Calculation Cover Sheet

Project/Task Performance Assessment for F-Area Tank Farm	Calculation No. T-CLC-F-00421	Project/Task No. N/A
Title Structural Assessment of F-Area Tank Farm After Final Closure	Functional Classification GS	Sheet 1 of 69
	Discipline Structural Mechanics	
Calc Level ☒ Type 1 ☐ Type 2	Type 1 Calc Status ☐ Preliminary ☒ Confirmed	
Computer Program No. NA ☐ ANSYS	Version/Release No. 11.0 (EA1C-V-222V)	

Purpose and Objective

The purpose of this calculation is to evaluate the long term structural behavior/integrity of grout-filled high level waste (HLW) tanks in F-Area.

DC/RO Stamp Date

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Date: 12/18/07

Summary of Conclusion

Using a "worst-case" methodology, the long term structural behavior/integrity of grout-filled high level waste (HLW) tanks in F-Area was evaluated. Comparison of maximum tensile stresses to the modulus of rupture indicates at a very high-confidence level that the grout-filled HLW tanks of FTF will not crack due to long term settlement and material degradation. The intent of this conclusion is to provide a high confidence level solution, a monte carlo simulation is recommended if a more precise determination of cracking probability is required.

Revisions

Rev. No.	Revision Description
0	Initial Issue

Sign Off

Rev. No.	Originator (Print) Sign/Date	Verification/Checking Method	Verifier/Checker (Print) Sign/Date	Manager (Print) Sign/Date
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Security Classification of the Calculation (U)		

Calculation Continuation Sheet

Calculation No.	Sheet No.	Rev.
T-CLC-F-00421	2	0

Revision History

Revision	Description	Sheets Affected
0	Initial Issue	All

Open Items

There are no open items.

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1 PURPOSE AND OBJECTIVE

The purpose of this calculation is to evaluate the long term structural behavior/integrity of grout-filled high level waste (HLW) tanks in F-Area. Final closure of the F-Area Tank Farm (FTF) will consist of filling the tanks with grout and constructing a closure cap on top of the tanks. In support of a performance assessment (PA) for final closure documentation, it is necessary to evaluate the long-term (10,000 year) structural behavior of the grout-filled tanks. Due to the length of time involved, material degradation and the potential for low probability seismic events becomes significant. As the grout-filled tanks are essentially monoliths of grout in the ground, structural collapse cannot occur, but cracking could occur. The results of this calculation may be used in hydraulic waste transport modeling as part of the PA.

The F-Area Tank Farm (FTF) is located within F-Area in the General Separations Area (GSA) of the Savannah River Site (SRS). The FTF is a nearly rectangular shaped area and comprises approximately 20 acres. The FTF includes twenty-two waste tanks, which were constructed between 1951 and 1976. An aerial view of the FTF, looking southwest, is shown in Figure 1.1.

The tanks were installed during four separate construction episodes, with a different tank design for each episode, leading to the designation of the following four different tank groups. Detailed descriptions of each tank type are contained in the body of this calculation:

- The first group of eight tanks (tanks 1 through 8), designated Type I Waste Tanks, was constructed in 1951. The backfill around this group of tanks extends approximately 9 ft above the flat topped tanks.
- The second group of four tanks (tanks 17-20), designated Type IV Waste Tanks, was constructed in 1956. Approximately 2-ft 8-in of backfill was placed over the domed tank tops. Additionally, the concrete 242-F evaporator building was built in the center of this grouping of four tanks.
- The third group of two tanks (tanks 33 and 34), designated Type III Waste Tanks, was constructed in 1969. The backfill around this group of tanks extends to the top of the tank perimeter walls but does not cover the sloping tank top itself.
- The fourth and final group of eight tanks (tanks 25-28 and 44-47), designated Type IIIA Waste Tanks, was constructed in two phases in 1975 and 1976, respectively. The backfill around this group of tanks also extends to the top of the tank perimeter walls but does not cover the

sloping tank top itself. Additionally, the concrete 242-16F evaporator building was built in the middle of this grouping of eight tanks.

The current closure concept for all of the FTF waste tanks is to fill the majority of the tank interior with reducing fill grout and fill the very top of certain tanks with a strong grout to protect against inadvertent intrusion after closure. After the tanks have been filled, a closure cap will be constructed on top of the tanks. The FTF closure cap is primarily intended to provide physical stabilization of the site, to minimize infiltration, and to provide an intruder deterrent. It is anticipated that the closure cap will be installed over all twenty-two waste tanks and associated ancillary equipment at the end of the operational period.

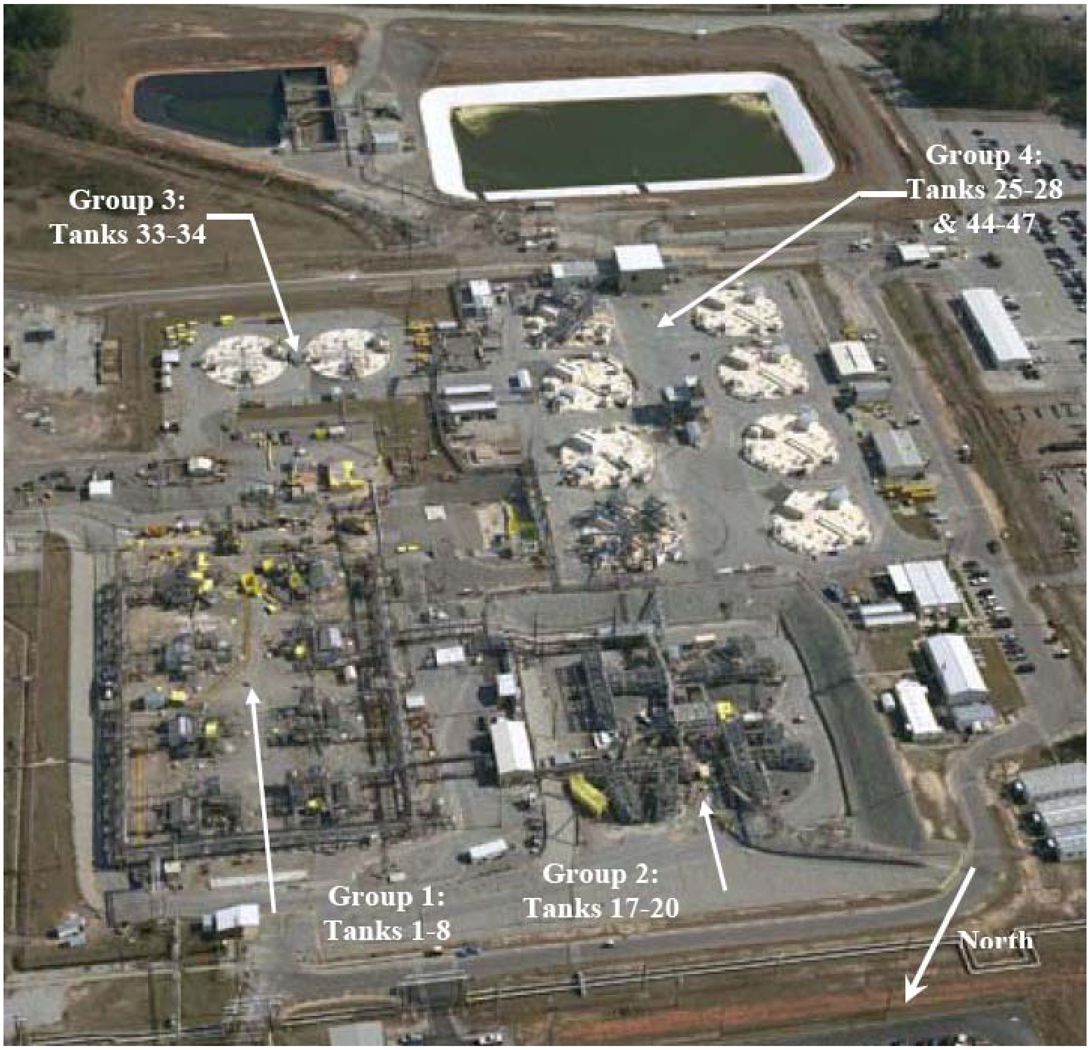

Figure 1.1 FTF Aerial View

2 INPUT AND ASSUMPTIONS

2.1 Input

Material Strengths

Grout compressive strength is taken as 1800 psi [3]. Modulus of rupture is calculated as $f_r = 7.5\sqrt{f_c^{'}}$ [1, Sec. 9.5.2.3].

Loads

Grout unit weight is taken as 130 pcf [8].

Soil unit weight is taken as 120 pcf.

Capacities

Capacities are calculated per ACI 349 [1]. Load factors and ϕ factors are taken as 1.0 to provide a median approach to the expected loading and material properties.

Miscellaneous

Closure cap details, used to estimate the cap thickness over the grout-filled tanks are taken from WSRC-STI-2007-00184 Rev 2 [10].

Tank dimensions and details are taken from the drawings listed in the References Section.

2.2 Assumptions

1) It is assumed that both the tank and any annular space will be filled with grout and no large voids will be present that could collapse.

2) All steel is assumed to have degraded, so no credit is taken for reinforcing steel in the tank vault.

3) Due to thermal degradation it is assumed the tank vault concrete strength is the same as the grout fill.

4) Soil bearing failures are not considered as this would provide more support for the tank after failure.

3 METHODOLOGY

In the final closure state, the HLW tanks will be filled with grout. No large voids will be present, so there is no potential for structural collapse. The tanks will essentially be monoliths of grout in the ground. While collapse will not occur, over the long-term, material degradation and seismic events may cause cracking to occur in these monoliths. This cracking creates the potential for contaminants to leach out of the tank grout and into the environment.

This type of problem easily lends itself to a Monte Carlo simulation. Such a calculation has been completed for the Z-Area saltstone vaults [9]. Another way to approach the problem is to make bounding assumptions regarding the problem, essentially looking at the worst-case scenario. If the results from such a worst-case scenario are acceptable, then there is no need for a Monte Carlo simulation. The latter approach will be followed in the present calculation. The primary assumption in this approach is that all tanks are considered to be monoliths of grout. All reinforcing steel and the steel tank itself are neglected in the analysis. The entire volume is considered to have the properties of the hardened grout.

The following sections discuss the mechanisms that could lead to cracking, the applicability to the grout filled tanks, and methodology for analysis.

3.1 Material Degradation

Due to the length of time the tank will be buried, many degradation mechanisms can affect the grout-filled tank. Concrete degradation mechanisms include sulfate and magnesium attack, alkali and calcium hydroxide leaching, and carbonation. These degradation mechanisms were examined for the Low-Activity Waste (LAW) vaults in E-Area and determined to only degrade the outermost inch or two of the concrete over several thousand years [7]. As the grout-filled tanks have dimensions on the order of 30-ft tall by 85-ft in diameter, the degradation of the outermost few inches will have a negligible affect on the overall structural integrity of the grout monolith.

Steel encased in concrete, such as the concrete vault reinforcing steel or the steel tank itself, is subject to oxic and anoxic corrosion. Cracking may directly expose steel to the soil, leading to rapid deterioration. Both oxic and anoxic corrosion were considered in the LAW vault analysis, primary due to the impact on time to structural collapse. As a bounding assumption, for the FTF all steel, reinforcing steel and the tank itself, will be neglected and the entire volume will be considered to have the properties of the hardened grout.

Thermal degradation of the concrete tank vaults may have occurred while the tanks were in operation due to excessive heat from the HLW. Strength reduction factors for concrete exposed to high temperatures can be found in reference [2]. In lieu of a detailed analysis of thermal degradation, it is assumed that the concrete has the same strength as the grout, about 1800 psi. For tanks with a specified concrete strength of 2500 psi, this amounts to a reduction of about 70%. At the 84% confidence level, this is equivalent to exposure to a temperature of 300°F.

3.2 Seismic Loads

The grout-filled tanks will be covered with a soil cap in the final closure state. As a buried structure, loads other than just seismic inertial loads will act on the grout-filled tank. Dynamic earth pressure and wave passage effects also need to be considered.

As the grout-filled tank is essentially a monolithic block, inertial loading and dynamic earth pressures will have negligible effects. Only extremely large accelerations would have significant effects. As a buried monolith, no amplification will occur in the structure, so the ZPA applies. Only extremely large accelerations would have significant effects. Based on extrapolation from SRS PC-3 and PC-4 site specific spectra, the horizontal and vertical ZPA's for an event with a probability of exceedance of 1e-6 are 0.45g and 2.0 g respectively [7]. By inspection the grout monolith will not crack from these accelerations.

Wave passage effects can be significant in long buried structures, such as pipes. Since the grout-filled tanks are squat cylinders, not long structures, wave passage effects will be negligible.

Significant, soil-structure interaction (SSI) effects are not expected since the grout-filled tanks will be completely buried. However, differential lateral movements in the soil due to seismic events could produce large forces if the grout-filled tank tries to deform with the soil. Calculations performed in T-CLC-E-00018 [7] show that the maximum differential lateral displacement from the top of the LAW vault to the bottom (about 28-ft, similar to a Type I tank) is only about 0.05 inches for a PC-3 event and 0.09 inches for a PC-4 event. Since the grout-filled tank is much stiffer than the surrounding soil, it is reasonable to assume the soil will deform locally around the tank by this small amount rather than the tank deform with the soil. Therefore, large shear forces are not expected to occur from differential lateral movements in the soil column.

Settlement due to seismic events is discussed in the next section.

3.3 Settlement

Settlement can occur due to static load and seismic loads. Static settlement is likely to occur due to that large overburden load from the soil cap. This settlement is expected to be fairly uniform. Any static differential settlement would be small in magnitude and cause the grout-filled tank to rotate as a rigid body. Small magnitudes of rigid body rotation will induce only small lateral forces that can be neglected. Static differential settlement is not considered further.

Seismic differential settlement can occur due to liquefaction and soft zone settlement. Typically for F-Area facilities, liquefaction settlement is expected to be small for PC-3 events. For example, for the planned Waste Solidification Building (WSB) in F-Area, the conclusion of calculation K-CLC-F-00059 [4] states “appropriate settlement for design [PC-3] would be 1 inch with *little differential settlement* as the settlements are relatively small and are expected to distribute due to the depth at which the settlements occur.” For larger seismic events, the magnitude may be larger, but again due to the depth, the settlement would be expected to distribute with little differential settlement.

Soft zones are areas of underconsolidated material in a stronger matrix material that essentially forms a soil arch, allowing the soft zones to remain underconsolidated. A large seismic event could cause the soil arch to fail resulting in settlement as consolidation occurs in the underconsolidated material until it is normally consolidated. Accordingly, soft zone settlement, if it occurs, is essentially independent of seismic event magnitude. The presence of soft zones beneath the FTF is unknown, but will be considered in order to bound the problem. The closest facilities to the FTF for which soft zone settlements have been calculated are the FAMS and WSB facilities.

Two approaches are used to evaluate the grout-filled tanks for seismic differential settlement. The first method consists of essentially hand calcs using plate and beam theory. Three settlement conditions are considered to bound the problem for each type of tank:

1) A circular depression, resulting in loss of support at the tank center: This is idealized as a plate simply supported at the edges.
2) A trough aligned with the center of the tank: This is idealized as a simply supported beam.
3) A trough aligned with the edge of the tank: This is idealized as a cantilevered beam.

In the second approach, a grout-filled tank is modeled using 3D finite elements in ANSYS. The tank is supported on compression-only soil springs which allow for the soil to lose contact with the tank base. The same three settlement cases are used in this approach with the added variables of settlement extent and depth. For both approaches, the maximum stresses resulting from the settlement are compared to the cracking stress of the concrete. The tanks will not crack unless the cracking stress is exceeded. The combined results of these two approaches will provide insight into the likelihood of the grout-filled tanks cracking due to seismic differential settlement.

3.4 Summary of Methodology

A review of degradation mechanisms and effects of large seismic events indicates seismic differential settlement primarily due to soft zone settlement is the only mechanism that has a significant potential to cause cracking in the grout-filled tanks. Two approaches, hand calcs and 3D finite element modeling, are used to calculate the maximum stresses due to three settlement cases. The maximum stresses are compared to the cracking stress of the grout to determine if cracking occurs.

4 RESULTS AND CONCLUSIONS

4.1 Results

The effects of settlement on the grout-filled tanks of FTF were analyzed using simple, idealized hands calcs and 3-D finite element models. Three bounding settlement cases were considered: 1) a circular depression, 2) a trough aligned with the center of the tank, and 3) a trough aligned with the edge of the tank. The maximum tensile stresses in the grout-filled tank for each case are summarized in Table 4.1 and Table 4.2. It can be seen that the maximum stress is less than the modulus of rupture (318 psi) for all cases except two. Both of these cases are for the trough centered on the Type I Tank center. As discussed in Section 6.4.4, this is a small overstress (4%) and occurs for a small depth. Since this occurs for an extreme settlement case and due to the many bounding assumptions made in this calculation, this small overstress is deemed acceptable and the tank will not crack.

Table 4.1 Max Tensile Stresses in Grout-Filled Tanks due to Settlement (Hand Calc)

	Max Tensile Stresses (psi)		
Settlement Type	**Tank Type I**	**Tank Type III/IIIA**	**Tank Type IV**
Circular Settlement	146	92.8	140.2
Trough centered on tank center (SS)	**330**	**209.8**	**317**
Trough centered on tank edge (cantilever)	244	154.7	233.8

Table 4.2 Max Tensile Stresses in Grout-Filled Tank (Type I) due to Settlement (3-D Model)

Settlement Type	Circular Settlement (SS plate)				Trough Centered on tank center (SS)				Trough centered on tank edge (Cantilever)			
Settlement Width (ft)	70		215		70		215		70		215	
Settlement Depth (in)	2.8	12	2.8	12	2.8	12	2.8	12	2.8	12	2.8	12
Maximum Stress (psi)	110	129	115	148	253	313	227	**330**	147	175	72	211

4.2 Conclusions

Using a "worst-case" methodology, the long term structural behavior/integrity of grout-filled high level waste (HLW) tanks in F-Area was evaluated. Several bounding assumptions were made, including neglecting all reinforcing steel, assuming grout properties throughout the grout-filled HLW tank, and extreme settlement loads. Comparison of maximum tensile stresses to the modulus of rupture indicates at a very high-confidence level that the grout-filled HLW tanks of FTF will not crack due to long term settlement and material degradation. The intent of this conclusion is to provide a high confidence level solution, a monte carlo simulation is recommended if a more precise determination of cracking probability is required.

