dof and all NT time steps.

Subroutine PLOT

Plots coupled and/or uncoupled acceleration time history for all NC attachment dof.

Subroutine SUPIN

Finds maximum and minimum response quantities and times of occurrence for the maximum and minimum values.

Subroutine INVERT

Calculates the inversion of matrix  $[B]$ .

When a consistent mass matrix is used to describe the uncoupled structure this equation is slightly different. In this case:

$$\sum_{k=1}^{\text{all dof}} \left( M_{j,k} \cdot \sum_{m=1}^{\text{all}} PF_{k,m} \cdot \phi_{k,m} \right) = \begin{cases} 1 & j=l \\ 0 & j \neq l \end{cases} \quad (2.26a)$$

where  $M_{j,k}$  represents all the masses associated with the attachment dof. Equations 2.26 or 2.26a can be used to estimate the number of structure modes  $M_S$  needed to fully describe the inertial effects of concentrated reaction loads applied at the attachment degrees-of-freedom.

Similarly, the static flexibility matrix (displacement at dof k due to a unit applied reaction force at dof j) can be related to the modal results by:

$$f_{k,j} = \sum_{m=1}^{\text{all}} \left( \frac{PF_{j,m} \cdot \phi_{k,m}}{\omega_m^2} \right) \quad (2.27)$$

where  $\omega_m$  is the m-th mode eigenvalue (angular natural frequency).

Equation 2.27 can be used to estimate the number of structure modes  $M_S$  needed to fully describe the static flexibility of the structure subjected to concentrated reaction loads applied at the attachment degrees-of-freedom. Fewer modes are required to satisfy Equation 2.27 than are required to satisfy Equation 2.26 or 2.26a.

Accurate coupled acceleration time histories and their coupled response spectra are always calculated throughout the entire frequency range when either of the following are satisfied:

1. Sufficient uncoupled structure and equipment modes are included to satisfy Equation 2.26 or 2.26a.

or

2. Sufficient uncoupled structure and equipment modes are included to satisfy Equation 2.27 and all significant modes with frequencies below the frequency at which the uncoupled acceleration spectrum returns to approximately the zero period acceleration (ZPA) are included.

Generally, Criterion 2 is easier to satisfy than Criterion 1.

In most cases of practical application, it is unnecessary to compute accurate coupled response spectrum above a cutoff frequency (generally in the range of 30 to 50 hz) because structural models are generally not accurate beyond this cutoff frequency and higher frequency responses are not damaging because they do not contain sufficient velocity or displacement content. When it is not necessary to compute accurate coupled response spectra beyond a cutoff frequency it is permissible to relax the above rules as discussed below and in Appendix A.

When a cutoff frequency is established above which the coupled acceleration response spectra do not have to be accurately determined, it is only necessary to accurately include structure and equipment modes with frequencies up to slightly beyond this cutoff frequency so long as Equation 2.27 is not violated. So long as Equation 2.27 is satisfied, modes with natural frequencies beyond about 1.3 to 1.6 times the cutoff frequency will not influence the coupled response spectra below the cutoff frequency. In other words, Equation 2.26 or 2.26a does not have to be satisfied to ensure accurate spectra below this cutoff frequency.

When Equation 2.27 is not satisfied, the displacement effects of the neglected modes are not included. Ignoring these displacement effects can influence the response spectra even below the cutoff frequency. The displacement effects of ignored higher frequency modes on the spectra below the cutoff frequency can be approximated by the use of a correction flexibility matrix defined as follows:

$$\Delta f_{k,j} = f_{k,j} - \sum_{m=1}^M \left( \frac{PF_{j,m} \cdot \phi_{k,m}}{\omega_m^2} \right) \quad (2.28)$$

where M is the number of modes included. This correction flexibility matrix can be introduced as a pseudo-mode with high damping (20 percent damping or higher is recommended) set at a natural frequency higher than 1.6 times the cutoff frequency. With the introduction of this high frequency pseudo-mode correction flexibility matrix to incorporate the displacement effects of the ignored actual high frequency modes, Equation 2.27 is satisfied.



Some results of a study of the impact of not satisfying Equation 2.27, and of satisfying Equation 2.27 through the use of a high frequency, high damped, pseudo-mode correction flexibility matrix for both the uncoupled structure and uncoupled equipment are presented in Appendix A. Guidelines for the use of this pseudo-mode correction flexibility matrix developed as a result of this study, are presented in Chapter 6.

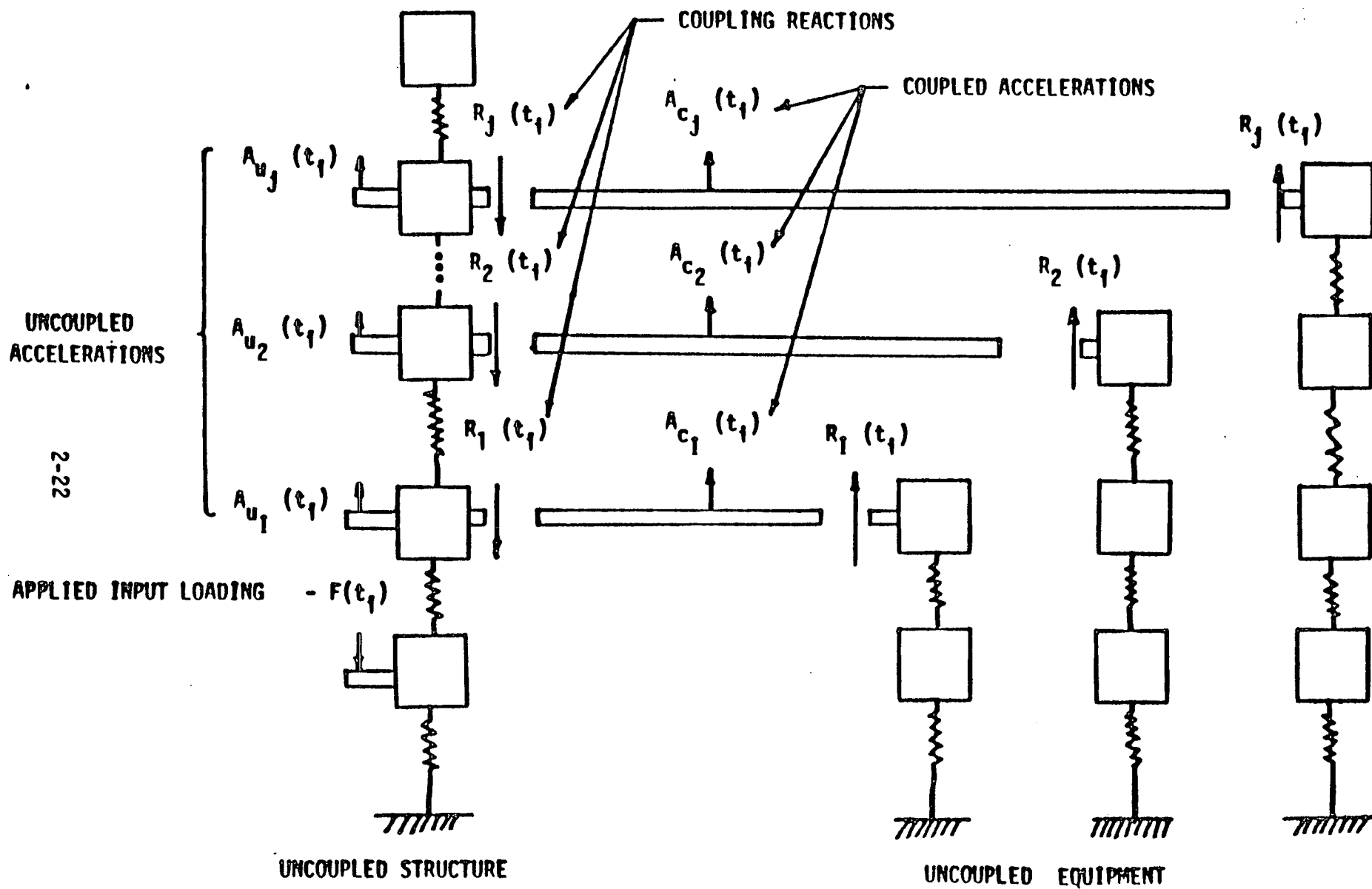


FIGURE 2-1: UNCOUPLED STRUCTURE AND EQUIPMENT MODELS SHOWING EQUILIBRIUM AND COMPATIBILITY REQUIREMENTS

### 3. PROGRAM INPUT

#### 3.1 GENERAL INSTRUCTIONS

This section discusses the input to program CMDOF including the modal data required, input format and sequence. Definitions and explanations have been provided. Additional explanation of modelling techniques for developing consistent uncoupled structure and equipment modal data and uncoupled structure time histories may be found in Appendix B. Input card listing of Example #2 (refer to Section 5: Program Validation) is presented in Table 3-1. Program CMDOF was written in the CDC scope 3.4 FORTRAN, FTN language, and therefore, all FORTRAN rules must be observed.

UNIPLLOT and tape control cards for program CMDOF are shown in Table 3-1. These cards are computer operating system dependent, and will have to be changed for other systems. Program CMDOF outputs coupled and uncoupled time histories to a magnetic tape on Unit 11. A request card must be used to save file 11 on magnetic tape.

Figure 3-1 illustrates the data card setup. A card with 99 in Columns 1 and 2 is used to terminate the program execution. The input for a problem consists of:

1. Title Card (one card)
2. Control Card (one card)
3. Structure Mode Cards
4. Equipment Mode Cards
5. Correction Flexibility Cards for Equipment
6. Uncoupled Acceleration Cards

## 3.2

TITLE CARD

Number of Cards = 1

Format = I2, 7A10

Column 1-2: IOU - Output Control

3-72: TIT - Title

The program will always print out the problem title, all structures and equipment modal data, and the input uncoupled acceleration time histories for each dof considered in the coupling analysis. Tape 11 which contains the uncoupled acceleration time histories, the coupled acceleration time histories, and the coupled reaction time histories for each coupled dof is always created. The integer IOU on the title card is used to define printing and plotting capabilities and to terminate the job execution according to the following:

IOU	Output
0	No plot
1	Plot coupled acceleration time histories for each dof for all NC degrees of coupling
2	Plot uncoupled and coupled acceleration time histories for each dof for all NC degrees of coupling
3	Print uncoupled and coupled acceleration time histories for each dof for all NC degrees of coupling
4	Same as (3) but also plot both uncoupled and coupled time histories for each dof for all NC degrees of coupling
5 - 99	Stop Execution. This number is used on the last card after the last set of accelerations in data deck

## 3.3

CONTROL CARD

Number of Cards = 1

Format = F8.0, I2, 6I5, 4F10.0

Column 1-8: DT - Time step size for input uncoupled acceleration time histories.

9-10: NC - Total number of coupling degrees of freedom between the structure and all attached equipment (maximum = 80).

11-15: NP - Number of input time points.

16-20: MS - Number of structure modes.

21-25: ME - Number of equipment modes.

26-30: N - Interpolation interval for calculating coupled response.

31-35 IS - Structure damping control.

36-40 IE - Equipment damping control.

41-50 AS - Structure damping coefficient  $\lambda_s$  or  $\bar{\alpha}_s$  depending on setting of IS.

51-60 BS - Structure damping coefficient  $\bar{\beta}_s$  (if needed).

61-70 AE - Equipment damping coefficient  $\lambda_E$  or  $\bar{\alpha}_E$  depending on setting of IE.

71-80 BE - Equipment damping coefficient  $\bar{\beta}_E$  (if needed).

Program CMDOF allows the coupled responses to be calculated at a smaller time step size than that at which the input uncoupled acceleration time histories is specified. The coupled analysis time step size is:

$$t = \frac{DT}{N} \quad (3.1)$$

and the total number of time points at which the coupled response is calculated is:

$$NT = 1 + N (NP - 1) \quad (3.2)$$

The input uncoupled acceleration time history is linearly interpolated to obtain values for intermediate time points when N is set greater than one.

Program CMDOF allows three options for specifying modal damping values for either the structure or equipment. These options are as follows:

IS or IE	Option
0	Read modal damping fraction on each mode data card.
1	Use $\bar{\alpha}$ , $\bar{\beta}$ damping fraction for all modes. Thus, for mode m $\lambda_m = \frac{\bar{\alpha}}{2f_m} + \frac{\bar{\beta}f_m}{2}$ where $f_m$ is the modal circular frequency (Hz).
2	Use constant damping fraction $\lambda$ for all modes.

Dynamic allocation of memory has been implemented in this program in order to increase computational efficiency. The total amount of blank common required by the program may be determined by the following equation.

$$\begin{aligned}
 \text{LTOT} = & 19 \cdot \text{NC} + 6 \cdot \text{NC}^2 + 3 \cdot \text{NC} \cdot \text{NPTOT} \\
 & + 4 \cdot \text{NC} \cdot (\text{MS} \cdot \text{METOT}) + 2 \cdot \text{NPTOT} \\
 & + 10 \cdot (\text{MS} + \text{METOT}) + 2
 \end{aligned}
 \tag{3.3}$$

where

$$\begin{aligned}
 \text{METOT} &= \text{ME} + 1 \\
 \text{NPTOT} &= (\text{NP} - 1) \cdot \text{N} + 1
 \end{aligned}$$

The dimensions of variables NC, ME, MS, NP, AND N are defined by the Control Card (Section 3.3). CMDOF is currently programmed to execute without special memory allocation so long as LTOT is less than 68,000 words. Larger problems may be run by specifying the computer core requirements with an appropriate job control card before program execution.

## 3.4

STRUCTURE MODE CARDS

Minimum Number of Cards = 2\*MS

Format = 8E10.4

Card 1:

Column 1-10:	$f_{S_m}$	- Structure modal cyclic frequency (Hz) for mode m.
11-20:	$\lambda_{S_m}$	- Structure modal damping fraction for mode m (must input if IS = 0).
21-30:	$\phi_{S_{j,m}}$	- Uncoupled structural eigenvector value for all NC dof for mode m in sequential order. ( $\phi_{S_{1,m}}$ to $\phi_{S_{NC,m}}$ for $1 \leq NC \leq 6$ ).
31-40:	$\vdots$	
71-80:	$\vdots$	(If $NC > 6$ continue listing $\phi_{S_{K,m}}$ on the next card starting in Columns 1-10 (i.e., $\phi_{S_{7,m}}$ ) using an 8E10.4 format until all NC dof are input).

Required  
for each  
Mode m

Card 2:

Column 1-10:	$PF_{S_{j,m}}$	- Structural participation factors associated with a concentrated reaction at the coupled dof for all NC dof for mode m in sequential order ( $PF_{S_{1,m}}$ to $PF_{S_{NC,m}}$ ) for $(1 \leq NC \leq 8)$ . (If $NC > 8$ continue listing $PF_{S_{K,m}}$ on the next card starting in Columns 1-10 (i.e., $PF_{S_{9,m}}$ ) using an 8E10.4 format until all NC dof are input).
11-20:	$\vdots$	
71-80:	$\vdots$	

The participation factor  $PF_{S_{j,m}}$  for mode m due to a concentrated equipment reaction at DOF j is given by:

$$PF_{S_{j,m}} = \frac{\phi_{S_{j,m}}}{\left\{ \phi_{S_m} \right\}^T [M] \left\{ \phi_{S_m} \right\}}$$

where  $\{\phi_{S_m}\}$  are the m-th mode eigenvectors and  $[M]$  is the structure mass matrix.



### 3.5 EQUIPMENT MODE CARDS

Minimum Number of Cards =  $2 \times ME$

Format = 8E10.4

Card 1:

Column 1-10:	$f_{E_m}$	- Equipment modal frequency (Hz) for mode m.
11-20:	$\lambda_{E_m}$	- Equipment modal damping for mode m must input if IE = 0).
21-30:	$\phi_{E_{j,m}}$	- Uncoupled equipment eigenvector value for all NC dof for mode m in sequential order. ( $\phi_{E_{1,m}}$ to $\phi_{E_{NC,m}}$ for $1 \leq NC \leq 6$ ).
31-40:	$\vdots$	(If $NC > 6$ continue listing $\phi_{E_{K,m}}$ on the next card starting in Columns 1-10 (i.e., $\phi_{E_{7,m}}$ ) using an 8E10.4 format until all NC dof are input).
71-80:		

Required  
for each  
Mode m

Card 2:

Column 1-10:	$PF_{E_{j,m}}$	- Equipment participation factor associated with a concentrated reaction at the coupled dof for all NC dof for mode m in sequential order. ( $PF_{E_{1,m}}$ to $PF_{E_{NC,m}}$ ) for $(1 \leq NC \leq 8)$ . (If $NC > 8$ continue listing $PF_{E_{K,m}}$ on the next card starting in Columns 1-10 (i.e., $PF_{E_{9,m}}$ ) using an 8E10.4 format until all NC dof are input).
11-20:	$\vdots$	
71-80:		

Each coupling degree-of-freedom must be free from external constraint (no support) in the uncoupled equipment model from which the equipment natural frequencies and eigenvector values are determined. The equipment modal participation factors are determined using an equation analogous to Equation 3.4.

### 3.6 CORRECTION FLEXIBILITY CARDS FOR EQUIPMENT

It was noted in Section 2.7 that accurate coupled time histories and coupled response spectra below a cutoff frequency may not be computed unless Equation 2.27 is satisfied by the structure and equipment modes input to the coupling program. In some cases, it may not be practical to input sufficient structure and equipment modes to satisfy Equation 2.27 for each coupling DOF. In these cases, approximations need to be made.

It is shown in Appendix A and summarized in Section 6 that accurate coupled response spectra can be obtained up to a cutoff frequency  $f_c$  so long as:

1. All important structure modes up to frequency  $1.3 f_c$  and all important equipment modes up to frequency  $1.6 f_c$  are included.
2. Equation 2.27 is satisfied for the equipment through the use of one or more added high frequency, high damped pseudo-modes with properties selected so as to satisfy Equation 2.27. The natural frequency for these pseudo-modes should satisfy:

$$1.6 f_c \leq f_p \leq \frac{0.1}{\Delta t} \quad (3.5)$$

where  $\Delta t$  is the analysis time step size. The lower limit is established so that the pseudo-mode corrects displacement effects without introducing significant dynamic interaction coupling effects below the cutoff frequency  $f_c$ . The upper limit is selected to ensure an adequate time step size. Best results are obtained if these pseudo-modes are highly damped to minimize dynamic interaction coupling (20% damping is recommended).

The high frequency pseudo equipment modes needed to satisfy Equation 2.27 when the actual equipment modes are cutoff too low to satisfy this equation can be introduced by either of two ways. First, pseudo high frequency modes needed to satisfy Equation 2.27 can be computed and input as additional modes on the equipment mode cards (Section 3.5). Alternately, an equipment correction flexibility matrix can be computed from Equation 2.28 and input as described in this section.

The parametric study presented in Appendix A tends to indicate that it is unnecessary to satisfy Equation 2.27 for the uncoupled structure and that no improvement in the coupled spectra below the cutoff frequency is obtained when this equation is satisfied through the use of pseudo high frequency modes. For this reason, no provisions have been incorporated for inputting a structure correction flexibility matrix to account for ignored higher frequency structure modes. Any user who wishes to introduce pseudo high frequency structure modes to satisfy Equation 2.27 can do so using the structure mode cards (Section 2.4).

If a correction flexibility matrix is not being input for the equipment, a blank card should be inserted after the last equipment mode card (Section 3.5) and the user should proceed to Section 3.7 and disregard the rest of this section. When an equipment correction flexibility matrix is being input, the following format should be followed:

Minimum Number of Cards = NC+1

Card 1: Format I10, 2F10.0

- Column 1-10: IFLAG - 1 - Correction flexibility for equipment will be used in program.
- 11-20:  $f_p$  - Pseudo-mode cyclic frequency (Hz) at which correction flexibility matrix is assigned.
- 21-30:  $\lambda_p$  - Pseudo-mode damping ratio (0.2 recommended).

Card 2: Format 8E10.4

Column 1-80: (DOF K=1)

$\Delta f_{E_{K,j}}$

- Correction flexibility for equipment (DOF K=1) due to unit loads at dof j=1 to j=NC ( $1 \leq j \leq 8$ ). (If NC > 8 continue listing  $\Delta f_{E_{K,j}}$  on the next card starting in Columns 1-10 (i.e.,  $\Delta f_{E_{K,9}}$ ) using an 8E10.4 format until all NC dof are input).

Card NC+1: (DOF K=NC)

Column 1-80:

$\Delta f_{E_{K,j}}$

- Correction flexibility for equipment (DOF K=NC) due to unit loads at dof j=1 to j=NC ( $1 \leq j \leq 8$ ). (If NC > 8 continue listing  $\Delta f_{E_{K,j}}$  on the next card starting in Columns 1-10 (i.e.,  $\Delta f_{E_{K,9}}$ ) using an 8E10.4 format until all NC dof are input).

Input one card for each coupled dof

The equipment flexibility terms  $\Delta f_{E_{k,j}}$  are computed from Equation 2.28. Note that this matrix must be symmetric, i.e.:

$$\Delta f_{E_{k,j}} = \Delta f_{E_{j,k}} \quad (3.6)$$

If equation 3.6 is not satisfied, an error has been made.

UNCOUPLED ACCELERATION CARDS

Minimum Number of Cards = NP

Format = 6E10.4

Column 1-60:  $A_{u_j}(t_i)$

- Uncoupled acceleration for dof j, at time  $t_i$ . Uncoupled acceleration are input sequentially for all dof across the card from  $A_{u_{k=1}}(t_i)$  to  $A_{u_{k=NC}}(t_i)$  ( $1 \leq NC \leq 6$ ). (For time step  $t_i$  if  $NC > 6$  continue listing  $A_{u_j}(t_i)$  on the next card starting in Columns 1-10 (i.e.,  $A_{u_7}(t_i)$ ) using an 6E10.4 format until all NC dof are input).

TABLE 3-1: INPUT CARD LISTING FOR EXAMPLE PROBLEM 2

CUR,VIS,10500,P4,NT1.  
 USER(L1589LA,ENGINEE)  
 PROJECT(AMMA12101.03)  
 REQUEST,TAPE11,NT,PE,SV. ← Tape Control Card  
 FIN.

APPLIC,UNIPLUT. ← UNIPLUT Application Card  
 LGO.

UNIPLUT,P=PLUT,D=COPEUCC.  
 INFORM,PLUT,UNANGL.  
 INFORM. PLOT FOR  
 INFORM, RAY KINCAID OF SHA  
 INFORM.  
 INFORM. PLOT ON 10 INCH PAPER } ← UNIPLUT Post-processing Cards

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3 SAMPLE PROBLEM - 3 COUPLED DUF ← Card #1  
 0.005 3 161 5 9 1 1 1 0.05 0.018 0.05 0.018 ← Card #2

1.407		10.13	14.16	17.04	} ← Card Set #3 (Structure Cards)
10.13	14.16	17.04			
4.108		-18.54	-10.13	5.278	
-18.54	-10.13	5.278	-17.04	-10.13	
6.476		5.278			
5.278	-17.04	-10.13	5.278	-18.54	
8.319		14.16			
14.16	5.278	-18.54	-18.54	14.16	
9.489		17.04			
17.04	-18.54	14.16			



**3-14**



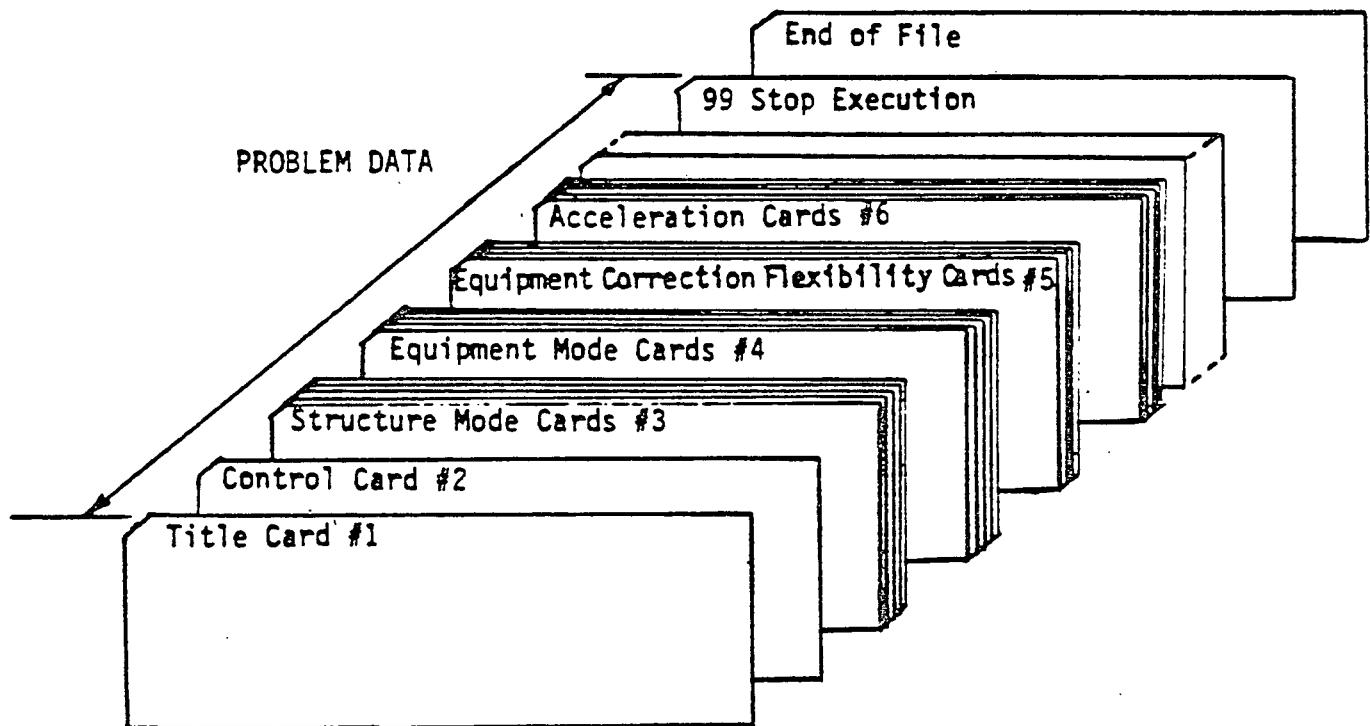


FIGURE 3-1. DATA DECK FORMAT

#### 4. PROGRAM OUTPUT

This section discusses the output capabilities of program CMDOF. The amount of output depends upon the value specified for integer IOU on the input title card as discussed in Section 3.2. Table 4-1 presents a sample output generated for Example Problem #2 (see Section 5) using the input contained in Table 3-1.

The minimum printed output consists of printing out the problem title, the control card information, the structure and equipment modal data, and the uncoupled input time histories for each dof considered. In addition, tables are printed out after the completion of the solution containing the following information in consistent problem units for each coupled degree-of-freedom:

1. The maximum and minimum coupled and uncoupled acceleration or the times at which they occur.
2. The maximum and minimum coupling reaction values and time of occurrence.
3. The maximum and minimum uncoupled and coupled structure displacements and velocities and the times of occurrence.

When IOU is set equal to 3 or 4, additional output will be generated. This output consists of printing out the uncoupled and coupled acceleration time histories for each coupled dof considered in the problem. When IOU is set to either 1 or 2, acceleration time history plots are generated for the coupled and uncoupled acceleration time histories for each dof.

Program CMDOF also generates a binary file, #11, which contains uncoupled and coupled acceleration time histories and reaction time histories. Appendix C presents a listing of subroutine WTTB which writes Tape 11 data. Program users may easily develop a routine to read this information by following the logic in WTTB.

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2

SAMPLE PROBLEM - 3 COUPLED DIF

INPUT TECH

UI =	.00500	NC =	3	WP =	161	MS =	5	ME =	9	N =	1	IS =	1	It =	1
AS =	.05000000	US =	.01000000	AL =	.05000000	HE =	.01000000								

TIME HISTORY CONTAINS 161 POINTS AT .005000 SECOND INCREMENT

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

EQUIPMENT HAS 4 MODES

MODE NUMBER	EQUIPMENT FREQUENCY (HZ)	EQUIPMENT DAMPING
1	1.717	.030013
2	4.405	.049501
3	7.576	.071400
4	9.293	.086327
5	2.201	.011167
6	6.166	.059548
7	8.910	.082996
8	3.056	.035605
9	8.000	.075130

EQUIPMENT EIGENVECTORS

EQUIPMENT PARTICIPATION FACTORS

MODE	DOF (1)	DOF (2)	DOF (3)	DOF (4)	DOF (5)	DOF (6)	DOF (1)	DOF (2)	DOF (3)	DOF (4)	DOF (5)	DOF (6)
1	0.	0.	.204E+02				0.	0.	.204E+02			
2	0.	0.	.179E+02				0.	0.	.179E+02			
3	0.	0.	.133E+02				0.	0.	.133E+02			
4	0.	0.	-.708E+01				0.	0.	-.708E+01			
5	0.	.229E+02	0.				0.	.229E+02	0.			
6	0.	-.184E+02	0.				0.	-.184E+02	0.			
7	0.	.102E+02	0.				0.	.102E+02	0.			
8	.264E+02	0.	0.				.264E+02	0.	0.			
9	-.163E+02	0.	0.				-.163E+02	0.	0.			

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

STRUCTURE HAS 5 MODES

MODE NUMBER	STRUCTURE FREQUENCY (HZ)	STRUCTURE DAMPING
1	1.407	.030431
2	4.108	.043058
3	6.476	.062144
4	8.514	.077076
5	9.449	.088036

STRUCTURE EIGENVECTORS

STRUCTURE PARTICIPATION FACTORS

MODE	DOF (1)	DOF (2)	DOF (3)	DOF (4)	DOF (5)	DOF (6)	DOF (1)	DOF (2)	DOF (3)	DOF (4)	DOF (5)	DOF (6)
1	.101E+02	.142E+02	.170E+02				.101E+02	.142E+02	.170E+02			
2	-.185E+02	-.101E+02	.528E+01				-.185E+02	-.101E+02	.528E+01			
3	.528E+01	-.170E+02	-.101E+02				.528E+01	-.170E+02	-.101E+02			
4	.142E+02	.528E+01	-.185E+02				.142E+02	.528E+01	-.185E+02			
5	.170E+02	-.185E+02	.142E+02				.170E+02	-.185E+02	.142E+02			

INPUT CONTAINS 161 POINTS AT .005000 SECOND INCREMENTS FOR EACH DEGREE OF FREEDOM

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

UNCOUPLED: ACCELERATION FOR DEGREE OF FREEDOM 1

0.	1.030E+00	5.005E+00	1.000E+01	5.030E+01	5.235E+01	0.000E+01	1.130E+02	5.500E+02	1.000E+02
2.270E+02	2.013E+02	2.020E+02	2.070E+02	2.712E+02	2.920E+02	1.713E+02	0.000E+01	-0.001E+00	-1.173E+02
-2.510E+02	-3.000E+02	-0.000E+02	-5.302E+02	-6.000E+02	-6.573E+02	-6.700E+02	-6.700E+02	-6.005E+02	-5.917E+02
-5.100E+02	-4.117E+02	-3.270E+02	-2.100E+02	-0.331E+01	0.000E+01	1.000E+02	3.155E+02	0.370E+02	5.072E+02
0.303E+02	7.101E+02	7.551E+02	7.715E+02	7.500E+02	7.167E+02	6.000E+02	5.500E+02	0.510E+02	5.350E+02
2.112E+02	9.027E+01	-2.115E+01	-1.200E+02	-2.000E+02	-2.733E+02	-3.170E+02	-3.023E+02	-3.070E+02	-3.373E+02
-3.130E+02	-2.010E+02	-2.053E+02	-2.000E+02	-1.750E+02	-1.002E+02	-1.303E+02	-1.225E+02	-1.253E+02	-1.300E+02
-1.500E+02	-1.050E+02	-2.150E+02	-2.050E+02	-2.720E+02	-2.926E+02	-3.037E+02	-3.030E+02	-2.903E+02	-2.630E+02
-2.230E+02	-1.710E+02	-1.000E+02	-3.001E+01	0.100E+01	1.220E+02	2.037E+02	2.012E+02	3.520E+02	0.107E+02
0.000E+02	5.005E+02	5.202E+02	5.300E+02	5.357E+02	5.101E+02	0.000E+02	0.000E+02	5.053E+02	3.367E+02
2.700E+02	2.050E+02	1.300E+02	0.732E+01	7.110E+01	-0.237E+01	-1.207E+02	-1.730E+02	-2.100E+02	-2.500E+02
-2.913E+02	-3.100E+02	-3.105E+02	-3.050E+02	-3.500E+02	-3.405E+02	-3.420E+02	-3.511E+02	-3.100E+02	-2.900E+02
-2.700E+02	-2.000E+02	-2.105E+02	-1.070E+02	-1.003E+02	-1.100E+02	-1.000E+01	-0.271E+01	-0.505E+00	2.027E+01
0.413E+01	9.750E+01	1.202E+02	1.503E+02	1.000E+02	2.070E+02	2.260E+02	2.021E+02	2.530E+02	2.503E+02
2.010E+02	2.001E+02	2.507E+02	2.301E+02	2.235E+02	2.000E+02	1.022E+02	1.570E+02	1.300E+02	1.020E+02
7.000E+01	0.003E+01	1.500E+01	-1.200E+01	-3.002E+01	-6.010E+01	-0.102E+01	-1.103E+02	-1.350E+02	-1.550E+02
-1.730E+02									

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

UNCOUPLED ACCELERATION FOR DEGREE OF FREEDOM 2

0.	6.071E-03	7.790E-02	3.760E-01	1.182E+00	2.898E+00	6.020E+00	1.112E+01	1.076E+01	2.942E+01
4.348E+01	6.110E+01	8.209E+01	1.058E+02	1.310E+02	1.554E+02	1.780E+02	1.968E+02	2.070E+02	2.074E+02
1.471E+02	1.731E+02	1.353E+02	8.340E+01	2.002E+01	-5.420E+01	-1.358E+02	-2.211E+02	-3.061E+02	-3.862E+02
-4.573E+02	-5.154E+02	-5.572E+02	-5.748E+02	-5.812E+02	-5.604E+02	-5.172E+02	-4.523E+02	-3.675E+02	-2.655E+02
-1.445E+02	-2.475E+01	1.072E+02	2.384E+02	3.646E+02	4.806E+02	5.810E+02	6.630E+02	7.211E+02	7.532E+02
7.575E+02	7.435E+02	6.819E+02	6.845E+02	5.845E+02	3.848E+02	2.537E+02	1.131E+02	-2.493E+01	-1.648E+02
-3.009E+02	-4.182E+02	-5.177E+02	-5.958E+02	-6.504E+02	-6.802E+02	-6.852E+02	-6.463E+02	-6.255E+02	-5.654E+02
-4.894E+02	-4.812E+02	-3.848E+02	-2.841E+02	-1.030E+02	-4.944E+00	8.688E+01	1.699E+02	2.421E+02	3.021E+02
3.484E+02	3.823E+02	4.022E+02	4.844E+02	4.846E+02	3.840E+02	3.641E+02	3.312E+02	2.421E+02	2.884E+02
2.017E+02	1.537E+02	1.868E+02	0.013E+01	1.754E+01	-2.049E+01	-5.285E+01	-7.863E+01	-4.716E+01	-1.080E+02
-1.111E+02	-1.067E+02	-9.523E+01	-7.765E+01	-5.509E+01	-2.847E+01	-4.096E+01	2.734E+01	5.398E+01	7.726E+01
4.556E+01	1.075E+02	1.120E+02	1.084E+02	9.646E+01	7.631E+01	4.865E+01	1.458E+01	-2.440E+01	-6.687E+01
-1.096E+02	-1.516E+02	-1.903E+02	-2.236E+02	-2.440E+02	-2.669E+02	-2.741E+02	-2.707E+02	-2.564E+02	-2.316E+02
-1.970E+02	-1.538E+02	-1.837E+02	-4.875E+01	9.826E+00	6.732E+01	1.238E+02	1.764E+02	2.231E+02	2.621E+02
2.421E+02	3.120E+02	3.214E+02	3.280E+02	3.083E+02	2.868E+02	2.564E+02	2.185E+02	1.745E+02	1.262E+02
7.519E+01	2.327E+01	-2.788E+01	-7.672E+01	-1.218E+02	-1.620E+02	-1.463E+02	-2.239E+02	-2.445E+02	-2.578E+02
-2.630E+02									



TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

UNCOUPLED ACCELERATION PER DEGREE OF FREEDOM 3

0.	2.461E-05	7.243E-04	5.933E-03	2.770E-02	9.415E-02	2.581E-01	6.076E-01	1.274E+00	2.437E+00
4.329E+00	7.224E+00	1.143E+01	1.726E+01	2.501E+01	3.490E+01	4.701E+01	6.127E+01	7.754E+01	9.483E+01
1.12ME+02	1.101E+02	1.457E+02	1.540E+02	1.654E+02	1.667E+02	1.605E+02	1.457E+02	1.217E+02	8.847E+01
4.639E+01	-3.523E+00	-5.46ME+01	-1.200E+02	-1.814E+02	-2.426E+02	-2.990E+02	-3.483E+02	-3.875E+02	-4.144E+02
-4.270E+02	-4.240E+02	-4.04ME+02	-3.649E+02	-3.202E+02	-2.576E+02	-1.847E+02	-1.047E+02	-2.114E+01	6.201E+01
1.40ME+02	2.115E+02	2.706E+02	3.151E+02	3.433E+02	3.534E+02	3.451E+02	3.192E+02	2.772E+02	2.215E+02
1.550E+02	8.122E+01	8.614E+00	-6.361E+01	-1.24ME+02	-1.853E+02	-2.272E+02	-2.527E+02	-2.604E+02	-2.496E+02
-2.210E+02	-1.760E+02	-1.172E+02	-4.809E+01	2.714E+01	1.04ME+02	1.797E+02	2.476E+02	3.046E+02	3.473E+02
3.732E+02	3.807E+02	3.692E+02	3.344E+02	2.428E+02	2.319E+02	1.544E+02	8.074E+01	-1.514E+00	-8.262E+01
-1.5ME+02	-2.254E+02	-2.803E+02	-3.206E+02	-3.450E+02	-3.527E+02	-3.440E+02	-3.200E+02	-2.824E+02	-2.337E+02
-1.767E+02	-1.145E+02	-5.037E+01	1.271E+01	7.197E+01	1.250E+02	1.700E+02	2.057E+02	2.313E+02	2.467E+02
2.523E+02	2.890E+02	2.379E+02	2.205E+02	1.4ME+02	1.730E+02	1.460E+02	1.186E+02	4.19ME+01	6.694E+01
4.407E+01	2.369E+01	5.89ME+00	-9.427E+00	-2.253E+01	-1.373E+01	-4.339E+01	-5.184E+01	-5.435E+01	-6.610E+01
-7.217E+01	-7.752E+01	-8.205E+01	-8.555E+01	-8.781E+01	-8.861E+01	-8.776E+01	-8.514E+01	-8.072E+01	-7.461E+01
-6.701E+01	-5.825E+01	-4.877E+01	-3.907E+01	-2.969E+01	-2.11ME+01	-1.406E+01	-8.75ME+00	-5.599E+00	-4.762E+00
-6.267E+00	-4.06ME+00	-1.556E+01	-2.257E+01	-3.045E+01	-3.852E+01	-4.60ME+01	-5.244E+01	-5.696E+01	-5.90ME+01
-5.840E+01									

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2

(CONTINUED)

TIME	AU( 1)	AC( 1)	AU( 2)	AC( 2)	AU( 3)	AC( 3)
0.0000	0.	0.	0.	0.	0.	0.
0.0050	.1030E+01	.5178E+00	.0071E-02	.2075E-03	.2461E-04	.1966E-04
0.0100	.5445E+01	.2747E+01	.7790E-01	.1473E-01	.7253E-03	.6229E-04
0.0150	.1488E+02	.7544E+01	.9760E+00	.8806E-01	.5933E-02	.2306E-03
0.0200	.3038E+02	.1550E+02	.1182E+01	.2815E+00	.2776E-01	.2282E-02
0.0250	.5235E+02	.2691E+02	.2898E+01	.7146E+00	.9415E-01	.9422E-02
0.0300	.8049E+02	.4174E+02	.0026E+01	.1523E+01	.2581E+00	.2861E-01
0.0350	.1134E+03	.5959E+02	.1112E+02	.2866E+01	.6076E+00	.7166E-01
0.0400	.1504E+03	.7990E+02	.1876E+02	.8427E+01	.1274E+01	.1569E+00
0.0450	.1896E+03	.1016E+03	.2942E+02	.7864E+01	.2437E+01	.3100E+00
0.0500	.2279E+03	.1238E+03	.4348E+02	.1183E+02	.4329E+01	.5669E+00
0.0550	.2613E+03	.1442E+03	.6110E+02	.1694E+02	.7224E+01	.9705E+00
0.0600	.2824E+03	.1592E+03	.8204E+02	.2322E+02	.1143E+02	.1573E+01
0.0650	.2974E+03	.1659E+03	.1058E+03	.3058E+02	.1726E+02	.2433E+01
0.0700	.2712E+03	.1621E+03	.1318E+03	.3877E+02	.2501E+02	.3612E+01
0.0750	.2324E+03	.1465E+03	.1559E+03	.4738E+02	.3490E+02	.5172E+01
0.0800	.1719E+03	.1105E+03	.1746E+03	.5598E+02	.4701E+02	.7151E+01
0.0850	.8987E+02	.7858E+02	.1964E+03	.6379E+02	.6127E+02	.9583E+01
0.0900	.8061E+01	.2813E+02	.2070E+03	.7020E+02	.7739E+02	.1248E+02
0.0950	.1117E+03	.3073E+02	.2079E+03	.7440E+02	.9483E+02	.1580E+02
0.1000	.2314E+03	.9518E+02	.1971E+03	.7565E+02	.1128E+03	.1950E+02
0.1050	.3448E+03	.1620E+03	.1731E+03	.7329E+02	.1301E+03	.2339E+02
0.1100	.4480E+03	.2274E+03	.1953E+03	.6603E+02	.1857E+03	.2745E+02
0.1150	.5392E+03	.2890E+03	.2390E+02	.5595E+02	.1560E+03	.3138E+02
0.1200	.6099E+03	.3474E+03	.8032E+02	.4033E+02	.1654E+03	.3491E+02
0.1250	.6573E+03	.3852E+03	.5420E+02	.2075E+02	.1667E+03	.3789E+02
0.1300	.6790E+03	.4152E+03	.1358E+03	.2944E+01	.1605E+03	.4002E+02
0.1350	.6744E+03	.4308E+03	.2211E+03	.2944E+02	.1457E+03	.4095E+02
0.1400	.6444E+03	.4315E+03	.3061E+03	.9433E+02	.1217E+03	.4046E+02
0.1450	.5917E+03	.4173E+03	.3862E+03	.8490E+02	.8847E+02	.3836E+02
0.1500	.5190E+03	.3892E+03	.4573E+03	.1205E+03	.8639E+02	.3448E+02
0.1550	.4319E+03	.3479E+03	.5154E+03	.1494E+03	.3523E+01	.2871E+02
0.1600	.3274E+03	.2932E+03	.5572E+03	.1766E+03	.5968E+02	.2104E+02
0.1650	.2106E+03	.2261E+03	.5794E+03	.1495E+03	.1200E+03	.1153E+02
0.1700	.8331E+02	.1483E+03	.5812E+03	.2173E+03	.1814E+03	.3786E+00
0.1750	.4996E+02	.6246E+02	.5604E+03	.2209E+03	.2426E+03	.1218E+02
0.1800	.1845E+03	.2850E+02	.5172E+03	.2335E+03	.2990E+03	.2574E+02
0.1850	.3155E+03	.1212E+03	.4523E+03	.2305E+03	.3483E+03	.3496E+02
0.1900	.4379E+03	.2121E+03	.3675E+03	.2194E+03	.3875E+03	.5423E+02
0.1950	.5472E+03	.2976E+03	.2655E+03	.2004E+03	.8144E+03	.6815E+02
0.2000	.6393E+03	.3744E+03	.1895E+03	.1736E+03	.4270E+03	.8111E+02
0.2050	.7101E+03	.4392E+03	.2375E+02	.1398E+03	.4240E+03	.9255E+02
0.2100	.7551E+03	.4844E+03	.1072E+03	.9988E+02	.4048E+03	.1019E+03
0.2150	.7715E+03	.5196E+03	.2384E+03	.9512E+02	.3699E+03	.1069E+03
0.2200	.7584E+03	.5313E+03	.3646E+03	.7863E+01	.3202E+03	.1129E+03
0.2250	.7167E+03	.5231E+03	.4806E+03	.4293E+02	.2576E+03	.1138E+03
0.2300	.6489E+03	.4959E+03	.5816E+03	.9107E+02	.1847E+03	.1114E+03
0.2350	.5589E+03	.4513E+03	.6030E+03	.1390E+03	.1047E+03	.1058E+03
0.2400	.4514E+03	.3920E+03	.7211E+03	.1820E+03	.2114E+02	.9688E+02
0.2450	.3339E+03	.3212E+03	.7582E+03	.2198E+03	.6281E+02	.8510E+02

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2

(CONTINUED)

TIME	AU( 1)	AC( 1)	AU( 2)	AC( 2)	AU( 3)	AC( 3)
.2500	.2112E+03	.2427E+03	.7579E+03	.2493E+03	.1408E+03	.7083E+02
.2550	.9027E+02	.1607E+03	.7335E+03	.2708E+03	.2115E+03	.5454E+02
.2600	.2315E+02	.7906E+02	.6819E+03	.2832E+03	.2706E+03	.3687E+02
.2650	.1240E+03	.1553E+01	.6045E+03	.2859E+03	.3153E+03	.1848E+02
.2700	.2082E+03	.6844E+02	.5045E+03	.2790E+03	.3433E+03	.1486E+02
.2750	.2733E+03	.12A3E+03	.3860E+03	.2631E+03	.3534E+03	.1746E+02
.2800	.3179E+03	.1762E+03	.2537E+03	.2388E+03	.3451E+03	.3358E+02
.2850	.3423E+03	.2108E+03	.1131E+03	.2074E+03	.3192E+03	.4768E+02
.2900	.3479E+03	.2321E+03	.2993E+02	.1706E+03	.2772E+03	.5919E+02
.2950	.3373E+03	.2405E+03	.1498E+03	.1298E+03	.2215E+03	.6766E+02
.3000	.3139E+03	.2373E+03	.3009E+03	.6720E+02	.1556E+03	.7298E+02
.3050	.2819E+03	.2244E+03	.4182E+03	.4448E+02	.8322E+02	.7500E+02
.3100	.2453E+03	.2039E+03	.5177E+03	.3369E+01	.8619E+01	.7382E+02
.3150	.2084E+03	.1785E+03	.5958E+03	.3437E+02	.6361E+02	.6968E+02
.3200	.1798E+03	.1506E+03	.6304E+03	.6744E+02	.1298E+03	.6299E+02
.3250	.1487E+03	.1228E+03	.6802E+03	.9470E+02	.1853E+03	.5435E+02
.3300	.1307E+03	.9724E+02	.8852E+03	.1155E+03	.2272E+03	.4428E+02
.3350	.1225E+03	.7562E+02	.6663E+03	.1293E+03	.2527E+03	.3361E+02
.3400	.1253E+03	.5931E+02	.6255E+03	.1363E+03	.2604E+03	.2296E+02
.3450	.1380E+03	.4901E+02	.5954E+03	.1367E+03	.2496E+03	.1311E+02
.3500	.1590E+03	.4487E+02	.4894E+03	.1313E+03	.2210E+03	.4647E+01
.3550	.1859E+03	.4643E+02	.4812E+03	.1210E+03	.1760E+03	.1780E+01
.3600	.2159E+03	.5283E+02	.3044E+03	.1070E+03	.1172E+03	.5742E+01
.3650	.2454E+03	.6280E+02	.2041E+03	.9039E+02	.4809E+02	.6999E+01
.3700	.2728E+03	.7879E+02	.1030E+03	.7260E+02	.2739E+02	.5386E+01
.3750	.2926E+03	.8713E+02	.4944E+01	.9485E+02	.1048E+03	.9687E+00
.3800	.3037E+03	.9613E+02	.8868E+02	.3835E+02	.1797E+03	.6081E+01
.3850	.3034E+03	.1061E+03	.1899E+03	.2409E+02	.2876E+03	.1534E+02
.3900	.2903E+03	.1099E+03	.2821E+03	.3046E+02	.3046E+03	.2634E+02
.3950	.2634E+03	.1044E+03	.3021E+03	.5153E+01	.3473E+03	.3844E+02
.4000	.2234E+03	.1011E+03	.3489E+03	.1316E+01	.3732E+03	.5096E+02
.4050	.1714E+03	.8773E+02	.3827E+03	.1277E+01	.3807E+03	.6315E+02
.4100	.1080E+03	.6865E+02	.4822E+03	.4900E+01	.3692E+03	.7425E+02
.4150	.3681E+02	.4463E+02	.8894E+03	.1150E+02	.3394E+03	.8363E+02
.4200	.4199E+02	.1670E+02	.4446E+03	.2075E+02	.2928E+03	.9066E+02
.4250	.1229E+03	.1372E+02	.3490E+03	.3159E+02	.2319E+03	.9485E+02
.4300	.2037E+03	.4513E+02	.3641E+03	.4313E+02	.1599E+03	.9581E+02
.4350	.2812E+03	.7584E+02	.3312E+03	.5457E+02	.8074E+02	.9340E+02
.4400	.3524E+03	.10A2E+03	.2921E+03	.6492E+02	.1519E+01	.8755E+02
.4450	.8147E+03	.12A6E+03	.2484E+03	.7339E+02	.8262E+02	.7841E+02
.4500	.4660E+03	.1480E+03	.2017E+03	.7930E+02	.1585E+03	.6625E+02
.4550	.5045E+03	.1612E+03	.1537E+03	.8208E+02	.2254E+03	.5155E+02
.4600	.5292E+03	.1677E+03	.1060E+03	.8135E+02	.2803E+03	.3482E+02
.4650	.5396E+03	.1672E+03	.8813E+02	.7694E+02	.3286E+03	.1679E+02
.4700	.5357E+03	.1598E+03	.1754E+02	.6883E+02	.3450E+03	.1932E+01
.4750	.5181E+03	.1462E+03	.2049E+02	.8721E+02	.3527E+03	.2057E+02
.4800	.4878E+03	.1271E+03	.5285E+02	.8245E+02	.3440E+03	.3842E+02
.4850	.4463E+03	.1037E+03	.7863E+02	.2503E+02	.3200E+03	.5485E+02
.4900	.3953E+03	.7743E+02	.9716E+02	.5582E+01	.2824E+03	.6921E+02
.4950	.3367E+03	.4959E+02	.1080E+03	.1624E+02	.2337E+03	.0104E+02

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2

(CONTINUED)

TIME	AU( 1)	AC( 1)	AU( 2)	AC( 2)	AU( 3)	AC( 3)
.5000	.2726E+03	.2174E+02	.1111E+03	.3669E+02	.1767E+03	.8996E+02
.5050	.2050E+03	.4730E+01	.1067E+03	.5801E+02	.1145E+03	.9574E+02
.5100	.1360E+03	.2851E+02	.9523E+02	.7855E+02	.9037E+02	.9838E+02
.5150	.6732E+02	.4859E+02	.7765E+02	.9759E+02	.1271E+02	.9777E+02
.5200	.7119E+00	.6416E+02	.3509E+02	.1146E+03	.7197E+02	.9419E+02
.5250	.6237E+02	.7470E+02	.2697E+02	.1290E+03	.1250E+03	.8793E+02
.5300	.1207E+03	.A000E+02	.9096E+00	.1405E+03	.1700E+03	.7933E+02
.5350	.1784E+03	.A021E+02	.2734E+02	.1408E+03	.2057E+03	.6883E+02
.5400	.2196E+03	.7562E+02	.5394E+02	.1537E+03	.2313E+03	.5699E+02
.5450	.2590E+03	.9693E+02	.7726E+02	.1552E+03	.2467E+03	.8432E+02
.5500	.2913E+03	.5491E+02	.9556E+02	.1533E+03	.2523E+03	.3132E+02
.5550	.3164E+03	.4046E+02	.1075E+03	.1882E+03	.2490E+03	.1847E+02
.5600	.3345E+03	.2462E+02	.1120E+03	.1399E+03	.2379E+03	.6262E+01
.5650	.3454E+03	.8332E+01	.1064E+03	.1290E+03	.2205E+03	.4955E+01
.5700	.3504E+03	.7447E+01	.9846E+02	.1155E+03	.1944E+03	.1489E+02
.5750	.3495E+03	.2194E+02	.7631E+02	.9999E+02	.1730E+03	.2322E+02
.5800	.3424E+03	.3445E+02	.9805E+02	.8272E+02	.1460E+03	.2990E+02
.5850	.3311E+03	.4488E+02	.1858E+02	.6410E+02	.1186E+03	.3480E+02
.5900	.3109E+03	.5174E+02	.2440E+02	.4455E+02	.9188E+02	.3796E+02
.5950	.2944E+03	.5619E+02	.6047E+02	.2446E+02	.6694E+02	.3982E+02
.6000	.2708E+03	.5706E+02	.1094E+03	.4214E+01	.4407E+02	.3931E+02
.6050	.2440E+03	.5697E+02	.1516E+03	.1501E+02	.2369E+02	.3781E+02
.6100	.2148E+03	.5397E+02	.1903E+03	.3526E+02	.5894E+01	.3511E+02
.6150	.1824E+03	.4926E+02	.2236E+03	.5379E+02	.9427E+01	.3143E+02
.6200	.1493E+03	.4337E+02	.2494E+03	.7185E+02	.2253E+02	.2698E+02
.6250	.1146E+03	.3668E+02	.2669E+03	.8604E+02	.3373E+02	.2190E+02
.6300	.7888E+02	.2979E+02	.2741E+03	.1408E+03	.4339E+02	.1664E+02
.6350	.4271E+02	.2302E+02	.2707E+03	.1127E+03	.5184E+02	.1114E+02
.6400	.6505E+01	.1667E+02	.2564E+03	.1225E+03	.5935E+02	.9626E+01
.6450	.2927E+02	.1093E+02	.2316E+03	.1299E+03	.6610E+02	.2485E+00
.6500	.6413E+02	.5860E+01	.1970E+03	.1349E+03	.7217E+02	.4885E+01
.6550	.9759E+02	.1428E+01	.1538E+03	.1373E+03	.7752E+02	.9677E+01
.6600	.1292E+03	.2444E+01	.1837E+03	.1373E+03	.8205E+02	.1407E+02
.6650	.1583E+03	.8192E+01	.8875E+02	.1389E+03	.8555E+02	.1799E+02
.6700	.1844E+03	.9861E+01	.9026E+01	.1302E+03	.8781E+02	.2140E+02
.6750	.2076E+03	.1381E+02	.9732E+02	.1234E+03	.8861E+02	.2427E+02
.6800	.2269E+03	.1833E+02	.1238E+03	.1147E+03	.8776E+02	.2658E+02
.6850	.2421E+03	.2368E+02	.1764E+03	.1043E+03	.8514E+02	.2828E+02
.6900	.2530E+03	.3000E+02	.2231E+03	.9255E+02	.8072E+02	.2937E+02
.6950	.2593E+03	.3745E+02	.2621E+03	.7984E+02	.7461E+02	.2983E+02
.7000	.2610E+03	.4596E+02	.2921E+03	.6644E+02	.6701E+02	.2965E+02
.7050	.2581E+03	.5541E+02	.3120E+03	.5277E+02	.5825E+02	.2884E+02
.7100	.2507E+03	.8557E+02	.3214E+03	.3907E+02	.4877E+02	.2741E+02
.7150	.2391E+03	.7609E+02	.3201E+03	.2571E+02	.3907E+02	.2538E+02
.7200	.2235E+03	.8660E+02	.3883E+03	.1380E+02	.2969E+02	.2280E+02
.7250	.2044E+03	.9657E+02	.2868E+03	.1126E+01	.2110E+02	.1972E+02
.7300	.1822E+03	.1055E+03	.2564E+03	.9698E+01	.1806E+02	.1623E+02
.7350	.1576E+03	.1129E+03	.2185E+03	.1911E+02	.8758E+01	.1243E+02
.7400	.1309E+03	.1103E+03	.1745E+03	.2722E+02	.5599E+01	.8427E+01
.7450	.1029E+03	.1211E+03	.1262E+03	.2897E+02	.4762E+01	.4366E+01

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

TIME	AU( 1)	AC( 1)	AU( 2)	AC( 2)	AU( 3)	AC( 3)
.7500	.7000E+02	.1211E+03	.7510E+02	.3930E+02	.6267E+01	.3869E+00
.7550	.4491E+02	.1181E+03	.2327E+02	.4329E+02	.9968E+01	.3362E+01
.7600	.1594E+02	.1118E+03	.2788E+02	.4603E+02	.1556E+02	.6731E+01
.7650	.1246E+02	.1024E+03	.7672E+02	.4768E+02	.2257E+02	.9588E+01
.7700	.3992E+02	.9991E+02	.1218E+03	.4830E+02	.3045E+02	.1180E+02
.7750	.6618E+02	.7464E+02	.1620E+03	.4812E+02	.3852E+02	.1326E+02
.7800	.9102E+02	.5696E+02	.1963E+03	.4732E+02	.4688E+02	.1392E+02
.7850	.1143E+03	.3736E+02	.2239E+03	.4609E+02	.5244E+02	.1371E+02
.7900	.1359E+03	.1638E+02	.2445E+03	.4493E+02	.5696E+02	.1265E+02
.7958	.1558E+03	.5341E+01	.2578E+03	.4280E+02	.5908E+02	.1077E+02
.8000	.1738E+03	.2726E+02	.2638E+03	.4100E+02	.5880E+02	.0143E+01

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

DOF	UNCOUPLED ACCELERATIONS				COUPLED ACCELERATIONS				EQUIPMENT REACTION			
	AMAX.	TMAX. (SEC)	AMIN.	TMIN. (SEC)	AMAX.	TMAX. (SEC)	AMIN.	TMIN. (SEC)	RMAX.	TMAX. (SEC)	RMIN.	TMIN. (SEC)
1	.7719E+03	.2100	-.0790E+03	.1300	.9313E+03	.2200	-.4019E+03	.1400	.5377E+00	.2330	-.4638E+00	.3100
2	.7578E+03	.2200	-.0692E+03	.3300	.2039E+03	.2650	-.2338E+03	.1000	.0728E+00	.5350	-.3931E+00	.4290
3	.3007E+03	.4000	-.0279E+03	.2000	.9301E+02	.0300	-.1130E+03	.2250	.8076E+00	.4000	-.2337E+00	.9050

TABLE 4-1: PUTOUT FOR EXAMPLE PROBLEM 2

(CONTINUED)

4-14

DOF	UNCOUPLED DISPLACEMENTS				COUPLED DISPLACEMENTS			
	D <sub>MAX.</sub>	T <sub>MAX.</sub> (SEC)	D <sub>MIN.</sub>	T <sub>MIN.</sub> (SEC)	D <sub>MAX.</sub>	T <sub>MAX.</sub> (SEC)	D <sub>MIN.</sub>	T <sub>MIN.</sub> (SEC)
1	.6667E+00	.1250	-.8994E+00	.4650	.4014E+00	.1250	-.3791E+00	.2250
2	.6000E+00	.1590	-.6637E+00	.4180	.2891E+00	.6700	-.3307E+00	.2700
3	.6062E+00	.1990	-.5319E+00	.4000	.1775E+00	.8150	-.2420E+00	.4050

TABLE 4-1: OUTPUT FOR EXAMPLE PROBLEM 2  
(CONTINUED)

UNCOUPLED VELOCITIES					COUPLED VELOCITIES			
DOF	VHAR,	THAN, (SEC)	VMIN,	VMIN, (SEC)	VHAR,	THAN, (SEC)	VMIN,	VMIN, (SEC)
1	.1757E+02	.0600	-.2310E+02	.1790	.1284E+02	.2690	-.1609E+02	.1000
2	.1020E+02	.0900	-.2039E+02	.2030	.0690E+01	.3100	-1.9832E+01	.2200
3	.9193E+01	.0400	-.1367E+02	.2400	.0097E+01	.0700	-.0503E+01	.2700

TAPR11 WAS GENERATED FOR THIS PROB.



## 5. PROGRAM VALIDATION - EXAMPLE PROBLEMS

### 5.1 GENERAL

This section presents the results for four of the sample problems used in validating Post-Processor Program CMDOF. In order to check the accuracy of this program, all uncoupled structure and equipment modes were included in the post-processor program for all cases studied in the validation runs. Example Problem 1 (Figures 5-1 and 5-2) represents a multi-degree-of-freedom (mdof) structure with three locations of cross-coupling between the structure and attached equipment. Example Problem 2 (Figures 5-3 and 5-4) is a mdof structure coupled at three locations with three different equipment subsystems. Example Problem 3 (Figures 5-5 and 5-6), is a mdof structure coupled at two different locations with three coupled degrees-of-freedom at each location.

Program CMDOF is intended to primarily be used for multiple supported piping systems attached at one or more locations to an uncoupled vibrating structure but with at least one quiet support away from this vibrating structure. Thus, the program has been formulated with the uncoupled equipment model being attached to an at-rest support independent of the uncoupled structure model (see Figures 5-1, 5-3, and 5-5). However, in some cases the uncoupled equipment is only attached to the uncoupled structure such as shown in Figure 5-7. A previous program CBR (Reference 4) was written specifically for this type problem. However, this problem can also be solved using program CMDOF. The fourth example problem (Figure 5-7) is shown to illustrate the use of program CMDOF for problems in which the uncoupled equipment is only attached to the uncoupled structure.

Program CMDOF has been validated for many additional problems including all of the example problems contained in Reference 4. The four example problems presented in this section was selected because in each case the uncoupled equipment has similar mass, stiffness, and frequency characteristics as the uncoupled structure. Thus the equipment introduces very severe coupling

effects such that the coupled time history and spectra have very little resemblance to the uncoupled time history and spectra. These examples were considered to be especially severe tests of the coupling program. For example problems 1, 2, and 4 both the coupled and uncoupled models have damping specified as  $\bar{\alpha}$ ,  $\bar{\beta}$  damping (see Section 3.3 for definition of terms). For Example Problem 3 constant modal damping has been used for all modes for both the uncoupled and coupled models.

For each example problem, the uncoupled structure model was subjected to the triangular pulse force time history shown in Figure 5-8. This force time history was applied to both the uncoupled and coupled structures as shown on the respective figures for all four example problems. Computer Program HDYNE (Reference 2), a well-tested program which uses the Nigam-Jennings integration scheme, was used to generate the uncoupled acceleration time histories for all the equipment attachment nodes. In addition, Program HDYNE was used to directly perform a coupled analysis for each example problem in order to test the accuracy of program CMDOF.

## 5.2 EXAMPLE PROBLEM #1

Figure 5-1 illustrates the uncoupled structure and equipment models used in Example Problem #1. This figure shows a structure with cross-coupling at three dof to an attached piece of equipment. The coupled structure and equipment model is shown in Figure 5-2.

Damping for both the uncoupled and coupled structure and equipment models was specified using mass and stiffness proportional damping (Rayleigh Damping). The damping for any mode is given by:

$$\lambda_m = \frac{\bar{\alpha}}{2f_m} + \frac{\bar{\beta}f_m}{2} \quad (5.1)$$

In this problem,  $\alpha = 0.05$  and  $\beta = 0.018$  were used and result in the modal damping values tabulated in Figures 5-1 and 5-2. All analyses were performed using a time step size of 0.005 seconds.

Computer Program HDYNE uses a diagonal mass matrix and a normalized eigenvector defined by  $\phi^T M \phi = 1.0$  for any mode so that:

1. The participation factor  $PF_{j,m}$  equals the eigenvector  $\phi_{j,m}$  for any attachment dof  $j$  for mode  $m$ .

2. The  $\sum_{m=1}^{all} \phi_{j,m} \cdot PF_{j,m}$  for dof  $j$  must be equal to  $1/M_j$   
(one divided by the mass at dof  $j$ ).

Figure 5-1 lists the input frequencies and eigenvector data required for both the uncoupled structure and equipment needed in order to perform a coupling analysis using Post-Processor Program CMDOF.

Plots showing the comparison between the input uncoupled acceleration time history, the coupled acceleration time history results using Post-Processor Program CMDOF, and the coupled acceleration time history calculated from the fully coupled model (Figure 5-2) are shown in Figures 5-9 and 5-10 for dof 1 and 2, respectively.

Note the excellent agreement between the HDYNE fully coupled results and the coupled results using Post-Processor Program CMDOF. Even though the difference between peak coupled and peak uncoupled acceleration is substantial (a ratio of 0.4 for dof 2), the fully coupled and the Post-Processor Program CMDOF coupled accelerations are essentially identical (less than 2 percent different at the peaks). Similarly, the fully coupled and the CMDOF post-processed coupled response spectra (Figures 5-11, 5-12 for dof 1 and 2) are essentially identical throughout the frequency range even though the coupled spectra are substantially different from the uncoupled spectra. Thus, it can be concluded for this problem that the Post-Processor Program CMDOF accurately calculates the coupled accelerations for all attachment dof even though the coupled accelerations are very different from the input uncoupled acceleration.

### 5.3 EXAMPLE PROBLEM #2

The second example problem consists of a five dof uncoupled structure with three separate equipment subsystems attached at three different locations. The uncoupled structure and equipment models are illustrated in Figure 5-3 while the fully coupled model used in HDYNE to check the results of Post-Processor Program CMDOF is shown in Figure 5-4. This problem is quite different from Example Problem #1 since, in this case, there is not any coupling between the uncoupled equipment subsystems. Mass and stiffness proportional damping with  $\bar{\alpha} = 0.05$  and  $\bar{\beta} = 0.018$  were again used for both the uncoupled and coupled models.

Figures 5-13, 5-14, and 5-15 present a comparison of the input uncoupled acceleration time history with the coupled acceleration time history results found using the Post-Processor Program CMDOF for dof 1, 2, and 3 respectively. Also shown in these figures are the coupled acceleration time history results found using a fully coupled model (Figure 5-4) with program HDYNE. The results show that the program CMDOF correctly calculates the coupled acceleration time history for all the attachment dof with negligible errors ( $\approx 2\%$  max. error). The ratio of peak coupled to uncoupled acceleration is 0.69, 0.33, and 0.26 for dof 1, 2, and 3 respectively. This demonstrates that coupling effects can significantly reduce structural response below the levels predicted by an uncoupled analysis. This reduction in structural response can also be seen in Figure 5-16 which is a comparison of response spectra generated for dof 2 using both the uncoupled and coupled acceleration time histories. Note the significant reduction in the predicted response when the true coupled acceleration time history is used. This is representative of the type of reduction which result from using a coupled analysis or compared to an uncoupled analysis.

### 5.4 EXAMPLE PROBLEM #3

Example Problem #3, couples a structure and an equipment subsystem together at two locations with coupling between three different dof (translation, rotation, and torsion) at each location. The uncoupled structure and equipment models are shown in Figure 5-5. The fully coupled model used in

HDYNE to check the results of the program CMDOF is shown in Figure 5-6. Table 5-1 presents the element stiffness and masses for all dof used in the uncoupled and coupled models. Table 5-2 presents the uncoupled structures and equipment frequencies, and eigenvectors. Constant modal damping or 2% of critical was assumed for all modes for both the uncoupled and coupled models.

Figure 5-17 and 5-18 present comparisons of the input uncoupled acceleration time history, coupled results using the Post-Processor Program CMDOF, and coupled acceleration calculated using the fully coupled model shown in Figure 5-6 for dof 1 and 2. Response spectra comparing Post-Processor and fully coupled results for the two dof are shown in Figures 5-19 and 5-20. Excellent agreement between the Post-Processor results and response from the fully coupled analysis can be noted in all figures. Similar results were obtained at the other attachment dof. The maximum error found in a peak coupled acceleration at any of the six coupled dof was approximately 3%.

Results for this example problem showed that program CMDOF calculates the coupled accelerations well for all dof even when there is multiple degrees of coupling at one or more attachment nodes. Again, it can be concluded that program CMDOF accurately calculates coupled response from uncoupled structure and equipment models for all attachment dof when all equipment and structure modes are included.

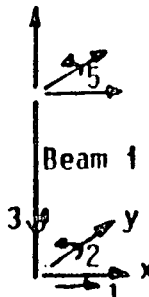
#### 5.5 EXAMPLE PROBLEM #4

Example Problem #4 represents a simple uncoupled equipment model coupled only to a simple uncoupled structure model. In other words, the equipment is not also coupled to the base. Program CMDOF cannot handle this equipment model directly. However, a pseudo equipment model in which a very light weight is used at the attachment point which is then connected to the base by a very soft spring can be developed. This pseudo equipment model can be used in Program CMDOF. The pseudo spring must have low enough stiffness and the pseudo attachment weight must be low enough to not significantly influence the solution. On the other hand, these properties

must not be so low as to prevent inaccurate model results to be computed for the pseudo equipment model.

Figure 5-21 shows the uncoupled acceleration time history for the structure attachment mode together with the coupled acceleration time histories from the fully coupled model and from the coupling Pos-Processor Program CMDOF. Again, one notes excellent agreement of the CMDOF coupling results versus the fully coupled model results.

TABLE 5-1: ELEMENT STIFFNESSES AND INERTIAS - EXAMPLE PROBLEM 3

Element Designation On Models	Complete Element Stiffness about Shear Center							Comments
	$K_{11}$ (k/ft)	$K_{12}$ (K/Rad)	$K_{15}$ (K/Rad)	$K_{22}$ (ft-K/Rad)	$K_{55}$ (ft-K/Rad)	$K_{25}$ (ft-K/Rad)	$K_{22}$ (ft-K/Rad)	
1) $K_s$ - (Uncoupled Structure)	12.	60.	60.	400.	400.	200.	50.	<p><b>ELEMENT STIFFNESS DEFINITION</b></p> <p>Far End</p> <p>1 = x Translation 2 = Near End y-Rotation 5 = Far End y-Rotation 3 = Torsion</p> 
2) $K_e$ - (Uncoupled Equipment)	12.	60.	60.	400.	400.	200.	50.	
3) $K_{c1}$ - (Coupled Model)	12.	60.	60.	400.	400.	200.	50.	
4) $K_{c2}$ - (Coupled Model)	24.	120.	120.	800.	800.	400.	100.	

5-7

Weight Designation On Models	Element Inertias About Mass Center (CG)		
	Translational (Kips)	Rotational (ft <sup>2</sup> -Kip)	Torsional (ft <sup>2</sup> -Kip)
1) $W_s$ - (Uncoupled Structure)	0.4	1000.	1000.
2) $W_e$ - (Uncoupled Equipment)	0.4	1000.	1000.
3) $W_{c1}$ - (Coupled Model)	0.4	1000.	1000.
4) $W_{c2}$ - (Coupled Model)	0.8	2000.	2000.

