OSR 31-688 (Rev 11-20-97)
Stores: 26-8910.00

WESTINGHOUSE SAVANNAH RIVER COMPANY
INTEROFFICE MEMORANDUM

BNFL

January 27, 1999

HLW-STE-99-0023

TO: E. J. Freed, 742-5G

FROM: T. B. Caldwell, 742-4G

TANK 8F WASTE REMOVAL
Pump Tank Concentrations During Dilution Operations

Pump Tanks FPT-1 and HPT-7 will be used during the transfer of Tank 8F sludge slurry from F-Tank Farm to H-Tank Farm. The two pump tanks are schematically connected to each other by the interarea line (IAL). During steady state operations, the sludge concentration in the pump tanks and the transfer line are equivalent. During dilution operations, the sludge transfer from Tank 8 is halted and clean dilution water is supplied to FPT-1 at the same flow rate as during the sludge transfer. Therefore, the sludge concentration leaving HPT-7 is a time dependent value.

An analytical model was prepared to estimate the time it takes for the sludge concentration exiting HPT-7 to drop to less than 5% of the steady state concentration. Several assumptions were made to facilitate this determination.

- Concentration of the sludge is uniform in both pump tanks and equivalent to the concentration of the incoming sludge from Tank 8 prior to stoppage of the transfer.
- Inlet sludge concentration is zero.
- Liquid volumes of the two tanks are not equal and considered constant.
- Liquid volumes of the two tanks are well mixed with no concentration gradients.
- IAL volume shall be considered constant and included as part of the FPT-1 volume.

A material balance was conducted first on FPT-1. The solution from that material balance was used as an input in HPT-7. Refer to the attachment for the derivation of the following equation:

$$\boxed{C(t) = \frac{\alpha}{(\beta-\gamma)}e^{-\gamma t} + e^{-\beta t}\left(C_{O2} - \frac{\alpha}{(\beta-\gamma)}\right),}$$

with :

$$\alpha = \frac{QC_{O1}}{V_2},\ \ \beta = \frac{Q}{V_2},\ \ \text{and}\ \gamma = \frac{Q}{V_1},$$

with $V_1 \neq V_2$.

where,

$C(t)$ = Sludge Concentration at time t (weight%)
t = Time (minutes)

Q	=	Flowrate (ft^3 per minute)
C_{O1}	=	Initial sludge concentration of FPT-1 (weight%)
C_{O2}	=	Initial concentraton of HPT-7 (weight%)
V_1	=	FP1 + IAL liquid volume (ft^3)
V_2	=	HPT-7 liquid volume (ft^3)

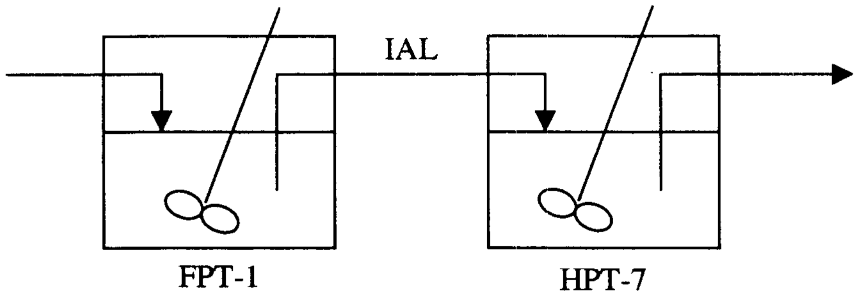
For discussion purposes, set C_{O1} = 15 wt%, C_{O2} = 15 wt%, Q = 13.367 ft^3/min (100 gpm), V_1 = 856.76 ft^3, and V_2 = 374.28 ft^3. Setting $C(t)$ to $0.05C_{O2}$, and solving for t iteratively yields $t \approx 229$ minutes.

Therefore with a 100 gpm dilution flow, the sludge concentration exiting HPT-7 is estimated to drop below 95% of the original concentration of 15 wt% in about 3 hours 50 minutes.

C: N. R. Davis, 703-H
M. J. Green, 742-14G
G. M. Johnson, 742-4G
E. Saldivar, 742-4G
M. J. Turek, 703-H

ATTACHMENT

Derivation Of Equation That Predicts Sludge Concentration At The HPT-7 Pump Outlet



Assumptions

1. Concentration of sludge in FPT-1 is uniform and equivalent to the concentration of the incoming sludge from Tank 8 prior to stoppage of the transfer.
2. Inlet sludge concentration is zero.
3. Liquid volumes of the two tanks are not equal and considered constant.
4. Liquid volumes of the two tanks are well mixed.
5. IAL volume shall be considered constant and part of FPT-1 volume.