5 REFERENCES

1. ACI 349-06, Code Requirements for Nuclear Safety-Related Concrete Structures and Commentary
2. BNL 52384, Thermal Degradation of Concrete in the Temperature Range from Ambient to 315°C (600°F), M.K. Kassir, K.K. Bandyopadhyay, and M. Reich. October, 1996.
3. C-SPP-F-00047, Rev 2. F Tank Farm, Tanks 18F and 19F, Furnishing and Delivery of Tank Closure Mixes (U). March, 2003.
4. K-CLC-F-00059, Revision 1. Liquefaction Settlement for Waste Solidification Building (U). July, 2007
5. K-CLC-F-00076, Revision 0. Waste Solidification Building (WSB) Soft Zone Settlement. July, 2007.
6. K-ESR-G-00011, Revision 0. Static Modulus of Subgrade Reaction for Large Structures at Savannah River Site (SRS).
7. T-CLC-E-00018, Revision 1. Low Activity Waste (LAW) Vault Structural Degradation Prediction. June, 2006.
8. T-CLC-F-00373 Revision 0. Tanks 18 and 19 Closure Structural Calculation (U). May, 2006.
9. T-CLC-Z-00006 Revision 0. Saltstone Vault Structural Degradation Prediction. July, 2003.
10. WSRC-STI-2007-00184, Revision 2. FTF Closure Cap Concept and Infiltration Estimates. October, 2007.
11. WSRC-STI-2007-00369, Revision 0. Hydraulic and Physical Properties of Tank Grouts and Base Mat Surrogate Concrete for FTF Closure. October, 2007.
12. Young, W.C. "Roark's Formulas for Stress and Strain". 6th Ed. McGraw-Hill. 1989.
13. Tank Drawings:

    W145225 Rev. 4. Waste Storage Tanks - 241 F&H – Design of Concrete Tank – Concrete.
    W167477 Rev. 21. Additional Waste Storage Tanks – Dome Plan & Details – Concrete.
    W238163 Rev. 10. High Level Waste Storage Facilities, General Arrangement & Construction Details – Concrete.

6 CALCULATIONS

6.1 Tank Type Descriptions

There are three primary types of tanks in the FTF. Type I, and III/IIIA tanks consist of cylindrical primary steel tanks with a full or partial secondary liner in a concrete vault with a flat roof. An annular space exists between the primary and secondary liners. Type IV tanks are post-tensioned cylindrical primary steel tanks with a reinforced concrete spherical dome. A concrete vault was constructed by spraying shotcrete on the post-tensioned steel tank.

Since a grout-filled tank is considered as a monolith of grout the primary parameters of concern are the tank diameter and height which are tabulated in Table 6.1 below. Also included is the lowest tank top elevation which will be used to calculate the surcharge load from the closure cap.

Table 6.1 Characteristics of FTF tanks

Tank Type	**I**	**III/IIIA**	**IV**
Tank Numbers	1-8	25-28, 33-34, 44-47	17-20
Outside Diameter (ft) [1]	80	90	85
Height (ft) [1]	29	40.5	34.25 [2]
Tank Top Elevation [3] (ft-msl)	268 Tank 7	286.33 Tank 47	264.2 Tank 20

Notes: 1. Including vault
2. Does not include 11-ft height of dome
3. Taken from Table 2, Ref [10]

6.2 Soil Closure Cap

The preferred closure cap concept [10] is shown schematically in Figure 6.1. In order to meet various slope and drainage requirements, a single cap over the entire FTF is required. The peak of the cap, at an elevation of 316.67 ft-msl, would occur roughly over the 241-21F pump pits #2 and #3. The maximum top slope is 1.5%. Based on the information contained in Figure 6.1, the maximum cap elevation over each type of tank is established in Table 6.2.

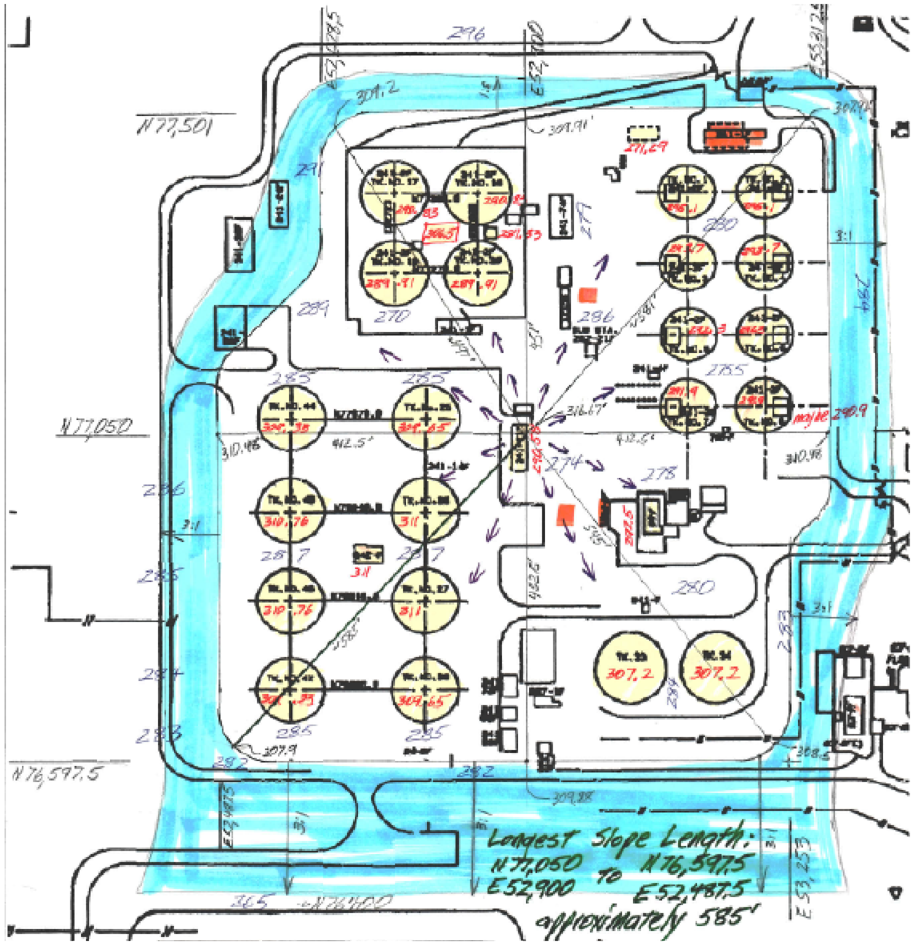

Figure 6.1 FTF Closure Cap Concept [10]

Table 6.2 Maximum Closure Cap Thickness

Tank Type	I	III/IIIA	IV
Min. distance from cap peak to tank center (ft)	218.25 (Tank 7)	97 (Tank 25)	194 (Tank 20)
Max cap elev. Over tank (ft-msl)	313.4	315.2	313.8
Max cap thickness (ft)	45.4	28.9	49.8

Sample calc: Type I: Cap EL = 316.67 – 0.015(218.28) = 313.4
Thickness = 313.4 – 268 = 45.4

6.3 Grout Properties

Several kinds of grout will be used to fill the waste tanks. Reference compressive strengths provided in specification C-SPP-F-00047 [3] indicate the 90-day compressive strength of any grout type will be at least 1800 psi. These expected strengths were verified in reference [11]. However, it should be noted that the grout mixtures contain large amounts pozzolanic material. The pozzolanic materials increase the workability of the grout but hydrate very slowly. Therefore the long-term (>> 90-day) compressive strength of the grout is expected to be much greater than 1800 psi. In order to bound the problem, the compressive strength of 1800 psi is used. The problem is also bounded by assuming the any concrete vaults to be the same strength as the grout.

The tensile strength of the grout can be related to compressive strength through an ACI equation, $f_r = 7.5\sqrt{f_c^{'}}$, [1, Sec. 9.5.2.3]. This is actually the modulus of rupture, not the pure tensile strength. The pure tensile strength is usually lower than the modulus of rupture, but the higher value is justified because the stress state of the grout block will be closer to a beam in flexure than a beam in pure tension.

Based on a compressive strength of 1800 psi, the maximum tensile stress before cracking occurs is $f_r = 7.5\sqrt{1800} = 318.2\,psi$. The grout unit weight is taken as 130 pcf [8]

6.4 Settlement Characteristics

As noted in the methodology, the closest facilities to the FTF for which soft zone settlements have been calculated are the FAMS and WSB facilities. In lieu of additional geotechnical investigation, the soft-zone settlement curve for WSB is adopted for this calculation. The maximum settlement is 2.8 inches. The settlement profile is tabulated in Table 6.3[5]. The settlement curve can be approximated by an inverted log-normal curve of the form shown in Equation 6.1 The settlement curve data in Table 6.3 and an approximated curve with extent, *D*, equal to 215 are shown in Figure 6.2.

$$\delta = \frac{1}{\sqrt{2\pi}} e^{z_c^2/2} \qquad (6.1)$$

where: $z = \frac{x - x_c}{\sigma}$ and $\sigma = \frac{D}{5}$,

D is the extent of settlement (ft), x_c is the coordinate of the center of settlement (0-ft in this case), and x is the distance from the center of settlement (ft)

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Table 6.3 WSB Soft-Zone Settlement Profile

Distance From Centerline (ft)	Settlement (in)
0	-2.8
12	-2.7
20.7	-2.5
29.9	-2.2
35.3	-2.0
42.9	-1.7
47.5	-1.5
55.8	-1.2
61.4	-1.0
71.2	-0.7
79.3	-0.5
90.2	-0.3
97.9	-0.2
109.9	-0.1

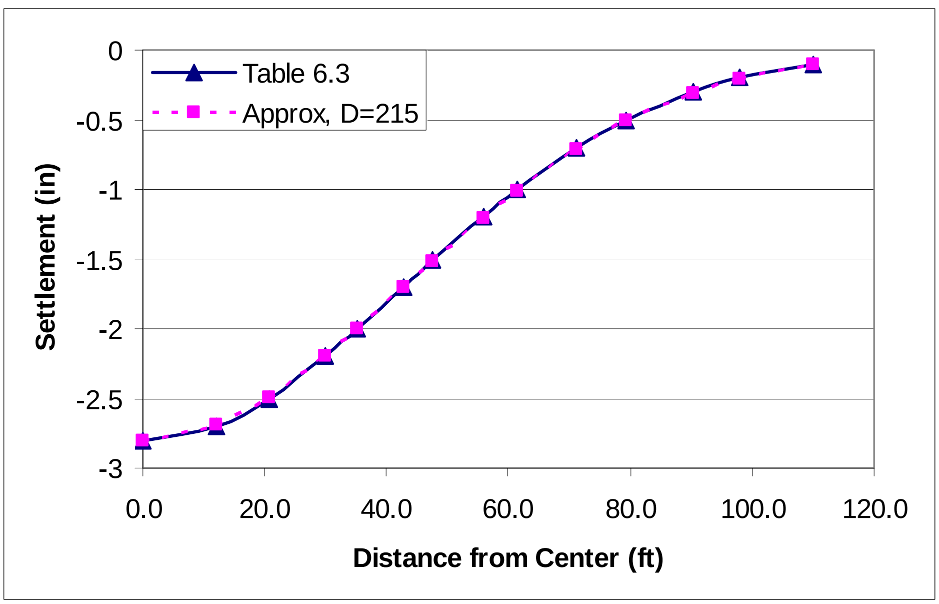

Figure 6.2 Soft Zone Settlement Profile

6.5 Hand Calculations

Settlement under a grout-filled tank is approximated by considering three scenarios: 1) a circular depression, 2) a trough aligned with the center of the tank, and 3) a trough aligned with the edge of the tank. Using plate and beam theory, the maximum stress in each tank type is found due to each settlement case. To bound the analysis, boundary conditions are taken as simply supported or fixed at edges, so the settlement extent and depth do not matter.

6.5.1 Circular Depression

For the settlement case of a circular depression, the settlement is assumed to extend to the edge of the tank. The stress in the tank can be approximated by a solid circular plate, simply supported at the edges, and under uniform loading. The solution for maximum stress can be found using the Roark solution [12] in Mathcad.

Given the magnitude of loads involved, it is unlikely the soil could support the grout-filled tank and the surcharge load only at the tank edges. However, any soil bearing failure would provide additional support to the tank bottom, thus lowering the maximum stresses. Therefore, soil bearing failures are not considered further.

6.5.1.1 Type I Tank

Assume settlement occurs directly beneath tank. Solve for maximum stress by using Roark's formulas

Mathcad e-book Roark's Formulas for stress and strain, 6th Ed:

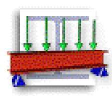

Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Case 10a Solid Circular Plate Simply Supported; Uniformly Distributed Pressure from r_o to a

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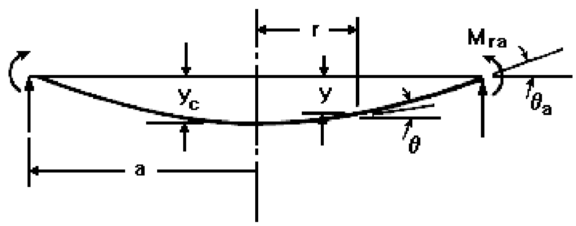
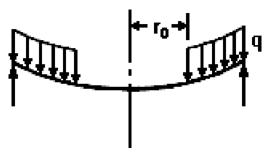
Solid circular plate **Uniformly distributed pressure from r_o to a**





Enter dimensions, properties and loading

Grout Properties: $f_c \equiv 1800\text{psi}$

grout unit weight: $\rho \equiv 130\text{pcf}$

Mod. of Rupture: $f_r := 7.5\sqrt{f_c \cdot \text{psi}}$ $f_r = 318.198\,\text{psi}$

soil unit weight: $\rho_s \equiv 120\text{pcf}$

Tank or "Plate" dimensions:

thickness/height: $t \equiv 29\text{ft}$

radius: $a \equiv 40\text{ft}$

Applied uniform pressure: $q \equiv t \cdot \rho + 45.5\text{ft} \cdot \rho_s$ $q = 9.23\,\text{ksf}$

(29-ft tank height plus 45.5 ft soil cover)

Modulus of elasticity: $E \equiv 57000\sqrt{f_c \cdot \text{psi}}$ $E = 2.418 \times 10^6\,\text{psi}$

Poisson's ratio: $\nu \equiv 0.2$

Radial location of applied load: $r_o \equiv 1 \cdot 10^{-10} \cdot \text{in}$ (uniform load over entire tank)

Constants

Shear modulus:

$$G \equiv \frac{E}{2\cdot(1+\nu)}$$

D is a plate constant used in determining boundary values; it is also used in the general equations for deflection, slope, moment and shear. K_{sro} is the tangential shear constant used in determining the deflection due to shear.

$$D \equiv \frac{E\cdot t^3}{12\cdot(1-\nu^2)} \qquad D = 8.847\cdot 10^{12}\cdot lbf\cdot in$$

$$K_{sro} \equiv -0.30\cdot\left[1-\left(\frac{r_o}{a}\right)^2\cdot\left(1+2\cdot\ln\left(\frac{a}{r_o}\right)\right)\right] \qquad K_{sro} = -0.3$$

Boundary values

The G_n and L_n functions used in the equations below are defined at the end of this document.

M_r is radial moment, Q is shear, y is deflection and θ is slope.

Due to bending:

At the edge of the plate (a):

$$M_{ra} := 0\cdot\frac{lbf\cdot in}{in} \qquad M_{ra} = 0\cdot\frac{lbf\cdot in}{in}$$

$$Q_a := \frac{-q}{2\cdot a}\cdot\left(a^2-r_o^2\right) \qquad Q_a = -15.383\cdot\frac{kip}{in}$$

$$y_a := 0\cdot in \qquad y_a = 0\cdot in$$

$$\theta_a := \frac{q}{8\cdot D\cdot a\cdot(1+\nu)}\cdot\left(a^2-r_o^2\right)^2 \qquad \theta_a = 0.005\cdot deg$$

At the center of the plate (c):

$$M_c := q\cdot a^2\cdot L_{17} \qquad M_c = 2.954\cdot 10^6\cdot\frac{lbf\cdot in}{in}$$

$$y_c := \frac{-q\cdot a^4}{2\cdot D}\cdot\left(\frac{L_{17}}{1+\nu}-2\cdot L_{11}\right) \qquad y_c = -0.026\cdot in$$

Due to tangential shear stresses:

$$y_{sro} := \frac{K_{sro}\cdot q\cdot a^2}{t\cdot G} \qquad y_{sro} = -0.0126\cdot in$$

General formulas for deflection, slope, moment, shear and stress as a function of r

Define r, the range of the radius:

$$r \equiv \frac{a}{100}, \frac{a}{50} .. a$$

Deflection

$$y(r) := y_c + \frac{M_c \cdot r^2}{2 \cdot D \cdot (1+\nu)} + LT_y(r)$$

Deflections at the center and outer radius:

$y(r_o) = -0.026 \cdot in$ $\qquad$ $y(a) = 0 \cdot in$

Maximum deflection (magnitude):

$Y_{(r) \cdot \frac{100}{in}} := y(r)$ $\qquad$ $A := \max(Y)$ $\qquad$ $B := \min(Y)$

$y_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\qquad$ $y_{max} = -0.026 \cdot in$

Large deflection condition check

Check to verify that the absolute value of the maximum deflection is less than one-half the plate thickness (an assumption stated in the Notation file which must hold true). If $|y_{max}|$ is greater than t/2 (large deflection check = 0), the equations in this table used for plates with small deflections are subject to large errors.