TABLE 5-2: UNCOUPLED STRUCTURE AND EQUIPMENT DATA - EXAMPLE PROBLEM 3

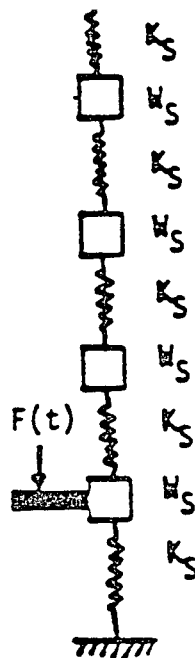
Uncoupled Structure Data

Mode	$f_s$ (Hz)	$\phi_s$ (1) (PF <sub>s</sub> (1))	$\phi_s$ (2) (PF <sub>s</sub> (2))	$\phi_s$ (3) (PF <sub>s</sub> (3))	$\phi_s$ (4) (PF <sub>s</sub> (4))	$\phi_s$ (5) (PF <sub>s</sub> (5))	$\phi_s$ (6) (PF <sub>s</sub> (6))	$\lambda_s$
1	0.184	.2138E+01	.2330E-01	-.1132E-01	.1470E+02	.4918E-01	-.1734E-01	0.02
2	0.215	-.1758E+01	.4681E-02	-.3410E-01	-.3697E+01	.1020E-01	-.9009E-01	0.02
3	0.519	-.9195E+01	-.8470E-02	.4217E-01	-.1165E+02	.6642E-01	.8524E-02	0.02
4	0.610	.9178E+01	-.3070E-02	-.8842E-01	-.6140E+01	-.4613E-02	-.5537E-02	0.02
5	0.791	.5031E+01	.2460E-01	-.5721E-01	-.1428E+01	.2208E-01	.9037E-01	0.02
6	0.938	-.1824E+01	-.8591E-01	-.5403E-01	-.3213E-01	-.1042E-01	-.8765E-02	0.02
7	1.339	-.4612E+01	-.9826E-01	-.2863E-01	.1719E+01	.8596E-01	.2009E-01	0.02
8	1.680	.3593E+01	.6099E-01	.6998E-02	-.1331E+01	.9729E-01	.3538E-02	0.02
9	4.934	.2004E+02	-.6781E-01	.8278E-01	.1715E+01	-.5553E-02	.1504E-01	0.02
10	7.341	-.7864E+01	-.1975E-02	.1060E-01	-.2656E+01	-.8758E-01	.8732E-01	0.02
11	8.773	-.1502E+02	-.1175E-01	.3094E-01	.1806E+02	.5152E-02	.1813E-01	0.02
12	9.992	.1900E+02	.1381E-01	-.3883E-01	.1401E+02	-.9223E-02	.3947E-01	0.02

Uncoupled Equipment Data

Mode	$f_e$ (Hz)	$\phi_e$ (1) (PF <sub>e</sub> (1))	$\phi_e$ (2) (PF <sub>e</sub> (2))	$\phi_e$ (3) (PF <sub>e</sub> (3))	$\phi_e$ (4) (PF <sub>e</sub> (4))	$\phi_e$ (5) (PF <sub>e</sub> (5))	$\phi_e$ (6) (PF <sub>e</sub> (6))	$\lambda_e$
1	0.257	-.3951E+01	-.2383E-01	.4080E-01	-.1915E+02	-.4517E-01	.8305E-01	0.02
2	0.287	.8862E-01	.2861E-01	.2847E-01	.1071E+02	.5355E-01	.8156E-01	0.02
3	0.625	-.4252E+01	.1150E-01	.7549E-01	-.7013E+01	.9553E-01	-.5312E-02	0.02
4	0.718	.5884E+01	.1408E-01	-.8377E-01	-.1051E+02	.3559E-01	.7834E-01	0.02
5	1.106	.3971E+01	.1141E+00	.5107E-01	-.2437E+01	-.8360E-01	-.8008E-04	0.02
6	1.599	.4853E+01	.8461E-01	.1239E-01	-.1220E+01	.6043E-01	-.7073E-02	0.02
7	4.937	.2011E+02	-.6783E-01	.8264E-01	.2376E+01	-.3476E-02	.1208E-01	0.02
8	8.201	-.1464E+02	-.8972E-02	.2689E-01	.1501E+02	-.2481E-01	.7095E-01	0.02
9	9.775	.1945E+02	.1543E-01	-.3616E-01	.9253E+01	-.1560E-01	.8321E-01	0.02





ATTACHMENT DOF (3)

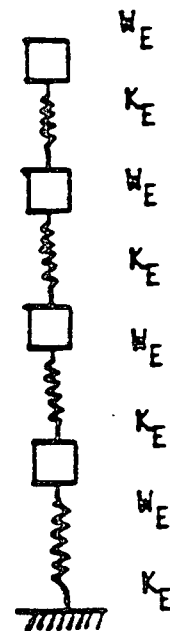
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ATTACHMENT DOF (2)

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ATTACHMENT DOF (1)

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UNCOUPLED STRUCTURE

UNCOUPLED EQUIPMENT

Uncoupled Structure Data

$W_S = 0.4 \text{ Kip}; K_S = 12 \text{ K/ft}$

Mode	$f_S$ ( $H_z$ )	$\phi_S(1)$ ( $PF_S(1)$ )	$\phi_S(2)$ ( $PE_S(2)$ )	$\phi_S(3)$ ( $PF_S(3)$ )	$\lambda_S$
1	1.407	10.13	14.16	17.04	0.030
2	4.108	-18.54	-10.13	5.28	0.043
3	6.476	5.28	-17.04	-10.13	0.062
4	8.319	14.16	5.28	-18.54	0.078
5	9.489	17.04	-18.54	14.16	0.088

For Any Dof:  $\sum_{M=1}^{all} \phi_S \cdot PF_S = 965. = \frac{1}{M_S} = \frac{386.088 \text{ in/sec}^2}{0.4 \text{ Kip}}$

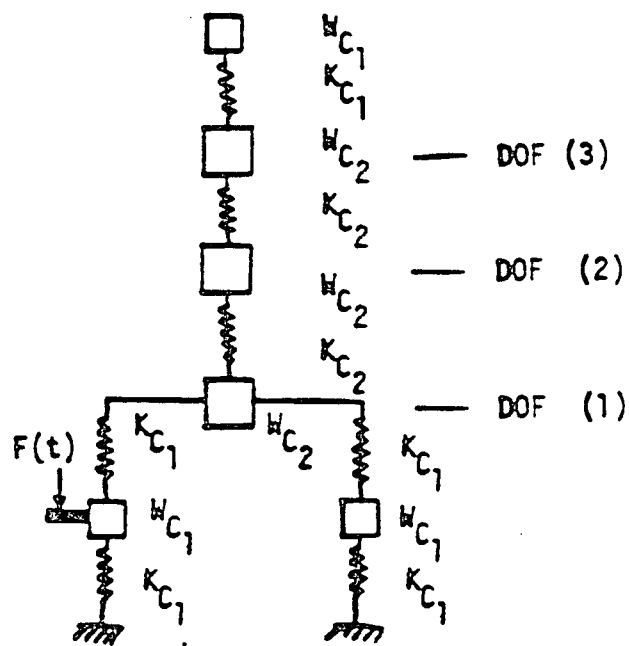
Uncoupled Equipment Data

$W_E = 0.4 \text{ Kip}; K_E = 12 \text{ K/ft}$

Mode	$f_E$ ( $H_z$ )	$\phi_E(1)$ $PF_E(1)$	$\phi_E(2)$ $PF_E(2)$	$\phi_E(3)$ $PF_E(3)$	$\lambda_E$
1	1.717	13.31	17.94	20.40	0.030
2	4.945	-17.94	0.0	17.94	0.050
3	7.576	-7.08	17.94	13.31	0.071
4	9.293	-20.40	17.94	-7.08	0.086

For Any Dof:  $\sum_{M=1}^{all} \phi_E \cdot PF_E = 965. = \frac{1}{M_E} = \frac{386.088 \text{ in/sec}^2}{0.4 \text{ Kip}}$

FIGURE 5-1: EXAMPLE PROBLEM 1 - UNCOUPLED STRUCTURE AND EQUIPMENT MODELS



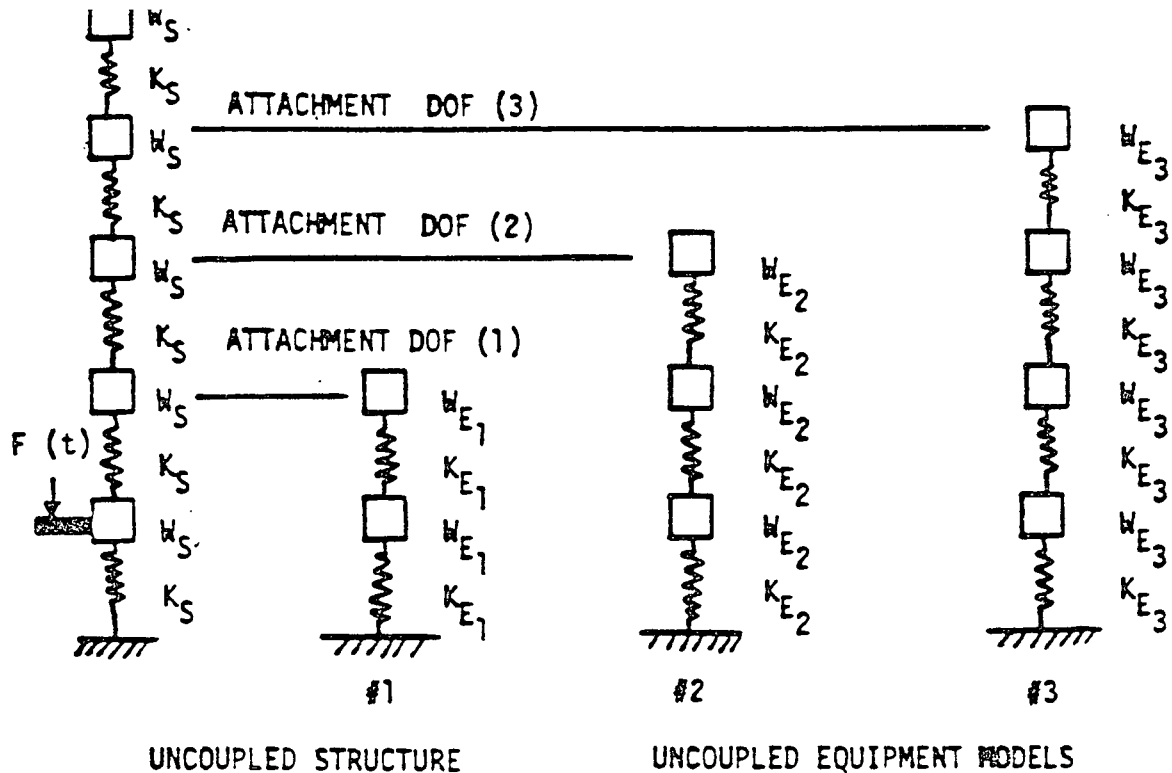
### COUPLED STRUCTURE AND EQUIPMENT

$$W_{C_1} = 0.4 \text{ Kip}; \quad W_{C_2} = 0.8 \text{ Kip}$$

$$K_{C_1} = 12 \text{ K/ft}; \quad K_{C_2} = 24 \text{ K/ft}$$

Mode	$f_C$ (Hz)	$\lambda_C$
1	1.539	0.030
2	4.249	0.044
3	6.109	0.059
4	6.993	0.066
5	7.905	0.074
6	9.358	0.087

FIGURE 5-2: EXAMPLE PROBLEM 1 - COUPLED STRUCTURE AND EQUIPMENT



#### Uncoupled Structure Data

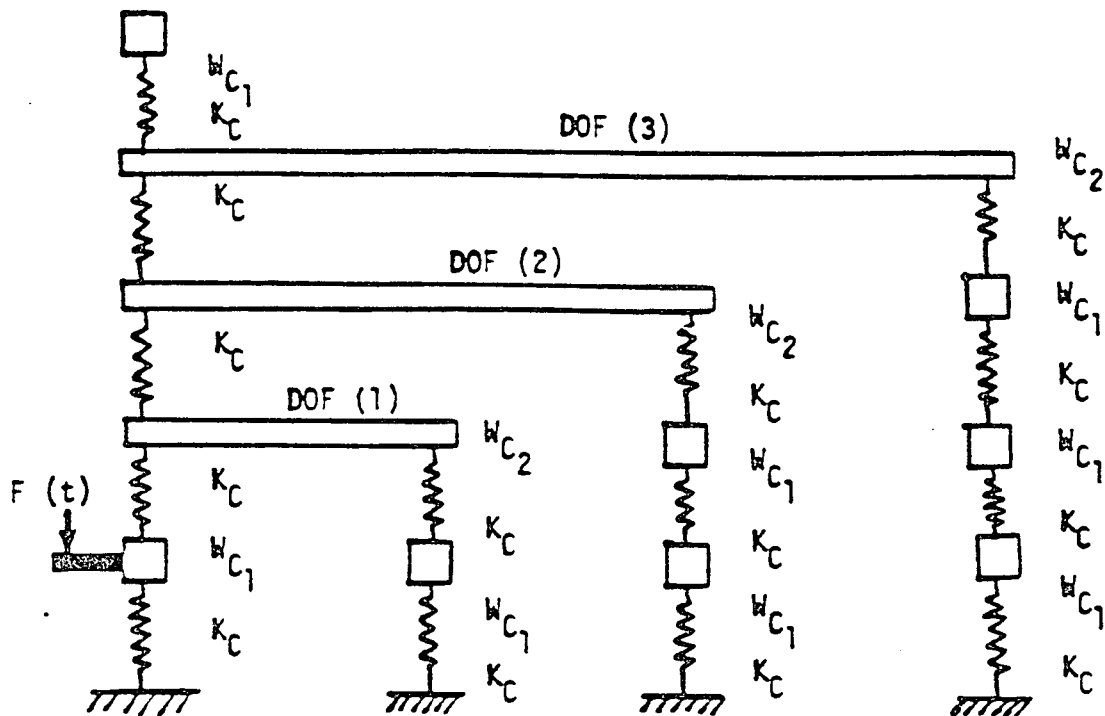
All structural data for Example Problem 2 is the same as the data used in Example Problem 1

#### Uncoupled Equipment Data

$$W_{E1} = W_{E2} = W_{E3} = 0.4 \text{ Kip}; \quad K_{E1} = K_{E2} = K_{E3} = 12 \text{ Kip/ft.}$$

	Mode	$f_E$ (Hz)	$\phi_E(1)$ ( $PF_E(1)$ )	$\phi_E(2)$ ( $PF_E(2)$ )	$\phi_E(3)$ ( $PF_E(3)$ )	$\lambda_E$
Model 1	1	3.056	26.43	0.0	0.0	0.036
	2	8.001	-16.33	0.0	0.0	0.075
Model 2	1	2.201	0.0	29.90	0.0	0.031
	2	6.166	0.0	-18.36	0.0	0.060
	3	8.910	0.0	10.19	0.0	0.083
Model 3	1	1.717	0.0	0.0	20.40	0.030
	2	4.945	0.0	0.0	17.94	0.050
	3	7.576	0.0	0.0	13.31	0.071
	4	9.293	0.0	0.0	-7.08	0.086

FIGURE 5-3: Example Problem 2 - Uncoupled Structure and Equipment Models



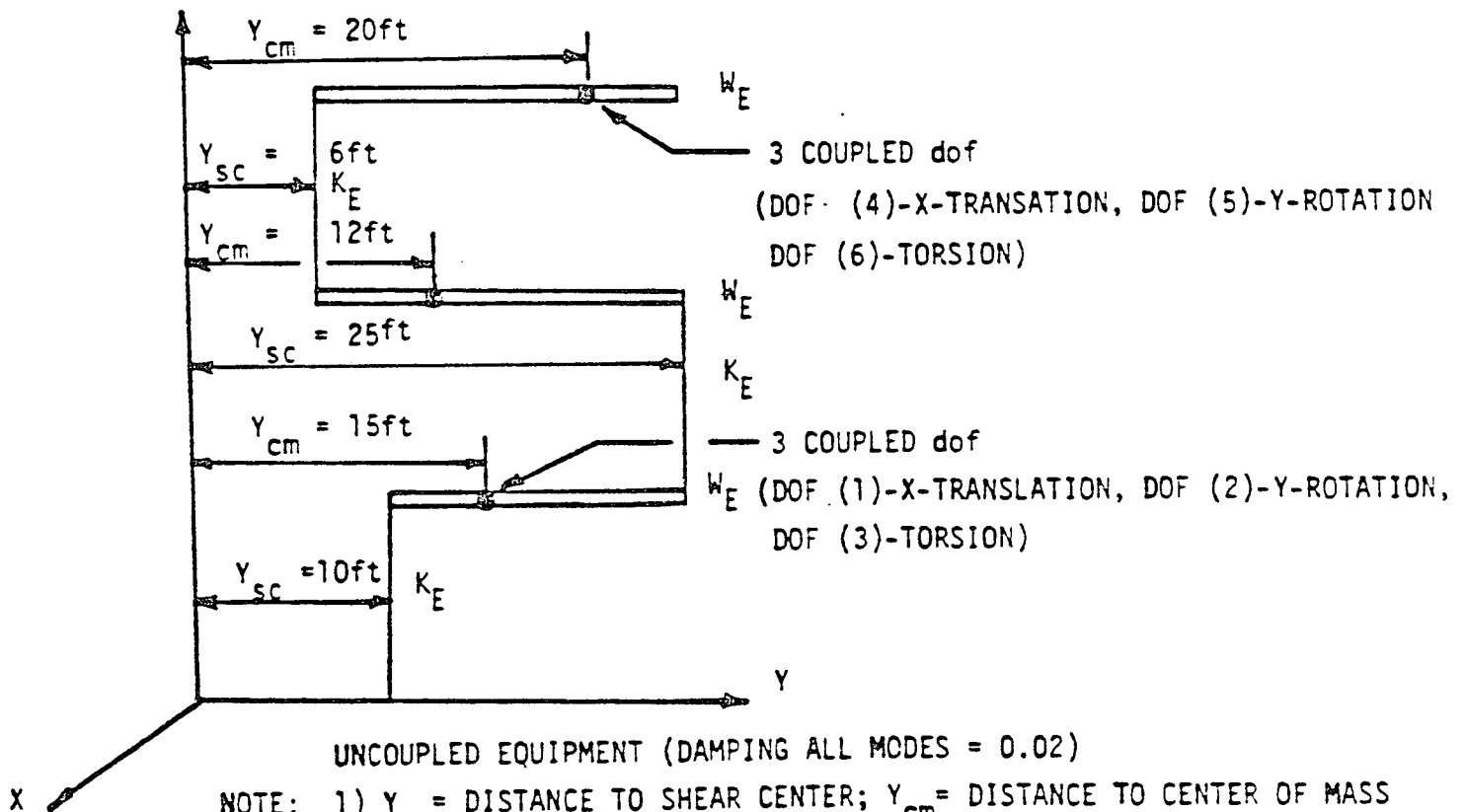
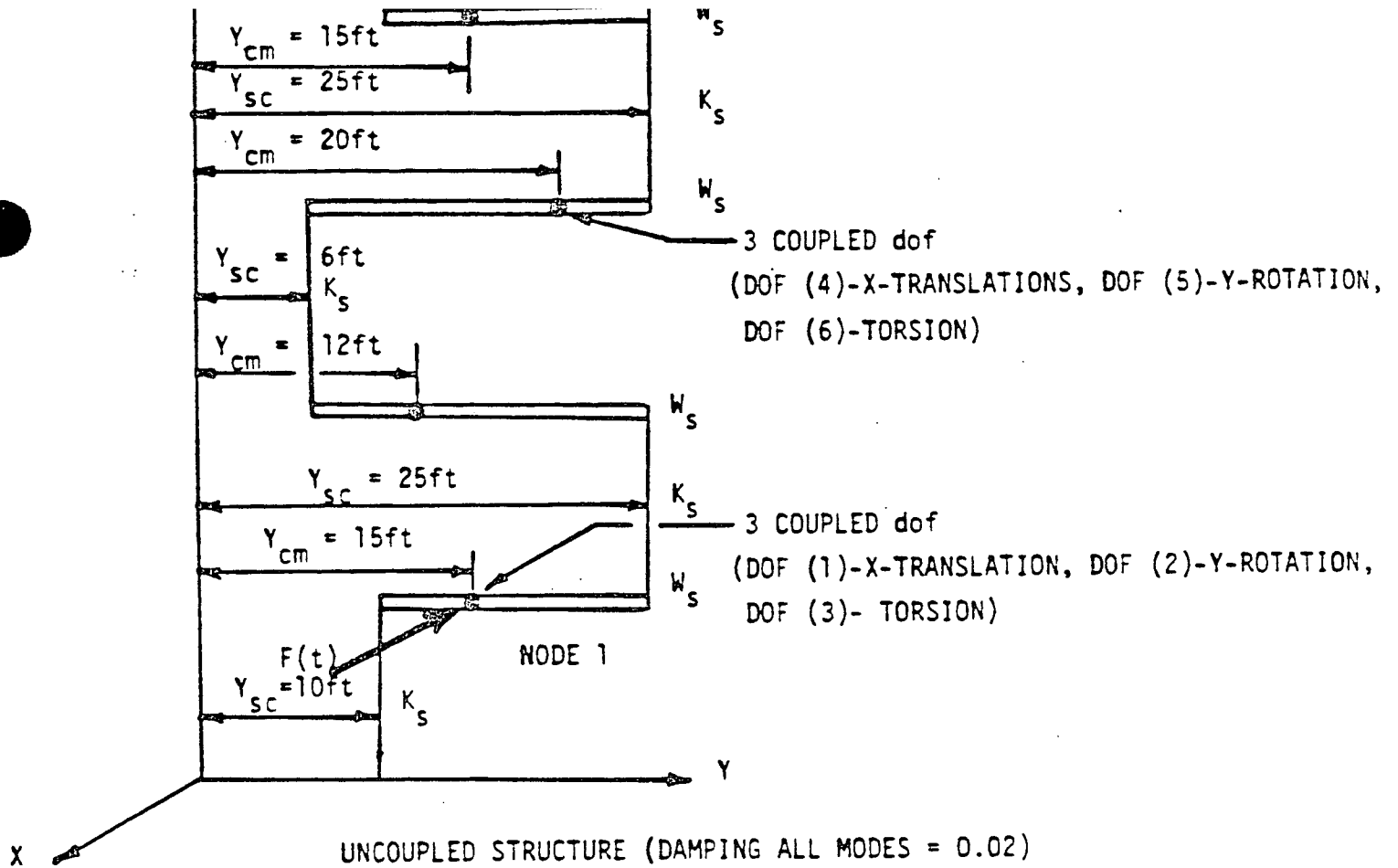
# COUPLED STRUCTURE AND EQUIPMENT

$$W_{C1} = 0.4 \text{ Kip} ; W_{C2} = 0.8$$

$$K_C = 12 \text{ Kip/ft.}$$

MODE	$f_c$ (Hz)	$\lambda_c$
1	1.762	0.030
2	3.392	0.038
3	4.140	0.043
4	4.571	0.047
5	6.063	0.059
6	6.694	0.064
7	6.993	0.066
8	7.773	0.073
9	8.308	0.078
10	8.861	0.083
11	9.256	0.086

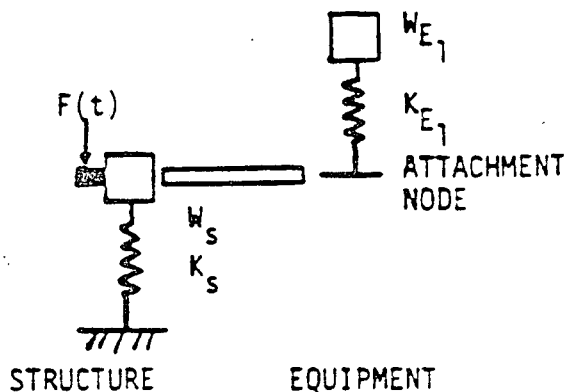
FIGURE 5-4: EXAMPLE PROBLEM 2 - COUPLED STRUCTURE AND EQUIPMENT MODELS



- NOTE: 1)  $Y_{sc}$  = DISTANCE TO SHEAR CENTER;  $Y_{cm}$  = DISTANCE TO CENTER OF MASS  
 2) SEE TABLE 5-1 FOR ELEMENT STIFFNESSES AND WEIGHTS  
 3) INPUT APPLIED IN X-DIRECTION

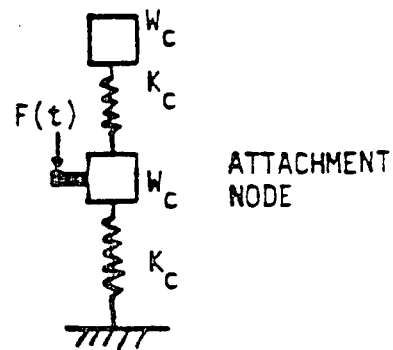
FIGURE 5-5: EXAMPLE PROBLEM 3- UNCOUPLED STRUCTURE AND EQUIPMENT MODELS  
 5-13





### UNCOUPLED MODEL

(MODEL USED IN REF.4)



### COUPLED MODEL

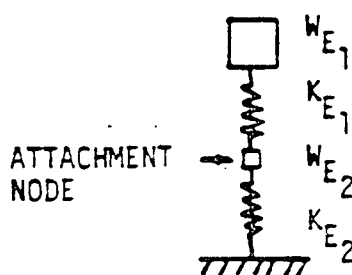
### WEIGHTS & STIFFNESSES

$$W_c = W_{E1} = W_s = 0.4 \text{ KIP}$$

$$K_c = K_{E1} = K_s = 12 \text{ K/ft}$$

$$W_{E2} = 0.01 \text{ KIP}$$

$$K_{E2} = 1 \text{ K/ft}$$



### STRUCTURE DATA

MODE	$f_s$ (Hz)	$PF_s$ $\phi_s$	$\lambda_s$
1	4.945	31.08	0.0496

### COUPLED DATA

MODE	$f_c$ Hz	$\lambda_c$
1	3.06	0.0357
2	8.00	0.0751

### FIXED BASE EQUIPMENT DATA (REF 4)

MODE	$f_E$ (Hz)	$\lambda_E$
1	4.945	0.0496

### CMDOF EQUIPMENT DATA

MODE	$f_E$ (Hz)	$PF_E$ $\phi_E$	$\lambda_E$
1	0.045	30.69	0.56
2	31.66	194.1	0.28