Approach

1. Perform material balance on FPT-1.
2. Perform material balance on HPT-7 using solution from FPT-1 balance.
3. Solve ordinary differential equation on HPT-7.

Terms

V_1 =Liquid volume of FPT-1 (ft^3)
V_2 =Liquid volume of HPT-7 (ft^3)
t =Time (min)
Q = Volumetric flowrate (ft^3/min)
C = Sludge concentration (wt%)
C' = Concentration leaving FPT-1 (wt%)
C_{O1} = FPT-1 initial concentration (wt%)
C_{O2} = HPT-7 initial concentration (wt%)

FPT-1 Material Balance

$$Q_{in}C_{in} \longrightarrow \boxed{V_2 \frac{dC}{dt}} \longrightarrow Q_{out}C_{out}$$

FPT-1

$$Q_{in}C_{in} = V_1\frac{dC}{dt} + Q_{out}C_{out} \tag{1}$$

From assumption 2, $C_{in} = 0$.

Equation (1) reduces to:

$$\frac{dC}{dt} = -\frac{QC}{V_1} \tag{2}$$

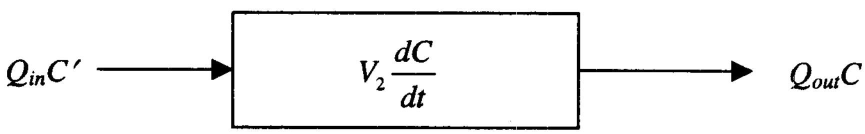
Separate and integrate (2),

$$-\frac{V_1\,dC}{QC} = dt \;\Rightarrow\; -\frac{V_1}{Q}\int_{C_{O1}}^{C'}\frac{dC}{C} = \int_0^t dt \tag{3}$$

$$t = -\frac{V_1}{Q}\ln\frac{C'}{C_O} = \frac{V_1}{Q}\ln\frac{C_O}{C'} \tag{4}$$

$$C'(t) = C_{O1}e^{-Qt/V_1} \tag{5}$$

<u>HPT-7 Material Balance</u>



HPT-7

$$Q_{in}C' = V_2\frac{dC}{dt} + Q_{out}C_{out} \tag{6}$$

From (5), (7) becomes,

$$Q_{in}C_{O1}e^{-Qt/V_1} = V_2\frac{dC}{dt} + Q_{out}C \tag{7}$$

Let $Q = Q_{in} = Q_{out}$, then (7) reduces to,

$$\frac{dC}{dt} = \frac{QC_{O1}}{V_2}e^{-Qt/V_1} - \frac{Q}{V_2}C \tag{8}$$

Let $\alpha = \frac{QC_{O1}}{V_2}$, $\beta = \frac{Q}{V_2}$, and $\gamma = \frac{Q}{V_1}$, then (8) simplifies to,

$$\frac{dC}{dt} + \beta\,C = \alpha\,e^{-\gamma\,t} \tag{9}$$

Equation (9) is in the form $\frac{dy}{dx}+P(x)y=Q(x)$, which is a first order linear differential equation where the general solution is $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+A$. If $P(x)\equiv\beta$, $Q(x)\equiv\alpha e^{-\gamma t}$, $y\equiv C$, and $x\equiv t$, then the general solution for (9) is,

$$Ce^{\int^{t}\beta\,dt}=\int\alpha\,e^{-\gamma t}e^{\int^{t}\beta\,dt}dt+A \tag{10}$$

Simply (10),

$$\begin{aligned}Ce^{\beta t}&=\int\alpha\,e^{-\gamma t}e^{\beta t}dt+A\\&=\int\alpha\,e^{t(\beta-\gamma)}dt+A\\&=\frac{\alpha}{(\beta-\gamma)}e^{t(\beta-\gamma)}+A\end{aligned}$$

So $A=Ce^{\beta t}-\frac{\alpha}{(\beta-\gamma)}e^{t(\beta-\gamma)}$, at $t=0$, $C(0)=C_{O2}$, therefore, $A=C_{O2}-\frac{\alpha}{(\beta-\gamma)}$.

So (10) becomes:

$$\boxed{\begin{aligned}&C(t)=\frac{\alpha}{(\beta-\gamma)}e^{-\gamma t}+e^{-\beta t}\left(C_{O2}-\frac{\alpha}{(\beta-\gamma)}\right),\\&\text{where:}\\&\alpha=\frac{QC_{O1}}{V_2},\ \beta=\frac{Q}{V_2},\ \text{and } \gamma=\frac{Q}{V_1},\\&\text{with } V_1\neq V_2\,.\end{aligned}} \tag{11}$$