$check := if\left(|y_{max}| > \frac{t}{2}, 0, 1\right)$ $\qquad$ $check = 1$

Slope

$$\theta(r) := \frac{M_c \cdot r}{D \cdot (1+\nu)} + LT_\theta(r)$$

Slope at center and outer radius:

$\theta(r_o) = 0 \cdot deg$ $\qquad$ $\theta(a) = 0.005 \cdot deg$

Maximum slope (magnitude):

$S_{(r) \cdot \frac{100}{in}} := \theta(r)$ $\qquad$ $A := \max(S)$ $\qquad$ $B := \min(S)$

$\theta_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\qquad$ $\theta_{max} = 0.005 \cdot deg$

Moment; radial and tangential

$M_r(r) := M_c + LT_M(r)$

$M_t(r) := \frac{\theta(r)\cdot D\cdot\left(1-\nu^2\right)}{r} + \nu\cdot M_r(r)$

Radial and tangential moment at center and outer radius:

$M_r\left(r_o\right) = 2.954\cdot 10^6\cdot\frac{lbf\cdot in}{in}$ $M_r(a) = 0\cdot\frac{lbf\cdot in}{in}$

$M_t(0.01\cdot in) = 2.954\cdot 10^6\cdot\frac{lbf\cdot in}{in}$ $M_t(a) = 1.477\cdot 10^6\cdot\frac{lbf\cdot in}{in}$

Maximum radial and tangential moment (magnitude):

$Mr_{(r)\cdot\frac{100}{in}} := M_r(r)$ $Ar := max(Mr)$ $B := min(Mr)$

$Mt_{(r)\cdot\frac{100}{in}} := M_t(r)$ $At := max(Mt)$ $Bt := min(Mt)$

$Mr_{max} := (Ar > -B)\cdot Ar + (Ar \le -B)\cdot B$ $Mr_{max} = 2.953\cdot 10^6\cdot\frac{lbf\cdot in}{in}$

$Mt_{max} := (At > -Bt)\cdot At + (At \le -Bt)\cdot Bt$ $Mt_{max} = 2.953\cdot 10^6\cdot\frac{lbf\cdot in}{in}$

Shear

$Q(r) := LT_Q(r)$

Shear at center and outer radius:

$Q(0.01\cdot in) = -0.32\cdot\frac{lbf}{in}$ $Q(a) = -1.538\cdot 10^4\cdot\frac{lbf}{in}$

Maximum shear (magnitude):

$V_{(r)\cdot\frac{100}{in}} := Q(r)$ $A := max(V)$ $B := min(V)$

$Q_{max} := (A > -B)\cdot A + (A \le -B)\cdot B$ $Q_{max} = -1.538\cdot 10^4\cdot\frac{lbf}{in}$

Bending stresses; radial and tangential

$\sigma_r(r) := \frac{6 \cdot M_r(r)}{t^2}$ $\sigma_t(r) := \frac{6 \cdot M_t(r)}{t^2}$

Radial and tangential stress at center and outer radius:

$\sigma_r(0.01 \cdot in) = 146.334 \cdot psi$ $\sigma_r(a) = 0 \cdot psi$

$\sigma_t(0.01 \cdot in) = 146.334 \cdot psi$ $\sigma_t(a) = 73.167 \cdot psi$

Maximum radial and tangential stresses:

$\sigma r_{r \cdot \frac{100}{in}} := \sigma_r(r)$ $Ar := max(\sigma r)$ $Br := min(\sigma r)$

$\sigma t_{r \cdot \frac{100}{in}} := \sigma_t(r)$ $At := max(\sigma t)$ $Bt := min(\sigma t)$

$\sigma r_{max} := (Ar > -Br) \cdot Ar + (Ar \le -Br) \cdot Br$ $\sigma r_{max} = 146.319 \cdot psi$

$\sigma t_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\sigma t_{max} = 146.326 \cdot psi$

Review the maximum values for deflection, slope, moment, stress and shear

$y_{max} = -0.026 \cdot in$ $\theta_{max} = 0.005 \cdot deg$

$Mr_{max} = 2.953 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $Mt_{max} = 2.953 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

$\sigma r_{max} = 146.319 \cdot psi$ $\sigma t_{max} = 146.326 \cdot psi$

$Q_{max} = -1.538 \cdot 10^4 \cdot \frac{lbf}{in}$ $f_r = 318.198\,psi$

Total deflection of plate (bending induced plus shear induced):

$y_{ro.total} := y(r_o) + y_{sro}$ $y_{ro.total} = -0.039 \cdot in$

Compare max stress to modulus of rupture (cracking stress)

$\frac{\sigma t_{max}}{f_r} = 0.46$ **< 1, OK,** grout block does NOT crack

Check shear strength

$V_c := 2\sqrt{f_c \cdot psi}$ $V_c = 84.853\,psi$

Total circumferential area $C := 2\pi \cdot a \cdot t$ $C = 7288.495\,ft^2$

Total shear load $V_u := q \cdot \pi \cdot a^2$ $V_u = 4.64 \times 10^4\,kip$

Shear resistance $V_n := V_c \cdot C$ $V_n = 8.906 \times 10^4\,kip$

$\frac{V_u}{V_n} = 0.521$ **< 1, OK,** no shear failure

It should be noted that the bearing capacity at the edge of the tank may be far exceeded, so more settlement could occur. However, this would result in greater support and lower stresses.

The remainder of this page displays the general plate functions and constants used in the equations above.

$$L_{11} \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{a}\right)^2 - 5 \cdot \left(\frac{r_o}{a}\right)^4 \ldots + -\left[4 \cdot \left(\frac{r_o}{a}\right)^2 \cdot \left[2 + \left(\frac{r_o}{a}\right)^2\right] \cdot \ln\left(\frac{a}{r_o}\right)\right]\right]$$

$$L_{17} \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{a}\right)^4\right] - \left(\frac{r_o}{a}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{a}{r_o}\right)\right]\right]$$

$$G_{11}(r) \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{r}\right)^2 - 5 \cdot \left(\frac{r_o}{r}\right)^4 \ldots + -\left[4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \left[2 + \left(\frac{r_o}{r}\right)^2\right] \cdot \ln\left(\frac{r}{r_o}\right)\right]\right] \cdot (r > r_o)$$

$$G_{14}(r) \equiv \frac{1}{16} \cdot \left[1 - \left(\frac{r_o}{r}\right)^4 - 4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \ln\left(\frac{r}{r_o}\right)\right] \cdot (r > r_o)$$

$$G_{17}(r) \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{r}\right)^4\right] - \left(\frac{r_o}{r}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{r}{r_o}\right)\right]\right] \cdot (r > r_o)$$

$$LT_y(r) \equiv \frac{-q \cdot r^4}{D} \cdot G_{11}(r) \qquad LT_M(r) \equiv -q \cdot r^2 \cdot G_{17}(r)$$

$$LT_\theta(r) \equiv \frac{-q \cdot r^3}{D} \cdot G_{14}(r) \qquad LT_Q(r) \equiv \frac{-q}{2 \cdot r} \cdot \left(r^2 - r_o^2\right) \cdot (r > r_o)$$

6.5.1.2 Type III/IIIA Tank

Assume settlement occurs directly beneath tank. Solve for maximum stress by using Roark's formulas

Mathcad e-book Roark's Formulas for stress and strain, 6th Ed:

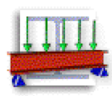


Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Case 10a Solid Circular Plate Simply Supported; Uniformly Distributed Pressure from r_o to a

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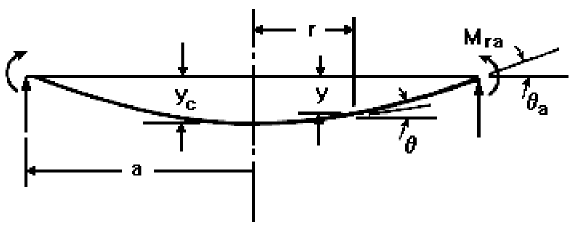
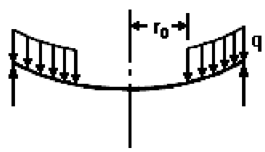
Solid circular plate



Uniformly distributed pressure from r_o to a



Enter dimensions, properties and loading

Grout Properties: $f_c \equiv 1800psi$

Soil/grout unit weight: $\rho \equiv 130pcf$

Mod. of Rupture: $f_r := 7.5\sqrt{f_c \cdot psi}$ $f_r = 318.198\,psi$

Tank "Plate" dimensions:

thickness/height: $t \equiv 40.5ft$

radius: $a \equiv 45ft$

Applied uniform pressure: $q \equiv (t + 28.9ft)\cdot\rho$ $q = 9.022\,ksf$

(40.5-ft tank height plus 28.9 ft soil cover)

Modulus of elasticity: $E \equiv 57000\sqrt{f_c \cdot psi}$ $E = 2.418 \times 10^{6} psi$

Poisson's ratio: $\nu \equiv 0.2$

Radial location of applied load: $r_o \equiv 1 \cdot 10^{-10} \cdot in$

Constants

Shear modulus:

$$G \equiv \frac{E}{2\cdot(1+\nu)}$$

D is a plate constant used in determining boundary values; it is also used in the general equations for deflection, slope, moment and shear. K_{sro} is the tangential shear constant used in determining the deflection due to shear.

$$D \equiv \frac{E\cdot t^3}{12\cdot(1-\nu^2)} \qquad D = 2.41\cdot 10^{13}\cdot lbf\cdot in$$

$$K_{sro} \equiv -0.30\cdot\left[1-\left(\frac{r_o}{a}\right)^2\cdot\left(1+2\cdot\ln\left(\frac{a}{r_o}\right)\right)\right] \qquad K_{sro} = -0.3$$

Boundary values

The G_n and L_n functions used in the equations below are defined at the end of this document.

M_r is radial moment, Q is shear, y is deflection and θ is slope.

Due to bending:

At the edge of the plate (a):

$$M_{ra} := 0\cdot\frac{lbf\cdot in}{in} \qquad M_{ra} = 0\cdot\frac{lbf\cdot in}{in}$$

$$Q_a := \frac{-q}{2\cdot a}\cdot\left(a^2-r_o^2\right) \qquad Q_a = -16.916\cdot\frac{kip}{in}$$

$$y_a := 0\cdot in \qquad y_a = 0\cdot in$$

$$\theta_a := \frac{q}{8\cdot D\cdot a\cdot(1+\nu)}\cdot\left(a^2-r_o^2\right)^2 \qquad \theta_a = 0.002\cdot deg$$

At the center of the plate (c):

$$M_c := q\cdot a^2\cdot L_{17} \qquad M_c = 3.654\cdot 10^6\cdot\frac{lbf\cdot in}{in}$$

$$y_c := \frac{-q\cdot a^4}{2\cdot D}\cdot\left(\frac{L_{17}}{1+\nu}-2\cdot L_{11}\right) \qquad y_c = -0.015\cdot in$$

Due to tangential shear stresses:

$$y_{sro} := \frac{K_{sro}\cdot q\cdot a^2}{t\cdot G} \qquad y_{sro} = -0.0112\cdot in$$

General formulas for deflection, slope, moment, shear and stress as a function of r

Define r, the range of the radius:

$$r \equiv \frac{a}{100}, \frac{a}{50} .. a$$

Deflection

$$y(r) := y_c + \frac{M_c \cdot r^2}{2 \cdot D \cdot (1 + \nu)} + LT_y(r)$$

Deflections at the center and outer radius:

$y(r_o) = -0.015 \cdot in$ $\quad$ $y(a) = 0 \cdot in$

Maximum deflection (magnitude):

$Y_{(r) \cdot \frac{100}{in}} := y(r)$ $\quad$ $A := max(Y)$ $\quad$ $B := min(Y)$

$y_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\quad$ $y_{max} = -0.015 \cdot in$

Large deflection condition check

Check to verify that the absolute value of the maximum deflection is less than one-half the plate thickness (an assumption stated in the Notation file which must hold true). If $|y_{max}|$ is greater than t/2 (large deflection check = 0), the equations in this table used for plates with small deflections are subject to large errors.

$check := if\left(|y_{max}| > \frac{t}{2}, 0, 1\right)$ $\quad$ $check = 1$

Slope

$$\theta(r) := \frac{M_c \cdot r}{D \cdot (1 + \nu)} + LT_\theta(r)$$

Slope at center and outer radius:

$\theta(r_o) = 0 \cdot deg$ $\quad$ $\theta(a) = 0.002 \cdot deg$

Maximum slope (magnitude):

$S_{(r) \cdot \frac{100}{in}} := \theta(r)$ $\quad$ $A := max(S)$ $\quad$ $B := min(S)$

$\theta_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\quad$ $\theta_{max} = 0.002 \cdot deg$

Moment; radial and tangential

$M_r(r) := M_c + LT_M(r)$

$M_t(r) := \frac{\theta(r) \cdot D \cdot \left(1 - \nu^2\right)}{r} + \nu \cdot M_r(r)$

Radial and tangential moment at center and outer radius:

$M_r(r_o) = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $M_r(a) = 0 \cdot \frac{lbf \cdot in}{in}$

$M_t(0.01 \cdot in) = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $M_t(a) = 1.827 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

Maximum radial and tangential moment (magnitude):

$Mr_{(r) \cdot \frac{100}{in}} := M_r(r)$ $Ar := max(Mr)$ $B := min(Mr)$

$Mt_{(r) \cdot \frac{100}{in}} := M_t(r)$ $At := max(Mt)$ $Bt := min(Mt)$

$Mr_{max} := (Ar > -B) \cdot Ar + (Ar \leq -B) \cdot B$ $Mr_{max} = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

$Mt_{max} := (At > -Bt) \cdot At + (At \leq -Bt) \cdot Bt$ $Mt_{max} = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

Shear

$Q(r) := LT_Q(r)$

Shear at center and outer radius:

$Q(0.01 \cdot in) = -0.313 \cdot \frac{lbf}{in}$ $Q(a) = -1.692 \cdot 10^4 \cdot \frac{lbf}{in}$

Maximum shear (magnitude):

$V_{(r) \cdot \frac{100}{in}} := Q(r)$ $A := max(V)$ $B := min(V)$

$Q_{max} := (A > -B) \cdot A + (A \leq -B) \cdot B$ $Q_{max} = -1.692 \cdot 10^4 \cdot \frac{lbf}{in}$

Bending stresses; radial and tangential

$$\sigma_r(r) := \frac{6 \cdot M_r(r)}{t^2} \qquad \sigma_t(r) := \frac{6 \cdot M_t(r)}{t^2}$$

Radial and tangential stress at center and outer radius:

$\sigma_r(0.01 \cdot in) = 92.819 \cdot psi$ $\sigma_r(a) = 0 \cdot psi$

$\sigma_t(0.01 \cdot in) = 92.819 \cdot psi$ $\sigma_t(a) = 46.409 \cdot psi$

Maximum radial and tangential stresses:

$\sigma r_{r \cdot \frac{100}{in}} := \sigma_r(r)$ $Ar := \max(\sigma r)$ $Br := \min(\sigma r)$

$\sigma t_{r \cdot \frac{100}{in}} := \sigma_t(r)$ $At := \max(\sigma t)$ $Bt := \min(\sigma t)$

$\sigma r_{max} := (Ar > -Br) \cdot Ar + (Ar \le -Br) \cdot Br$ $\sigma r_{max} = 92.81 \cdot psi$

$\sigma t_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\sigma t_{max} = 92.814 \cdot psi$

Review the maximum values for deflection, slope, moment, stress and shear

$y_{max} = -0.015 \cdot in$ $\theta_{max} = 0.002 \cdot deg$

$Mr_{max} = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $Mt_{max} = 3.654 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

$\sigma r_{max} = 92.81 \cdot psi$ $\sigma t_{max} = 92.814 \cdot psi$

$Q_{max} = -1.692 \cdot 10^4 \cdot \frac{lbf}{in}$ $f_r = 318.198 \, psi$

Total deflection of plate (bending induced plus shear induced):

$y_{ro.total} := y(r_o) + y_{sro}$ $y_{ro.total} = -0.026 \cdot in$

Compare max stress to modulus of rupture (cracking stress)

$\frac{\sigma t_{max}}{f_r} = 0.292$ **< 1, OK,** grout block does NOT crack

Check shear strength

$V_c := 2\sqrt{f_c \cdot psi}$ $V_c = 84.853\,psi$

Total circumferential area $C := 2\pi \cdot a \cdot t$ $C = 1.145 \times 10^4\,ft^2$

Total shear load $V_u := q \cdot \pi \cdot a^2$ $V_u = 5.74 \times 10^4\,kip$

Shear resistance $V_n := V_c \cdot C$ $V_n = 1.399 \times 10^5\,kip$

$\frac{V_u}{V_n} = 0.41$ **< 1, OK,** no shear failure

It should be noted that the bearing capacity at the edge of the tank may be far exceeded, so more settlement could occur. However, this would result in greater support and lower stresses.

The remainder of this page displays the general plate functions and constants used in the equations above.

$$L_{11} \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{a}\right)^2 - 5 \cdot \left(\frac{r_o}{a}\right)^4 \dots + -\left[4 \cdot \left(\frac{r_o}{a}\right)^2 \cdot \left[2 + \left(\frac{r_o}{a}\right)^2 \right] \cdot \ln\left(\frac{a}{r_o}\right) \right] \right]$$

$$L_{17} \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{a}\right)^4 \right] - \left(\frac{r_o}{a}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{a}{r_o}\right) \right] \right]$$

$$G_{11}(r) \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{r}\right)^2 - 5 \cdot \left(\frac{r_o}{r}\right)^4 \dots + -\left[4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \left[2 + \left(\frac{r_o}{r}\right)^2 \right] \cdot \ln\left(\frac{r}{r_o}\right) \right] \right] \cdot (r > r_o)$$

$$G_{14}(r) \equiv \frac{1}{16} \cdot \left[1 - \left(\frac{r_o}{r}\right)^4 - 4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \ln\left(\frac{r}{r_o}\right) \right] \cdot (r > r_o)$$

$$G_{17}(r) \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{r}\right)^4 \right] - \left(\frac{r_o}{r}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{r}{r_o}\right) \right] \right] \cdot (r > r_o)$$

$$LT_y(r) \equiv \frac{-q \cdot r^4}{D} \cdot G_{11}(r) \qquad LT_M(r) \equiv -q \cdot r^2 \cdot G_{17}(r)$$

$$LT_\theta(r) \equiv \frac{-q \cdot r^3}{D} \cdot G_{14}(r) \qquad LT_Q(r) \equiv \frac{-q}{2 \cdot r} \cdot \left(r^2 - r_o^2\right) \cdot (r > r_o)$$

6.5.1.3 Type IV Tank

Assume settlement occurs directly beneath tank. Solve for maximum stress by using Roark's formulas

Mathcad e-book Roark's Formulas for stress and strain, 6th Ed:

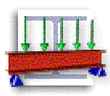


Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Case 10a Solid Circular Plate Simply Supported; Uniformly Distributed Pressure from r_o to a

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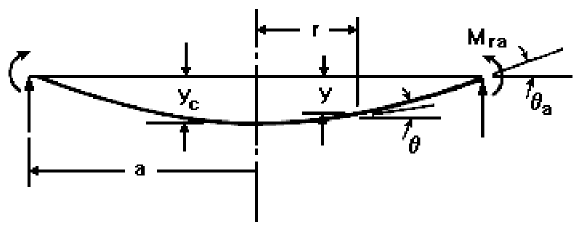
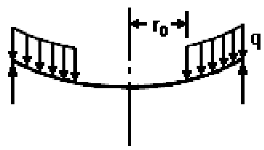
Solid circular plate



Uniformly distributed pressure from r_o to a



Enter dimensions, properties and loading

Grout Properties: $f_c \equiv 1800psi$

Soil/grout unit weight: $\rho \equiv 130pcf$

Mod. of Rupture: $f_r := 7.5\sqrt{f_c \cdot psi}$ $f_r = 318.198\,psi$

Tank "Plate" dimensions:

thickness/height: $t \equiv 34.25ft$ (do not consider dome to bound problem)

radius: $a \equiv 42.5ft$

Applied uniform pressure: $q \equiv (t + 49.8ft) \cdot \rho$ $q = 10.926\,ksf$

(34.25 tank height plus 49.8 ft soil cover)

Modulus of elasticity: $E \equiv 57000\sqrt{f_c \cdot psi}$ $E = 2.418 \times 10^6 psi$

Poisson's ratio: $\nu \equiv 0.2$

Radial location of applied load: $r_o \equiv 1 \cdot 10^{-10} \cdot in$

Constants

Shear modulus: $G \equiv \frac{E}{2\cdot(1+\nu)}$

D is a plate constant used in determining boundary values; it is also used in the general equations for deflection, slope, moment and shear. K_{sro} is the tangential shear constant used in determining the deflection due to shear.