FIGURE 5-7: EXAMPLE PROBLEM 4  
STRUCTURE AND EQUIPMENT MODELS

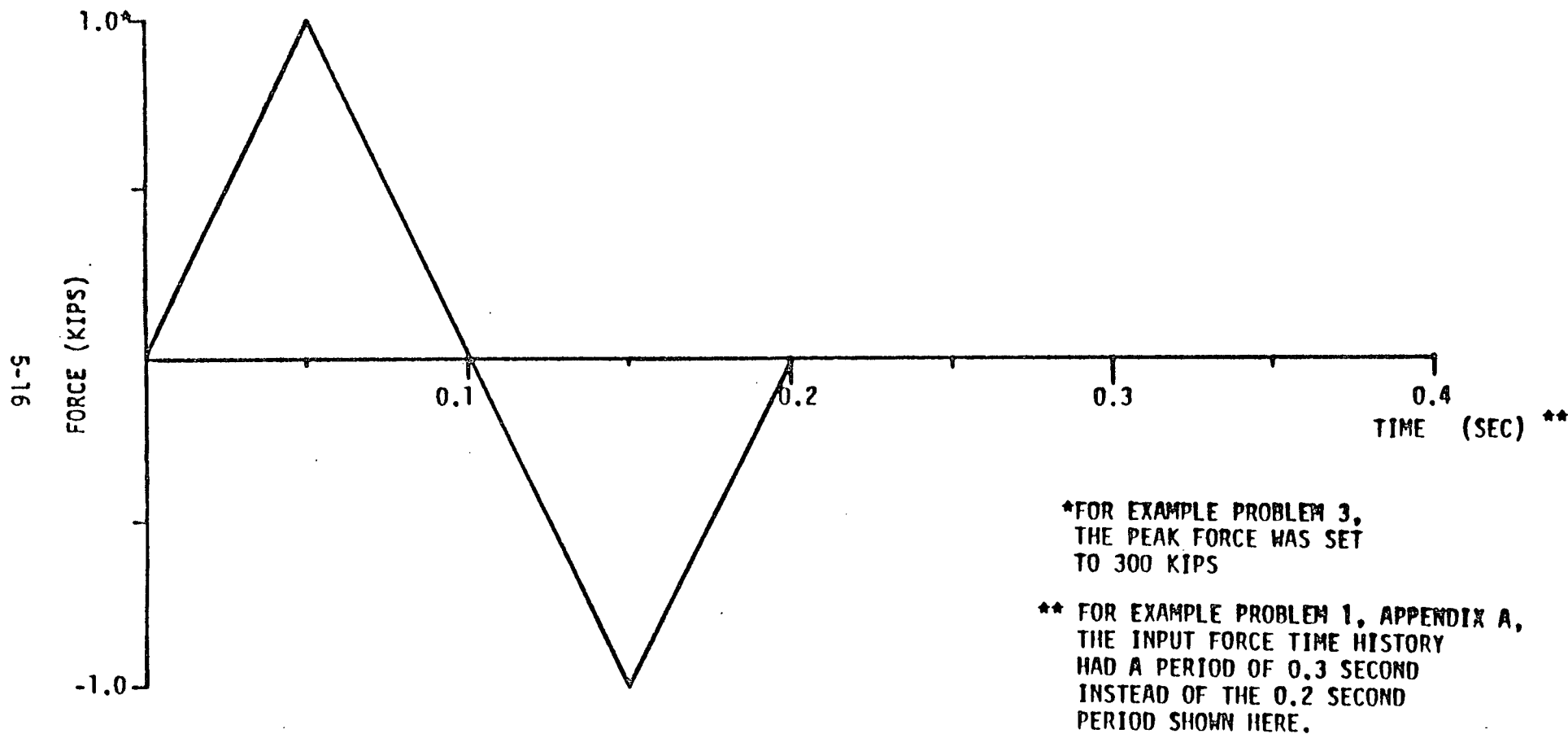


FIGURE 5-8: FORCE TIME HISTORY INPUT FOR EXAMPLE PROBLEMS



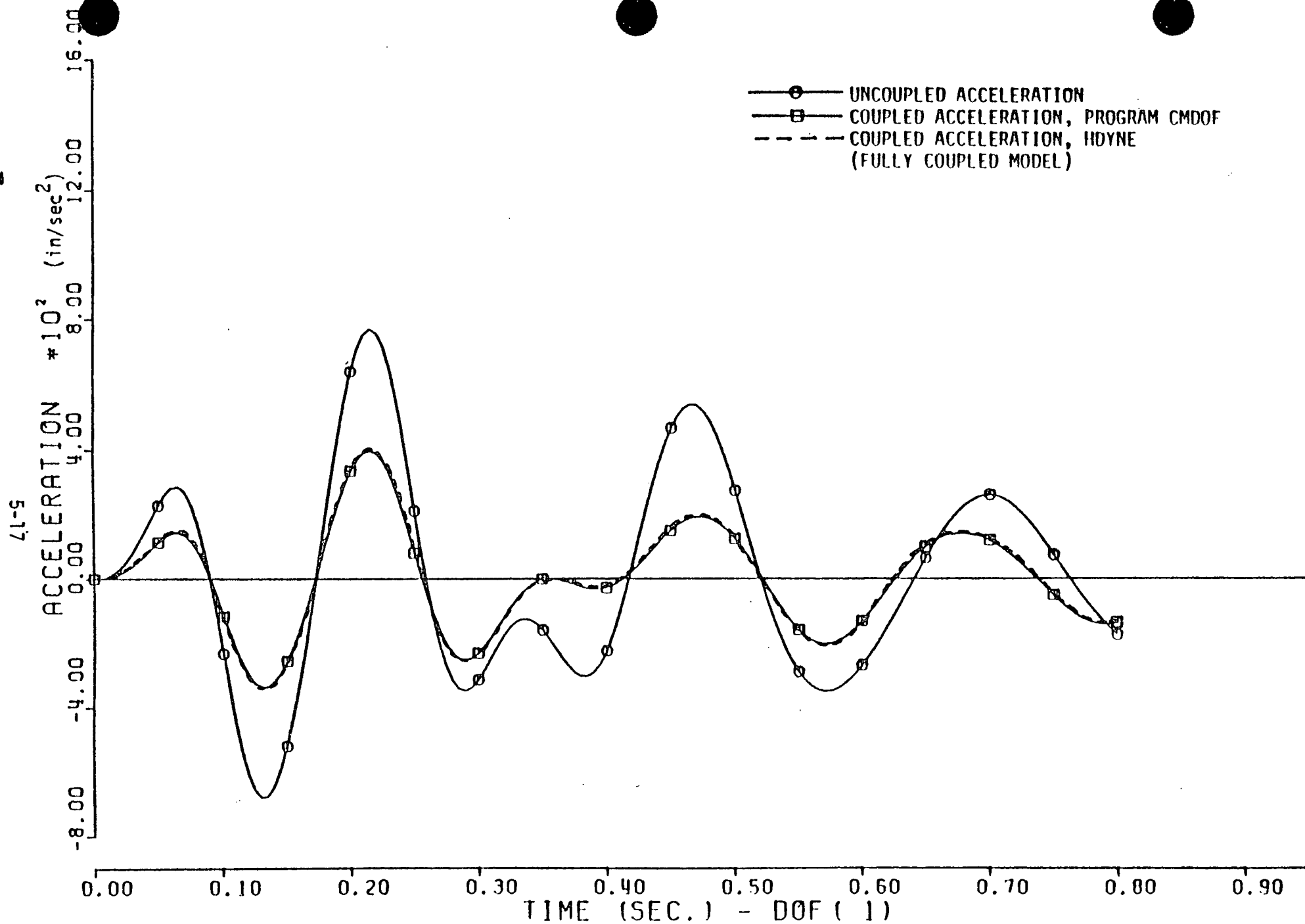


FIGURE 5-9: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 1 -DOF 1

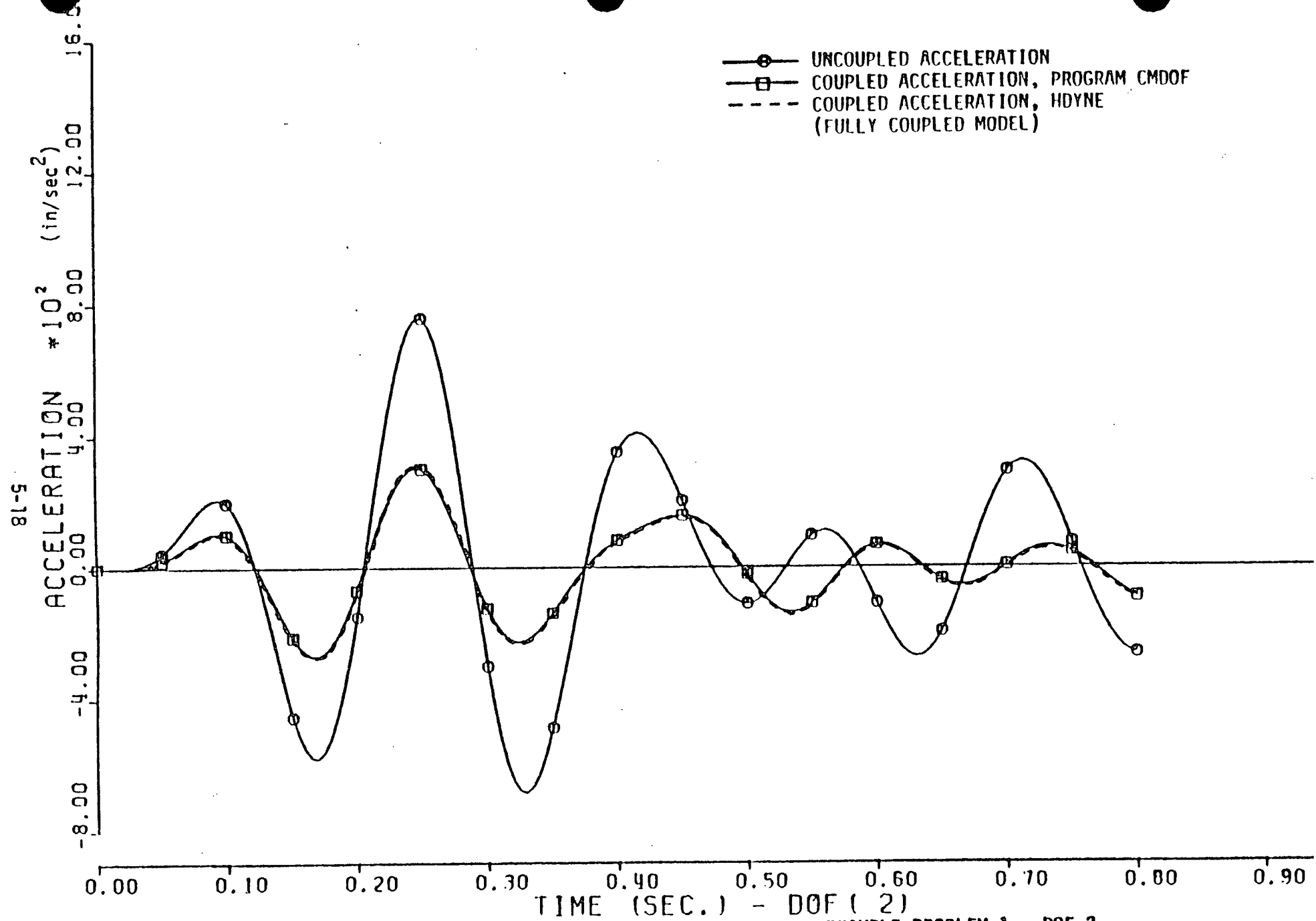


FIGURE 5-10: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 1 - DOF 2

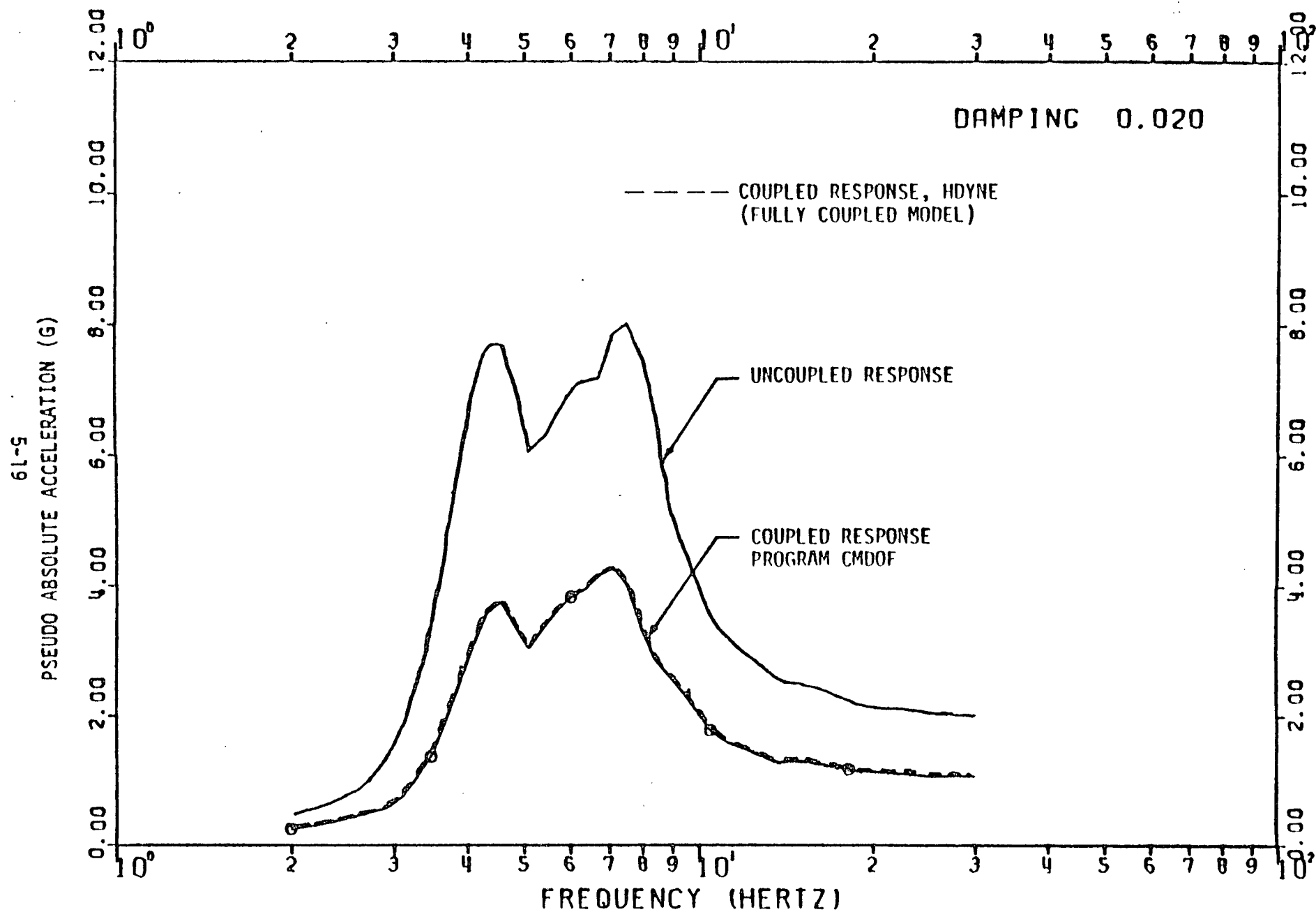


FIGURE 5-11: COMPARISON OF UNCOUPLED AND COUPLED RESPONSE - EXAMPLE PROBLEM 1 - DOF 1

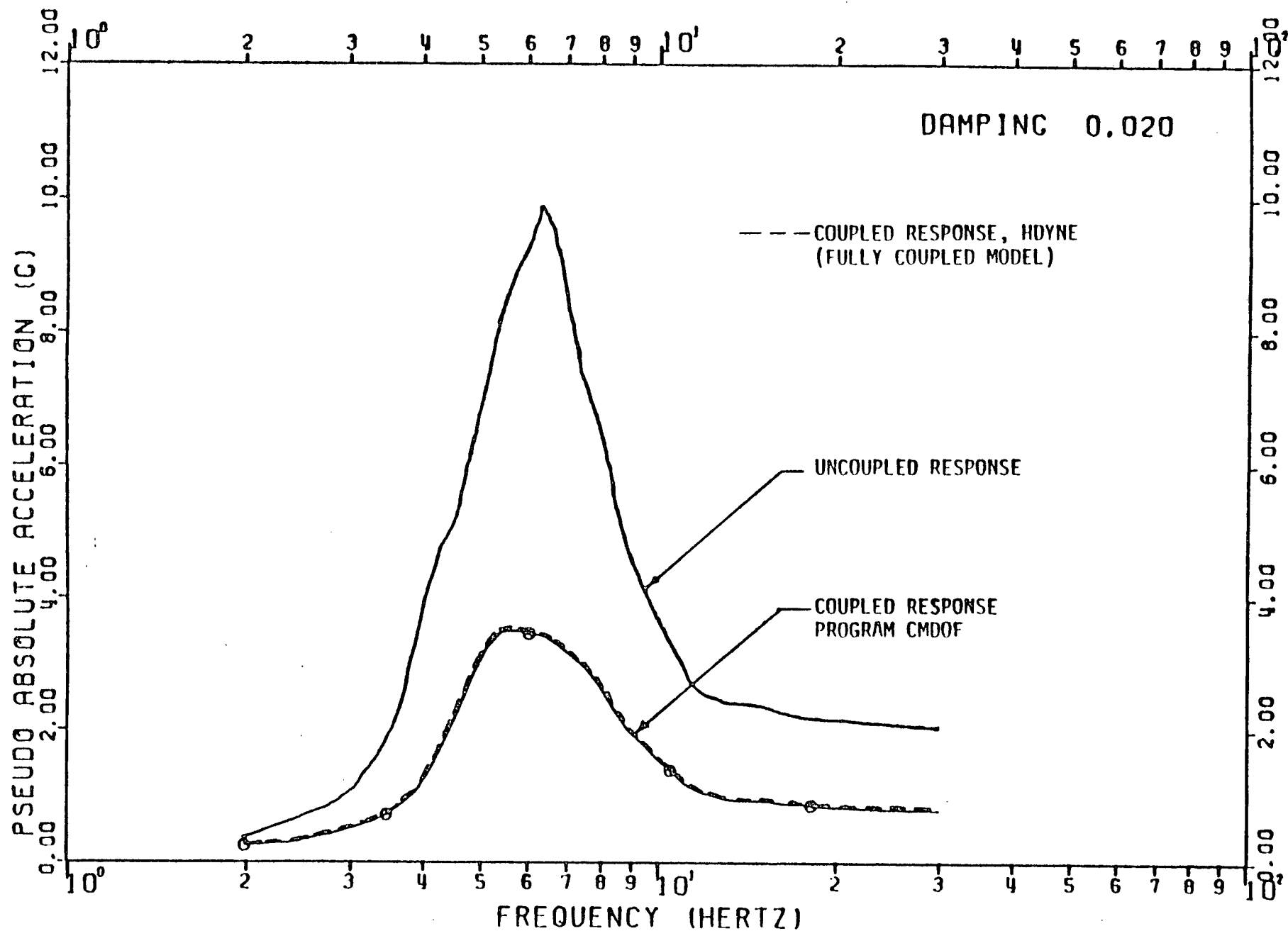


FIGURE 5-12: COMPARISON OF UNCOUPLED AND COUPLED RESPONSE - EXAMPLE PROBLEM 1 - DOF 2

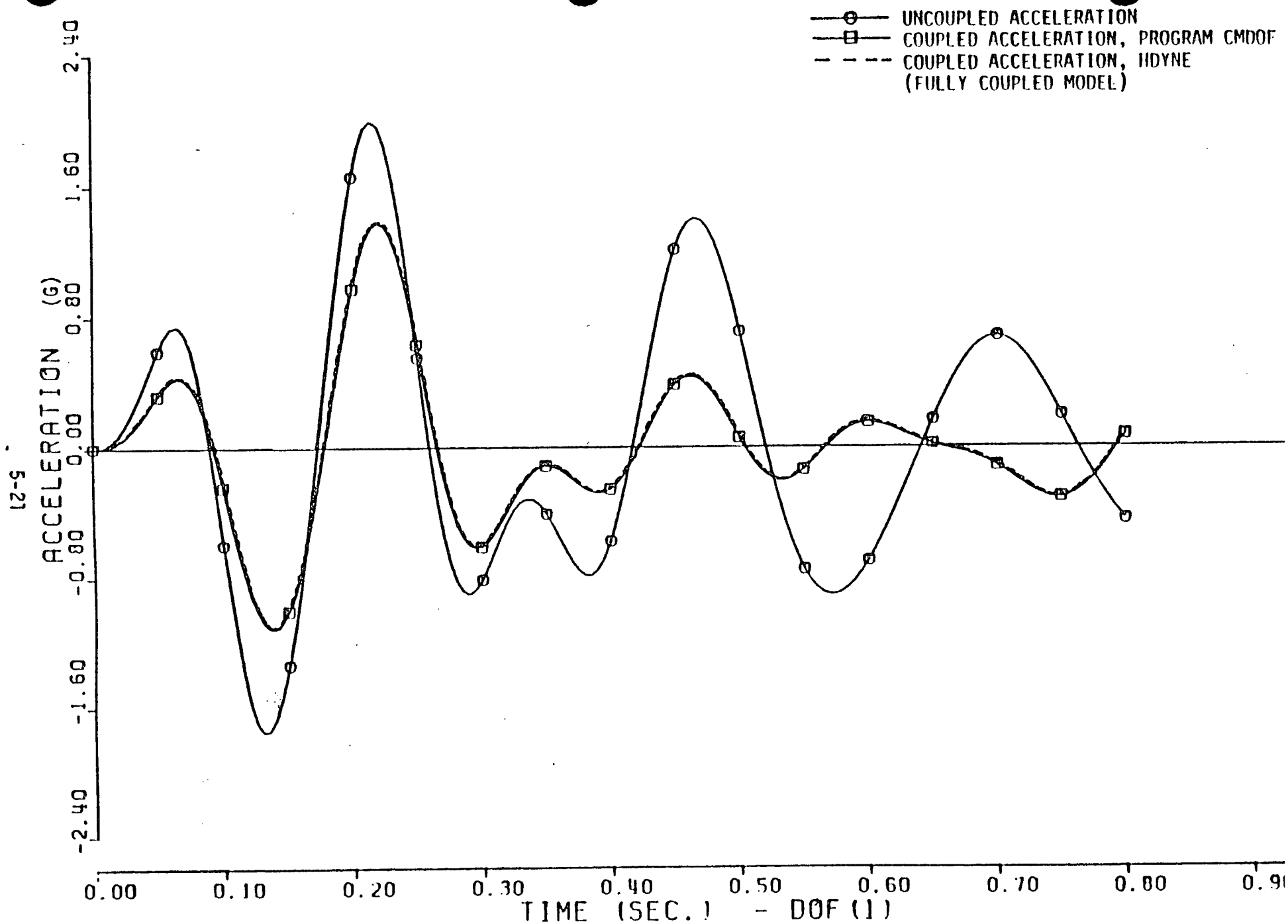


FIGURE 5-13: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 2 - DOF (1)

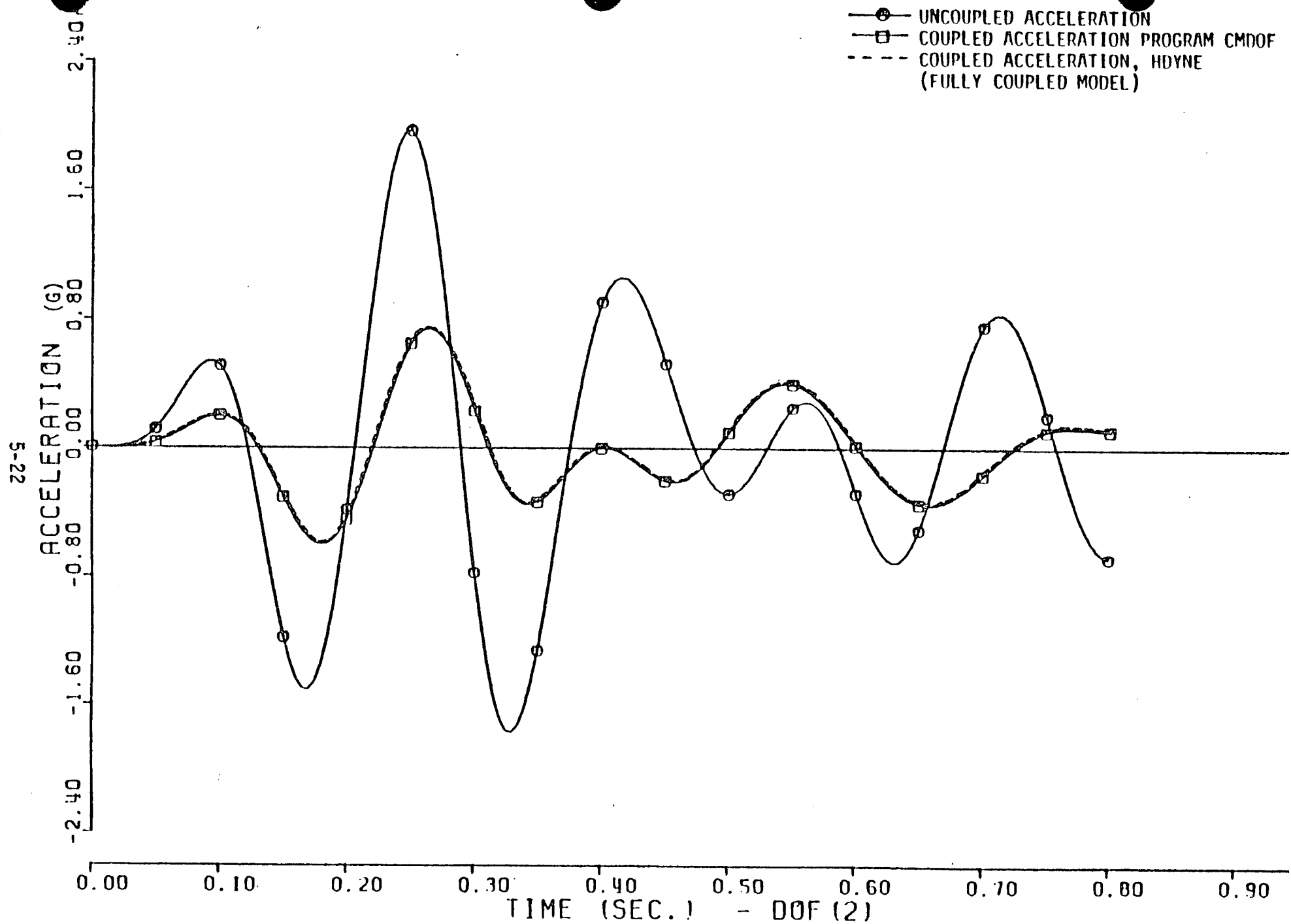


FIGURE 5-14: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 2 - DOF (2)

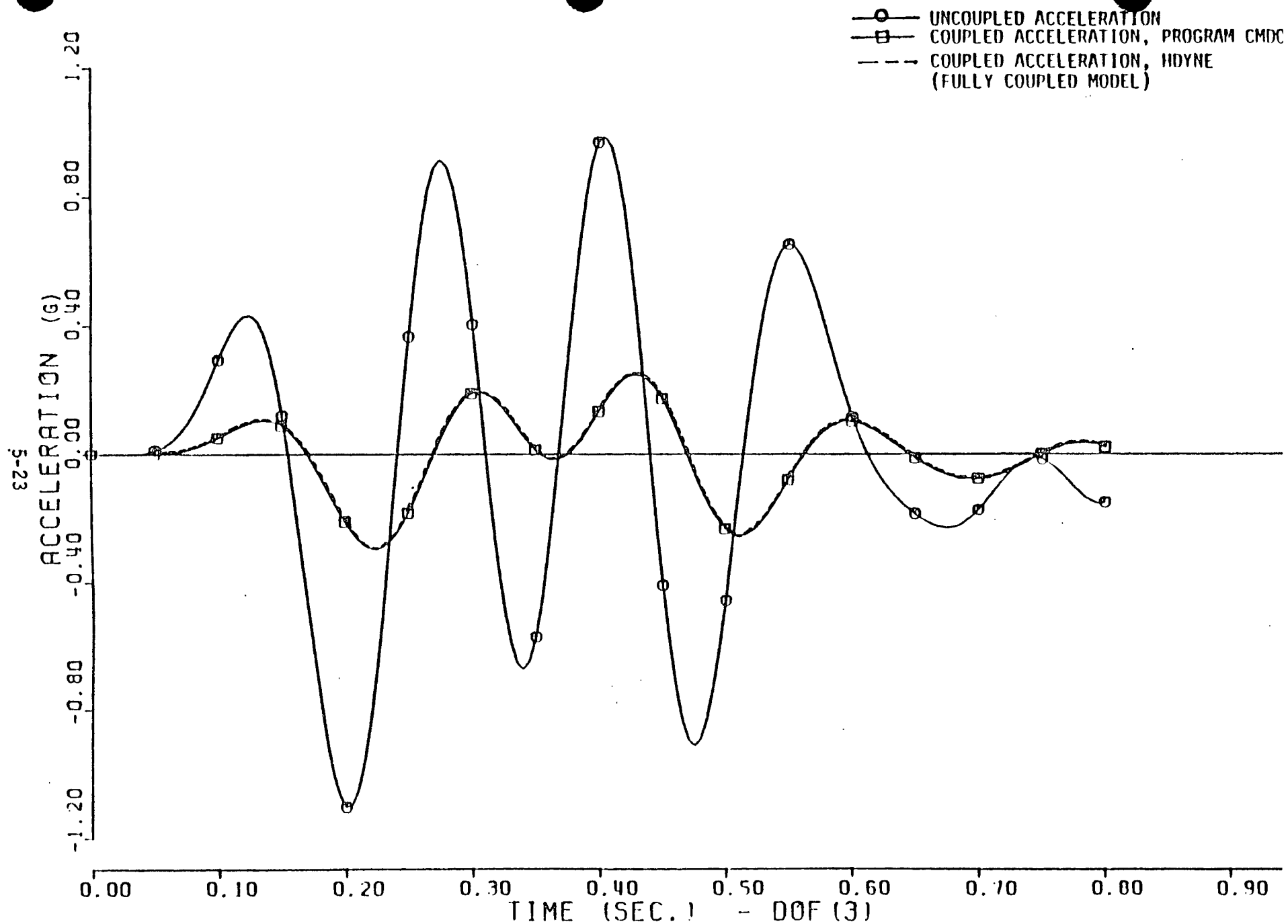


FIGURE 5-15: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 2 - DOF (3)

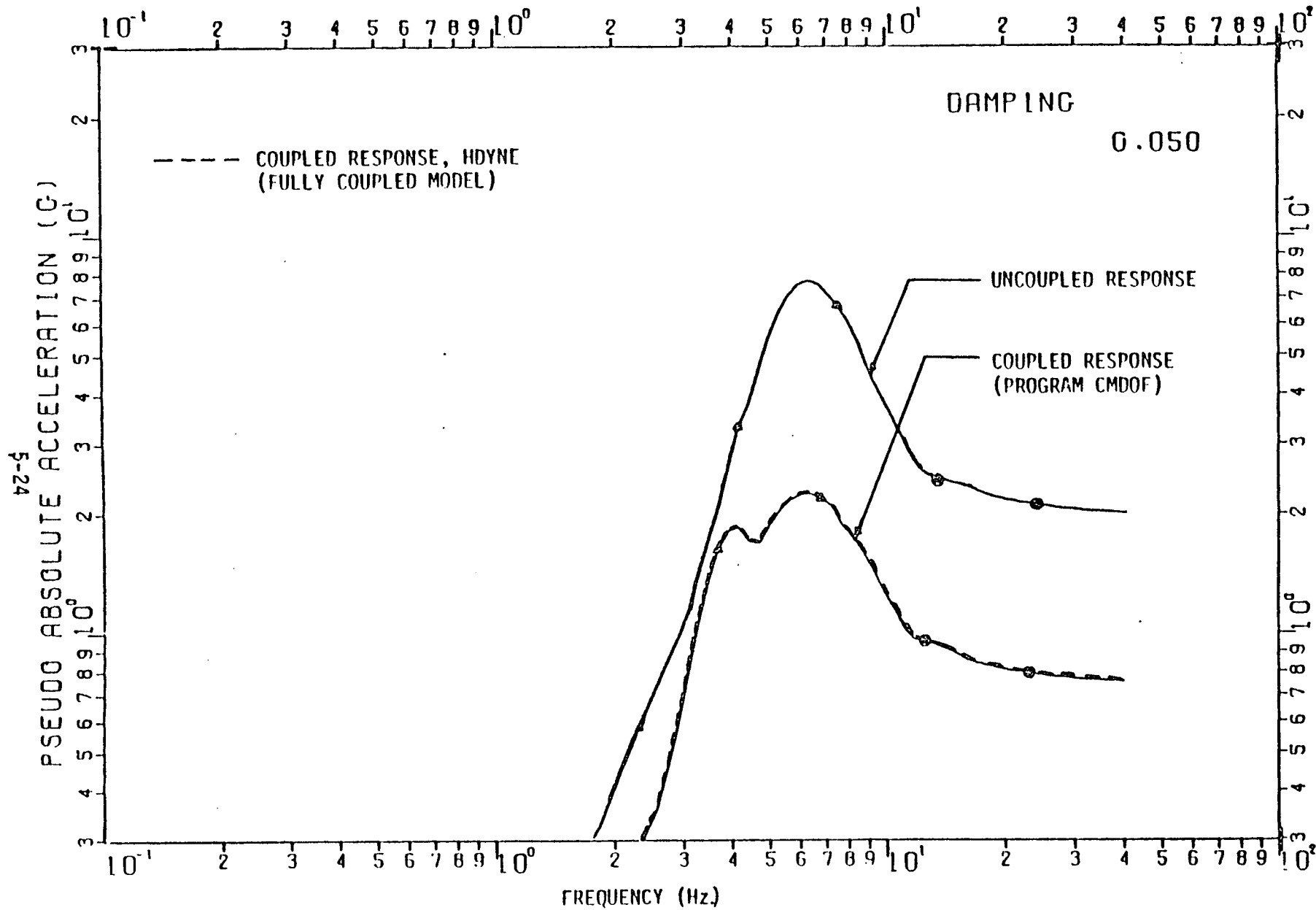


FIGURE 5-16: COMPARISON OF UNCOUPLED AND COUPLED RESPONSE SPECTRA -  
EXAMPLE PROBLEM 2 - DOF 2



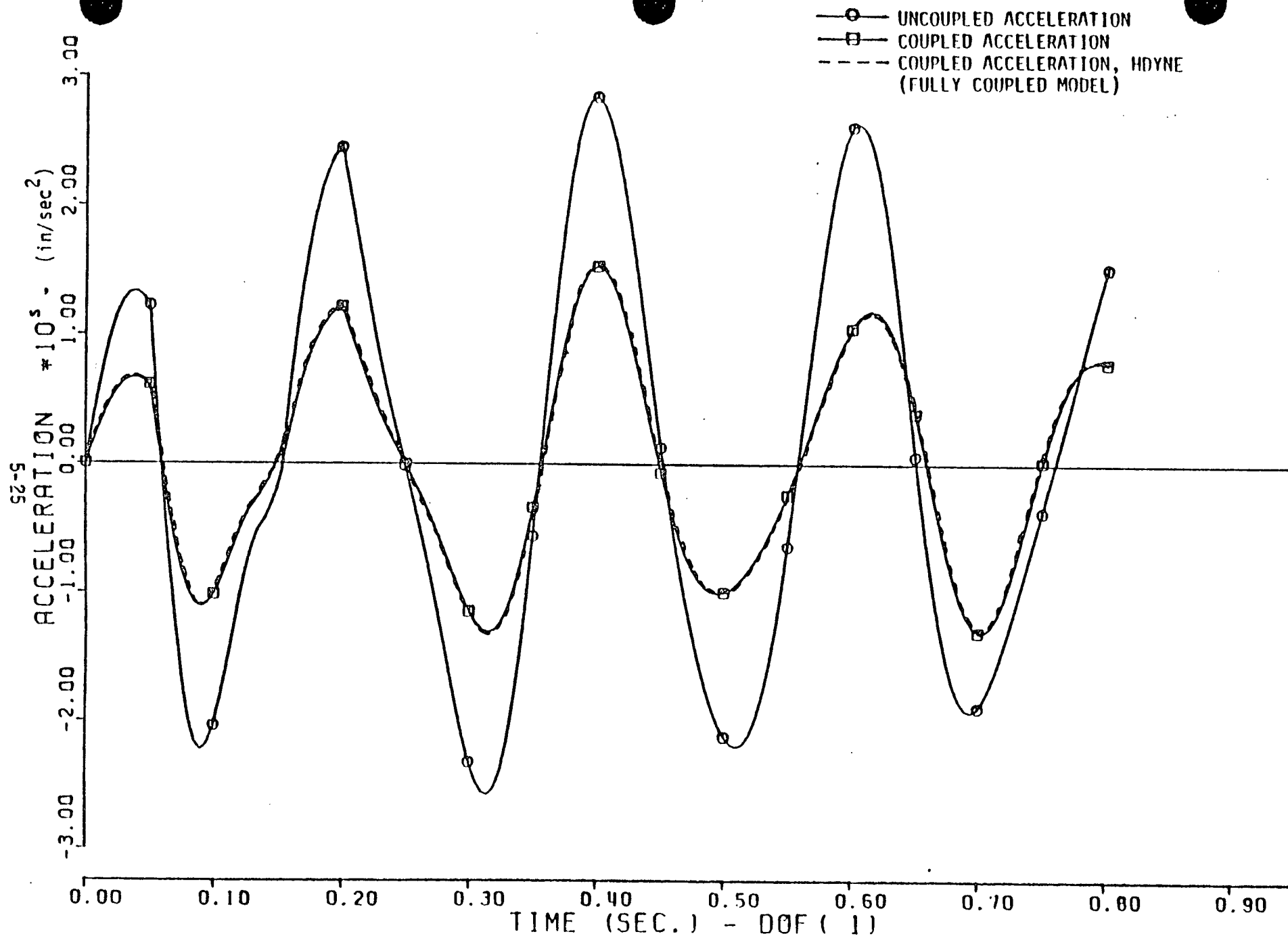


FIGURE 5-17: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 3 - DOF 1

5-26

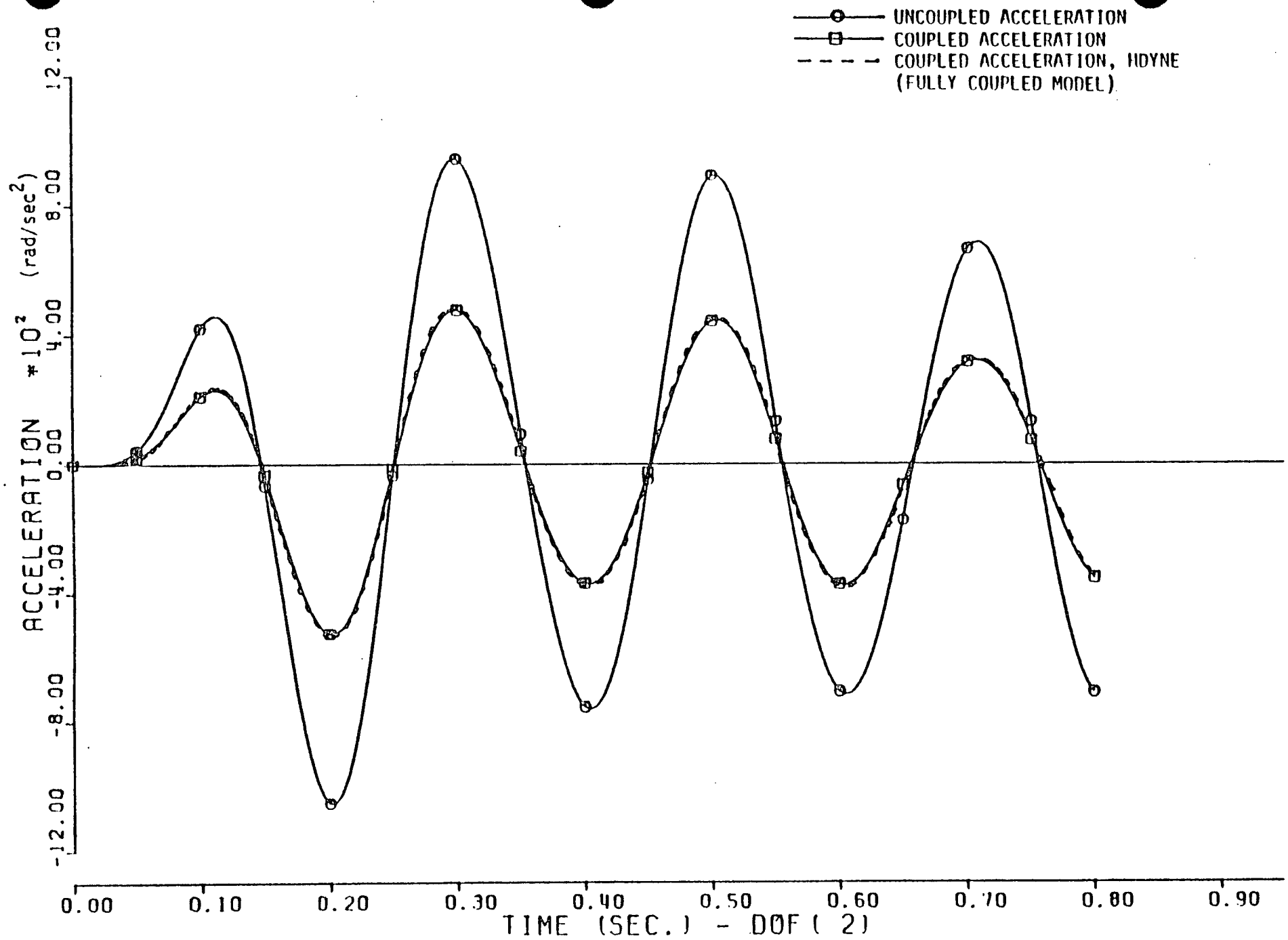


FIGURE 5-18: ACCELERATION TIME HISTORY RESULTS EXAMPLE PROBLEM 3 - DOF 2

5-27

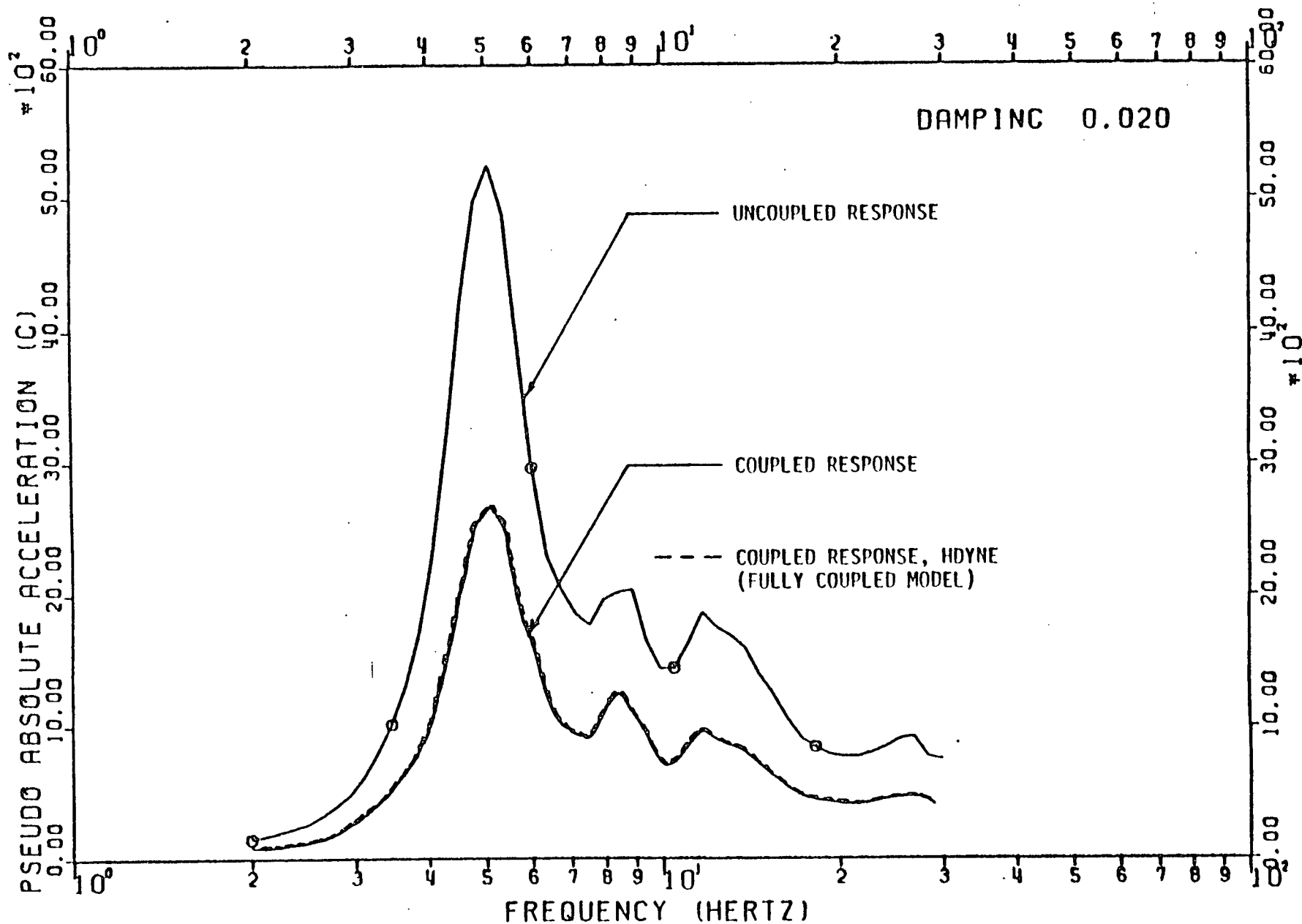


FIGURE 5-19: COMPARISON OF UNCOUPLED AND COUPLED RESPONSE EXAMPLE PROBLEM 3 - DOF 1

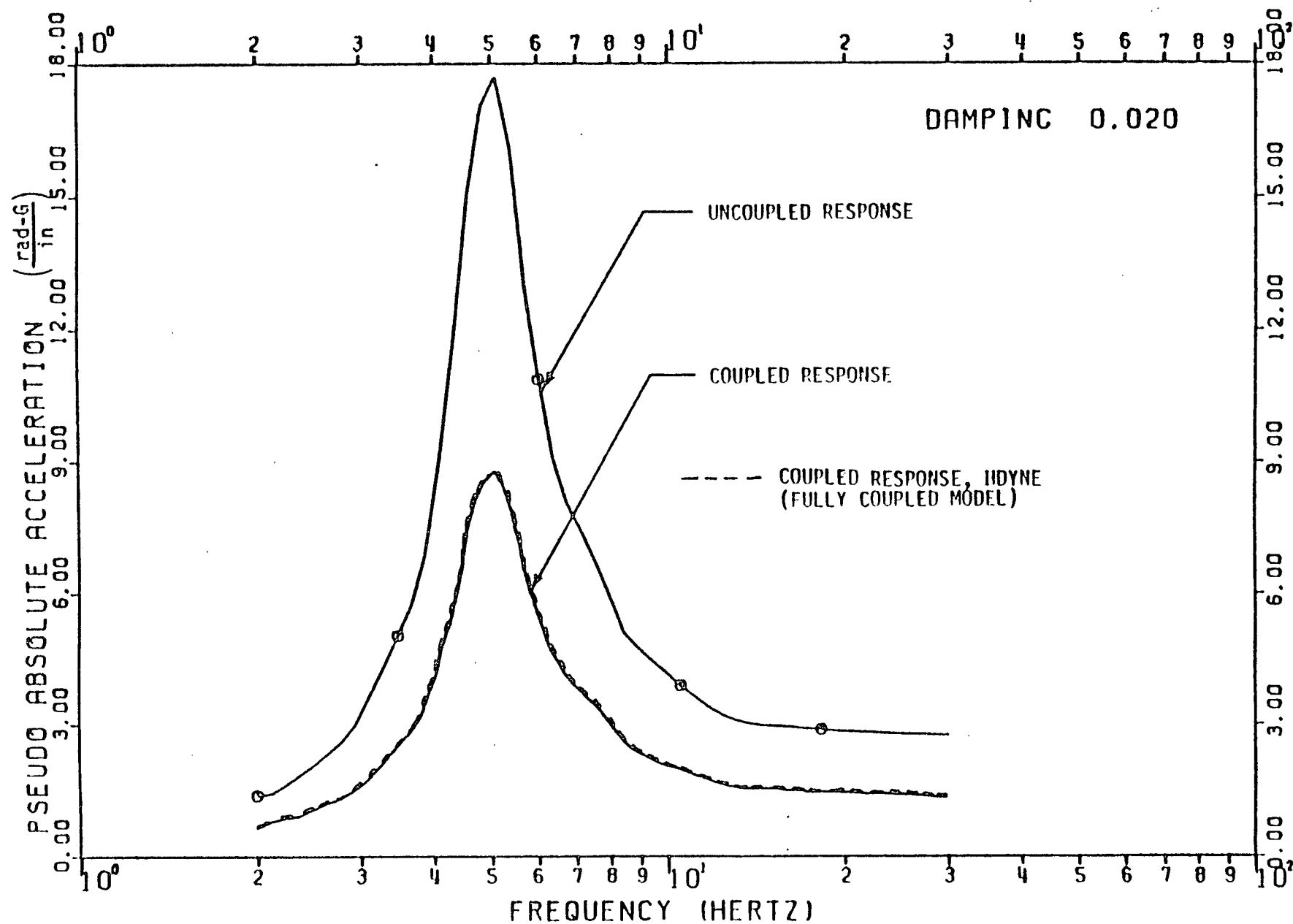


FIGURE 5-20: COMPARISON OF UNCOUPLED AND COUPLED RESPONSE - EXAMPLE PROBLEM 3 - DOF 2

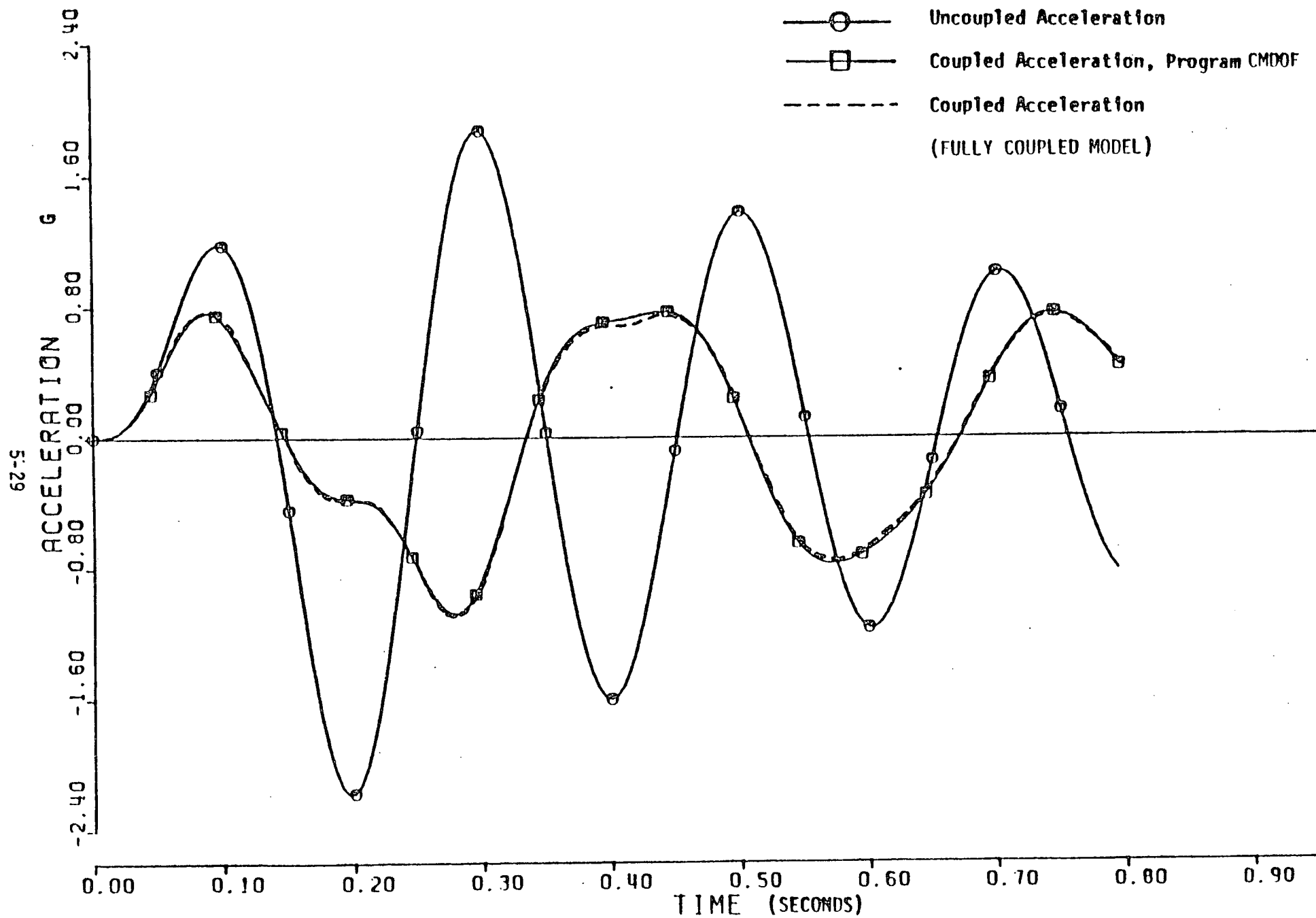


FIGURE 5-21: ACCELERATION TIME HISTORY RESULTS - EXAMPLE PROBLEM 4

## 6. GUIDELINES FOR USING POST-PROCESSOR COUPLING

### PROGRAM CMDOF

The post-processor coupling program CMDOF has been carefully checked for a number of different example problems. These validation problems (Section 5) compared the coupled response calculated by the post-processor program against the response calculated by a fully coupled analysis using Program HDYNE (Reference 2). In all cases studied, excellent agreement between the two different methods was obtained. However, these validation problems were idealized since all uncoupled structure and equipment modes were included in the post-processor program. All modes were included in the sample problems in order to ensure accurate calculation of coupled accelerations in all frequency ranges.