$D \equiv \frac{E\cdot t^3}{12\cdot(1-\nu^2)}$ $D = 1.457\cdot10^{13}\cdot lbf\cdot in$

$K_{sro} \equiv -0.30\cdot\left[1-\left(\frac{r_o}{a}\right)^2\cdot\left(1+2\cdot\ln\left(\frac{a}{r_o}\right)\right)\right]$ $K_{sro} = -0.3$

Boundary values

The G_n and L_n functions used in the equations below are defined at the end of this document.

M_r is radial moment, Q is shear, y is deflection and θ is slope.

Due to bending:

At the edge of the plate (a):

$M_{ra} := 0\cdot\frac{lbf\cdot in}{in}$ $M_{ra} = 0\cdot\frac{lbf\cdot in}{in}$

$Q_a := \frac{-q}{2\cdot a}\cdot\left(a^2 - r_o^2\right)$ $Q_a = -19.349\cdot\frac{kip}{in}$

$y_a := 0\cdot in$ $y_a = 0\cdot in$

$\theta_a := \frac{q}{8\cdot D\cdot a\cdot(1+\nu)}\cdot\left(a^2 - r_o^2\right)^2$ $\theta_a = 0.004\cdot deg$

At the center of the plate (c):

$M_c := q\cdot a^2\cdot L_{17}$ $M_c = 3.947\cdot10^6\cdot\frac{lbf\cdot in}{in}$

$y_c := \frac{-q\cdot a^4}{2\cdot D}\cdot\left(\frac{L_{17}}{1+\nu} - 2\cdot L_{11}\right)$ $y_c = -0.024\cdot in$

Due to tangential shear stresses:

$y_{sro} := \frac{K_{sro}\cdot q\cdot a^2}{t\cdot G}$ $y_{sro} = -0.0143\cdot in$

General formulas for deflection, slope, moment, shear and stress as a function of r

Define r, the range of the radius:

$$r \equiv \frac{a}{100}, \frac{a}{50} .. a$$

Deflection

$$y(r) := y_c + \frac{M_c \cdot r^2}{2 \cdot D \cdot (1 + \nu)} + LT_y(r)$$

Deflections at the center and outer radius:

$y(r_o) = -0.024 \cdot in$ $y(a) = 0 \cdot in$

Maximum deflection (magnitude):

$Y_{(r) \cdot \frac{100}{in}} := y(r)$ $A := max(Y)$ $B := min(Y)$

$y_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $y_{max} = -0.024 \cdot in$

Large deflection condition check

Check to verify that the absolute value of the maximum deflection is less than one-half the plate thickness (an assumption stated in the Notation file which must hold true). If $|y_{max}|$ is greater than t/2 (large deflection check = 0), the equations in this table used for plates with small deflections are subject to large errors.

$check := if\left(|y_{max}| > \frac{t}{2}, 0, 1\right)$ $check = 1$

Slope

$$\theta(r) := \frac{M_c \cdot r}{D \cdot (1 + \nu)} + LT_\theta(r)$$

Slope at center and outer radius:

$\theta(r_o) = 0 \cdot deg$ $\theta(a) = 0.004 \cdot deg$

Maximum slope (magnitude):

$S_{(r) \cdot \frac{100}{in}} := \theta(r)$ $A := max(S)$ $B := min(S)$

$\theta_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\theta_{max} = 0.004 \cdot deg$

Moment; radial and tangential

$M_r(r) := M_c + LT_M(r)$

$M_t(r) := \frac{\theta(r) \cdot D \cdot \left(1 - \nu^2\right)}{r} + \nu \cdot M_r(r)$

Radial and tangential moment at center and outer radius:

$M_r(r_o) = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $M_r(a) = 0 \cdot \frac{lbf \cdot in}{in}$

$M_t(0.01 \cdot in) = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $M_t(a) = 1.974 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

Maximum radial and tangential moment (magnitude):

$Mr_{(r) \cdot \frac{100}{in}} := M_r(r)$ $Ar := max(Mr)$ $B := min(Mr)$

$Mt_{(r) \cdot \frac{100}{in}} := M_t(r)$ $At := max(Mt)$ $Bt := min(Mt)$

$Mr_{max} := (Ar > -B) \cdot Ar + (Ar \le -B) \cdot B$ $Mr_{max} = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

$Mt_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $Mt_{max} = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

Shear

$Q(r) := LT_Q(r)$

Shear at center and outer radius:

$Q(0.01 \cdot in) = -0.379 \cdot \frac{lbf}{in}$ $Q(a) = -1.935 \cdot 10^4 \cdot \frac{lbf}{in}$

Maximum shear (magnitude):

$V_{(r) \cdot \frac{100}{in}} := Q(r)$ $A := max(V)$ $B := min(V)$

$Q_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $Q_{max} = -1.935 \cdot 10^4 \cdot \frac{lbf}{in}$

Bending stresses; radial and tangential

$$\sigma_r(r) := \frac{6 \cdot M_r(r)}{t^2} \qquad \sigma_t(r) := \frac{6 \cdot M_t(r)}{t^2}$$

Radial and tangential stress at center and outer radius:

$\sigma_r(0.01 \cdot in) = 140.203 \cdot psi$ $\sigma_r(a) = 0 \cdot psi$

$\sigma_t(0.01 \cdot in) = 140.203 \cdot psi$ $\sigma_t(a) = 70.101 \cdot psi$

Maximum radial and tangential stresses:

$\sigma r_{r \cdot \frac{100}{in}} := \sigma_r(r)$ $Ar := \max(\sigma r)$ $Br := \min(\sigma r)$

$\sigma t_{r \cdot \frac{100}{in}} := \sigma_t(r)$ $At := \max(\sigma t)$ $Bt := \min(\sigma t)$

$\sigma r_{max} := (Ar > -Br) \cdot Ar + (Ar \le -Br) \cdot Br$ $\sigma r_{max} = 140.189 \cdot psi$

$\sigma t_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\sigma t_{max} = 140.196 \cdot psi$

Review the maximum values for deflection, slope, moment, stress and shear

$y_{max} = -0.024 \cdot in$ $\theta_{max} = 0.004 \cdot deg$

$Mr_{max} = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$ $Mt_{max} = 3.947 \cdot 10^6 \cdot \frac{lbf \cdot in}{in}$

$\sigma r_{max} = 140.189 \cdot psi$ $\sigma t_{max} = 140.196 \cdot psi$

$Q_{max} = -1.935 \cdot 10^4 \cdot \frac{lbf}{in}$ $f_r = 318.198\, psi$

Total deflection of plate (bending induced plus shear induced):

$y_{ro.total} := y(r_o) + y_{sro}$ $y_{ro.total} = -0.038 \cdot in$

Compare max stress to modulus of rupture (cracking stress)

$\frac{\sigma t_{max}}{f_r} = 0.441$ **< 1, OK,** grout block does NOT crack

Check shear strength

$V_c := 2\sqrt{f_c \cdot psi}$ $V_c = 84.853\,psi$

Total circumferential area $C := 2\pi \cdot a \cdot t$ $C = 9145.962\,ft^2$

Total shear load $V_u := q \cdot \pi \cdot a^2$ $V_u = 6.2 \times 10^4\,kip$

Shear resistance $V_n := V_c \cdot C$ $V_n = 1.118 \times 10^5\,kip$

$\frac{V_u}{V_n} = 0.555$ **< 1, OK,** no shear failure

It should be noted that the bearing capacity at the edge of the tank may be far exceeded, so more settlement could occur. However, this would result in greater support and lower stresses.

The remainder of this page displays the general plate functions and constants used in the equations above.

$$L_{11} \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{a}\right)^2 - 5 \cdot \left(\frac{r_o}{a}\right)^4 \ldots + -\left[4 \cdot \left(\frac{r_o}{a}\right)^2 \cdot \left[2 + \left(\frac{r_o}{a}\right)^2\right] \cdot \ln\left(\frac{a}{r_o}\right) \right] \right]$$

$$L_{17} \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{a}\right)^4\right] - \left(\frac{r_o}{a}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{a}{r_o}\right)\right] \right]$$

$$G_{11}(r) \equiv \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{r}\right)^2 - 5 \cdot \left(\frac{r_o}{r}\right)^4 \ldots + -\left[4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \left[2 + \left(\frac{r_o}{r}\right)^2\right] \cdot \ln\left(\frac{r}{r_o}\right) \right] \right] \cdot (r > r_o)$$

$$G_{14}(r) \equiv \frac{1}{16} \cdot \left[1 - \left(\frac{r_o}{r}\right)^4 - 4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \ln\left(\frac{r}{r_o}\right) \right] \cdot (r > r_o)$$

$$G_{17}(r) \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{r}\right)^4\right] - \left(\frac{r_o}{r}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{r}{r_o}\right)\right] \right] \cdot (r > r_o)$$

$$LT_y(r) \equiv \frac{-q \cdot r^4}{D} \cdot G_{11}(r) \qquad LT_M(r) \equiv -q \cdot r^2 \cdot G_{17}(r)$$

$$LT_\theta(r) \equiv \frac{-q \cdot r^3}{D} \cdot G_{14}(r) \qquad LT_Q(r) \equiv \frac{-q}{2 \cdot r} \cdot \left(r^2 - r_o^2\right) \cdot (r > r_o)$$

6.5.2 Trough Aligned With Center of Tank

In this settlement case, the tank is idealized as a simply support beam with a loading as shown in Figure 6.3. While the true surcharge load on the tank is uniform, in the idealized case, the loading varies because the width (out-of-the-page) of the tank varies. For analysis, the center of settlement is held at the center of the tank, and the width of the settled zone is varied from zero to the edge of the tank. The maximum tensile stress will occur in the center of the tank at the bottom. Shear is not considered because due to the squat nature of the grout-filled tank, most of the shear load will be transferred directly into the supports, as shown in Figure 6.4.

Given the magnitude of loads involved, it is unlikely the soil could support the grout-filled tank and the surcharge load as the settlement width approaches the tank edges. However, any soil bearing failure would provide additional support to the tank bottom, thus lowering the maximum stresses. Therefore, soil bearing failures are not considered further.

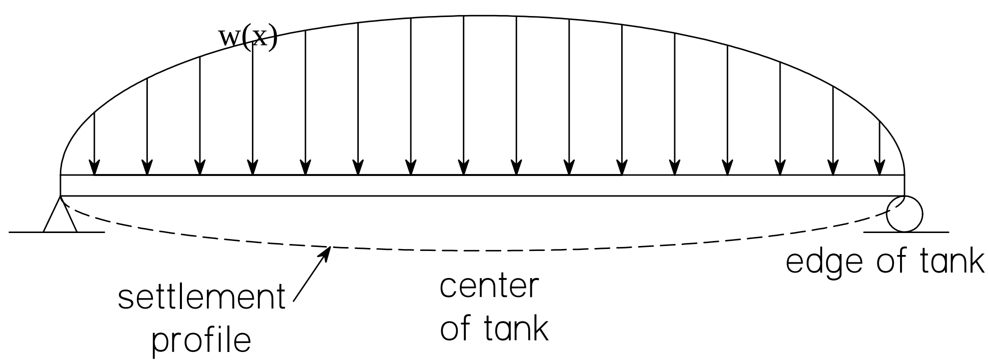


Figure 6.3 Idealized, Simply Supported Tank

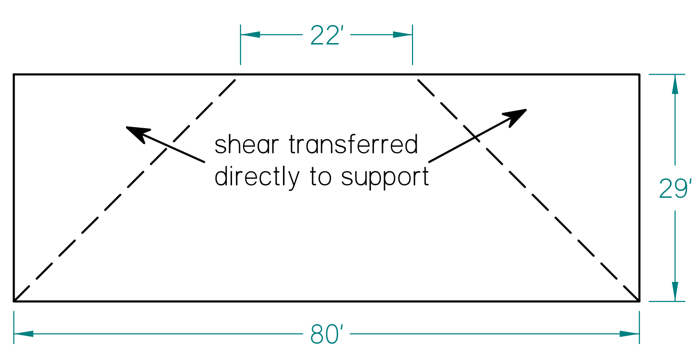


Figure 6.4 Shear Transferred Directly to Supports (Type I)

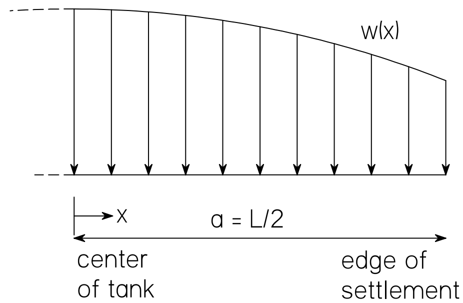
6.5.2.1 Type I Tank

Type I tank

$d \equiv 80ft$ $h \equiv 29ft$

$f_c := 1800psi$

$f_r := 7.5\sqrt{f_c \cdot psi}$ $f_r = 318.198\,psi$



surcharge: $q := 9.23ksf$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q \cdot \sqrt{d^2 - 4 \cdot x^2}$

x ranges from 0 to a (L/2)

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (simply supported span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (simply supported span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \dfrac{\int_0^x w(x) \cdot x\,dx}{\int_0^x w(x)\,dx}$

Max Moment at center w/ span (2a = L): $M(a) := V_r(a) \cdot a + \int_0^a w(x)\,dx \cdot x_{bar}(a)$ (simply supported span)

Max Moment: $M_{max} := M\left(\frac{L}{2}\right)$

Internal Moment: $M_i(x) := M_{max} - \int_0^x V_i(x)\,dx$ (simply supported span)

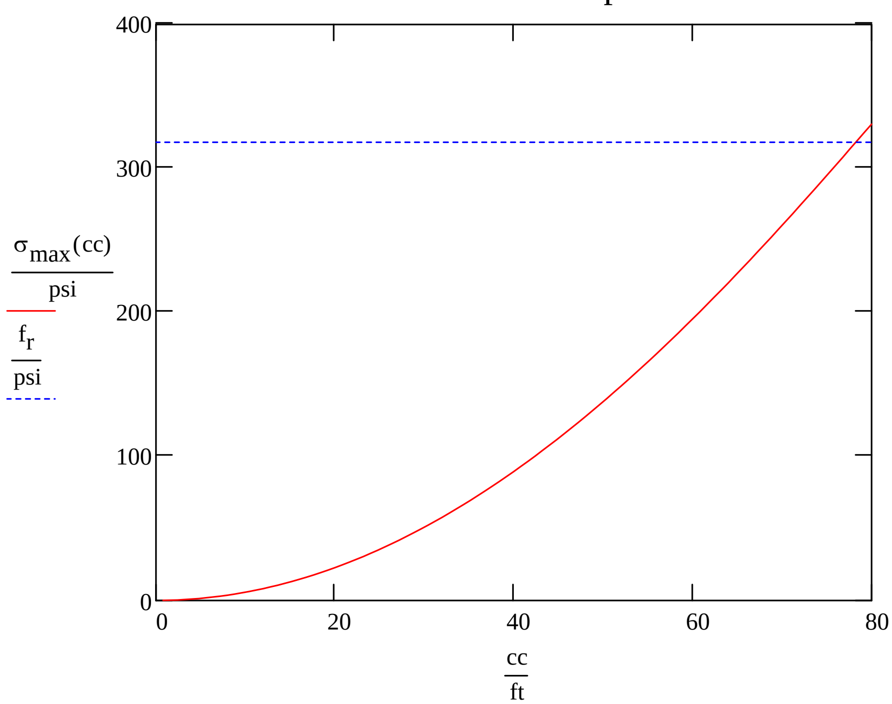
Moment of Inertia: $h = 29\,ft$ $L \equiv 50ft$

$d = 80\,ft$ $I := \dfrac{d \cdot h^3}{12}$

Max bending Stress w/ span L: $\sigma_{max}(L) := \dfrac{\left(M\left(\frac{L}{2}\right)\right)\cdot\frac{h}{2}}{I}$

$cc := 0d, 0.01d .. d$

Max Stress vs Span



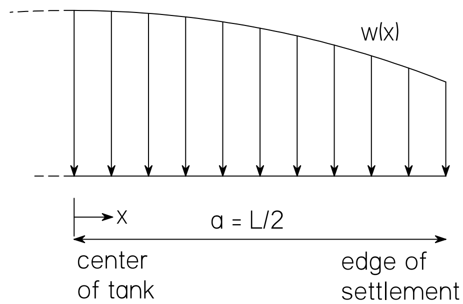
Compare max bending stress with cracking stress, f_r $\sigma_{max}(d) = 330.762\,psi$

$\dfrac{\sigma_{max}(d)}{f_r} = 1.039$ **> 1, NO GOOD!**

However, the overstress is small. Also, as seen from the above plot, the grout-filled tank must span almost the full tank diameter before the modulus of rupture is reached. At this span, the tank area in contact with the soil is very small, so the soil is likely to fail. This would result in more support, i.e. a smaller span and lower tensile stresses. As this is a idealized bounding case, the small overstress is judged acceptable and cracking will not occur.