Accurate coupled acceleration response spectra can be computed as long as the uncoupled acceleration time histories are accurate and sufficient uncoupled structure and equipment modes to satisfy either Criterion 1 or 2 of Section 2.7.2 are used. Whenever practical, enough modes to satisfy these criteria should be used. However, for very large and complex structural models, it may not be practical to compute sufficient modes to satisfy these criteria. In this case, a cutoff frequency should be established up to which accurate coupled response spectra are desired. Generally, this cutoff frequency would be in the range of 30 to 60 Hz since structural models are generally not defined in sufficient detail to be accurate beyond this range and because higher frequency vibrations do not contain sufficient velocity and displacement content to be damaging.

As a result of parametric studies presented in Appendix A, a series of guidelines have been established to enable the computer program CMDOF to compute reasonably accurate coupled acceleration time histories and coupled response spectra up to a cutoff frequency when insufficient modes are available to satisfy the criteria of Section 2.7.2. Program users are cautioned that these guidelines are the result of a limited study and may not be all inclusive.

### Rule 1

When the uncoupled acceleration time histories and their response spectra are only accurate up to a cutoff frequency  $f_c$ , the coupled results cannot be computed accurately beyond this frequency. Thus, if the uncoupled structure acceleration time histories have been determined by mode superposition including modes up to frequency  $f_m$ , no improvement is obtained by including higher frequency modes above  $f_m$  for the structure in the coupling program.

Reasonably accurate uncoupled and coupled response spectra can be determined up to a cutoff frequency  $f_c$  by including all structure modes up to a frequency of  $1.3 f_c$ . It appears to be unnecessary to satisfy Equation 2.27 for the structure modes or to incorporate the correction flexibility matrix (Equation 2.28) to correct for ignored higher modes for the structure.

### Rule 2

Inaccurate coupled response spectra result when insufficient uncoupled equipment modes to satisfy Equation 2.27 are used. Whenever practical, sufficient uncoupled equipment modes to satisfy Equation 2.27 should be included. However, essentially as good of results can be obtained up to the cutoff frequency when equipment modes up to  $1.6 f_c$  are included so long as a pseudo-mode is added to incorporate the correction flexibility matrix given by Equation 2.28. This pseudo-mode correction flexibility matrix (Equation 2.28) should be input for the equipment at a natural frequency greater than  $1.6 f_c$ . This pseudo-mode should be reasonably highly damped (20% damping is recommended).

Careful modelling of the uncoupled structure and equipment is also required in order to accurately calculate coupled response using the post-processor program. Sufficient dynamic dof must be included in the uncoupled models in order to retain all important dynamic characteristics. In addition, care must be taken to ensure consistent structure and piping dof at the piping attachment node. Appendix B of this report discusses modelling techniques used to ensure accurate coupled results.

## REFERENCES

1. Nigam, N. C., and Jennings, P. C., Digital Calculation of Response Spectra from Strong Motion Earthquake Records, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, 1968.
2. Kennedy, R. P., and Little, M. B., HDYNE - A Computer Program to Perform Modal and Time Dependent Dynamic Analyses of Vertical Beam Mass Systems With or Without Torsional Effects Subjected to Horizontal Ground Motion, HN-20-1034, Holmes & Narver, Inc., Las Vegas, Nevada, January, 1970.
3. Kennedy, R. P., and Kincaid, R. H., Effects of Dynamic Coupling Between Freestanding Steel Containment Structure and Attached Piping or Equipment, EDAC 134-281.02, EDAC, Inc., Irvine, California, February, 1979.
4. Kennedy, R. P., Vasudevan, R. V., and R. H. Kincaid, CBR - A Computer Program to Couple the Response of Structures and Supported Equipment Using the Results from Uncoupled Structure and Equipment Analyses, EDAC 134-281.01, Engineering Decision Analysis Company, Inc., Irvine, California, February, 1979.



## APPENDIX A

### PARAMETER STUDY ON IGNORING HIGH FREQUENCY MODES IN COUPLING ANALYSES

#### A.1 GENERAL

Section 2.7.2 presented criteria for ensuring that the post-processor coupling program CMDOF generates accurate coupled response spectra throughout the entire frequency range if the input uncoupled acceleration time histories are accurate. In some cases, these criteria might require more uncoupled structure and equipment modal data than is practical. Furthermore, it may not be necessary to generate accurate coupled response spectra beyond a frequency cutoff  $f_c$ . This Appendix presents the results of a parametric study to investigate the effects of ignoring high frequency uncoupled structure and equipment modes such that the criteria in Section 2.7.2 are not satisfied.

Several example problems were used in this study. The most extensive studies were conducted using Example Problem 1 (Figures 5-1 and 5-2) and Example Problem 2 (Figures 5-3 and 5-4) except that constant modal damping of 2% was used for all structure and equipment modes in lieu of the values shown in Chapter 5. Results are presented in this Appendix for Example Problem 1 subjected to the pulsive loading shown in Figure 5-8. Results are also presented for Example Problem 2 subjected to the pulsive loading shown in Figure A-1.

Example Problem 1 contains five (5) uncoupled structure modes and four (4) uncoupled equipment modes. Table A-1 presents the products  $PF_{j,m} \cdot \phi_{j,m}$  and  $(PF_{j,m} \phi_{j,m})/\omega_m^2$  for each attachment degree-of-freedom for both the uncoupled structure and equipment. Equation 2.26 is very poorly approximated unless all five structure modes and all four equipment modes are included. The terms of the flexibility matrix are in error by as much as about 10% when three (3) structure modes and three (3)

equipment modes are included. Accurate 2% damped uncoupled and coupled response spectra are shown for degrees-of-freedom 1 and 2 in Figures A-2 and A-3. Note that the uncoupled spectra do not return to approximately the zero period acceleration (ZPA) until the frequency exceeds 10 hz. Neither Criterion 1 or 2 of Section 2.7.2 are satisfied unless all five structure modes and all four equipment modes are included.

Similar results were obtained for Example Problem 2. Figures A-4 and A-5 present accurate uncoupled and coupled spectra for degrees-of-freedom 1 and 2, respectively. For Example Problem 2, neither Criterion 1 or 2 of Section 2.7.2 are satisfied unless all structure and equipment modes are included.

Note that substantial differences exist between the coupled and uncoupled spectra shown in Figures A-2 through A-5 for attachment degrees-of-freedom 1 and 2 for Example Problems 1 and 2. Since neither Criterion 1 or 2 of Section 2.7.2 are satisfied and there are substantial differences between the accurate coupled and uncoupled spectra, these two example problems provide a good basis for investigating the effects of ignoring the higher uncoupled structure and equipment modes. The accurate spectra shown in Figures A-2 through A-5 will be used as the basis for comparison in all cases.

#### A.2 EFFECT OF IGNORING HIGH FREQUENCY STRUCTURAL MODES ON UNCOUPLED STRUCTURAL RESPONSE

The initial part of this investigation examined the effects of including uncoupled structure modes only up to the maximum frequency  $f_m$  of 6.5 hz in calculating the uncoupled structure response to the input force time history. A comparison of uncoupled response spectra generated including all structure modes and uncoupled response spectra generated including only modes up to  $f_m = 6.5$  Hz for Example Problem 1 is shown in Figures A-6 and A-7 for attachment dof 1 and 2, respectively. Figures A-8 and A-9 present the same comparison for Example Problem 2.

Examination of these figures leads to the conclusion that when modal data is only included up to a frequency level of  $f_m = 6.5$  Hz, uncoupled structural response is calculated accurately up to a frequency of at least 5.5 hz or 85% of the maximum frequency  $f_m$ .

Based upon these examples and others, it is concluded that accurate uncoupled spectra can be computed up to a cutoff frequency  $f_c$  by mode superposition when all important structure modes up to  $1.3 f_c$  are included.

### A.3 EFFECT OF INACCURATE UNCOUPLED TIME HISTORY ABOVE CUTOFF FREQUENCY ON COUPLED SPECTRA

When the uncoupled time history has been obtained using modal superposition including only modes up to a maximum frequency  $f_m$ , this uncoupled time history may not be accurate above a cutoff frequency given by:

$$f_c \leq \frac{f_m}{1.3}$$

This conclusion was reached in the previous section.

Several questions are of interest:

1. What effect does an inaccurate uncoupled time history above the cutoff frequency  $f_c$  have on the coupled spectra?
2. Is there any improvement in accuracy obtained by including higher frequency structural modes in the post-processed coupling analyses when only structure modes up to a frequency  $f_m$  were included when the uncoupled time histories were generated?

Two cases were studied. In the first case, only uncoupled structure modes up to the maximum frequency,  $f_m$  of 6.5 hz were included in the coupling analysis using the post-processor program. In the second case studied, all uncoupled structure modes were included in the post-processor

coupling program. Both cases were analyzed including all uncoupled equipment modes and using an uncoupled structure time history based on mode superposition including only those uncoupled structure modes below the maximum frequency,  $f_m$  of 6.5 hz. A comparison of these two cases answers the question of whether the accuracy of the coupling program results can be improved by including uncoupled structure modes above the maximum frequency,  $f_m$ , used for the uncoupled structure time history.

Figures A-10 and A-11 present a comparison of the following cases for attachment dof 1 and dof 2 for Example Problem 1.

- (a) Post-processor program including uncoupled structure modes to the maximum frequency  $f_m = 6.5$  hz.
- (b) Post-processor program including all uncoupled structure modes.
- (c) Fully coupled analysis including all coupled structure and equipment modes.

Cases (a) and (c) were also run for Example Problem 2 and are compared in Figures A-12 and A-13.

When compared with case (c), one finds that both cases (a) and (b) provide reasonably accurate coupled response spectra up to a cutoff frequency of about 5 hz. Furthermore, Figures A-10 and A-11 show that case (b) coupled response spectra are no more accurate than case (a) coupled response spectra up to this cutoff frequency.

These plus other similar results lead to a proposed Rule 1 as follows:

### Rule 1

When the uncoupled acceleration time histories and their response spectra are only accurate up to a cutoff frequency  $f_c$ , the coupled results cannot be computed accurately beyond this frequency. Thus, if the uncoupled structure acceleration time histories have been determined by mode superposition including modes up to frequency  $f_m$ , no improvement is obtained by including higher frequency modes above  $f_m$  for the structure in the coupling program.

Reasonably accurate uncoupled and coupled response spectra can be determined up to a cutoff frequency  $f_c$  by including all structure modes up to a frequency of  $1.3 f_c$ . It appears to be unnecessary to satisfy Equation 2.27 for the structure modes or to incorporate the correction flexibility matrix (Equation 2.28) to correct for ignored higher modes for the structure.

#### A.4 EFFECT OF IGNORING HIGH FREQUENCY EQUIPMENT MODES

In Section A.3, it was shown that reasonably accurate coupled spectra could be determined up to a cutoff frequency  $f_c$  so long as:

1. The uncoupled time history is based upon inclusion of all uncoupled structure modes up to at least  $1.3 f_c$ .
2. All important uncoupled structure modes up to at least  $1.3 f_c$  are included in the post-processor coupling program.

These conclusions were reached based upon including all uncoupled equipment modes. Thus, for Example Problems 1 and 2, coupled spectra were reasonably accurate up to a cutoff frequency  $f_c$  of 5 hz when all structure modes up to a frequency  $f_m$  of 6.5 hz were included in generating the uncoupled time history and were included in the coupling program.

Studies were next conducted to evaluate the effect of ignoring high frequency equipment modes. For all of these studies, uncoupled structure modes were included up to a frequency  $f_m$  of 6.5 hz (i.e., 1.3 times a cutoff frequency  $f_c$  of 5 hz) in generating the uncoupled acceleration time histories and in treating structural coupling effects. Equipment modes were considered up to various frequency levels. The cases considered were as follows:

- Case (a): Structure modes up to  $1.3 f_c$  (6.5 hz) and all equipment modes.
- Case (c): Fully coupled with all structure and equipment modes.
- Case (d): Like case (a) except equipment modes above  $1.3 f_c$  (6.5 hz) ignored.
- Case (e): Like case (a) except equipment modes above  $1.52 f_c$  (7.6 hz) ignored.
- Case (f): Like case (e) except correction flexibility terms (Equation 2.28) included for ignored equipment modes. Correction flexibility matrix input at pseudo-mode frequency of  $1.52 f_c$  (7.6 hz) and 2% damping.
- Case (g): Like case (f) except 20% damping specified for correction flexibility matrix pseudo-mode for equipment.

Cases (a) and (c) were presented and discussed in Section A.3. Case (c) represents the accurate coupled spectra against which all other cases were judged. Case (a) represents the best achievable results for cases (d) through (g) since all of these cases also ignore structural modes above 6.5 hz ( $1.3 f_c$ ). Cases (d) through (g) were studied to determine

whether they could approximate case (c) below the cutoff frequency  $f_c$  of 5 hz nearly as well as case (a). Results are presented for degrees-of-freedom 1 and 2 for Example Problem 1 for all cases. Results are also presented for degrees-of-freedom 1 and 2 for Example Problem 2 for cases (c), and (g).

Coupled response spectra for cases (c), (d), and (e) are compared in Figures A-14 and A-15 for degrees-of-freedom 1 and 2 of Example Problem 1. One notes that neither case (d) or (e) are accurate even for frequencies below the cutoff frequency  $f_c$  of 5 hz. It is concluded that inaccurate coupled response spectra result even below the cutoff frequency when insufficient uncoupled equipment modes to satisfy Equation 2.27 are used.

Coupled response spectra for cases (a), (c), and (f) are compared in Figures A-16 and A-17 for these same degrees-of-freedom in Example Problem 1. It is seen that case (f) agrees with case (c) nearly as well as case (a) does for frequencies below the cutoff frequency  $f_c$  of 5 hz. Cases (c), (f), and (g) are compared in Figures A-18 and A-19. Case (g) shows some improvement over case (f) for frequencies below  $f_c$  of 5 hz. The conclusions are that including equipment modes up to  $1.6 f_c$  is sufficient so long as a pseudo-mode is added to incorporate the correction flexibility matrix as given by Equation 2.28. This pseudo-mode should be input for the equipment at a natural frequency greater than about  $1.6 f_c$  and should be reasonably highly damped for best results.

Cases (c) and (g) are also compared for Example Problem 2 degrees-of-freedom 1, and 2 in Figures A-20 and A-21. Again, it is shown that case (g) provides a reasonably accurate coupled response spectra up to the cutoff frequency of 5 hz.

Thus, proposed Rule 2 is established as follows:

## Rule 2

Inaccurate coupled response spectra result when insufficient uncoupled equipment modes to satisfy Equation 2.27 are used. Whenever practical, sufficient uncoupled equipment modes to satisfy Equation 2.27 should be included. However, essentially as good of results can be obtained up to the cutoff frequency when equipment modes up to  $1.6 f_c$  are included so long as a pseudo-mode is added to incorporate the correction flexibility matrix given by Equation 2.28. This pseudo-mode correction flexibility matrix (Equation 2.28) should be input for the equipment at a natural frequency greater than  $1.6 f_c$ . This pseudo-mode should be reasonably highly damped (20% damping is recommended).



TABLE A-1: EIGENVECTOR PRODUCTS FOR EXAMPLE PROBLEM 1

Uncoupled Structure

Mode	$PF_{j,m} \cdot \phi_{j,m}$ $\left( \frac{\text{in}}{\text{Kip-sec}^2} \right)$			$\left( \frac{PF_{j,m} \cdot \phi_{j,m}}{\omega_m^2} \right)$ (in/Kip)		
	j = 1	j = 2	j = 3	j = 1	j = 2	j = 3
1	102.6	200.5	290.4	1.313	2.566	3.715
2	343.7	102.6	27.9	0.516	0.154	0.042
3	27.9	290.4	102.6	0.017	0.175	0.062
4	200.5	27.9	343.7	0.073	0.010	0.126
5	290.4	343.7	200.5	0.082	0.097	0.056
$\Sigma$	965.1	965.1	965.1	2.001	3.002	4.001

Uncoupled Equipment

Mode	$PF_{j,m} \cdot \phi_{j,m}$ $\left( \frac{\text{in}}{\text{Kip-sec}^2} \right)$			$\left( \frac{PF_{j,m} \cdot \phi_{j,m}}{\omega_m^2} \right)$ (in/Kip)		
	j = 1	j = 2	j = 3	j = 1	j = 2	j = 3
1	177.2	321.8	416.2	1.522	2.765	3.576
2	321.8	0.0	321.8	0.333	0.0	0.333
3	50.1	321.8	177.2	0.022	0.142	0.078
4	416.2	321.8	50.1	0.122	0.094	0.015
$\Sigma$	965.3	965.4	965.3	1.999	3.001	4.002

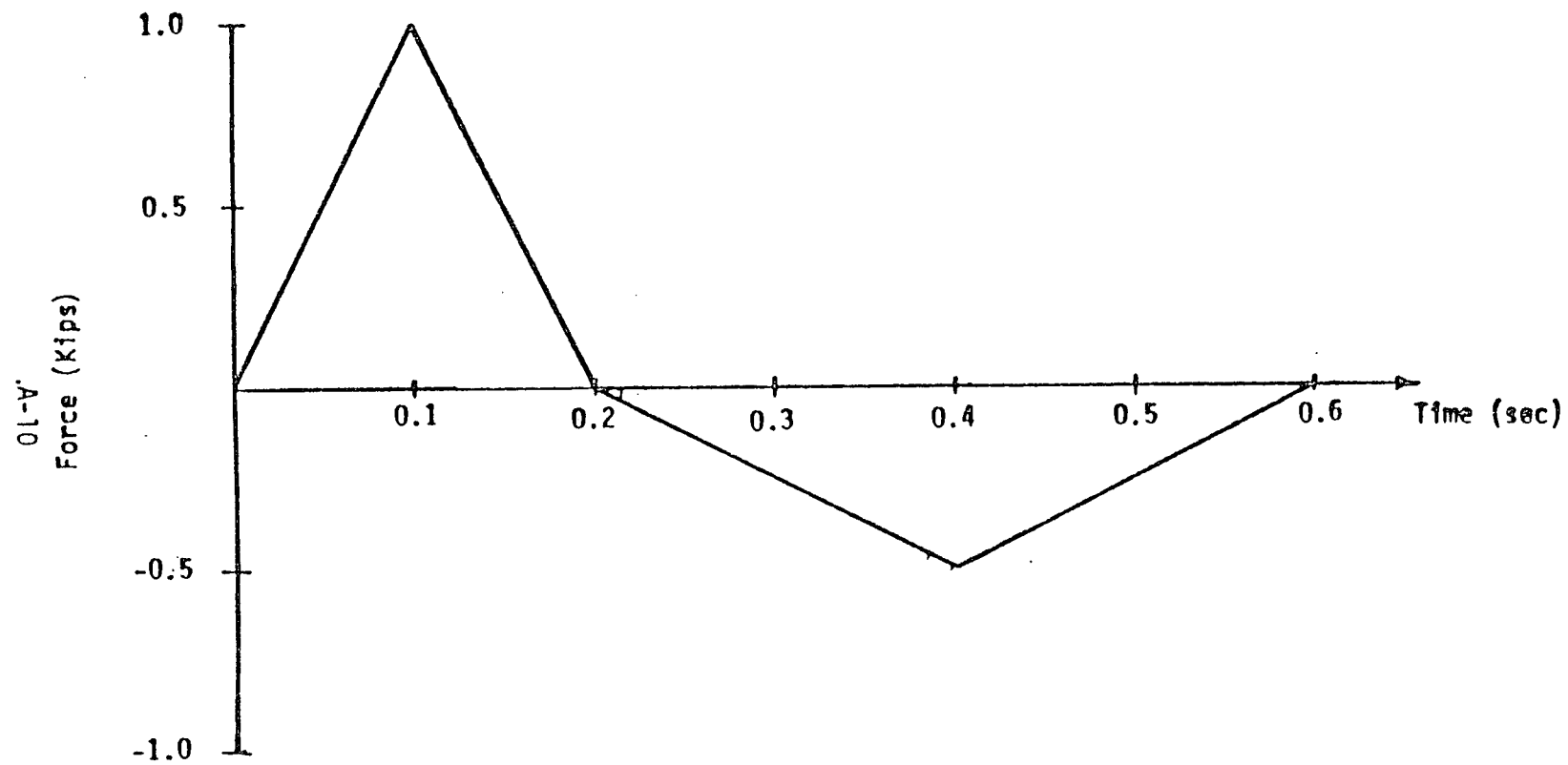


FIGURE A-1: FORCE TIME HISTORY INPUT FOR EXAMPLE PROBLEM 2

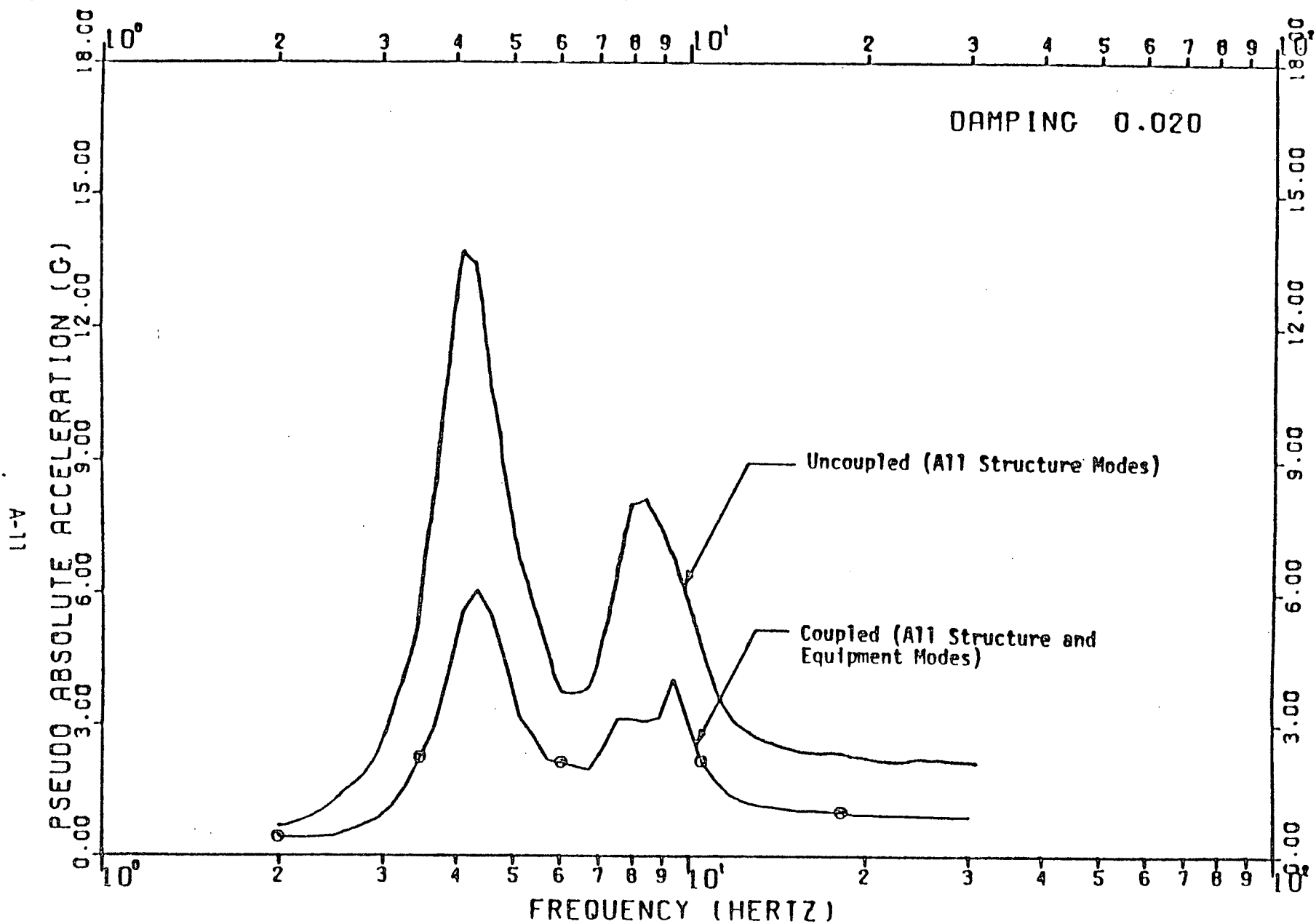


FIGURE A-2: EXAMPLE PROBLEM 1 - DOF 1 - UNCOUPLED AND COUPLED RESPONSE SPECTRA

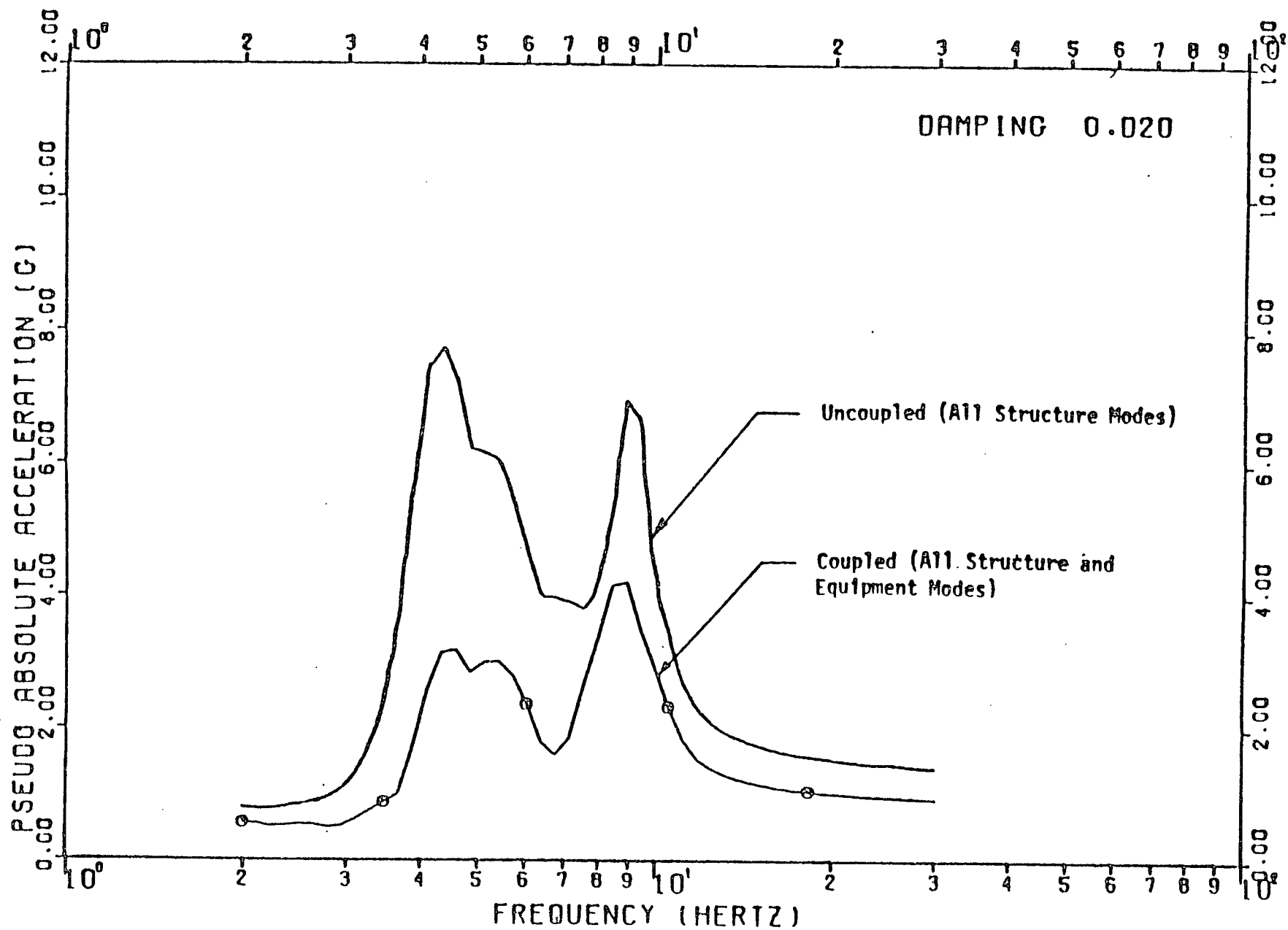


FIGURE A-3: EXAMPLE PROBLEM 1 - DOF 2 - UNCOUPLED AND COUPLED RESPONSE SPECTRA

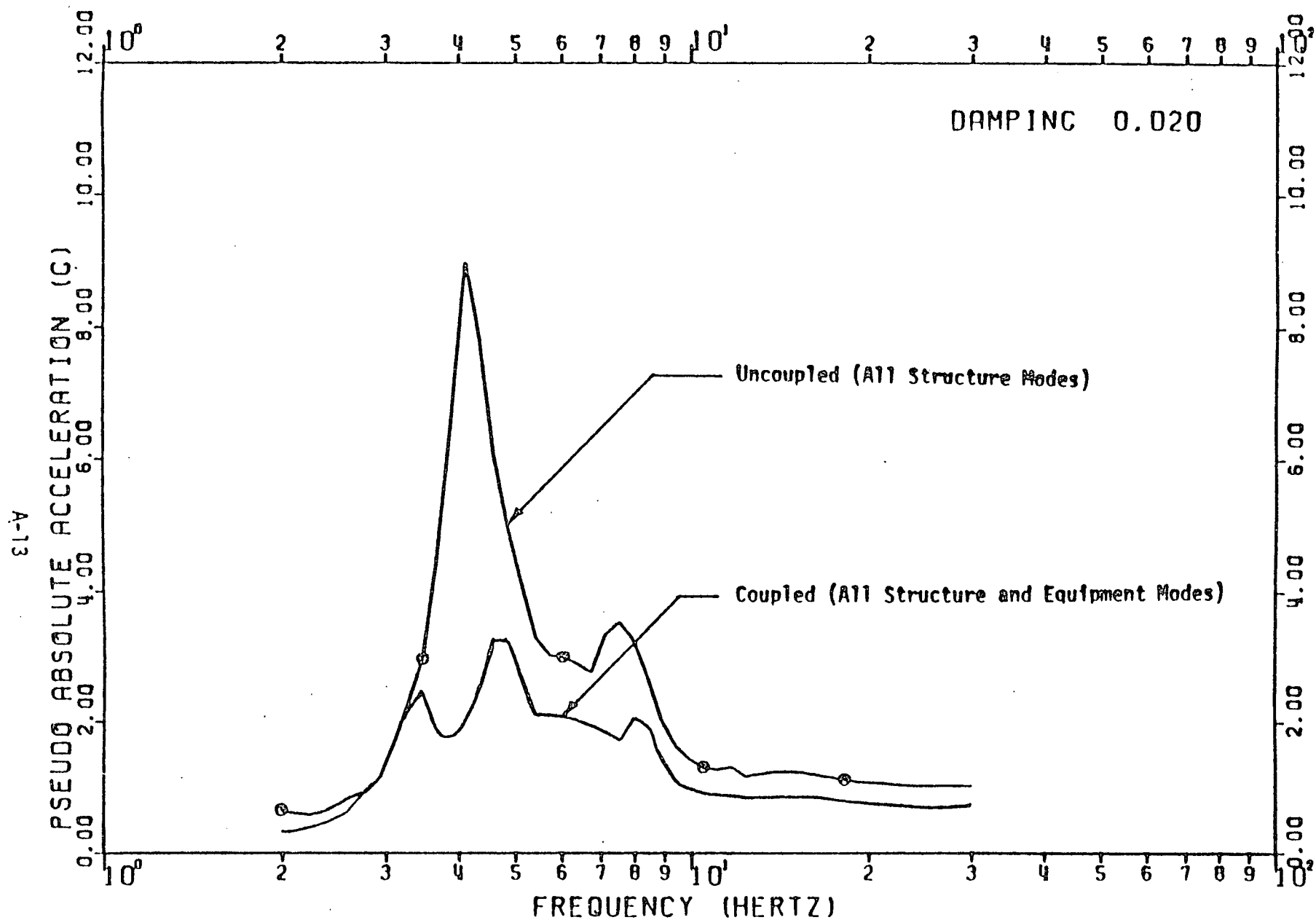


FIGURE A-4: EXAMPLE PROBLEM 2 - DOF 1 - UNCOUPLED AND COUPLED RESPONSE SPECTRA

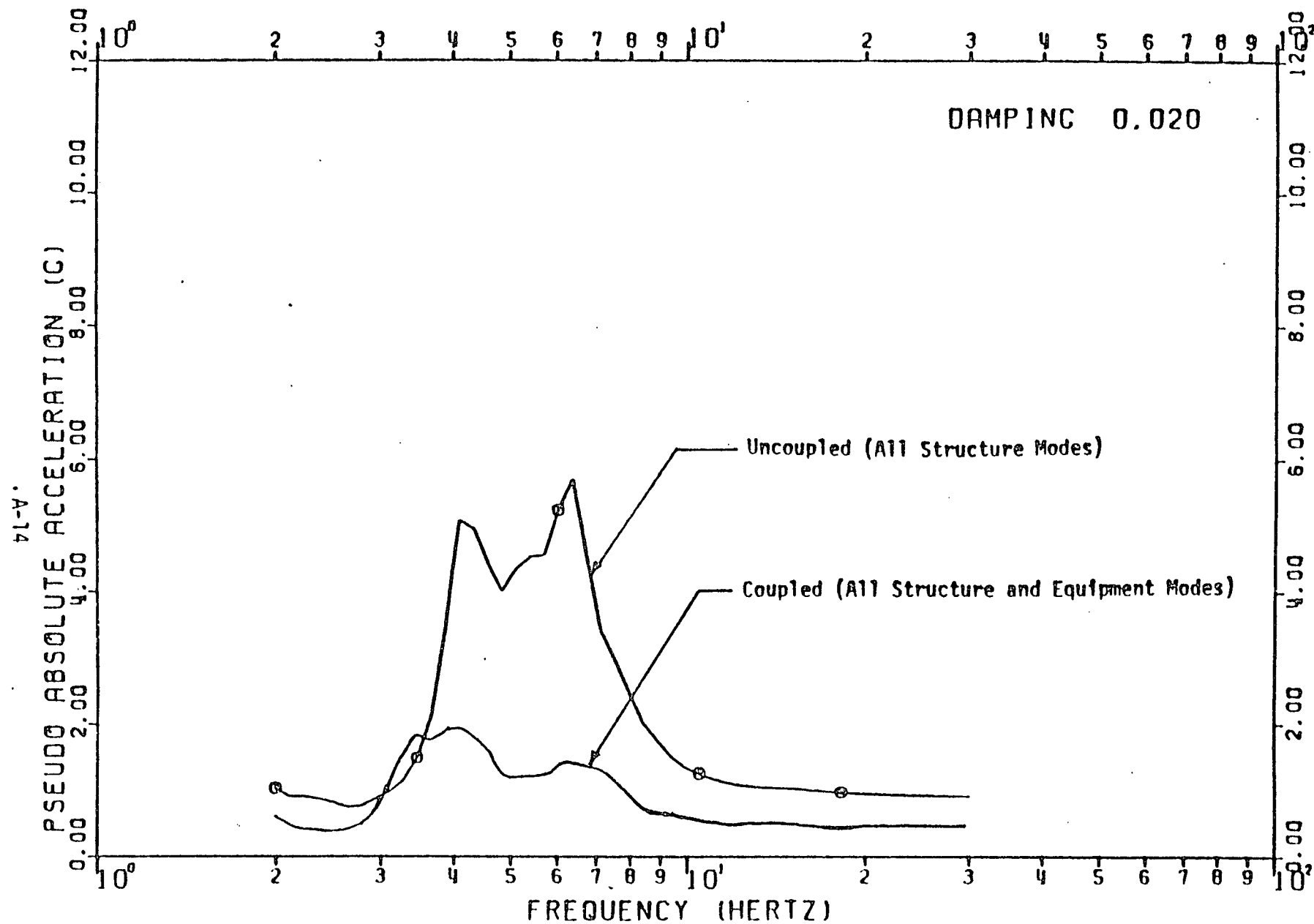


FIGURE A-5: EXAMPLE PROBLEM 2 - DOF 2 - UNCOUPLED AND COUPLED RESPONSE SPECTRA

A-15

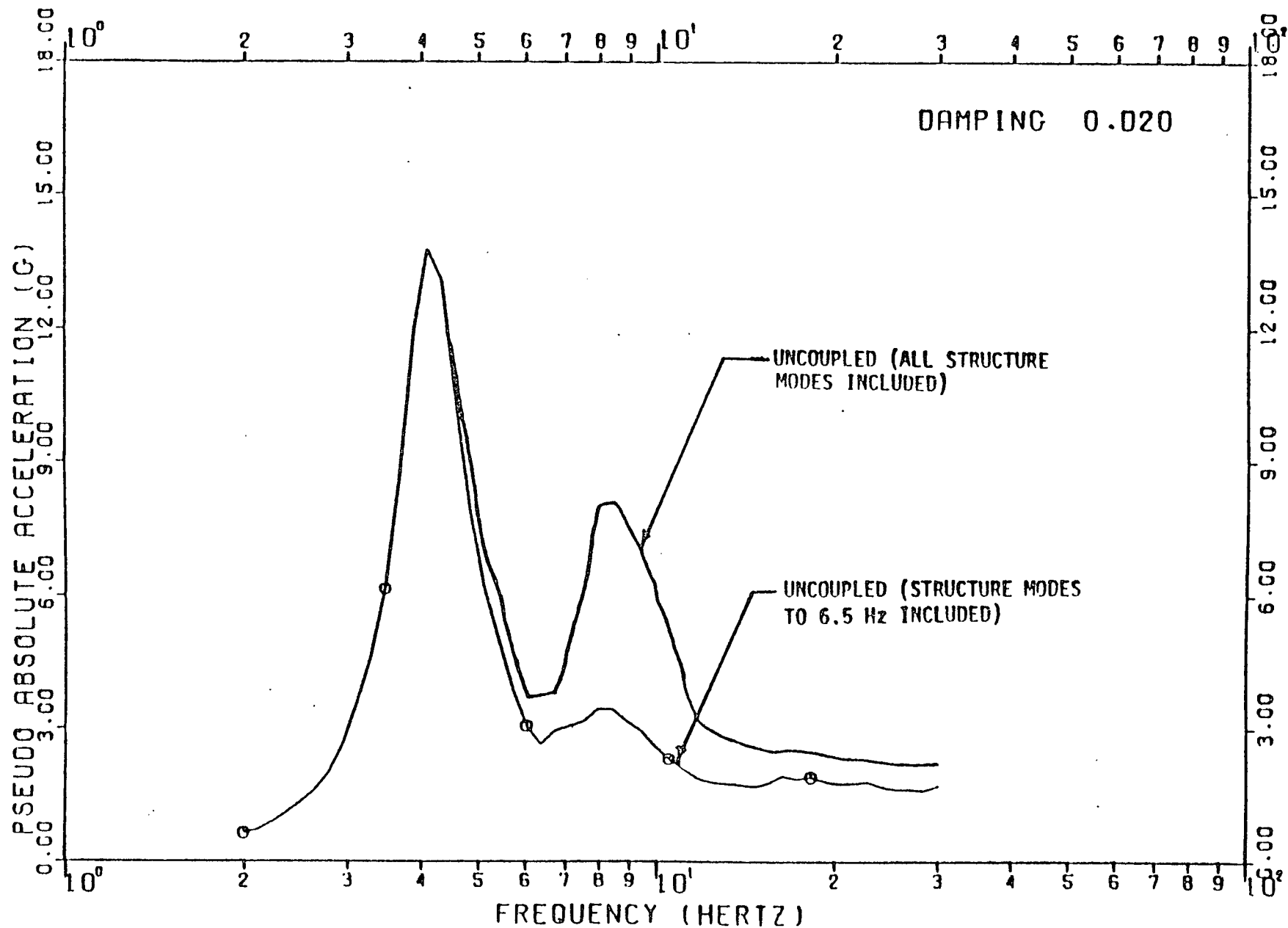


FIGURE A-6: EXAMPLE PROBLEM 1 - DOF 1 - COMPARISON OF UNCOUPLED STRUCTURE RESPONSE

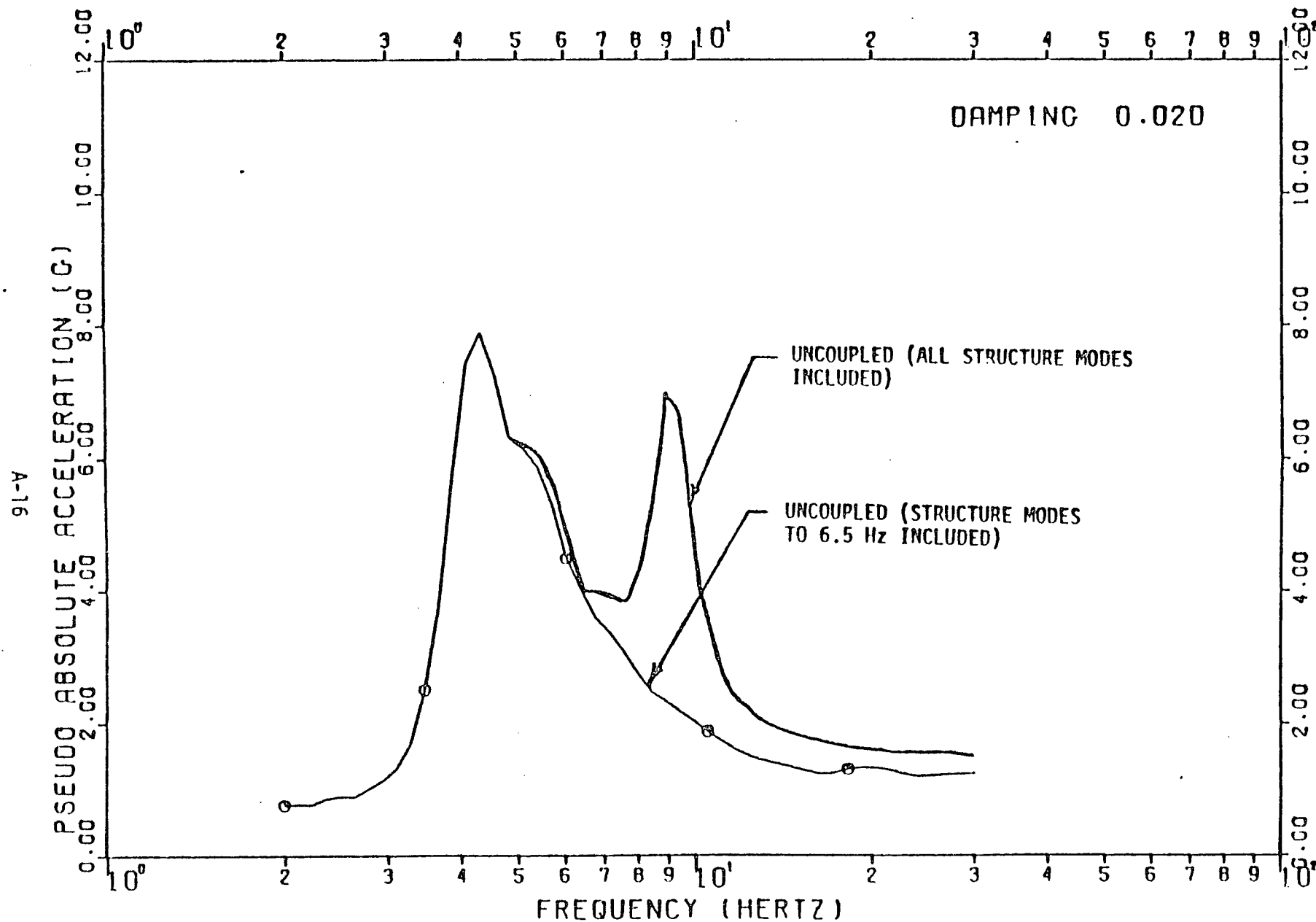


FIGURE A-7: EXAMPLE PROBLEM 1 - DOF 2 - COMPARISON OF UNCOUPLED STRUCTURE RESPONSE



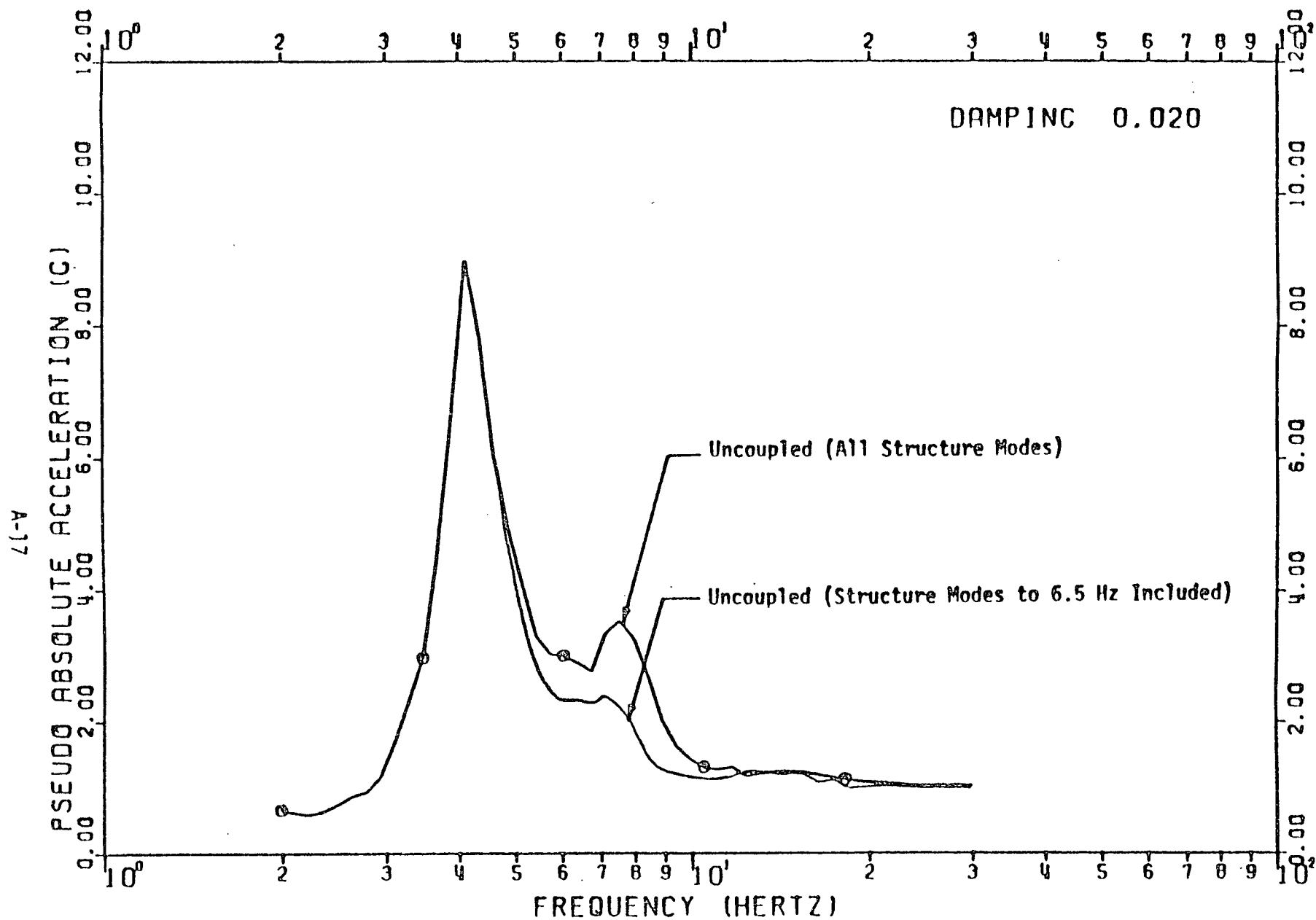


FIGURE A-8: EXAMPLE PROBLEM 2 -- DOF 1 - COMPARISON OF UNCOUPLED STRUCTURE RESPONSE

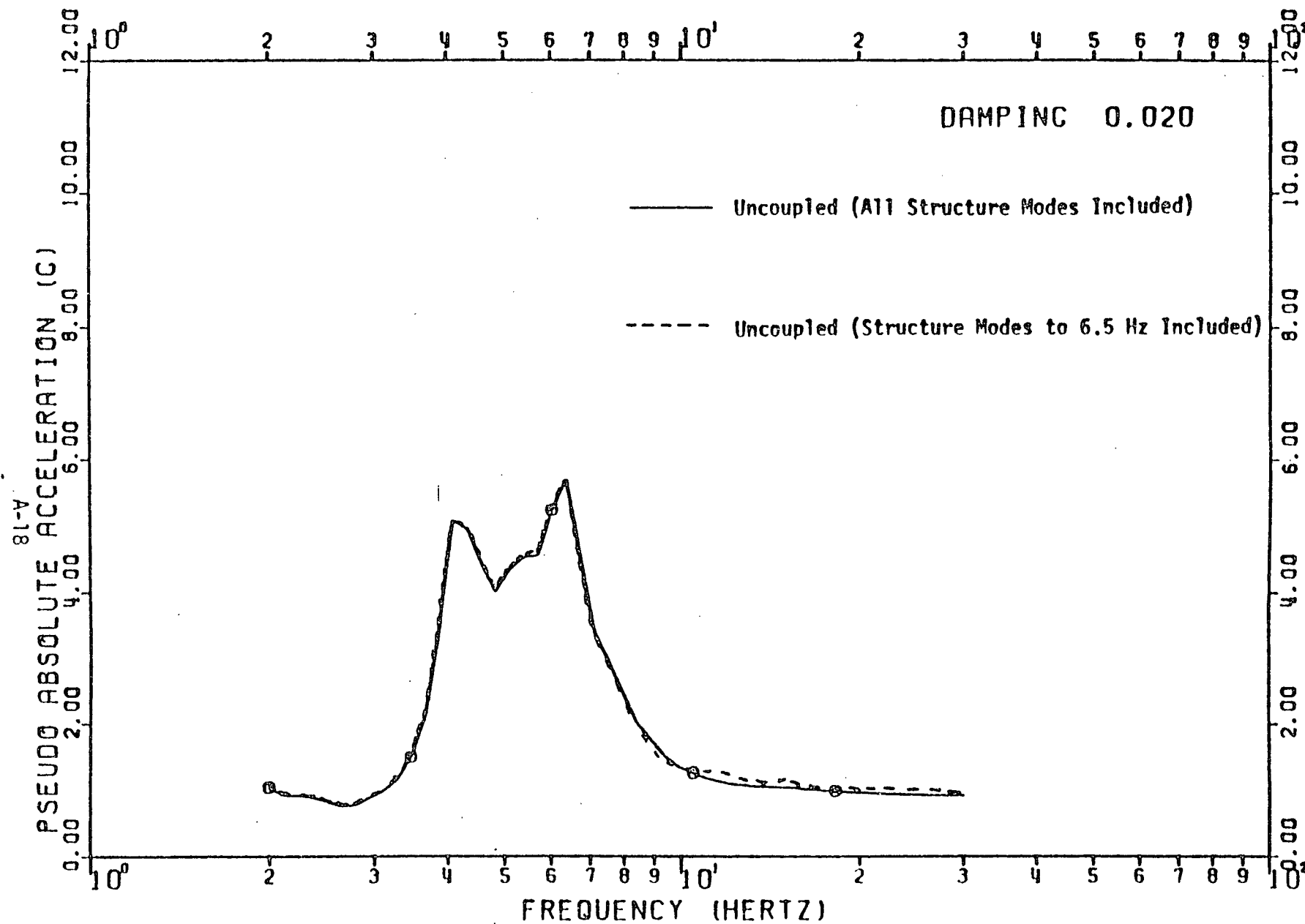


FIGURE A-9: EXAMPLE PROBLEM 2 - DOF 2 - COMPARISON OF UNCOUPLED STRUCTURE RESPONSE

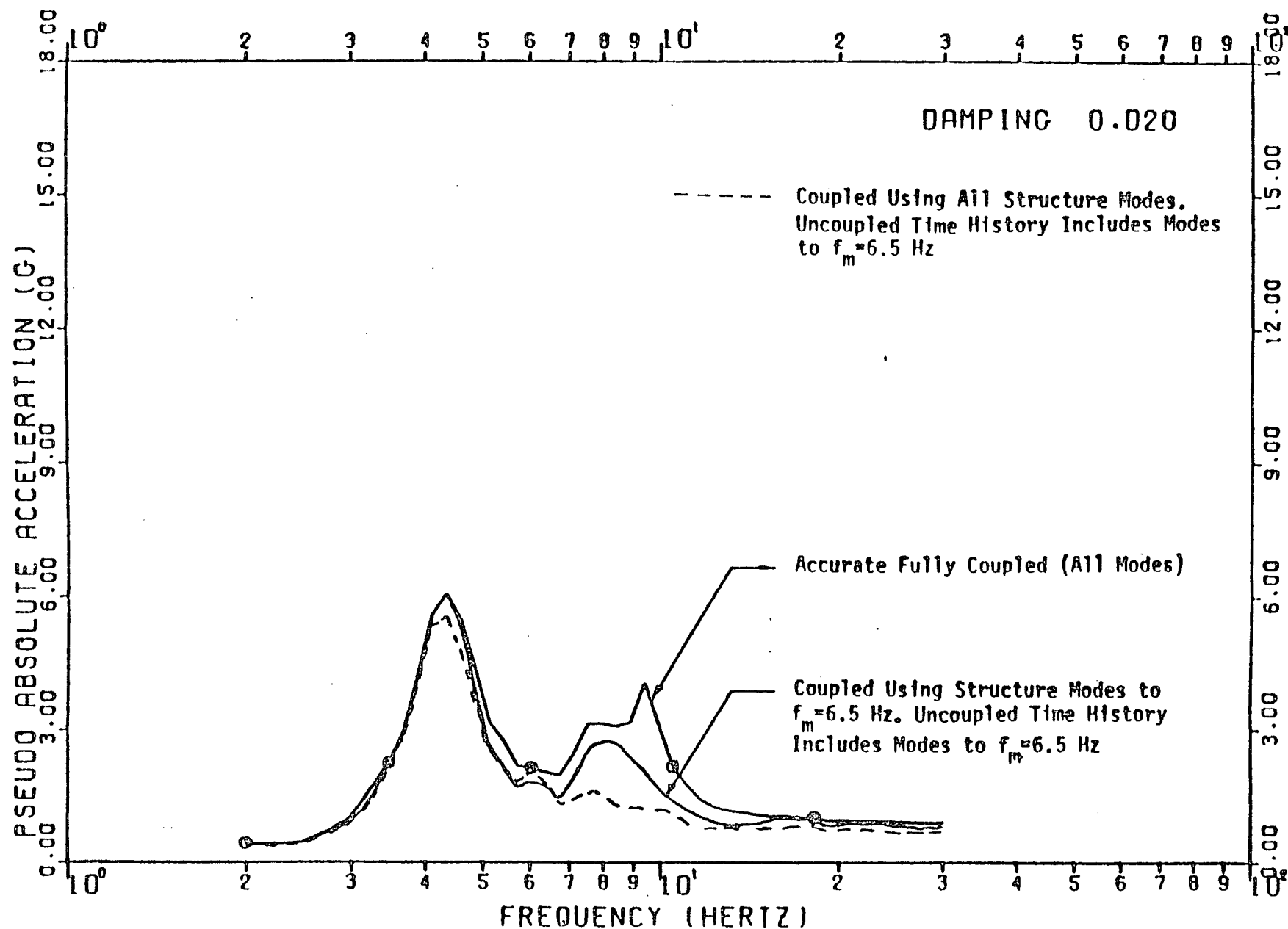


FIGURE A-10: EXAMPLE PROBLEM 1 - DOF 1 - ILLUSTRATION OF RULE 1

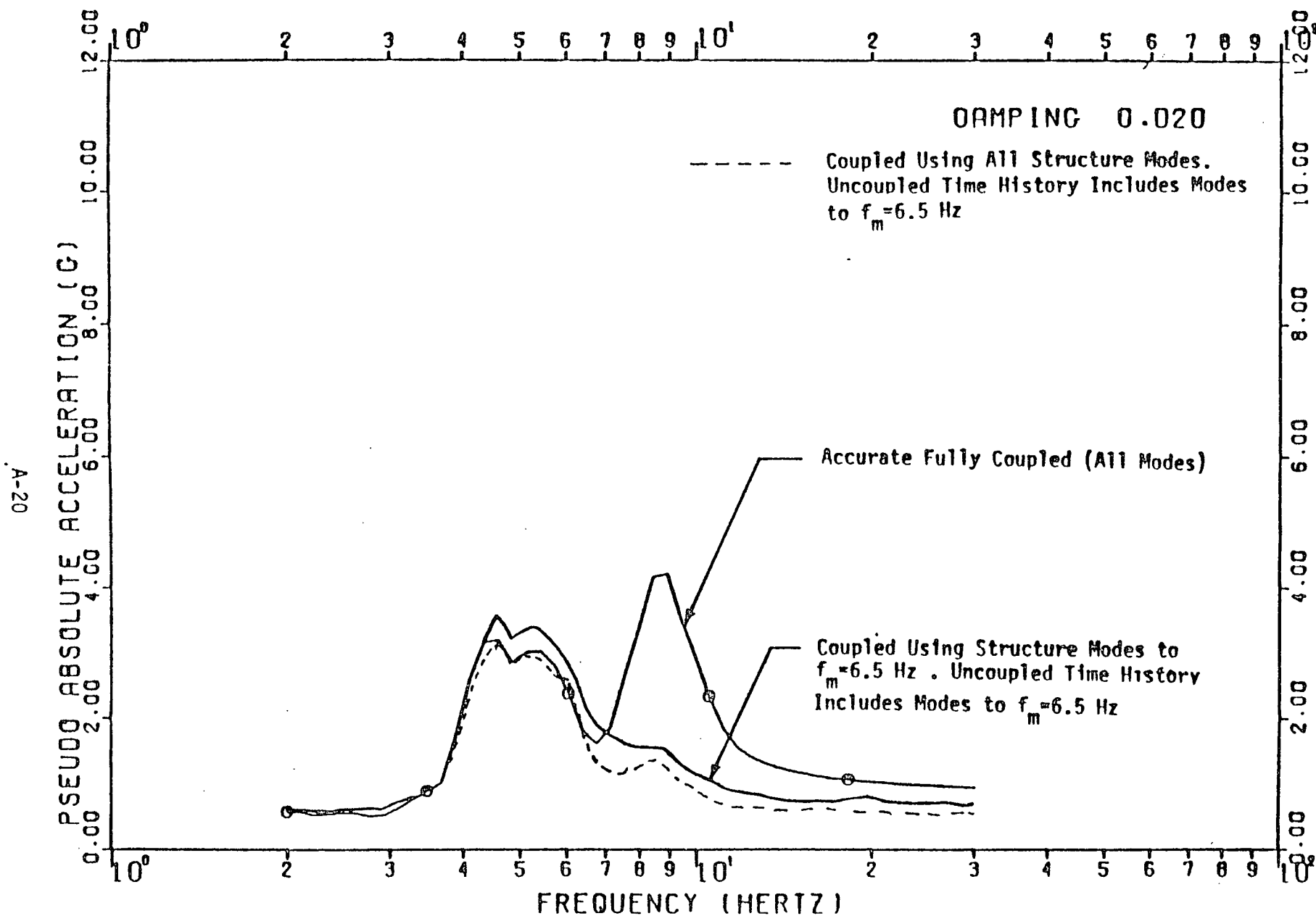


FIGURE A-11: EXAMPLE PROBLEM 1 - DOF 2 - ILLUSTRATION OF RULE 1

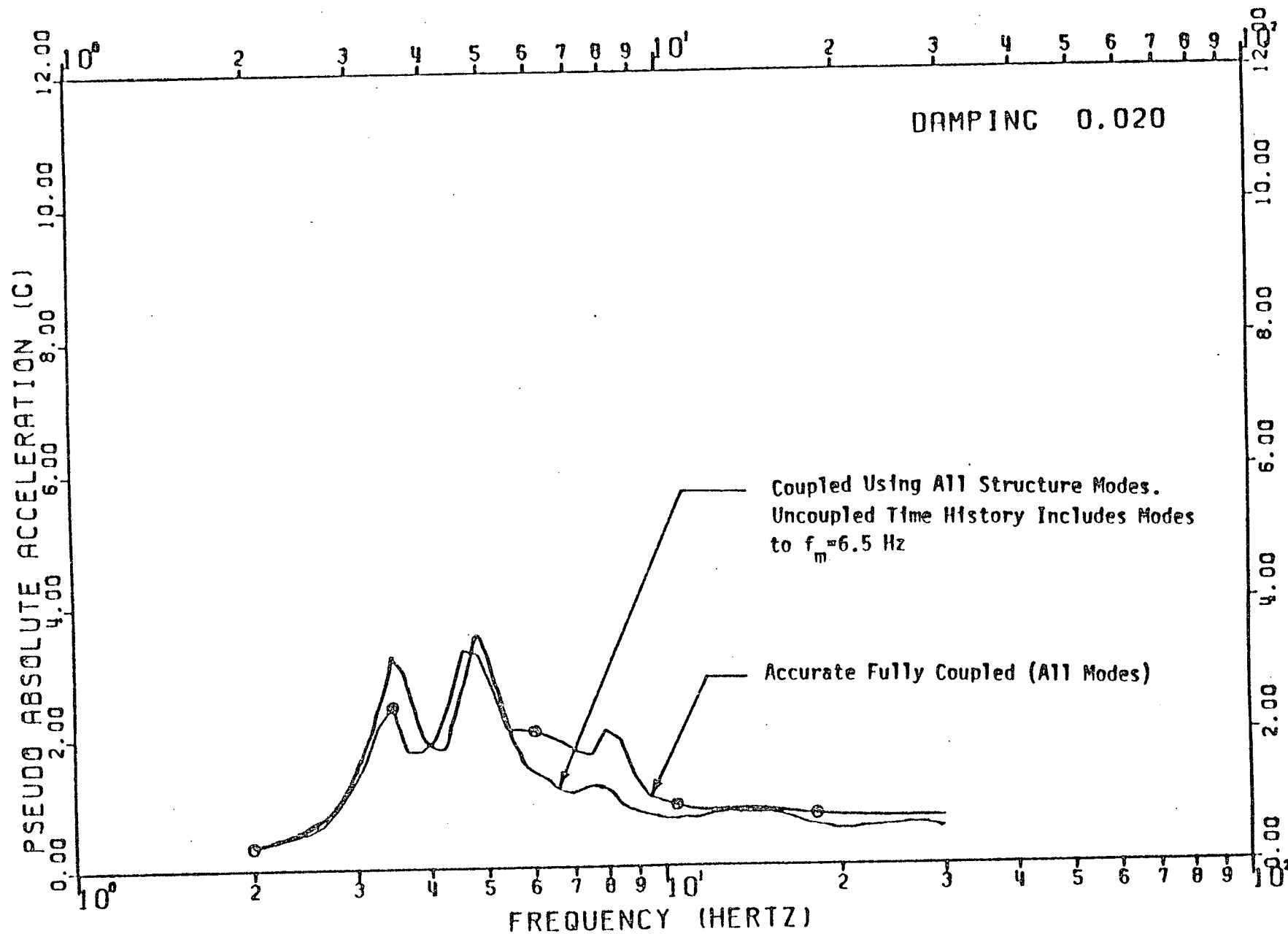


FIGURE A-12: EXAMPLE PROBLEM 2 - DOF 1 - ILLUSTRATION OF RULE 1

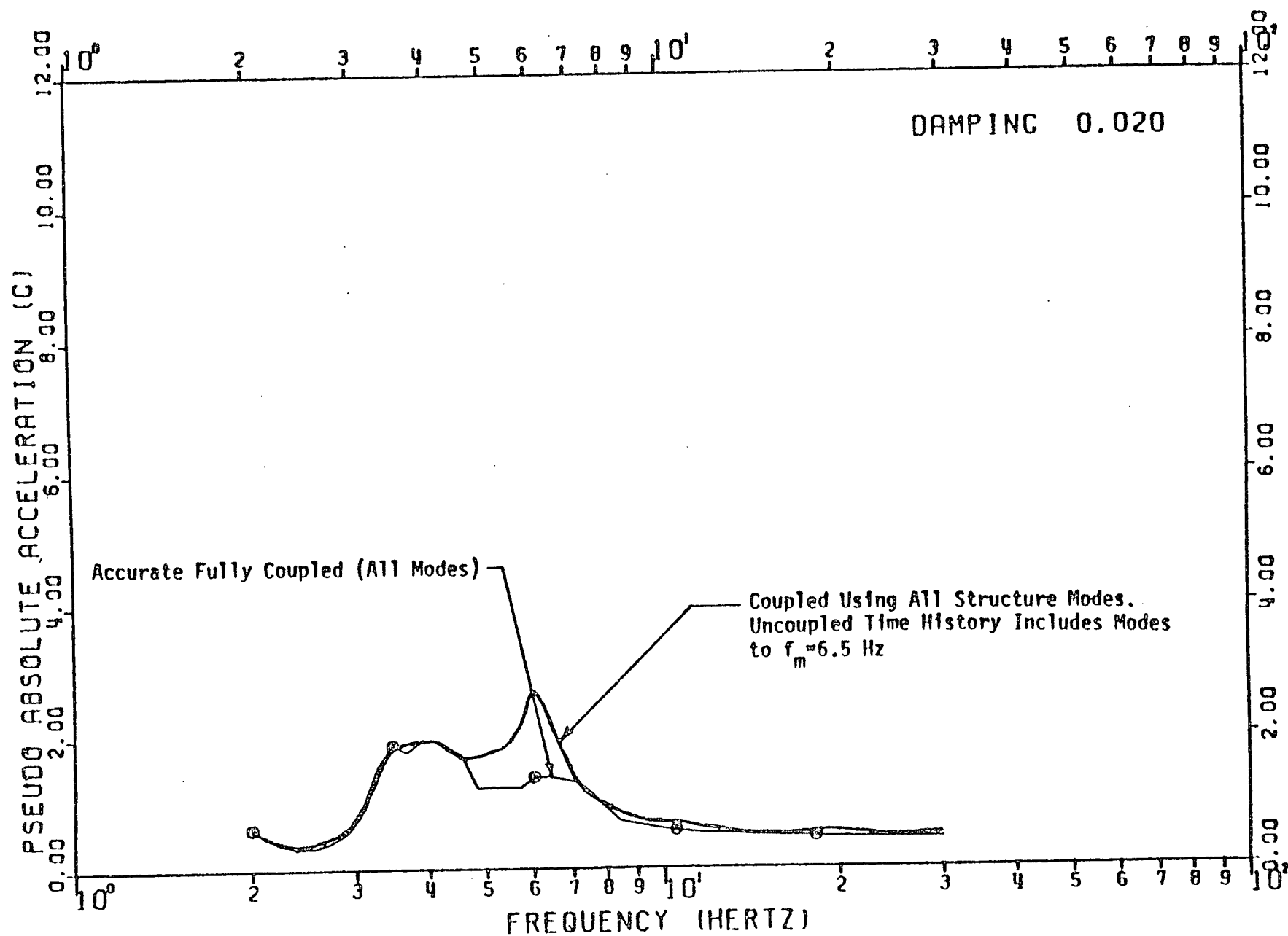


FIGURE A-13: EXAMPLE PROBLEM 2 - DOF 2 - ILLUSTRATION OF RULE 1

A-23

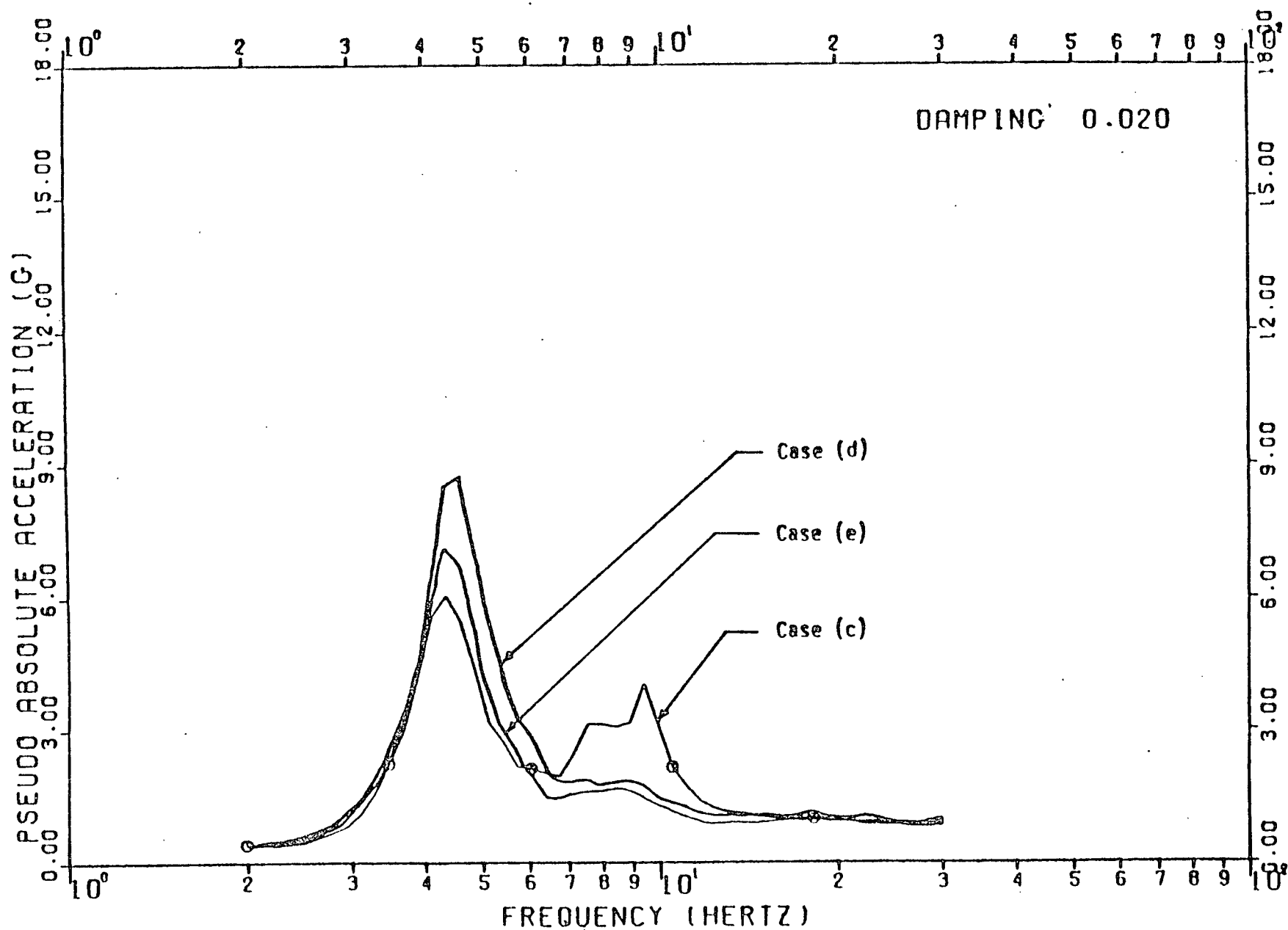


FIGURE A-14: EXAMPLE PROBLEM 1 - DOF 1 - COMPARISON OF CASES (c), (d), (e)