Shear is not a concern because the section is so deep

6.5.2.2 Type III/IIIA Tank



$d \equiv 90\text{ft}$ $h \equiv 40.5\text{ft}$

$f_c := 1800\text{psi}$

$f_r := 7.5\sqrt{f_c \cdot \text{psi}}$ $f_r = 318.198\,\text{psi}$

surcharge: $q := 9.022\text{ksf}$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q\cdot\sqrt{d^2 - 4\cdot x^2}$

x ranges from 0 to a (L/2)

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (simply supported span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (simply supported span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \dfrac{\int_0^x w(x)\cdot x\,dx}{\int_0^x w(x)\,dx}$

Max Moment at center w/ span (2a = L): $M(a) := V_r(a)\cdot a + \int_0^a w(x)\,dx\cdot x_{bar}(a)$ (simply supported span)

Max Moment: $M_{max} := M\left(\frac{L}{2}\right)$

Internal Moment: $M_i(x) := M_{max} - \int_0^x V_i(x)\,dx$ (simply supported span)

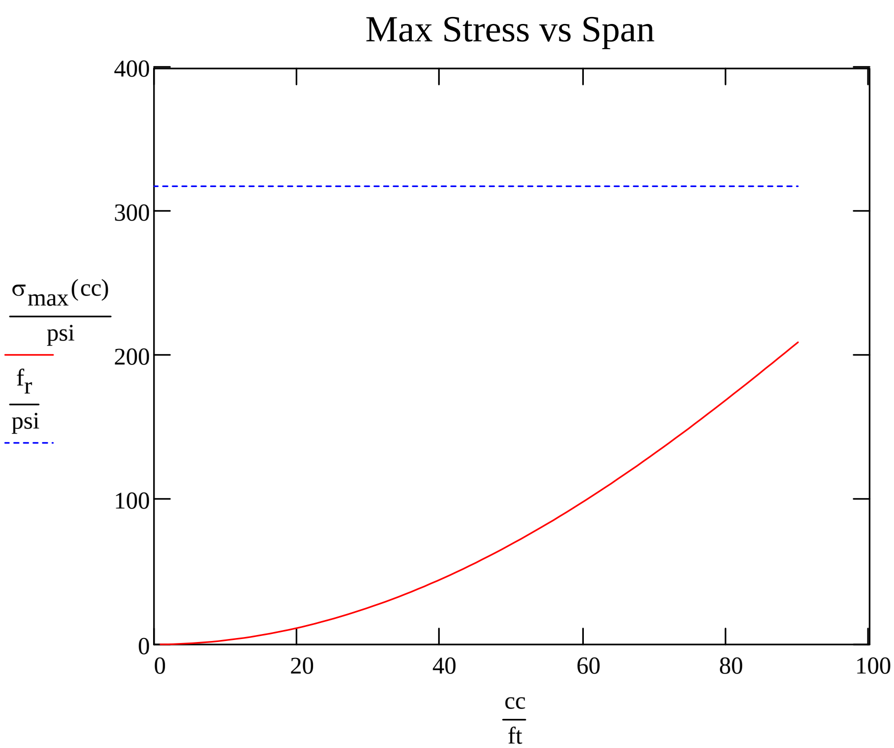
Moment of Inertia: $h = 40.5\,ft$ $L \equiv 50ft$

$d = 90\,ft$

$I := \frac{d \cdot h^3}{12}$

Max bending Stress w/ span L: $\sigma_{max}(L) := \frac{\left(M\left(\frac{L}{2}\right)\right) \cdot \frac{h}{2}}{I}$

$cc := 0d, 0.01d .. d$



Compare max bending stress with cracking stress, f_r

$\frac{\sigma_{max}(d)}{f_r} = 0.659$ **< 1, OK**: Cracking does NOT occur

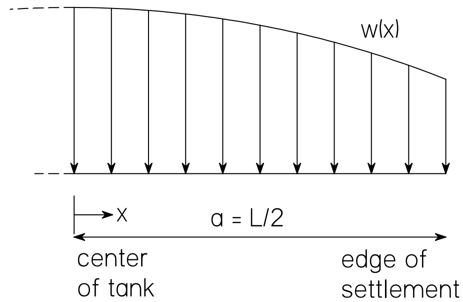
Shear is not a concern because the section is so deep

6.5.2.3 Type IV Tank

$d \equiv 85ft \qquad h \equiv 34.25ft$

$f_c := 1800psi$

$f_r := 7.5\sqrt{f_c \cdot psi} \qquad f_r = 318.198\,psi$



surcharge: $q := 10.93ksf$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q\cdot\sqrt{d^2 - 4\cdot x^2}$

x ranges from 0 to a (L/2)

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (simply supported span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (simply supported span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \dfrac{\int_0^x w(x)\cdot x\,dx}{\int_0^x w(x)\,dx}$

Max Moment at center w/ span (2a = L): $M(a) := V_r(a)\cdot a + \int_0^a w(x)\,dx\cdot x_{bar}(a)$ (simply supported span)

Max Moment: $M_{max} := M\left(\dfrac{L}{2}\right)$

Internal Moment: $M_i(x) := M_{max} - \int_0^x V_i(x)\,dx$ (simply supported span)

Moment of Inertia: $h = 34.25\,ft$ $L \equiv 50ft$

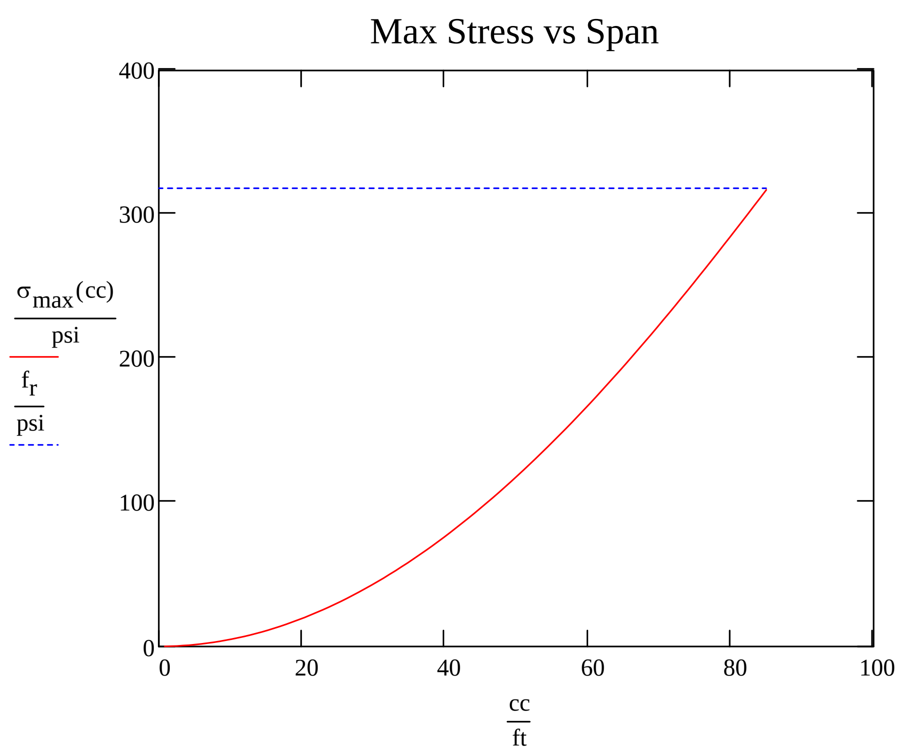
$d = 85\,ft$

$$I := \frac{d \cdot h^3}{12}$$

Max bending Stress w/ span L:

$$\sigma_{max}(L) := \frac{\left(M\left(\frac{L}{2}\right)\right) \cdot \frac{h}{2}}{I}$$

$$cc := 0d, 0.01d .. d$$



Compare max bending stress with cracking stress, f_r

$$\frac{\sigma_{max}(d)}{f_r} = 0.996$$

< 1, OK: Cracking does NOT occur

Shear is not a concern because the section is so deep

6.5.3 Trough Aligned With Edge of Tank

In this settlement case, the tank is idealized as a cantilevered beam as shown in Figure 6.5. As with the previous settlement case, while the true surcharge load on the tank is uniform, in the idealized case, the loading varies because the width (out-of-the-page) of the tank varies. For analysis, the center of settlement is held at the edge of the tank, and the width of the settled zone is varied from zero to the diameter of the tank. The maximum tensile stress will occur at the edge of the settlement zone at the top. While the analysis considers the settlement zone to increase such that the entire tank is cantilevered, in reality by the time the settlement zone reaches the center of the tank, the rigid body rotation will occur (tank will fall into settlement zone), providing more support. Cracking will not occur as long as the maximum tensile stress when the edge of the settlement zone is at the center of the tank is less than the modulus of rupture. As with the previous settlement case, shear is not considered because due to the squat nature of the grout-filled tank, most of the shear load will be transferred directly into the supports.

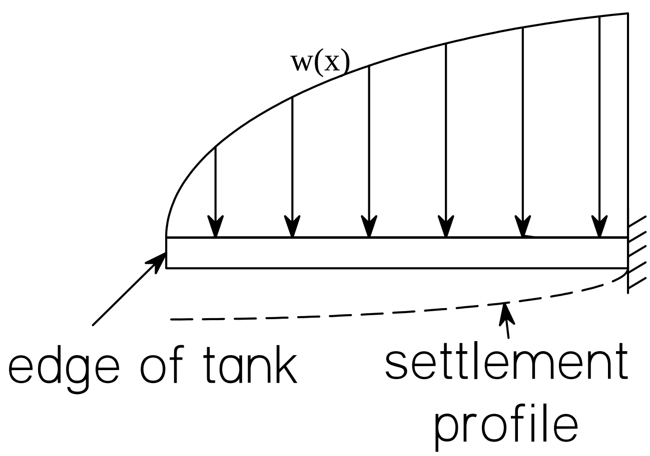


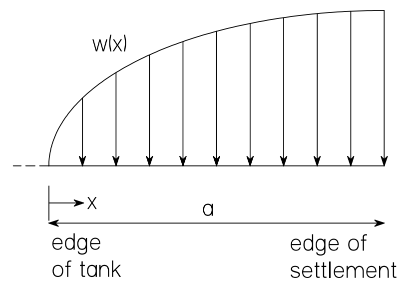
Figure 6.5 Idealized, Cantilevered Tank

6.5.3.1 Type I Tank

$d \equiv 80ft \qquad h \equiv 29ft$

$f_c := 1800psi$

$f_r := 7.5\sqrt{f_c \cdot psi} \qquad f_r = 318.198\,psi$



surcharge: $q := 9.23ksf$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q \cdot \sqrt{d^2 - 4 \cdot \left(\frac{d}{2} - x\right)^2}$

x ranges from 0 to a

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (cantilevered span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (cantilevered span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \dfrac{\int_0^x w(x) \cdot x\,dx}{\int_0^x w(x)\,dx}$

Max moment (cantilever support) $M_{max}(a) := -\left(\int_0^a w(x)\,dx\right) \cdot \left(a - x_{bar}(a)\right)$ (cantilevered span)

$M_{max}(L) = 3.938 \times 10^5\,kip \cdot ft$

Internal moment $M_i(x) := -\left(\int_0^x w(x)\,dx\right) \cdot \left(x - x_{bar}(x)\right)$ (cantilevered span)

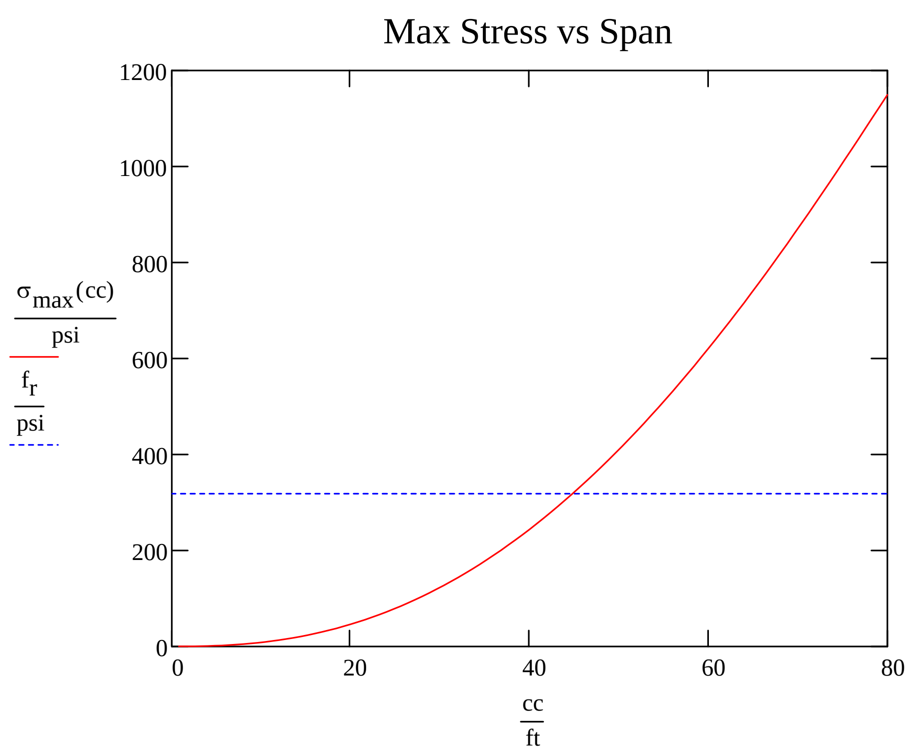
Moment of Inertia: $h = 29\,ft$ $L \equiv 40ft$

$d = 80\,ft$

$I := \frac{d \cdot h^3}{12}$

Max bending Stress w/ span L: $\sigma_{max}(L) := \frac{\left(M_{max}(L)\right) \cdot \frac{h}{2}}{I}$

$cc := 0d, 0.01d .. d$



Compare max bending stress at d/2 with cracking stress, f_r

$\frac{\sigma_{max}\left(\frac{d}{2}\right)}{f_r} = 0.766$ **< 1, OK**: Cracking does NOT occur

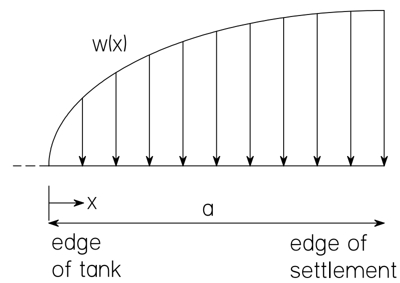
Shear is not a concern because the section is so deep

6.5.3.2 Type III/IIIA Tank

$d \equiv 90ft \quad h \equiv 40.5ft$

$f_c := 1800psi$

$f_r := 7.5\sqrt{f_c \cdot psi} \quad f_r = 318.198\,psi$



surcharge: $q := 9.022ksf$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q \cdot \sqrt{d^2 - 4 \cdot \left(\frac{d}{2} - x\right)^2}$

x ranges from 0 to a

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (cantilevered span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (cantilevered span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \dfrac{\int_0^x w(x) \cdot x\,dx}{\int_0^x w(x)\,dx}$

Max moment (cantilever support) $M_{max}(a) := -\left(\int_0^a w(x)\,dx\right) \cdot \left(a - x_{bar}(a)\right)$ (cantilevered span)

$M_{max}(L) = 4.147 \times 10^5 kip \cdot ft$

Internal moment $M_i(x) := -\left(\int_0^x w(x)\,dx\right) \cdot \left(x - x_{bar}(x)\right)$ (cantilevered span)

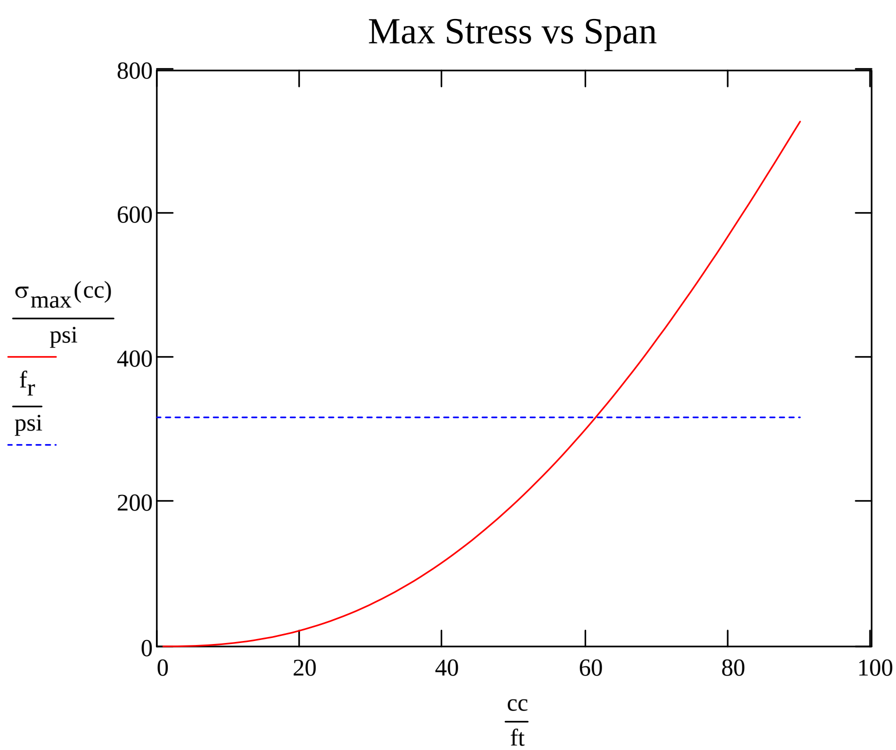
Moment of Inertia: $h = 40.5\,ft$ $L \equiv 40ft$

$d = 90\,ft$

$I := \frac{d \cdot h^3}{12}$

Max bending Stress w/ span L: $\sigma_{max}(L) := \frac{\left(M_{max}(L)\right) \cdot \frac{h}{2}}{I}$

$cc := 0d, 0.01d .. d$



Compare max bending stress at d/2 with cracking stress, f_r

$\frac{\sigma_{max}\left(\frac{d}{2}\right)}{f_r} = 0.486$ **< 1, OK**: Cracking does NOT occur

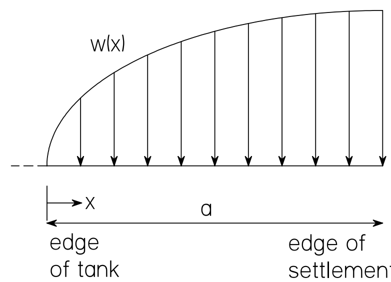
Shear is not a concern because the section is so deep

6.5.3.3 Type IV Tank

$d \equiv 85ft$ $h \equiv 34.25ft$ (do not consider dome)

$f_c := 1800psi$

$f_r := 7.5\sqrt{f_c \cdot psi}$ $f_r = 318.198\,psi$



surcharge: $q := 10.93ksf$

L is total span, can range from 0 to d

Distributed Load: $w(x) := -q \cdot \sqrt{d^2 - 4 \cdot \left(\frac{d}{2} - x\right)^2}$

x ranges from 0 to a

Shear Reaction w/ span (2a = L): $V_r(a) := -\int_0^a w(x)\,dx$ (cantilevered span)

Internal shear force $V_i(x) := -\int_0^x w(x)\,dx$ (cantilevered span)

centroid of loaded area (from 0 to x): $x_{bar}(x) := \frac{\int_0^x w(x) \cdot x\,dx}{\int_0^x w(x)\,dx}$

Max moment (cantilever support) $M_{max}(a) := -\left(\int_0^a w(x)\,dx\right) \cdot \left(a - x_{bar}(a)\right)$ (cantilevered span)

$M_{max}(L) = 4.847 \times 10^5 kip \cdot ft$

Internal moment $M_i(x) := -\left(\int_0^x w(x)\,dx\right) \cdot \left(x - x_{bar}(x)\right)$ (cantilevered span)

Moment of Inertia: $h = 34.25\,ft$ $L \equiv 40ft$

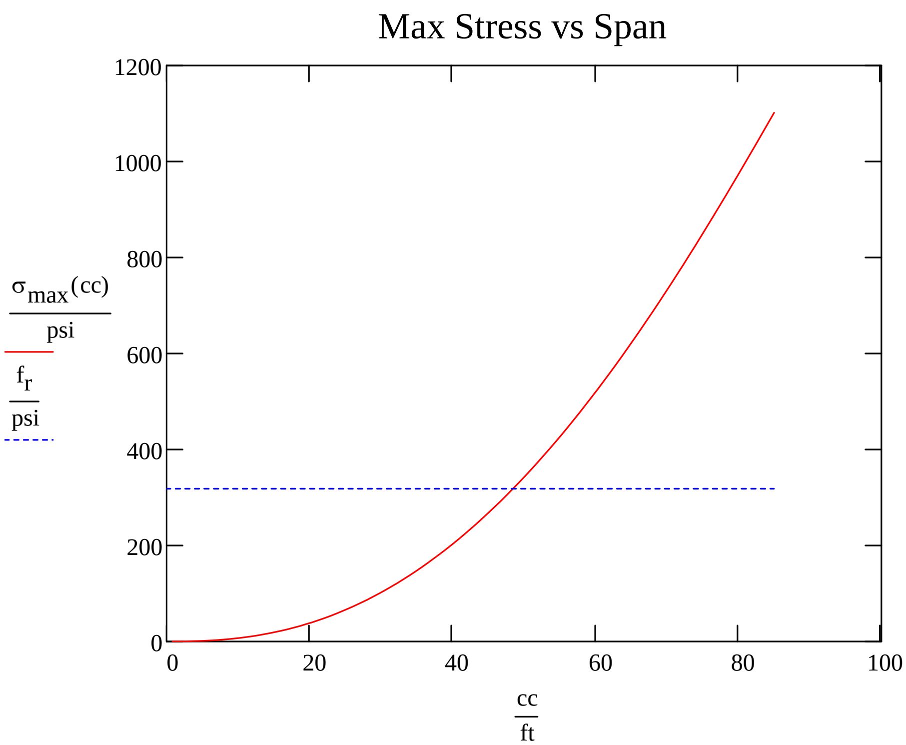
$d =$ [redacted] ft

$$I := \frac{d \cdot h^3}{12}$$

Max bending Stress w/ span L:

$$\sigma_{max}(L) := \frac{(M_{max}(L)) \cdot \frac{h}{2}}{I}$$

$$cc := 0d, 0.01d .. d$$



Compare max bending stress with cracking stress, f_r

$$\frac{\sigma_{max}\left(\frac{d}{2}\right)}{f_r} = 0.735$$

< 1, OK: Cracking does NOT occur

Shear is not a concern because the section is so deep

6.6 ANSYS Modeling

Settlement under a grout-filled tank is also examined using 3-D finite element models in ANSYS. This approach provides a more realistic model of the support conditions by using elements at the base of the structure that can be described as compression-only soil springs. This allows the ability to model the soil losing contact with the tank bottom and loads redistributing accordingly. The thickness of the tank is taken into account more appropriately than with the assumption of plate theory.

6.6.1 Settlement Characteristics

In this approach, the settlement extent and depth will affect the results. Four cases are considered as described below:

1) Settlement extent of 215-ft, depth of 2.8 inches. This is the settlement profile recommended for use in the design of WSB.

2) Settlement extent of 70-ft, depth of 2.8 inches. This case adjusts the settlement extent so almost the entire settlement occurs below the grout-filled tank. By holding the depth constant, the slope of the settled zone is higher. This may increase the likelihood of the soil losing contact with the tank bottom.

3) Settlement extent of 215-ft, depth of 12 inches. This case increases the settlement depth. The WSB settlement profile was recommended for PC-3 design. While soft zone settlement is mostly independent of seismic event size, this case increases the settlement depth to account for the possibility of larger settlements that could occur with a seismic event large than PC-3.

4) Settlement extent of 70-ft, depth of 12 inches. As with case 2, this case may increase the likelihood of the soil losing contact with the tank bottom.

All four cases are applied at the three locations used previously, for a total of twelve settlement profiles.

6.6.2 Type I Tank Model

A Type I tank is modeled in ANSYS as shown in Figure 6.6. The diameter is 80-ft and the height is 29-ft. The grout-filled tank is modeled as a monolith of grout, both reinforcing steel and the steel tank are neglected to bound the problem. The grout-filled tank is modeled using ANSYS SOLID185 elements. These are 3-D eight-node structural solid elements with three translational degrees of freedom at each node. SOLID185 element inputs are summarized in Table 6.4. The ANSYS COMBIN37 elements are used at the base of the model to represent soil spring elements, which transfer loads to the structure resulting from the differential support displacement conditions. These control elements were selected for this application due to adequately model the potential for complete loss of contact between the soil and the structure due to settlement. This element has the ability to turn off and on depending upon the input conditions. COMBIN37 element inputs are summarized in Table 6.5 and Figure 6.7.

The element edge length was set equal to 3-ft when the grout-filled tank was meshed. As seen in Figure 6.6, this produces regular elements near the outer edges of the tank. Elements become more irregular towards the center of the tank but still have edge lengths approximately equal to 3-ft. This is used to assign a constant soil spring stiffness to all soil spring elements. As the element areas vary slightly, the tributary area and thus the soil spring stiffness should vary as well. However, for the present case, since the element sizes are similar, it is judged that using a constant soil spring stiffness is a reasonable.

The soil spring stiffness is calculated from the modulus of subgrade reaction, k_s. Values suggested by Terzaghi can be found in Table 1 of K-ESR-G-00011, Rev 0 [6]. For several H-Area HLW tanks, values of k_s were back-calculated based on measured settlements. The values were on the order of 32-37 kcf [6]. However, for the grout-filled tanks, a subgrade modulus of 300 tcf is adopted to bound the analysis. From Terzaghi's table, this is appropriate for dense sand, submerged or not. A large value of value of k_s indicates a stiffer support and results in a greater chance the soil will lose contact with the tank base due to settlement; which will result in larger stresses in the tank. For this same reason, values of k_s are not adjusted for foundation size. Adjustments for foundation size result in lower value of stiffness. Given the area of each element face is 3' x 3' = 9-ft^2, a soil spring stiffness of 5400 kip/ft (600 kcf x 9-ft^2) is assigned to interior springs at support nodes. Exterior springs at support nodes have half the tributary area, so are assigned a stiffness of 2700 kip/ft.

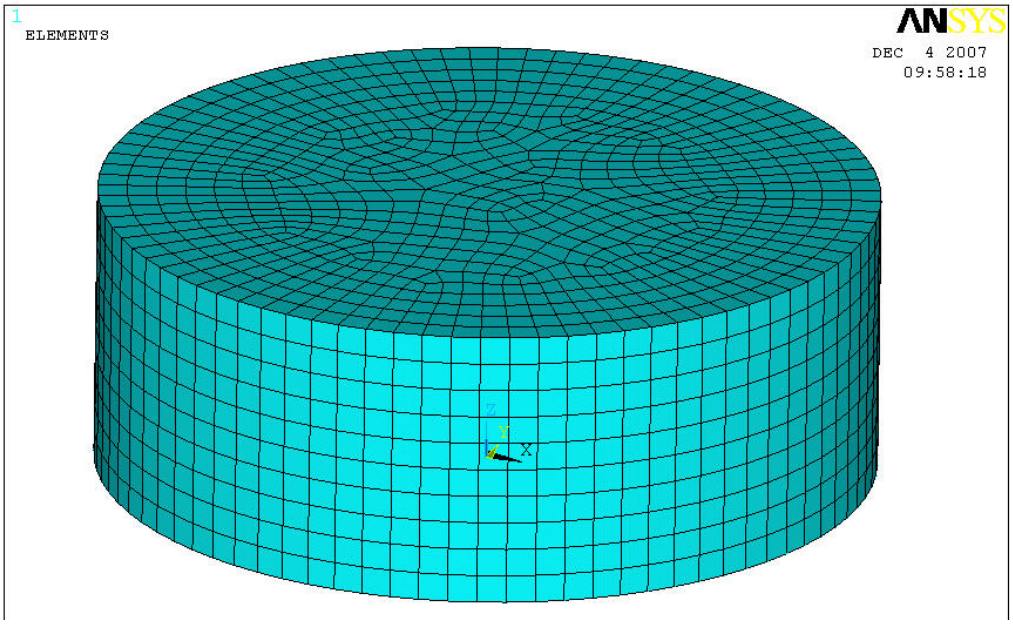


Figure 6.6 Type I Tank Model

Table 6.4 SOLID185 Element Inputs

Input	Item	Parameter	Value	Value Description
Element Options Element Type = 1	Keyopt(2)	Element Technology	3	Enhanced Strain Formulation
	Keyopt(3)	Layer Construction	1	Structural Solid
MP	EX	Young's Modulus	348200 kip/ft^2	--
	PRXY	Poisson's Ratio	0.2	--

Note: Default values are used for options not shown

Table 6.5 COMBIN37 Element Inputs

Input	Item	Parameter	Value	Value Description
Element Options Element Type = 3	Keyopt(1)	Control basis	0	UK-UL
	Keyopt(2)	DOF for control nodes	3	Displacement along Z-axis
	Keyopt(3)	DOF for active nodes	3	UZ displacement
	Keyopt(4)	ON-OFF range behavior	0	Overlapping
	Keyopt(5)	ON-OFF position behavior	0	OFF-EITHER-ON
	Keyopt(6)	Real Constant for RVMOD	0	Use STIF
	Keyopt(9)	Nonlinear behavior	0	Use RVMOD
Real Constants (R) Set 20 – Interior Elements Set 21 Exterior Elements	STIF	Element Spring Rate (kip/ft)	Set 20: 5400 Set 21: 2700	Soil Stiffness
	CPAR-ONVAL	Control Variable – see Figure 6.7	0	Displacement for element to turn on
	CPAR-OFFVAL	Control Variable– see Figure 6.7	-0.001	Displacement for element to turn off
	START	Control Variable– see Figure 6.7	1	Initial Status = ON

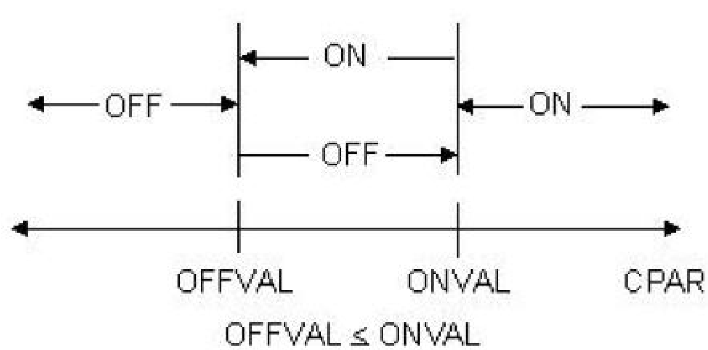


Figure 6.7 COMBIN37 Control Element Behavior

6.6.3 Loading and Boundary Conditions

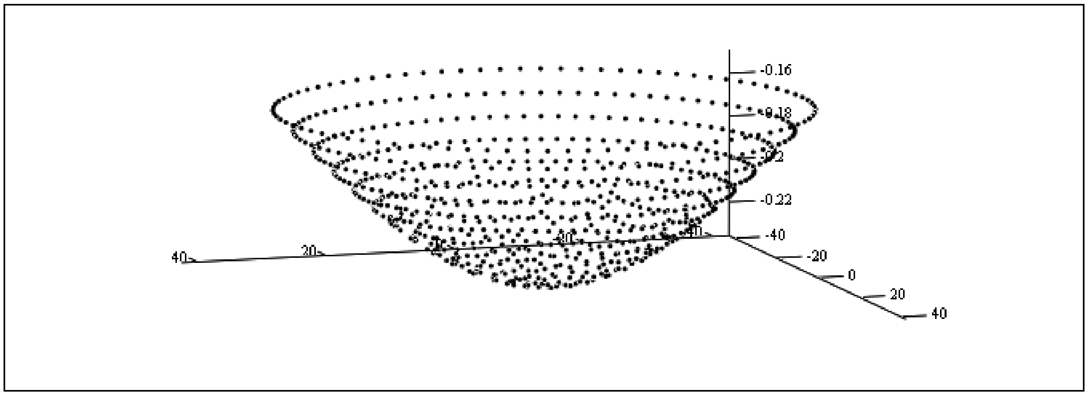
Vertical surcharge loads and self-weight are applied to the grout-filled tank model. To bound the problem, horizontal earth pressures are neglected. Otherwise they will induce additional compression stresses in the tank. A vertical surcharge load of 5.46 ksf is applied to the top area of the model. This is the load due to a minimum soil cap thickness of 45.5 ft (45.5-ft x 0.12 kcf = 5.46). Self weight loads are applied to each element. From Figure 6.6 Type I Tank Model, it can be seen that there are ten layers of elements in the model. Therefore, given a grout unit weight of 0.13 kcf, the self weight pressure applied to each element is 0.377 ksf (0.13 kcf x 29-ft/10).

Since the only DOF active in the soils springs is UZ, the tank itself must be restrained laterally. Nodes 1421 and 1463 are restrained in the UX direction and nodes 1442 and 1484 are restrained in the UY direction. To establish equilibrium, the UZ value of all support nodes are set equal to zero and a solution is found. This step establishes an initial compression in the soil springs to model the state immediately after closure. The model and results from this step are saved as *T1-Start.db*.

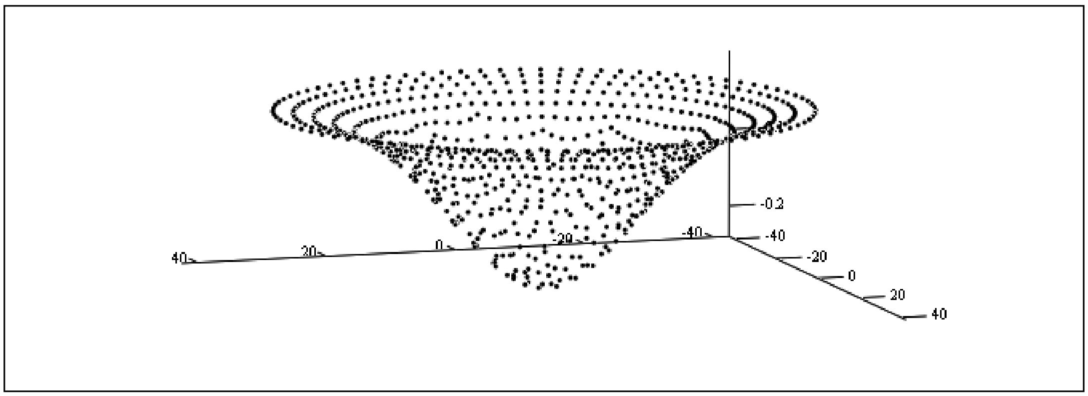
Soft zone settlement is applied as a change in boundary condition, i.e. a change in the UZ value of the support nodes. A settlement profile is generated for each of the twelve settlement cases described previously. Based on the location of each support node, the settlement at each node is calculated. Figure 6.8 shows the settlement profiles for a depth of 2.8 inches. The profiles for a settlement depth of 12 inches will look the same, only the vertical scale will change.

Calculation Continuation Sheet

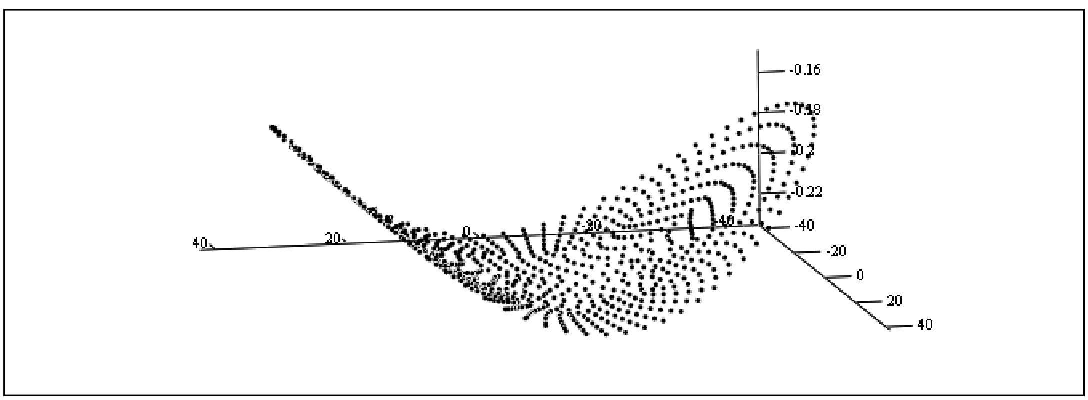
Calculation No.	Sheet No.	Rev.
T-CLC-F-00421	57	0



a) Circular Settlement, Extent = 215 ft



b) Circular Settlement, Extent = 70 ft

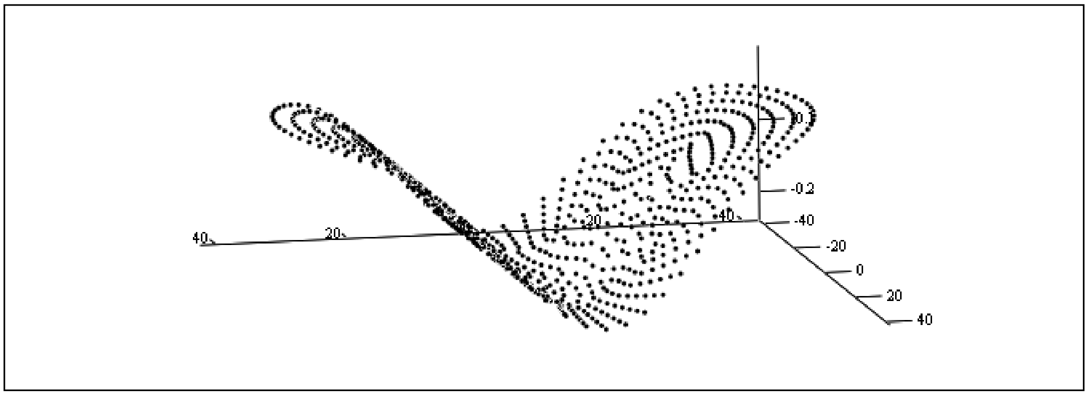


c) Center trough settlement, Extent = 215 ft

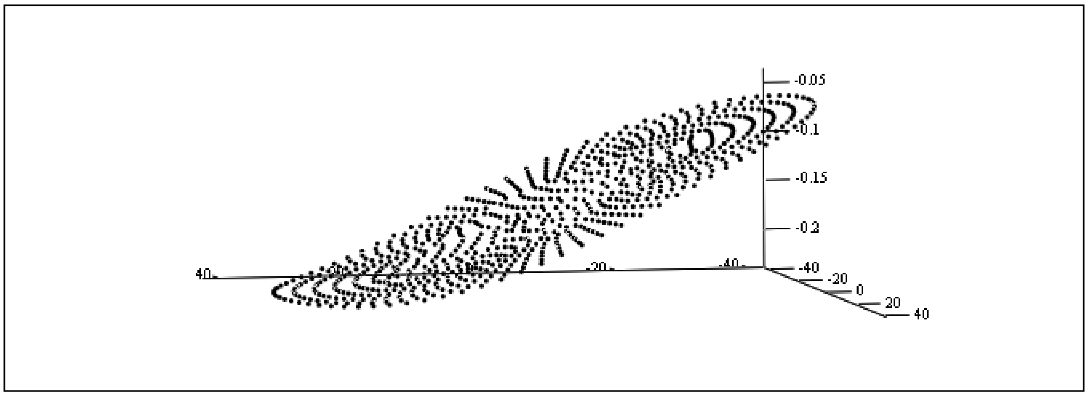
Figure 6.8 (cont.) Settlement Profiles (δ = 2.8 inches)

Calculation Continuation Sheet

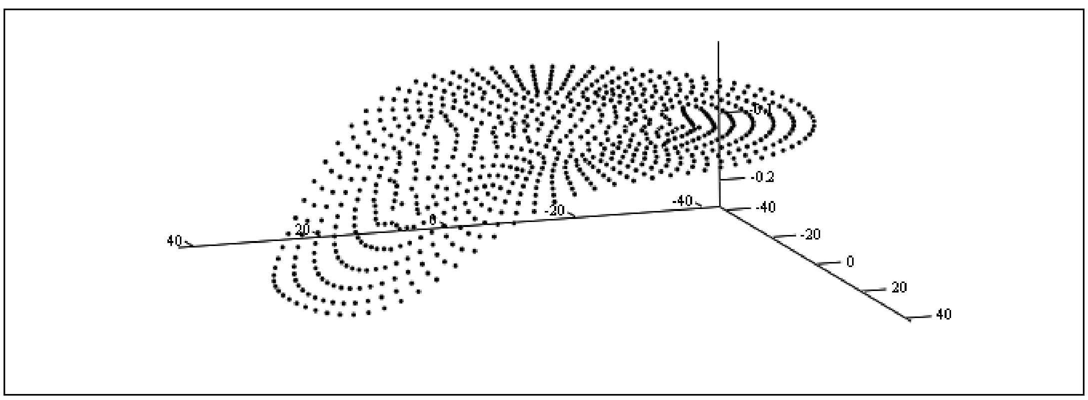
Calculation No.	Sheet No.	Rev.
T-CLC-F-00421	58	0



d) Center trough settlement, Extent = 70 ft



e) Edge trough settlement, Extent = 215 ft



f) Edge trough settlement, Extent = 70 ft

Figure 6.8 (cont.) Settlement Profiles (δ = 2.8 inches)

6.6.4 ANSYS Solutions and Results

The ANSYS solutions for each settlement case are found by restarting ANSYS from the equilibrium conditions (*T1-Start.db*) and applying the settlement profile. A non-linear static analysis is performed to find the solution.