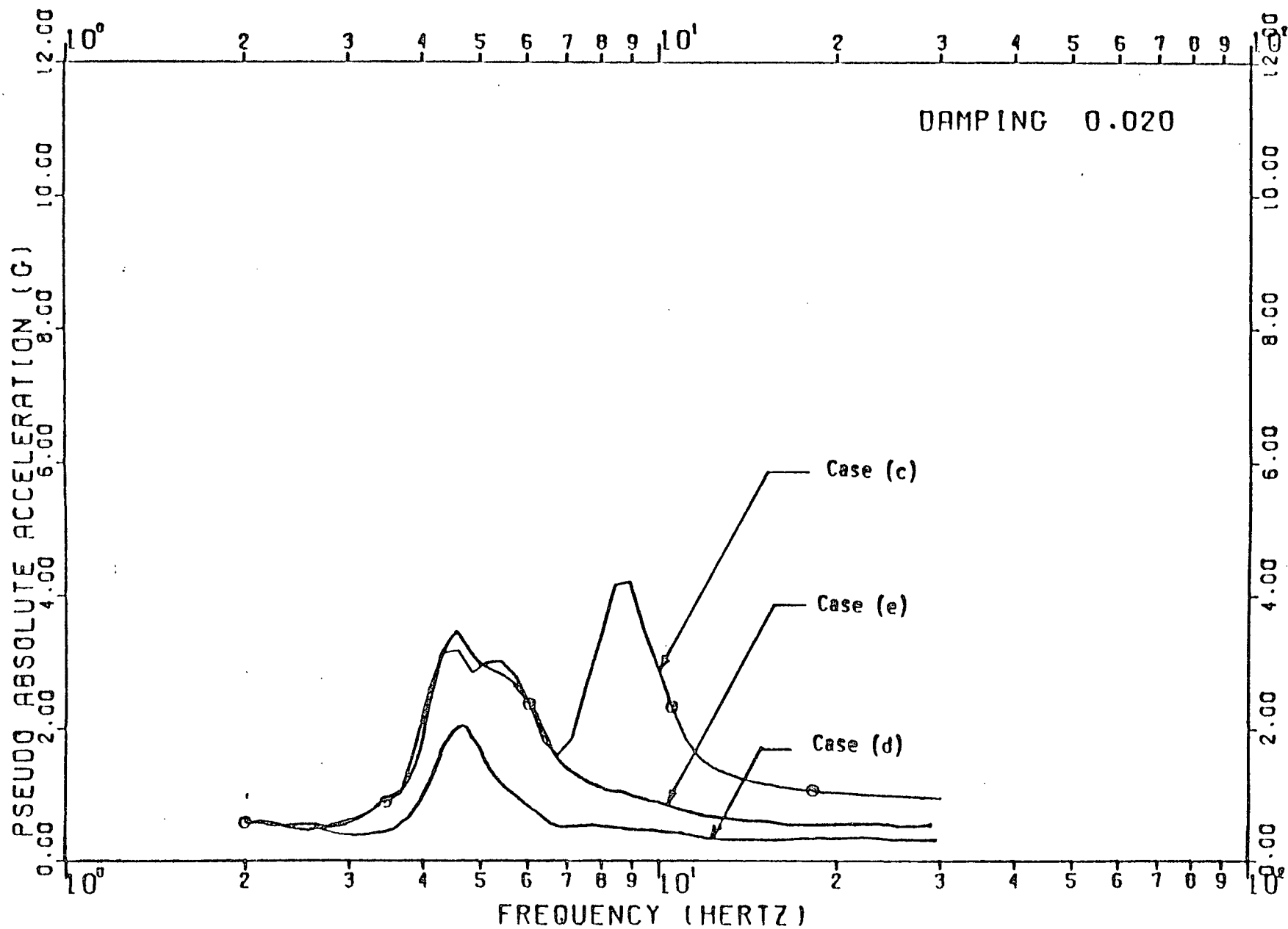


FIGURE A-15: EXAMPLE PROBLEM 1 - DOF 2 - COMPARISON OF CASES (c), (d), (e)



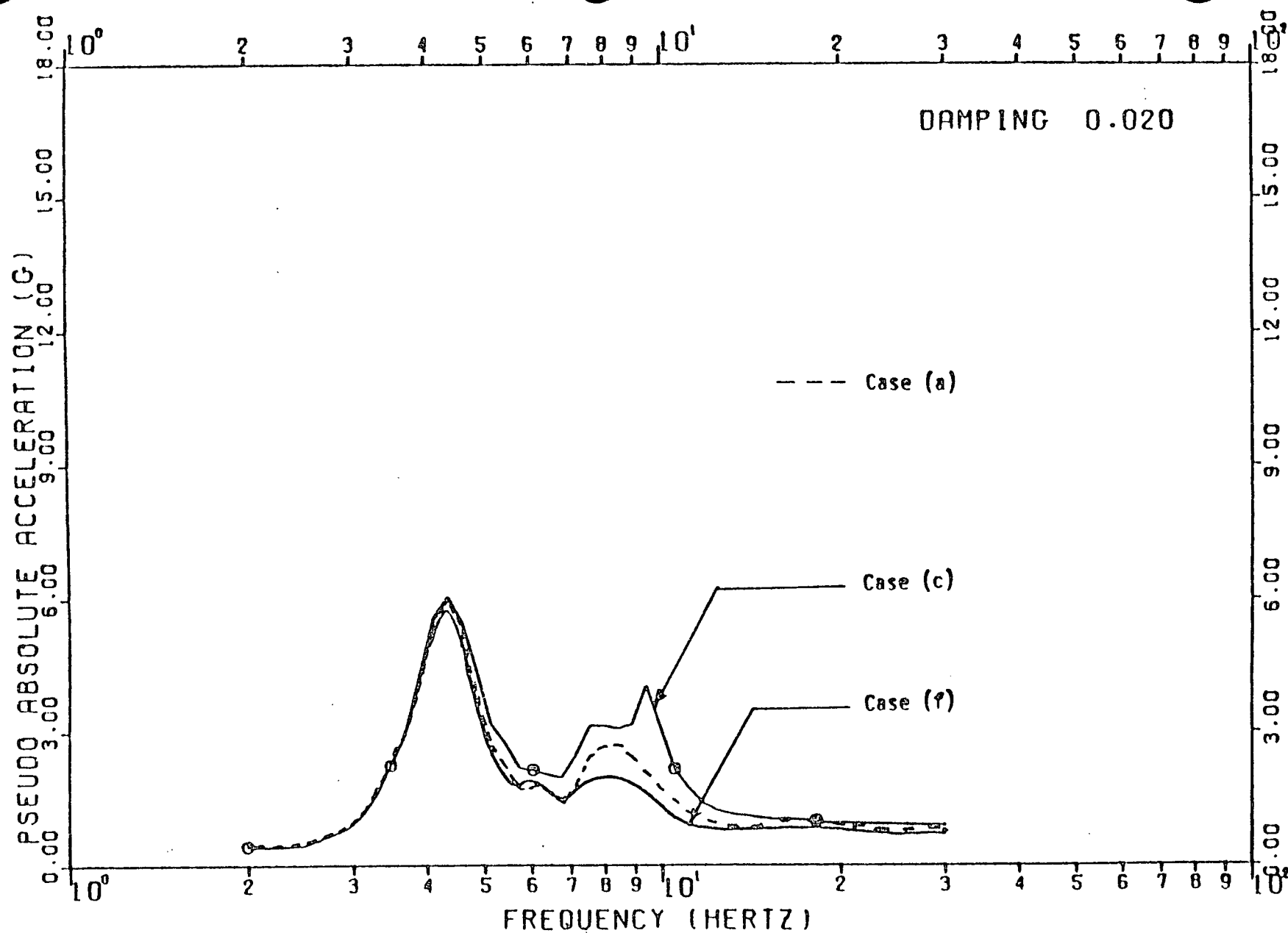


FIGURE A-16: EXAMPLE PROBLEM 1 - DOF 1 - COMPARISON OF CASES (a), (c), (f)

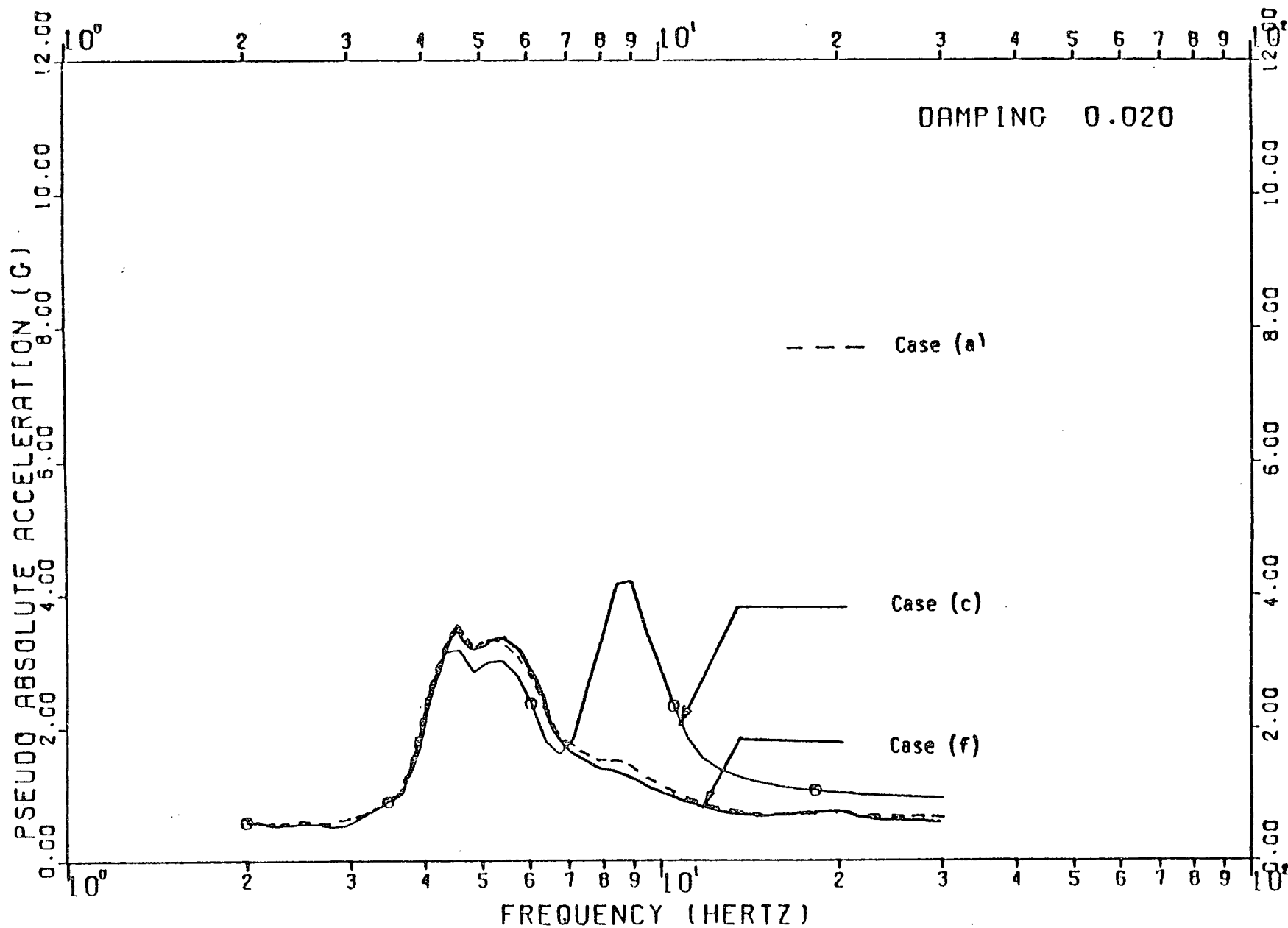


FIGURE A-17: EXAMPLE PROBLEM 1 - DOF 2 - COMPARISON OF CASES (a), (c), (f)

A-27

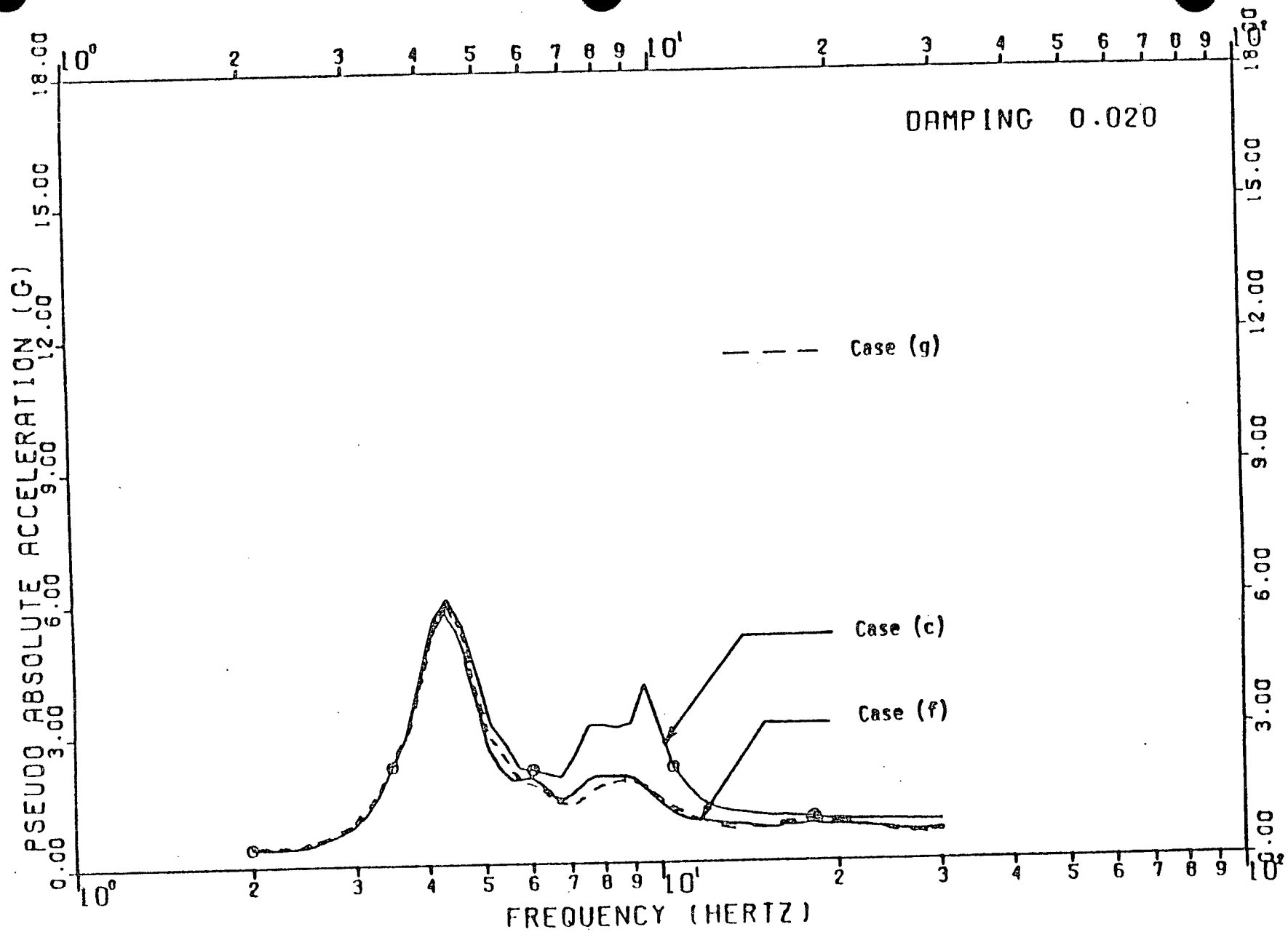


FIGURE A-18: EXAMPLE PROBLEM 1 - DOF 1 - COMPARISON OF CASES (c), (f), (g)

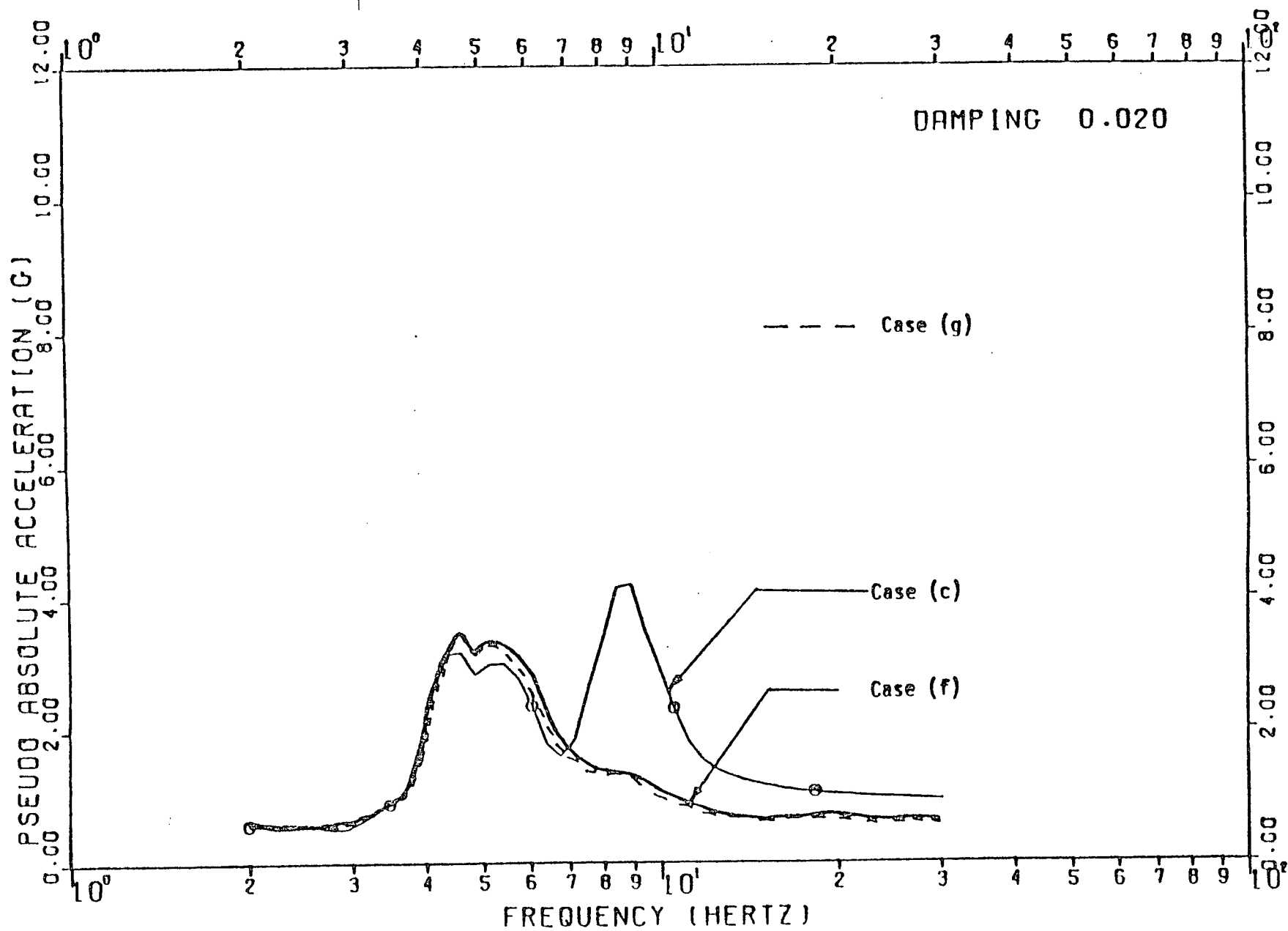


FIGURE A-19: EXAMPLE PROBLEM 1 - DOF 2 - COMPARISON OF CASES (c), (f), (g)

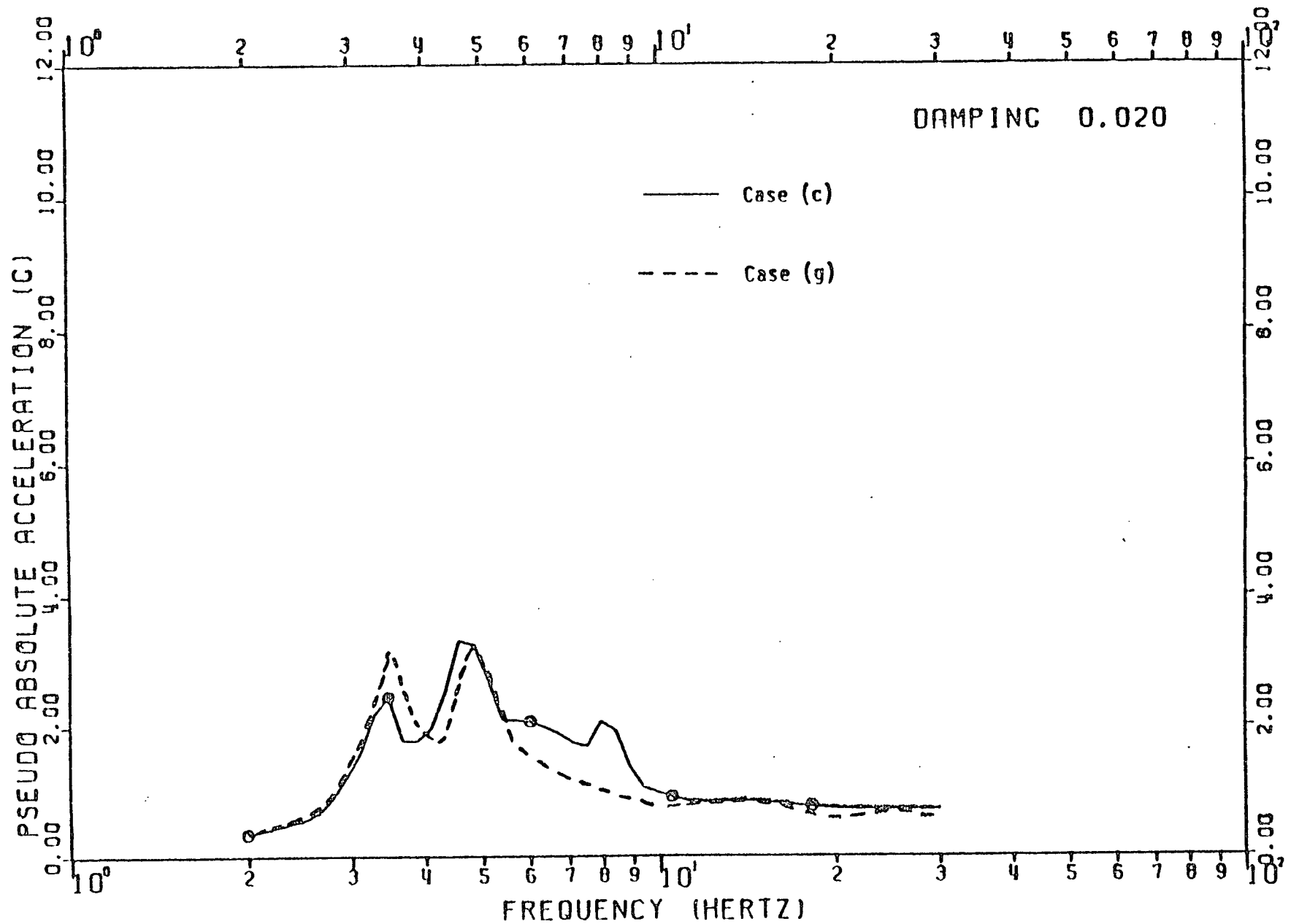


FIGURE A-20: EXAMPLE PROBLEM 2 - DOF 1 - COMPARISON OF CASES (c), (g)

A-30

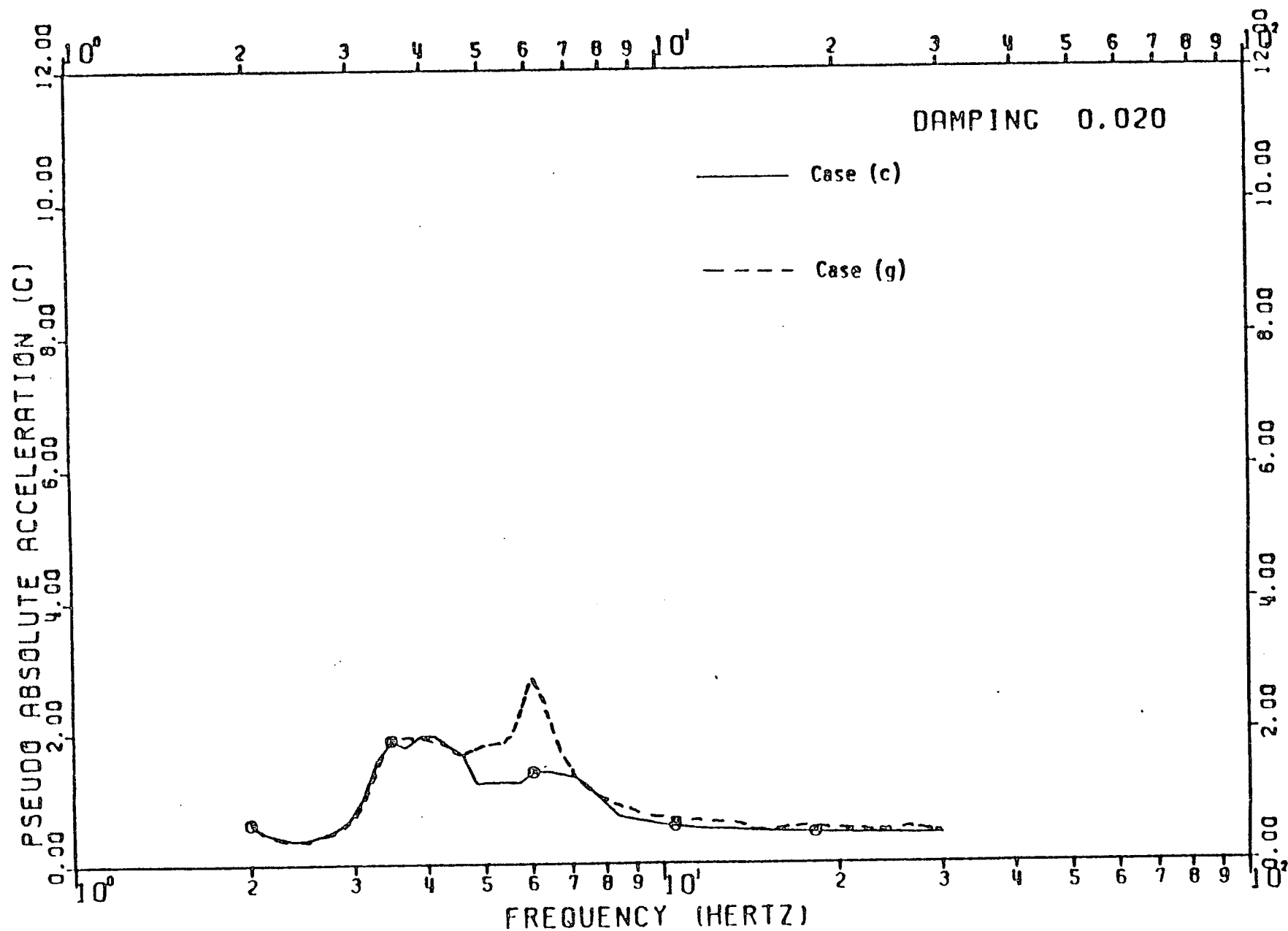


FIGURE A-21: EXAMPLE PROBLEM 2 - DOF 2 - COMPARISON OF CASES (c), (g)

## APPENDIX B

### PRACTICAL MODELLING CONSIDERATIONS WHEN USING POST-PROCESSOR PROGRAM CMDOF

#### B.1 GENERAL

When calculating coupled structural response with Post-Processor Coupling Program CMDOF, program users must fully understand the physical phenomenon being modelled and the mathematical models representing the uncoupled structure and attached equipment in order to obtain correct coupled results. One of the most important considerations to ensure accurate coupled results is to have uncoupled structure and equipment modal data and uncoupled structure time histories which fully describe the dynamic characteristics of the structure. Insufficient dynamic dof in the uncoupled structure and equipment model may give results which predict the overall structural dynamic response well but do not fully describe local response. Coupling Program CMDOF modifies local dynamic response of the structure and it is essential for correct calculation of coupled response to have local dynamic characteristics of the structure at equipment attachment nodes fully described. A complete description is accomplished by using sufficient nodal points and dynamic dof in the uncoupled structure and equipment models so all important dynamic characteristics are represented for all coupled dof. Care must also be taken not to couple undefined dof with the coupling program since incorrect coupled results will be obtained if this is attempted. The coupling program tacitly assumes all uncoupled structure and equipment modal data and uncoupled structure time histories are meaningful and represent actual physical behavior. However, in certain types of finite element formulation, one or more of the element dof are mathematically undefined and structure or equipment mathematical models created using

these finite elements will have corresponding undefined structural dof. Uncoupled structure modal data or uncoupled structure response time histories calculated by structural analysis programs such as NASTRAN or SAP may output results for these undefined dof. However, uncoupled results for these undefined dof have no physical meaning and must be disregarded when calculating coupled response.

A typical example of this type of problem occurs when a shell such as the Mark I torus is modelled using flat plate elements. Figure B-1 illustrates a finite element representation of the Peach Bottom Torus using flat plate finite elements. In this model, rotations about vectors normal to the shell at all nodal points are undefined. Figure B-2 illustrates this undefined dof for a typical flat plate element. Because plate elements are not coplanar at nodal points, rotations about vectors normal to the plate elements at nodal points must remain as unconstrained dof in the structural model. However, flat plate finite elements have no corresponding element stiffness for these rotations and any stiffness terms in the structural stiffness matrix for this dof are either due to the angle at which adjacent plate elements meet in the structural model or are due to an arbitrary stiffness term added by the structural analysis program. Therefore torsional characteristics of the uncoupled structure are inaccurately represented and corresponding torsional uncoupled structure modal data and acceleration time histories have no physical meaning and should be disregarded in the coupling analysis.

The following sections present methods for modelling uncoupled structure and equipment in order to ensure accurate coupled results using the post-processor program. In addition, the significance of undefined dof in finite element models is discussed and procedures for using data from these models in the coupling program outlined. Examples are presented to demonstrate coordinate system compatibility between structure and attached equipment.



## B.2 MODELLING THE UNCOUPLED STRUCTURE TO ENSURE ACCURATE COUPLED RESULTS

As discussed in the previous section, dynamic characteristics of the uncoupled structure must be accurately represented for all important coupled dof to ensure the model is consistent with desired coupled results. The discussion in this section will use the finite element model of the Mark I torus illustrated in Figure B-1 as a specific example but the remarks, in general, apply to any type of structural model.

Input for Coupling Program CMDOF requires uncoupled structure modal data and uncoupled structure acceleration time histories for all important coupled dof between the structure and attached piping. For the model in Figure B-1, significant coupling effects between structure and attached piping would be expected primarily radial to the shell with some coupling occurring for in-plane translations and longitudinal and circumferential rotations. Figure B-3 illustrates these dof for a typical piping-shell interface node.

Investigation of dynamic coupling for the Mark I Torus has shown that as a minimum requirement all translational dof must be included as dynamic dof. Rotational dynamic dof may be neglected in the uncoupled torus model when either of the following two conditions are met:

1. Coupling effects are desired for translational dof only.
2. Rotational dynamic characteristics of the uncoupled structure are accurately represented by using translational dynamic dof only.

The first condition represents a case where rotational accelerations at the piping attachment node are low. In this case, coupling of the rotational dof would have negligible impact on the overall piping response. Therefore, to reduce the number of dynamic dof in the structural model and increase cost effectiveness, rotational dynamic dof should not be considered when modelling the uncoupled structure.

In the second case, coupled response for both translation and rotational dof is desired. The uncoupled structure model for this case represents translational dof as dynamic dof and rotational dof as static dof. Modelling rotational dof as static dof typically is done to reduce the cost of dynamic analyses. This procedure is acceptable so long as rotational dynamic characteristics are accurately represented by static deformation patterns and no important dynamic interaction occurs for these dof. However, in our opinion, it is more accurate to include both translational and rotational dof as dynamic dof in the uncoupled model. This ensures an accurate dynamic description of the uncoupled structure which is essential to correctly calculate coupled response with the post-processor coupling program.

### B.3 UNCOUPLED PIPING MODELS CONSISTENT WITH UNCOUPLED STRUCTURE MODEL

When using the Coupling Program CMDOF, both uncoupled structure and uncoupled piping models must be mathematically consistent with each other to correctly calculate coupled response. In other words, there must be a one to one correspondence between fixed and unconstrained dof for the uncoupled structure and piping at the piping attachment point.

Figure B-4a schematically illustrates a cut section of a circular toroid with attached piping system. Coupling effects are desired for all translational and rotational dof. A force time history is applied to the structure as shown. Figure B-4b illustrates correspondence between attachment dof for the uncoupled structure and uncoupled piping models which are consistent with desired coupled results. In both uncoupled models, all translational and rotational dof are unconstrained at the

piping attachment node and are designated as dynamic dof in the uncoupled dynamic analyses. Coupling program input from these models would consist of frequencies, eigenvectors, and participation factors for all six attachment dof for both the uncoupled structure and equipment. In addition, uncoupled structure acceleration time histories are required for all attachment point dof. Coupled response would then be calculated by Post-Processor Program CMDOF for all dof.