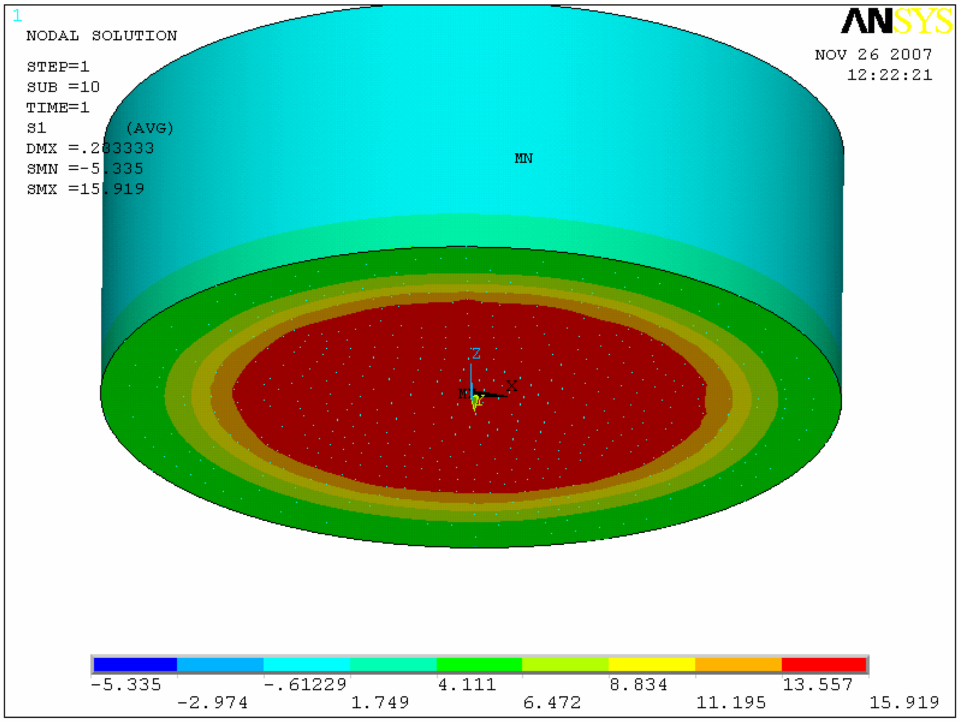
From the results, the first principle stresses are found for each settlement case. The maximum principle stress is equal to the maximum tensile stress in the grout-filled tank. Figures 6.9 thru 6.14 graphically show the principle stresses for each case. Note the stresses are in units of ksf. The maximum principle stress values can be compared to the modulus of rupture, $f_r = 7.5\sqrt{1800} = 318.2\,psi = 45.82\,ksf$. The conversion factor is 6.944 psi to 1 ksf.

In Figures 6.9 and 6.10, the stress distribution on the bottom of the tank is circular as would be expected for a circular depression. The maximum stress increases as the settlement magnitude increases. The maximum stress for the circular depression case is 21.32 ksf (148 psi). This compares favorably with the 146 psi calculated using Roark's in Section 6.5.1.1. Figures 6.11 and 6.12 are for the trough settlement case. As expected, the bottom of the tank is in compression at two locations (corresponding to the edges of the settlement) and in tension elsewhere. Again, the maximum stress increases as the settlement magnitude increases. The maximum stress for the trough centered with the tank is 47.5 ksf (330 psi). Again, this compares very favorably to the maximum stress of 330 psi calculated using beam theory in Section 6.5.2.1.

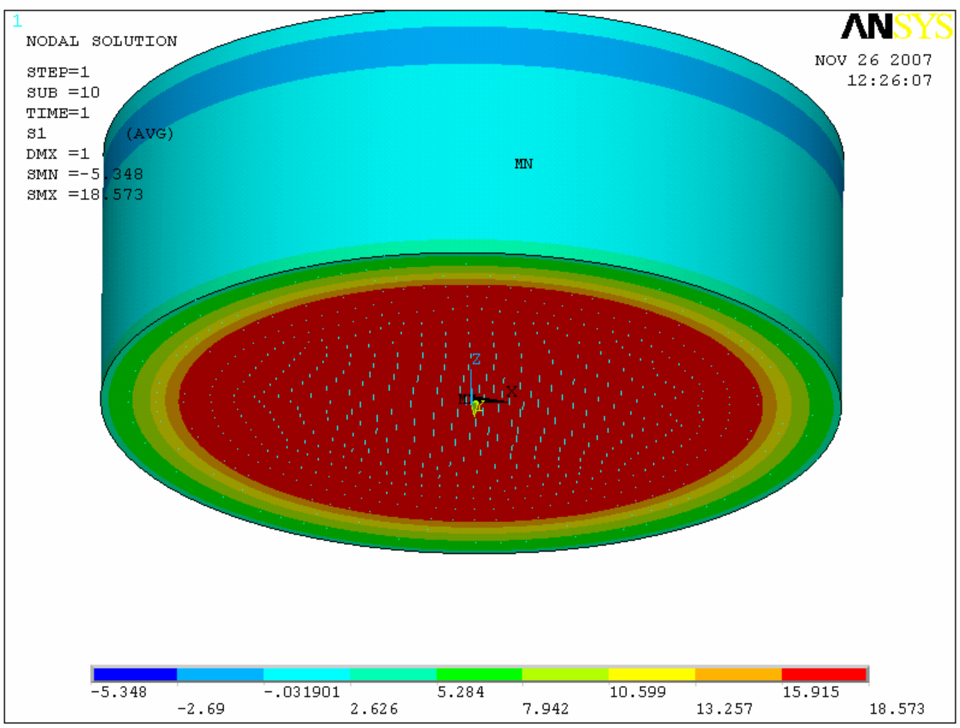
Figure 6.13 shows the stress distribution at the top of the tank due to a trough settlement centered at the tank edge. As expected, the tank behaves similar to a cantilevered beam and the maximum stress occurs at the tank center. Figure 6.14 is also for a trough settlement centered at the tank edge, but the maximum stresses occur at the bottom of the tank. This can be explained by comparing the two settlement profiles shown in Figure 6.8e and f. The results in Figure 6.14 are for Figure 6.8f, which is more like a true cantilever case. But the results shown in Figure 6.15 are from the settlement in Figure 6.8e. Due to the large width of the settlement, the tank rotates so it is not truly cantilevered, but supported at both ends. Therefore the maximum stress now occurs on the bottom of the tank. Note the stress concentrations at the edge of the tank in Figure 6.14a result from the lateral restraints on the model. For the case shown in Figure 6.14b, the outer ring of elements has been removed so the stress distribution shown is not biased due to the artificial stress caused by the constraints. The maximum stress for the trough centered on the tank edge is 30.44 ksf (211 psi). This also compares favorably with the maximum stress of 244 psi calculated using cantilevered beam theory.

It is observed that the maximum principle stresses are less than 45.82 ksf for all cases except one. For the case of a trough aligned with the center of the tank, extent 215-ft and depth of 12 inches, the maximum stress is 47.5 ksf, a D/C of 1.04. Figure 6.15 shows the principle stresses through a cross-section of the first layer of elements near the maximum stress location (Elements 3001, 3011, 3021, 3031, 3831, 3841, 3851, and 3861). It can be seen that the maximum stress occurs primarily on the surface and extends into the grout-filled tank only several inches, based on the 2.9-ft element height. Since the base slab of the concrete vault is 2’-6” thick, only the concrete vault would experience the high tensile stresses. Given the bounding assumptions made for this analysis and considering the extreme settlement in this case (12 inches) it is judged the Type I tank will not crack due to settlement.

For the hand calcs, the highest tensile stresses were observed in the Type I tanks. Given that the maximum tensile stresses as calculated using ANSYS are the same or lower than the stresses calculated using hand solutions, it is reasonable to assume the same would occur if ANSYS models of Type III/IIIA and IV tanks were analyzed. Since the Type I tanks do not crack under either analysis, it is judged the type III/IIIA and IV tanks will not crack as well, and no further ANSYS modeling is needed.

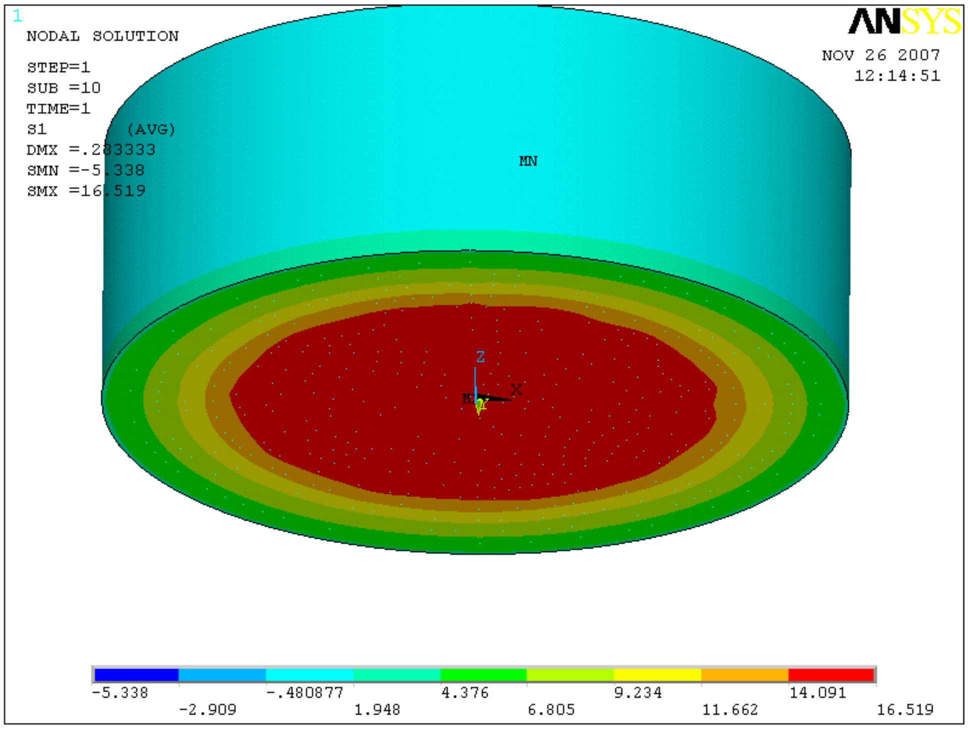


a) δ = 2.8 inches

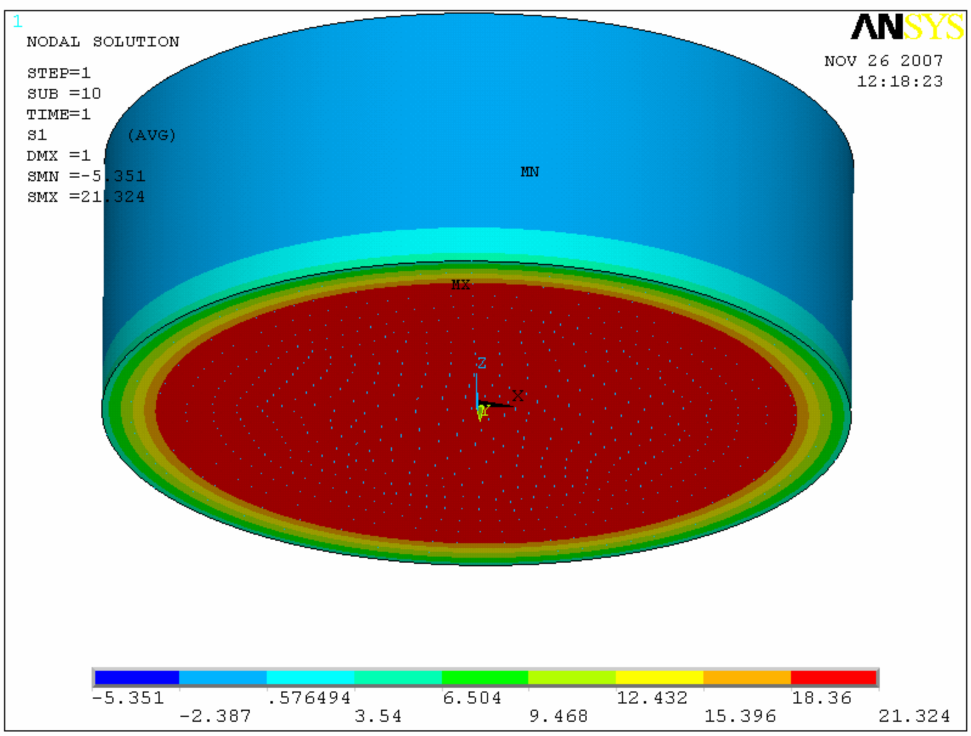


b) δ = 12 inches

Figure 6.9 Maximum Principle Stresses – Circular Settlement, Extent = 70-ft

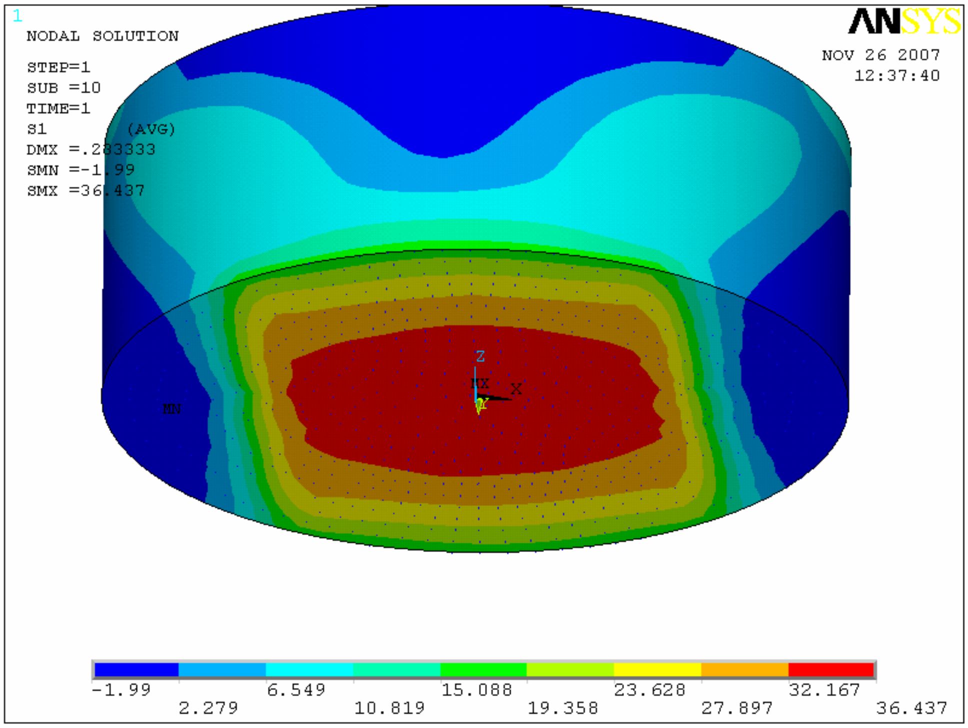


a) δ = 2.8 inches

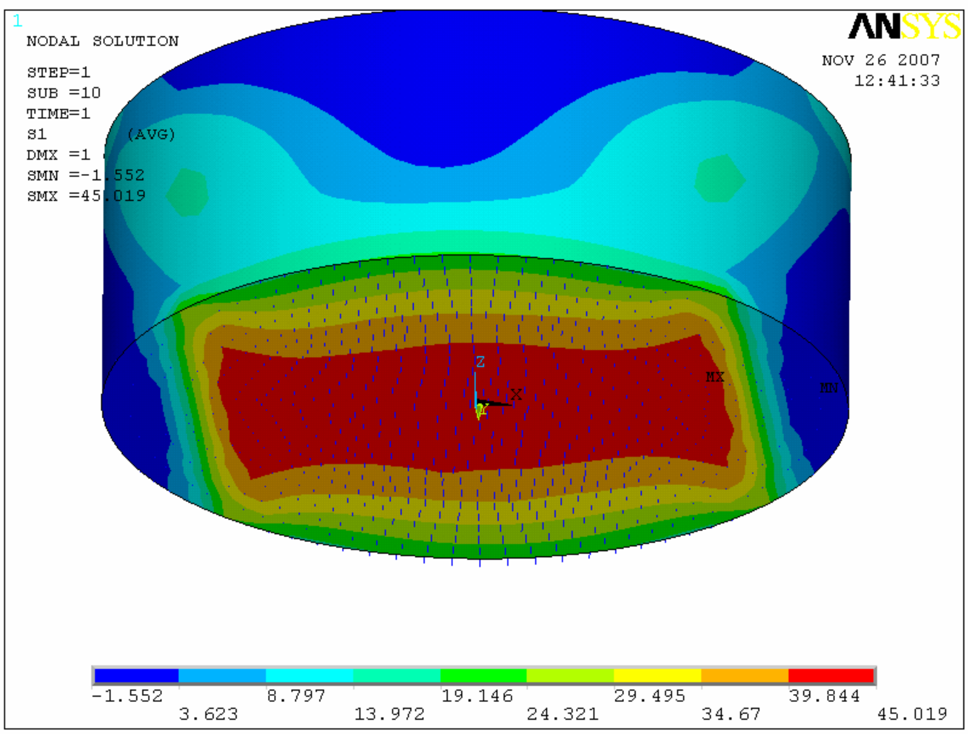


b) δ = 12 inches

Figure 6.10 Maximum Principle Stresses – Circular Settlement, Extent = 215-ft

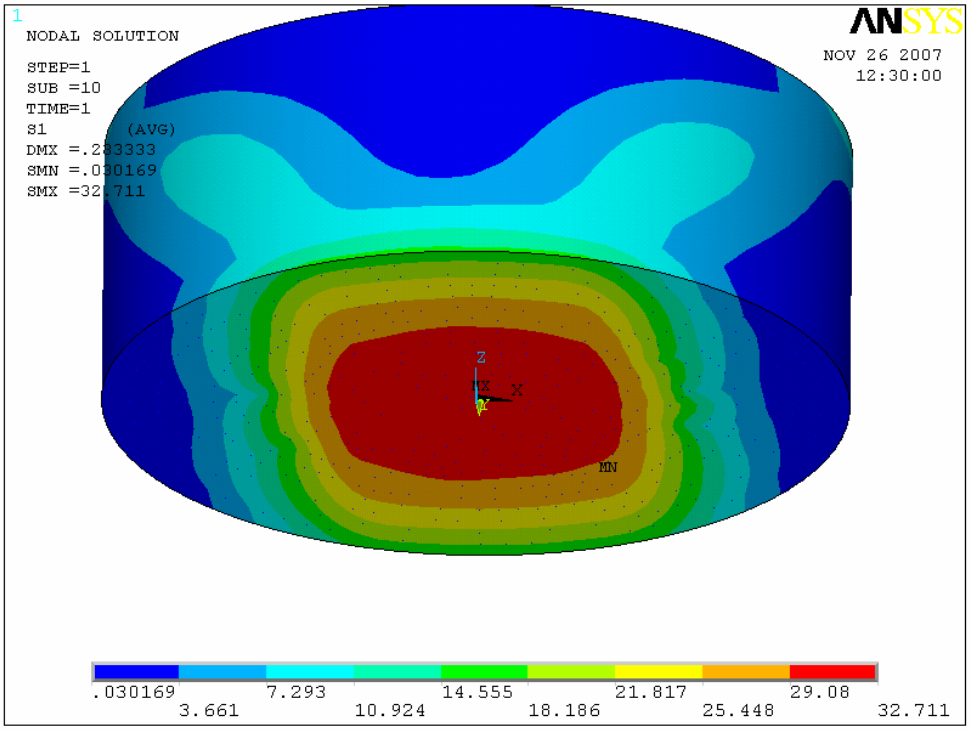


a) δ = 2.8 inches

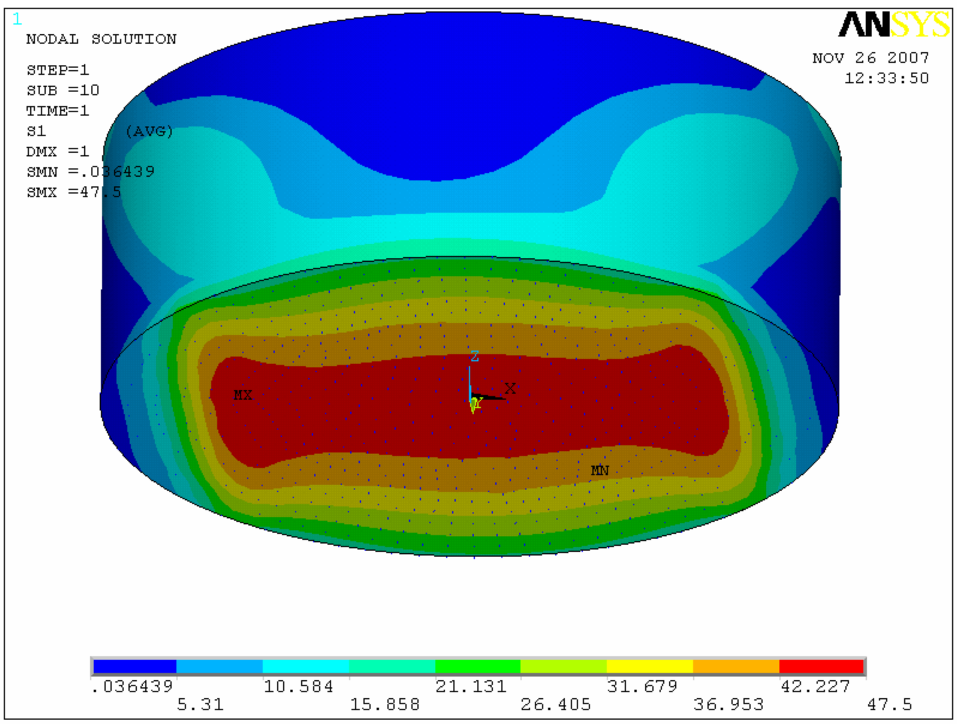


b) δ = 12 inches

Figure 6.11 Maximum Principle Stresses – Trough Settlement at Center, Extent = 70-ft



a) δ = 2.8 inches

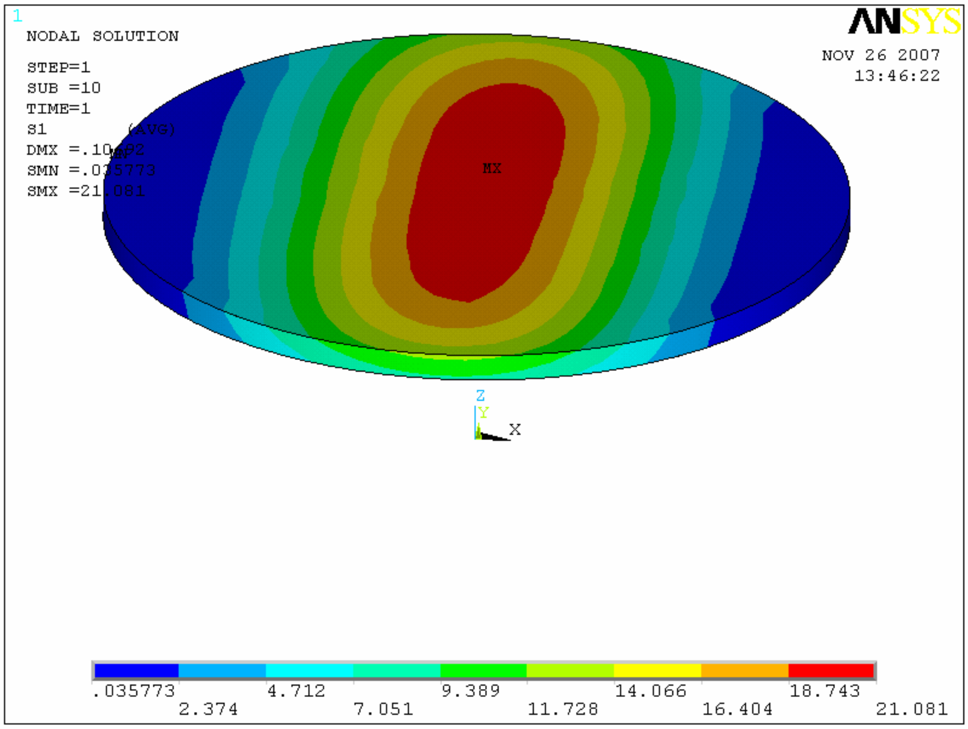


b) δ = 12 inches

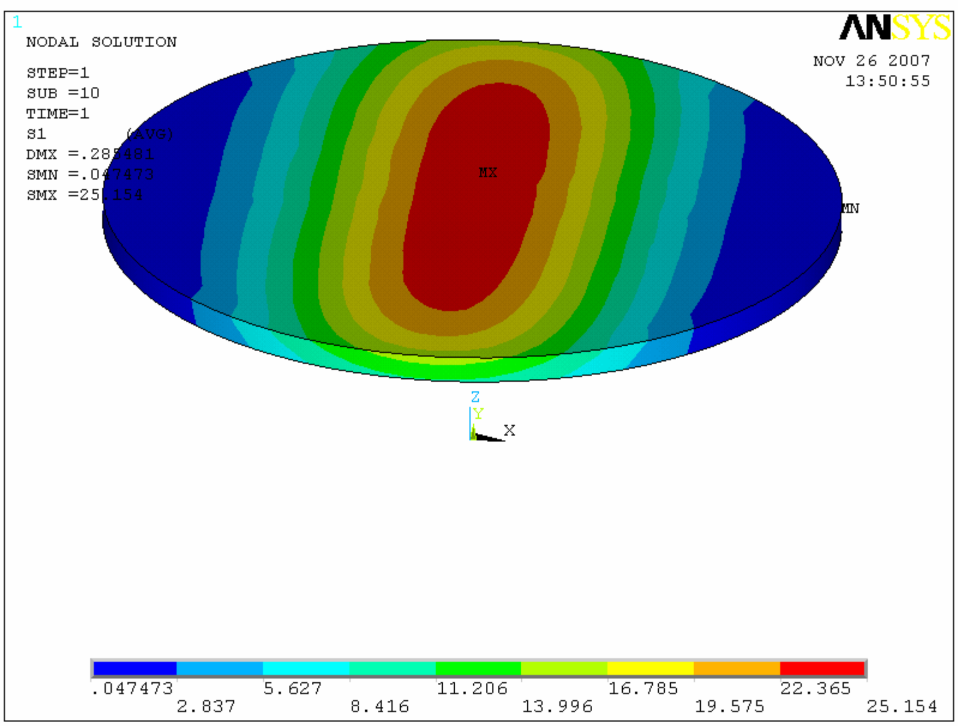
Figure 6.12 Maximum Principle Stresses – Trough Settlement at Center, Extent = 215-ft

Calculation Continuation Sheet

Calculation No.	Sheet No.	Rev.
T-CLC-F-00421	65	0

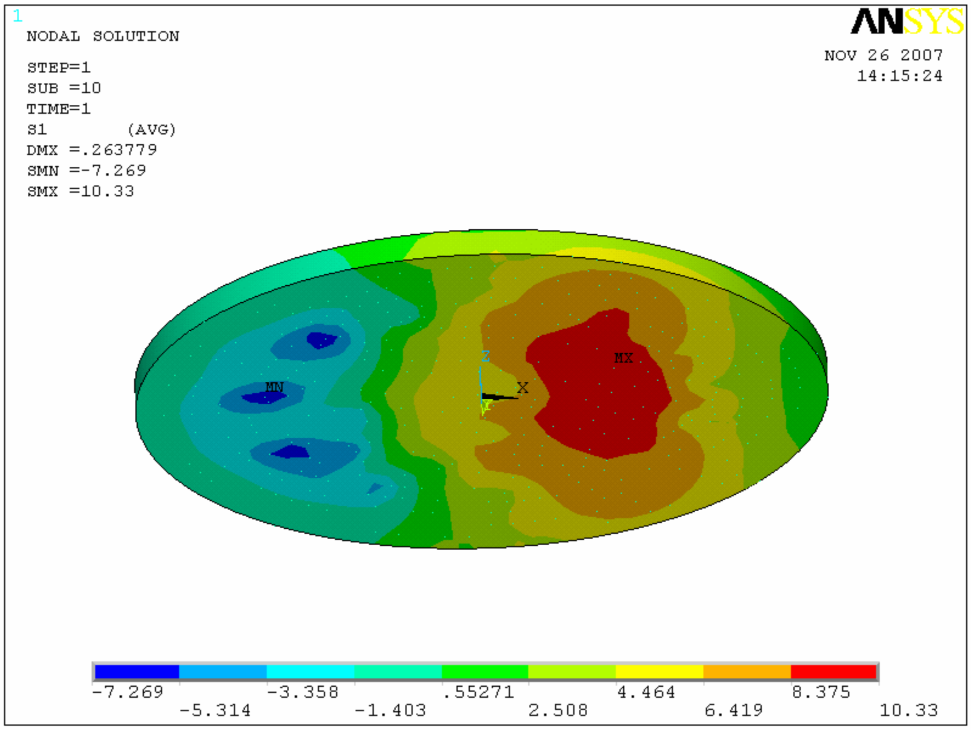


a) δ = 2.8 inches

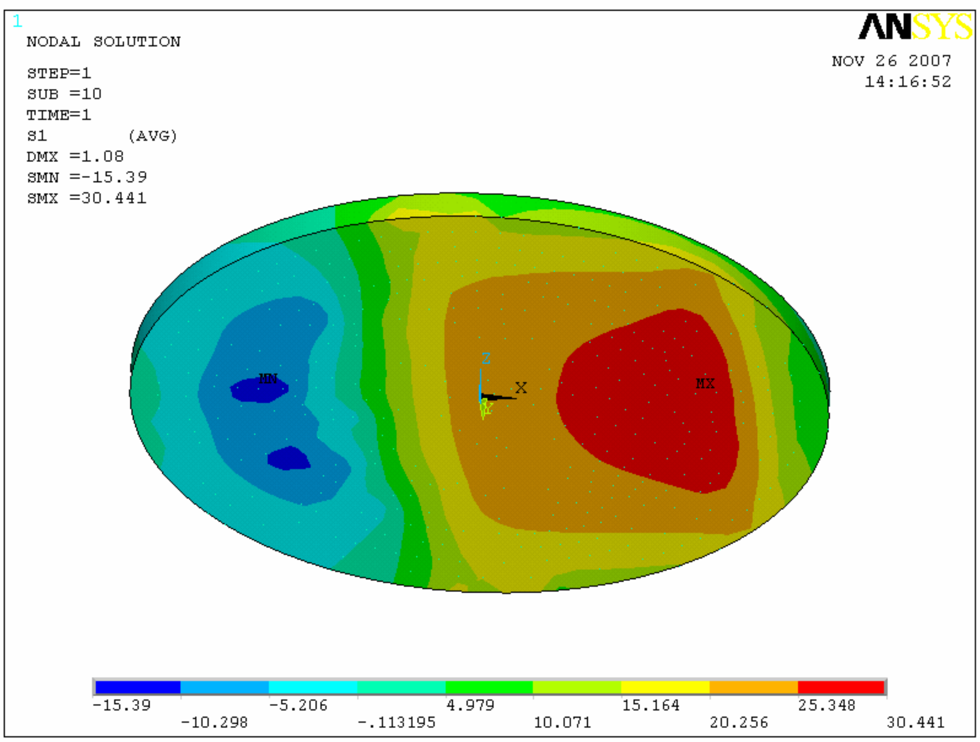


b) δ = 12 inches

Figure 6.13 Maximum Principle Stresses – Trough Settlement at Edge, Extent = 70-ft



a) δ = 2.8 inches



b) δ = 12 inches

Figure 6.14 Maximum Principle Stresses – Trough Settlement at Edge, Extent = 215-ft

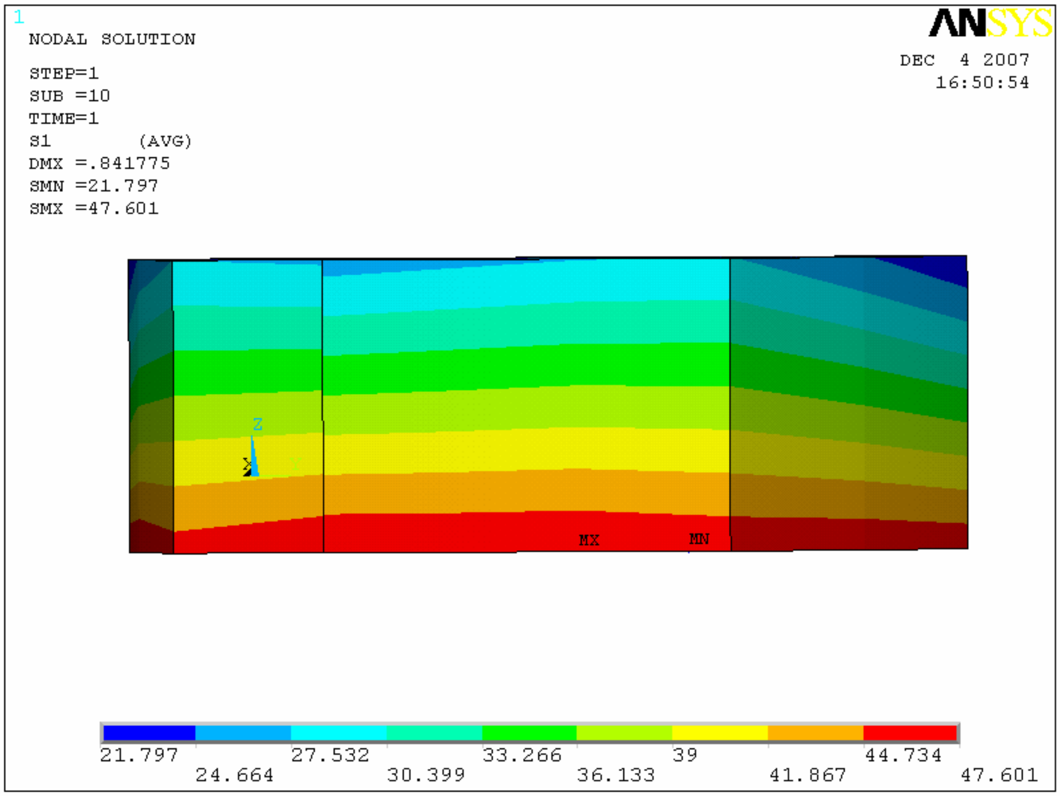


Figure 6.15 Distribution of Stress through Element Thickness (Figure 6.12b)

7 Attachment

Attachment A: ANSYS Files

The following files are included with the electronic media accompanying this calculation:

1) *T1-start.db*: ANSYS database file. The model can be resumed using this file and settlement loads applied.
2) *Batch Solve.txt*: This input file can be used to run and solve all settlement cases considered
3) *Disp70-2.8.csv*, *Disp70-12.csv*, *Disp215-2.8.csv*, *Disp215-12.csv*: Circular displacement field files.
4) *DispSS70-2.8.csv*, *DispSS70-12.csv*, *DispSS215-2.8.csv*, *Disp215-12.csv*: Displacement field files for trough centered on tank center
5) *DispC70-2.8.csv*, *DispC70-12.csv*, *DispC215-2.8.csv*, *DispC215-12.csv*: Displacement field files for trough centered on tank edge
6) *Support.xls*, *Simple Support.xls*, *Canti Support.xls*: Excel files used to create displacement fields

Attachment A: ANSYS Files

The following files are included with the electronic media accompanying this calculation:

1) *T1-start.db*: ANSYS database file. The model can be resumed using this file and settlement loads applied.
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3) *Disp70-2.8.csv*, *Disp70-12.csv*, *Disp215-2.8.csv*, *Disp215-12.csv*: Circular displacement field files.
4) *DispSS70-2.8.csv*, *DispSS70-12.csv*, *DispSS215-2.8.csv*, *Disp215-12.csv*: Displacement field files for trough centered on tank center
5) *DispC70-2.8.csv*, *DispC70-12.csv*, *DispC215-2.8.csv*, *DispC215-12.csv*: Displacement field files for trough centered on tank edge
6) *Support.xls*, *Simple Support.xls*, *Canti Support.xls*: Excel files used to create displacement fields