Another illustration of the importance of ensuring mathematical consistency may be shown by considering the coupled torus-piping system shown in Figure B-5a. The local coordinate system used at the piping attachment node is shown in Figure B-3. In this case, coupled response is required for translational dof only. Therefore, each translational dof must be designated as a dynamic degree of freedom. Rotational degrees of freedom should be unconstrained but included as static dof. Uncoupled structure eigenvectors and uncoupled acceleration time histories are input to Coupling Program CMDOF for translational dof only. All rotational eigenvectors and acceleration time histories should be ignored and not input to the coupling program since coupling effects for rotational dof are not considered.

A consistent uncoupled piping model used in calculating coupling effects is shown in Figure B-5b. In this model all translational dof ( $x$ ,  $y$ ,  $z$ ) are included as dynamic dof and are unconstrained at the piping attachment node. In addition, Bijlaard-type rotational springs approximating the longitudinal (dof  $R_2$ ) and circumferential (dof  $R_3$ ) rotational stiffness of the torus are connected to the uncoupled piping model at the piping attachment node. These springs approximate the torus rotational stiffness in the uncoupled piping eigenvalue analysis. The other rotational dof, torsion about a vector normal to the shell (dof  $R_1$ ), should be a fixed dof in the uncoupled piping model. As previously discussed in Section B.1, rotations about a vector normal to the shell are undefined when modelling a shell with flat plate elements. Due to these undefined rotations the torsional stiffness of the torus finite

element model is unrealistically low. Calculated torsional eigenvectors and uncoupled torsional accelerations correspond to a structure which is capable of high torsional response. This is not correct. In-plane rotations for the torus should be low since in reality, the torsional dof of a shell is extremely stiff. Since the coupling program forces compatibility and equilibrium between the structure and piping for all dof, coupling a piping system to a torus with high torsional response would incorrectly model the true physical behavior. A better assumption would be to consider the torsional dof between the piping and torus as a fixed dof in order to approximate the very low torsional response of the torus with attached piping.

Using this consistent piping model, uncoupled eigenvalues and eigenvectors are calculated for the translational dof and input to the coupling program. Coupling between the torus and attached piping is then calculated for translational dof only using the post-processor coupling program.

As a third and final example, consider the coupled torus-piping model shown in Figure B-6a. In this case, coupled response is desired for all translational and rotational dof. The uncoupled structure model used in this analysis has all translational dof and circumferential and longitudinal rotations defined as dynamic dof. These dof are unconstrained when calculating uncoupled structure modal properties and when calculating uncoupled acceleration time histories due to input dynamic loads. As previously noted, rotations about a vector normal to the shell are undefined in this type of model.

A consistent uncoupled piping model which should be used in calculating coupled response is shown in Figure B-6b. In this model, translational and rotational dof on the piping system are modelled as dynamic dof. However, at the piping attachment node, the torsional rotation (dof  $R_1$ ) of the piping system should be fixed. This ensures the piping system eigenvalues and eigenvectors correspond to a piping

system attached to a torsionally stiff structure which has low torsional response. Data from the eigenvalue analysis of this model is then used as input to Coupling Program CMDOF. In this case, only three translations and two rotations are directly coupled using this program since the torsional dof has been assumed to be fixed with essentially no important torsional response occurring. Therefore coupling program input for the uncoupled structure consists of eigenvalues, eigenvectors, and uncoupled acceleration time histories for the five defined dof. For the corresponding uncoupled piping model, eigenvalues and eigenvectors are input for the five unconstrained dof. Coupled acceleration time histories are then calculated for the five coupled dof by Post-Processor Program CMDOF.

#### B.4 SUBSYSTEM ATTACHED TO STRUCTURE AT MULTIPLE ATTACHMENT POINTS

Most coupling problems normally are concerned with equipment attached to the structure at a single location. However, in certain types of problems, such as a multi-branched piping system attached to a free-standing containment, the subsystem may be attached to the structure at two or more locations. This section presents an example of a multiple attached piping system and discusses the required coordinate system at each attachment point.

Figure B-7 schematically illustrates a Mark I Containment Structure with an attached piping system. The piping system is attached to the containment structure at two points (nodes 1 and 2). Coupled radial response of the structure and attached piping is desired at each of these locations.

Figure B-8 presents the local coordinate systems required at both piping attachment nodes. Program CMDOF will calculate response in any coordinate system the user specifies. For this problem, where coupled radial accelerations are desired, the natural choice for the coordinate system is one which is radial to the containment shell. In order to correctly calculate coupled radial response, the uncoupled piping model

must be free to move in the  $x$  and  $x'$  directions (nodes 1 and 2 respectively) in the eigenvalue analysis. All other dof at these attachment points on the piping system should be restrained. Eigenvectors for the uncoupled piping model probably will be calculated in the global  $X, Y, Z$ , coordinate system (Figure B-8). However, since eigenvector and participation factors for both the structure and equipment need be specified in a local coordinate system which is perpendicular to the shell at the pipe attachment node being studied, the modal data for the structure and equipment would need to be transformed to the two local coordinate systems (A and B) shown in Figure B-8. In addition, the uncoupled structure acceleration time histories at both attachment points need to be converted into local radial accelerations corresponding to coordinate systems A and B.

Coupled response from the coupling program is calculated in the specified local coordinate systems. These results may be transformed by the user to any other coordinate system for which results are desired by using a simple coordinate transformation.

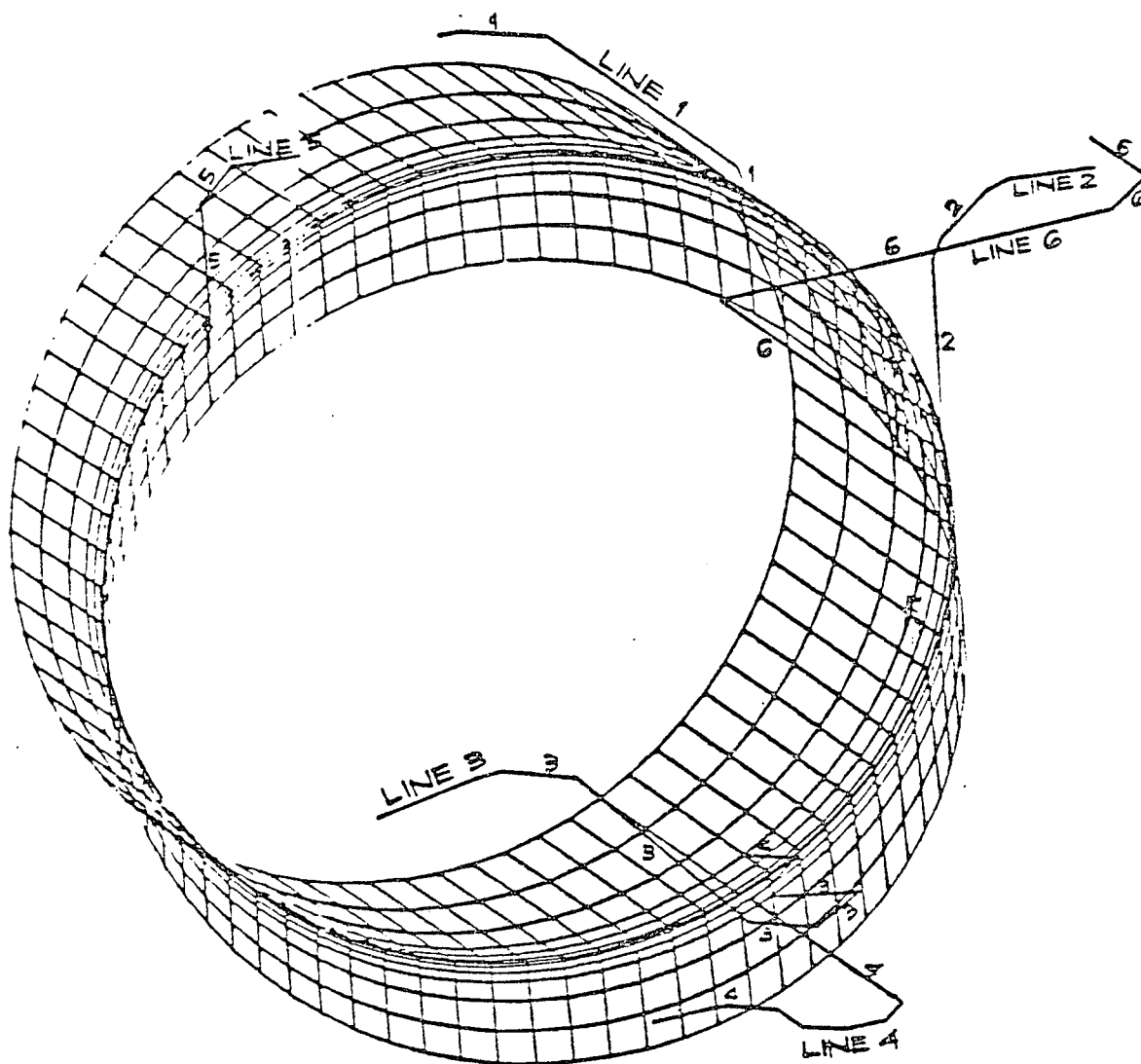


FIGURE B-1: TYPICAL COUPLED MARK I TORUS AND PIPING SYSTEMS

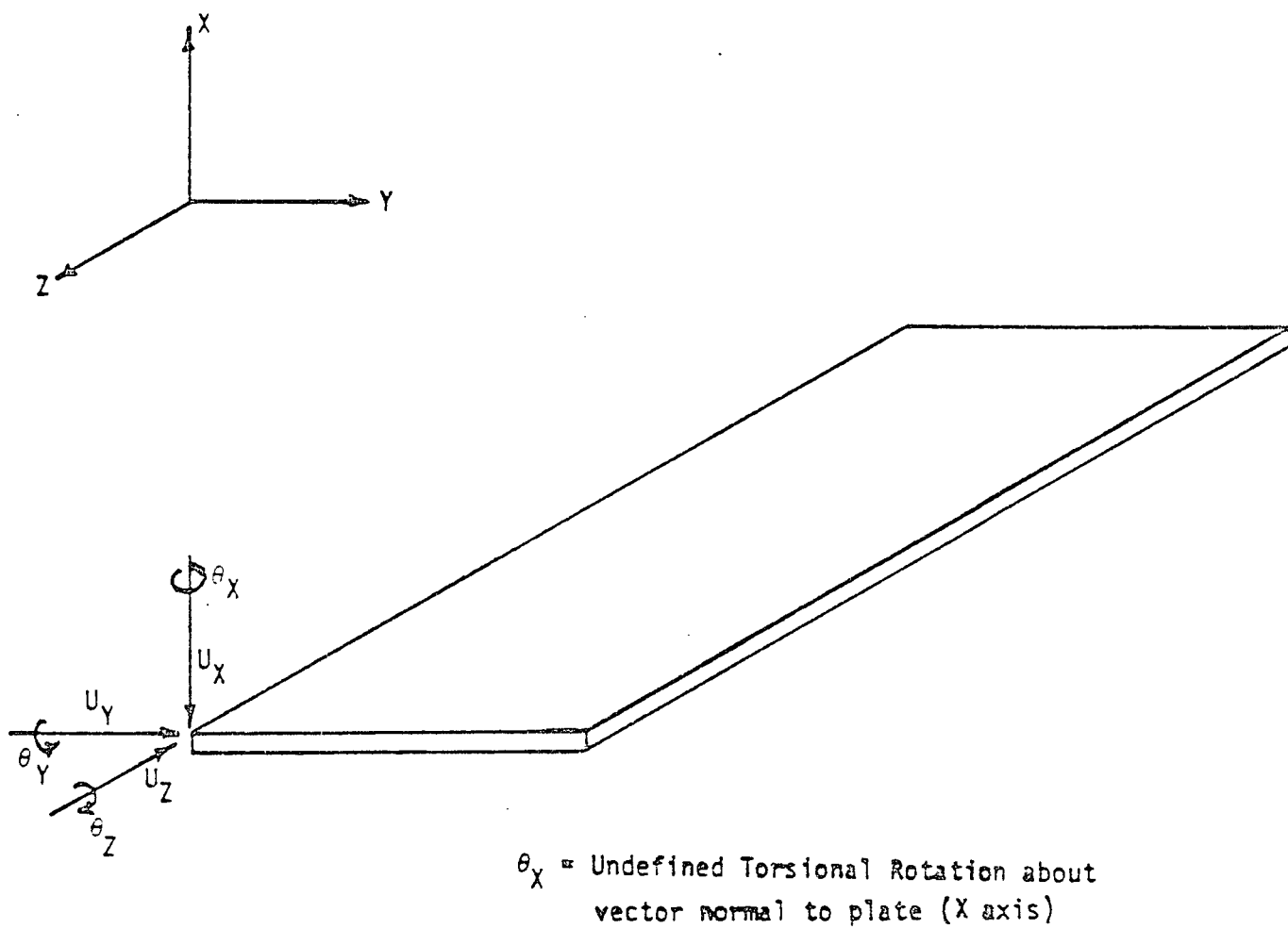
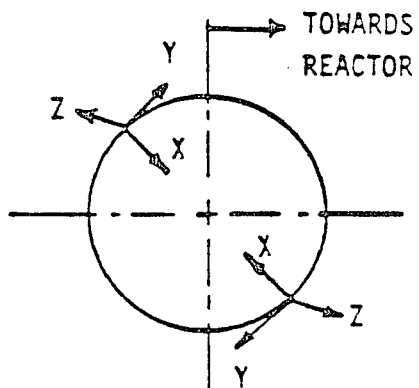
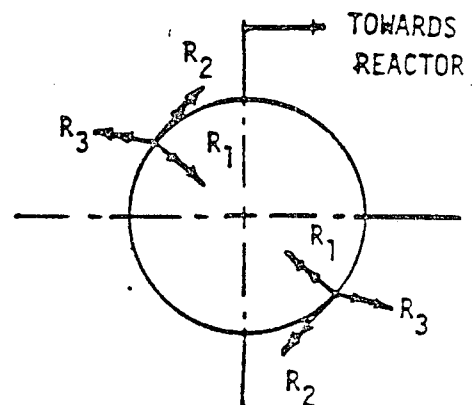


FIGURE B-2: FLAT PLATE FINITE ELEMENT SHOWING TRANSLATIONAL AND ROTATIONAL DOF



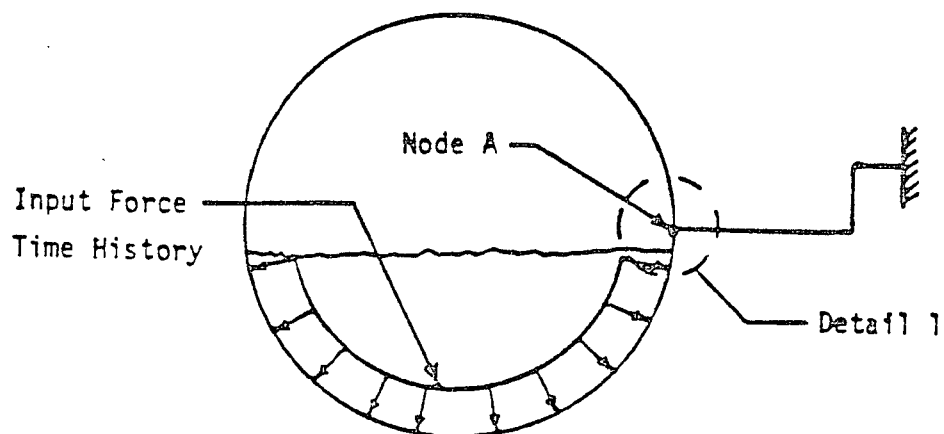


X = RADIAL TRANSLATION  
 Y = TANGENTIAL TRANSLATION  
 Z = LONGITUDINAL TRANSLATION  
 (PERPENDICULAR TO X,Y  
 IN R.H. COORDINATE  
 SYSTEM)

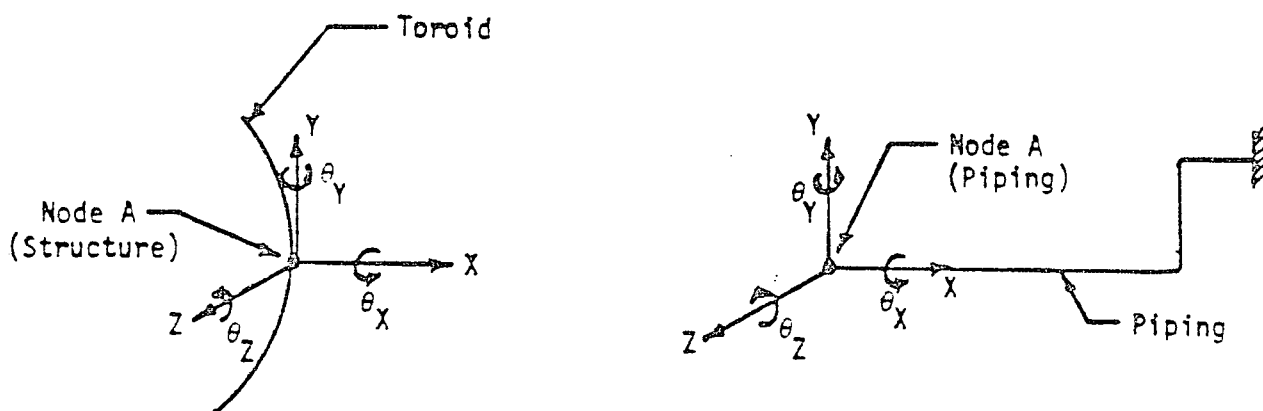


R<sub>1</sub> = ROTATION ABOUT X  
 R<sub>2</sub> = ROTATION ABOUT Y  
 R<sub>3</sub> = ROTATION ABOUT Z

FIGURE B-3: COORDINATE SYSTEM AT PIPING ATTACHMENTS

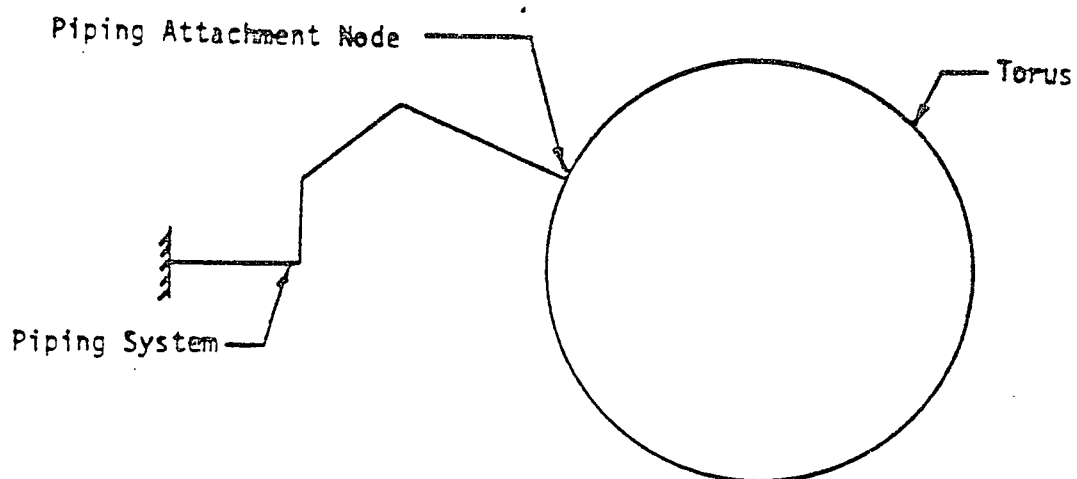


a) COUPLED STRUCTURE AND PIPING MODEL

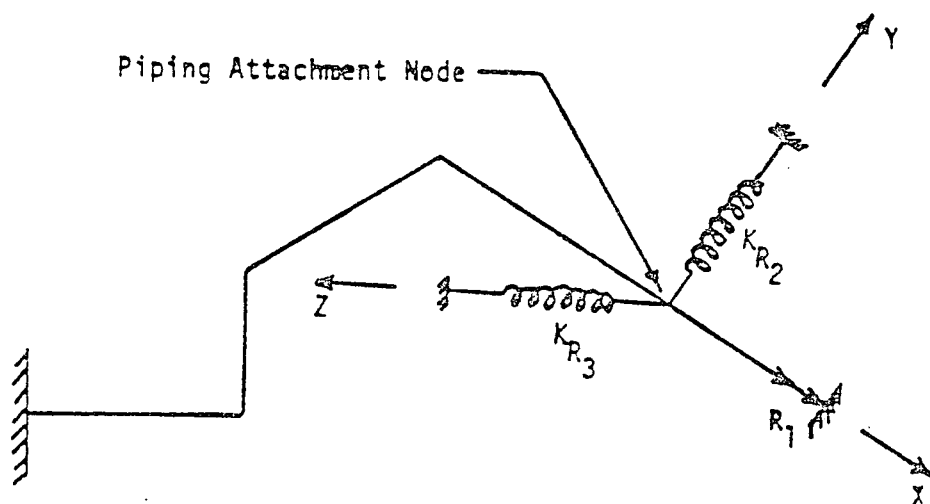


b) DETAIL 1 - CORRESPONDENCE OF LOCAL COORDINATE SYSTEMS AT ATTACHMENT NODE

FIGURE B-4: TOROID WITH ATTACHED PIPING SYSTEM

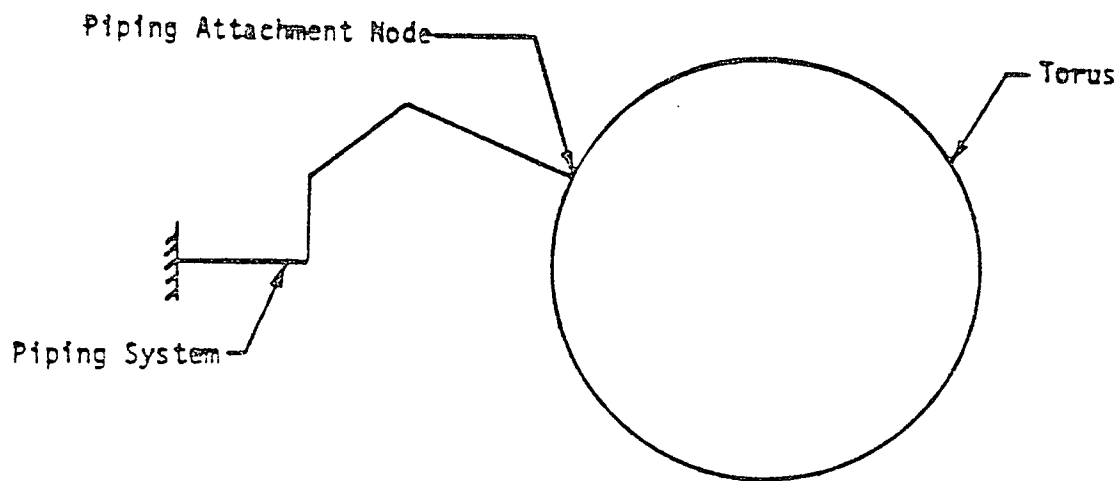


a) COUPLED TORUS - PIPING MODEL

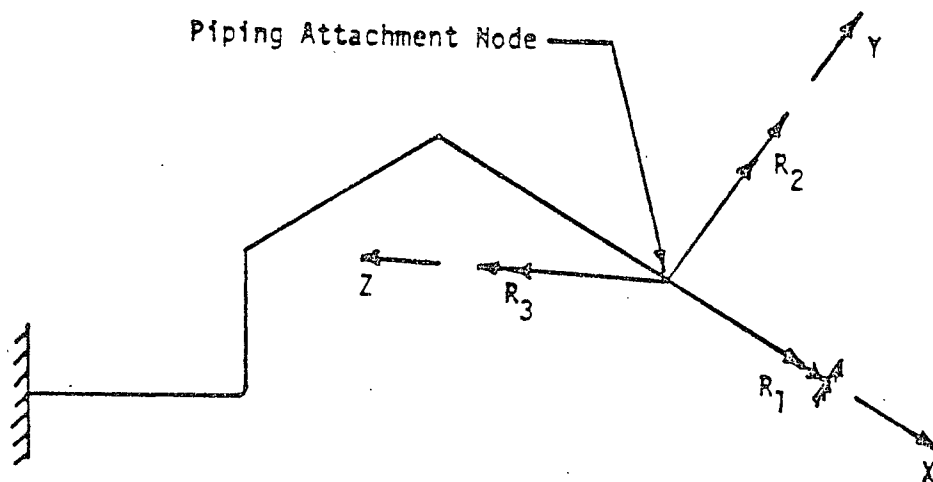


b) UNCOUPLED PIPING MODEL

FIGURE B-5: ILLUSTRATIVE EXAMPLE SHOWING FIXITY AT PIPING ATTACHMENT NODE - TRANSLATION DYNAMIC DOF ONLY

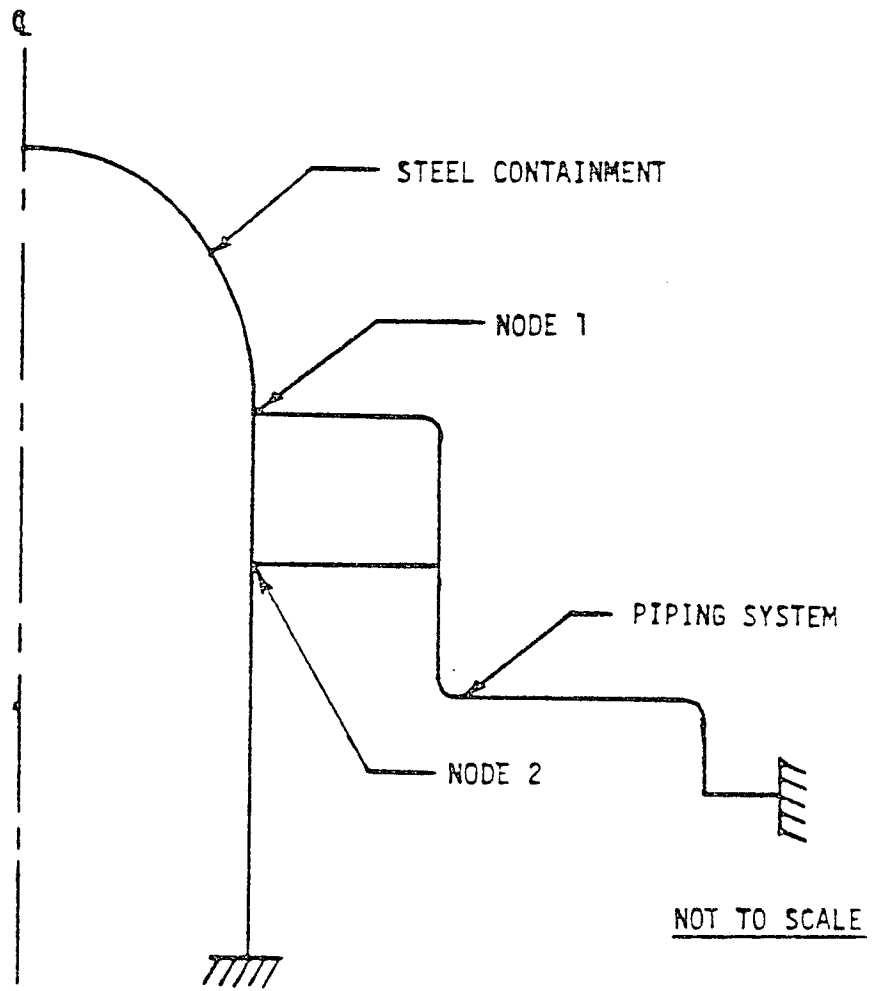


a) COUPLED TORUS - PIPING MODEL

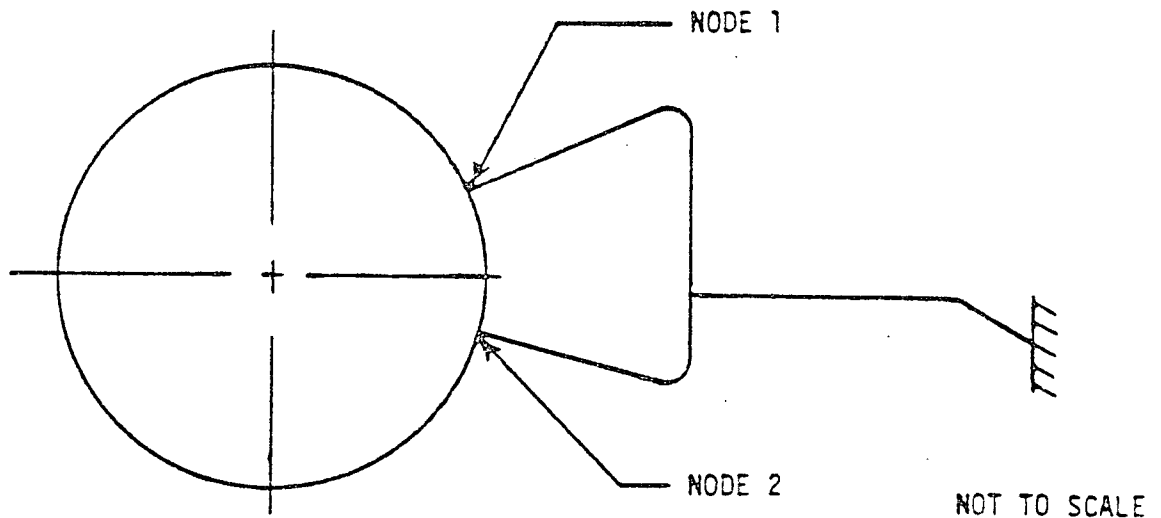


b) UNCOUPLED PIPING MODEL

FIGURE B-6: ILLUSTRATIVE EXAMPLE SHOWING FIXITY AT PIPING ATTACHMENT NODE - BOTH TRANSLATIONAL AND ROTATIONAL DYNAMIC DOF



a) ELEVATION



b) PLAN

FIGURE B-7: MARK I STEEL CONTAINMENT  
WITH ATTACHED PIPING

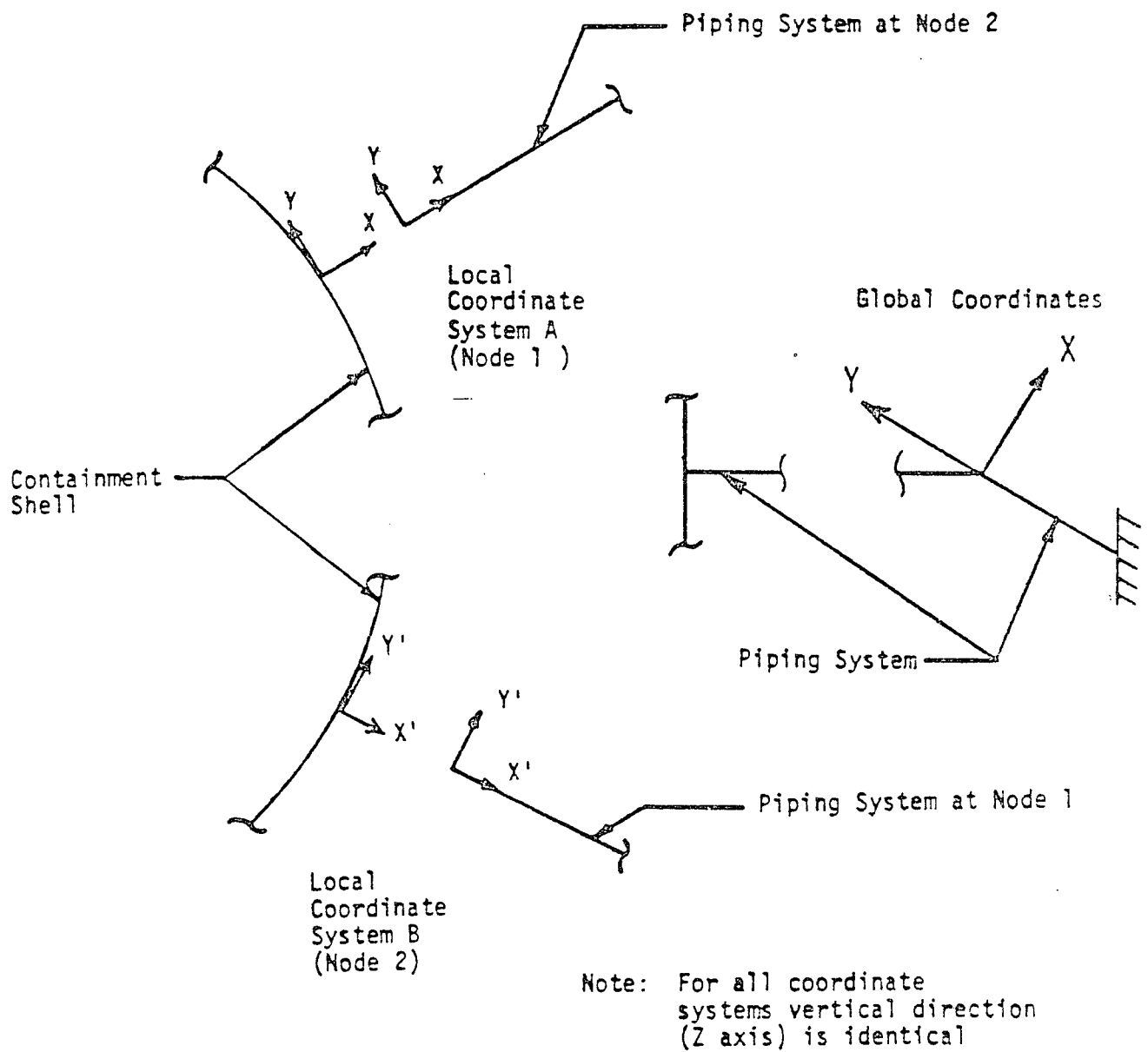


FIGURE B-8: LOCAL COORDINATE SYSTEMS  
AT ATTACHMENT POINTS

APPENDIX C

SUBROUTINE WTTB WRITES  
UNCOUPLED AND COUPLED ACCELERATION TIME HISTORIES  
AND REACTION TIME HISTORIES TO TAPE 11





Attachment C

FREQUENCY RESPONSE ANALYSIS

by

T. P. Khatua

## METHOD OF ANALYSIS

The equations of motion of a multi-degree-of-freedom system are

$$M\ddot{X} + C\dot{X} + KX = g_s F(t) \quad (1)$$

where  $M$ ,  $C$  and  $K$  are the mass, damping and stiffness matrices;  $\ddot{X}$ ,  $\dot{X}$  and  $X$  are the acceleration, velocity and displacement vectors; and  $g_s$  is the spatial distribution vector of the arbitrary loading time history.

If the external load is harmonic of frequency  $\omega$ , then equation (1) can be expressed as

$$X(\omega) = T(\omega) F(\omega) \quad (2)$$

where 
$$T(\omega) = [-\omega^2 M + i\omega C + K]^{-1} g_s \quad (3)$$

$T(\omega)$  is known as the transfer function of the structure.

For a system where classical modes exist, the displacements,  $X$ , can be expressed as

$$X = \sum \phi_j \xi_j \quad (4)$$

where  $\phi$  is the mode shape and  $\xi$  is the distributed coordinate.

The transfer function can be expressed as

$$T(\omega) = \sum_{j=1}^n \frac{\gamma_j \phi_j}{\omega_j^2 - \omega^2 + 2i\omega\beta_j\omega_j} \quad (5)$$

where  $\omega_j$ ,  $\phi_j$  and  $\gamma_j$  are the frequency, mode shape of the response quantity, and participation factor for the  $j^{\text{th}}$  mode, respectively;  $N$  is the number of modes;  $\beta_j$  is proportional modal damping and  $i = \sqrt{-1}$ .

For large models and models where the higher frequency response is important, the above two approaches of obtaining the transfer function are time consuming and expensive, as a wide range of frequencies is of interest. In such cases, the structure is analyzed for a band limited white-noise time history forcing function. Any arbitrary time history can be used, but to avoid numerical error in the high frequency range, the white-noise time history is preferred. The analytical expression for this time history is

$$f(t) = C \frac{\sin \omega t}{\omega t} \quad (6)$$

where  $C$  is a constant and  $\omega$  is the cut-off frequency parameter of the time history. The transfer functions are obtained as the ratio between the fourier transform of the

response and fourier transform of the forcing function for each discrete frequency. Therefore, the transfer function for the  $i^{\text{th}}$  component of the structural model can be expressed as

$$T_i(\omega) = R_i(\omega)/W(\omega) \quad (7)$$

The fourier transform can be obtained by using any standard subroutine.

Once the transfer functions are known for the structural model the response of a typical  $i^{\text{th}}$  component in the frequency domain due to any arbitrary load  $P(t)$  can be expressed as

$$r_i(\omega) = T_i(\omega) \times P(\omega) \quad (8)$$

in which  $P(\omega)$  is the fourier transform of the load. The response  $r_i(t)$  in the time domain can be obtained by performing the inverse fourier transform of  $r_i(\omega)$ .