

# The Dynamical System Scaling (DSS) Methodology

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Rev. 1

Nonproprietary

Prepared by

Jose N. Reyes, Jr. Chief Technology Officer

#### NuScale Power, Inc.

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#### Abstract

This document presents a framework for the design of separate effects and integral system test facilities. It is based on the Dynamical System Scaling (DSS) methodology as applied to single and coupled dynamical systems. This method will be used to determine the geometric dimensions and operating conditions for the NuScale Power (NuScale) Integral System Test (NIST) facility. It will guide the modification of an existing integral system test facility that was developed for the assessment of the multi-application small light water reactor (MASLWR). The NIST facility will be used to evaluate the important small break loss-of-coolant-accident (SBLOCA) phenomena in a single NuScale plant module. The data obtained from this facility will be used to support the NuScale plant design certification effort.

The report includes a detailed description of the DSS methodology, its application to a practical engineering problem for illustrative purposes, a description of the framework for designing of separate effects test and integral effects test facilities, and an overview of how the DSS methodology can be used to support the Evaluation Model Development and Assessment Program (EMDAP) of Regulatory Guide 1.203. F, Appendix A of the report presents the [[

#### ]]<sup>3(a)-(c)</sup>

The key findings of the report include the following:

[[

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

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#### 1.0 Introduction

This document presents a framework for the design of separate effects and integral system test facilities. It is based on the dynamical system scaling (DSS) methodology as applied to single and coupled dynamical systems. This method will be used to determine the geometric dimensions and operating conditions for the NuScale Power (NuScale) Integral System Test (NIST) facility. It will guide the modification of an existing integral system test facility that was developed for the assessment of the multi-application small light water reactor (MASLWR) (Reference 1). The NIST facility will be used to evaluate the important small break loss-of-coolant-accident (SBLOCA) phenomena in a single NuScale plant module. The data obtained from this facility will be used to support the NuScale plant design certification effort. A similar scaling analysis was successfully used to design the Advanced Plant Experiment at Oregon State University as part of the Westinghouse and U.S. Department of Energy programs conducted in support of the AP600 and AP1000 design certification efforts (References 2 and 3).

This section briefly discusses the purpose of design certification testing. The second chapter presents an overview of the DSS methodology with an application to a practical engineering problem for illustrative purposes. The third chapter presents the scaling analysis objectives and the design framework for an integral effects test facility. The fourth chapter presents the role of scaling analyses in the Evaluation Model Development and Assessment Process (EMDAP). Lastly, Appendix A presents the Theory of Process Space-Time Geometry, which serves as the basis for the DSS method.

Term	Definition
DSS	dynamical system scaling
ECC	emergency core cooling
EMDAP	evaluation model development and assessment process
GDF	general design framework
IET	integral effects test
LOCA	loss-of-coolant accident
LWR	light water reactor
MASLWR	multi-application small light water reactor
NIST	NuScale integral system test
NRC	U.S. Nuclear Regulatory Commission
PIRT	phenomena identification and ranking table
QA	quality assurance
SBLOCA	small break loss-of-coolant accident
SET	separate effects test

#### 1.1 Acronyms and Abbreviations

#### 1.2 Qualified Data for Plant Design Certification and Evaluation Model Development

Testing done in support of the certification of a nuclear power plant requires strict adherence to a quality assurance (QA) plan that has been developed, approved, and implemented prior to the start of testing. The NuScale Quality Management Plan (Reference 4) provides for the establishment of test procedures and test specifications as required to address the specific context in which the data will be used. The test procedures and technical specifications, in conjunction with a QA Plan, provide for the control of every

aspect of testing that can affect the quality of the data. Generally speaking, for reduced-scale thermal hydraulic test facilities, the data is the product, and its primary purpose is to benchmark a computer code that will be used in the safety analyses of the plant. Only *qualified data* is permitted to benchmark safety analysis codes in the certification process.

The DSS methodology supports the test program by identifying which features of the test facility (i.e., geometric and operational features) must be controlled to assure that the important thermal hydraulic phenomena are accurately simulated in the facility. The applicable QA criteria are presented in the NuScale Quality Management Plan, which is based on Appendix B of Part 50 in Title 10 of the Code of Federal Regulations (Reference 5) and ANSI/ASME NQA-1-2008 issued by the American Society of Mechanical Engineers (Reference 6).

The DSS methodology can also be used to address several key objectives of the elements of the Evaluation Model Development and Assessment Process (EMDAP) described in U.S. Nuclear Regulatory Commission (NRC) Regulatory Guide 1.203 (Reference 7). This approach is described in Chapter 4.

### 2.0 A Geometric Method for Dynamical System Scaling (DSS)

This chapter presents the dynamical system scaling (DSS) method for the scaling of single and coupled dynamical systems. It is based on the theory of process space-time geometry that describes dynamical system processes as [[  $]^{3(a)-(c)}$ . It also provides three important tools for

scaling dynamical systems. [[

ll<sup>3(a)-(c)</sup> The DSS

method is applied to the scaling of coupled dynamical systems at the component level and the integral system level.

Scaling is the means by which a process that evolves at one set of temporal and spatial scales is related to the same process at a different set of temporal and spatial scales. Reduced scaled models are often used for testing new design concepts prior to deploying a first-of-a-kind system for commercial use. A good example is the integral system testing conducted for the nuclear power industry (References 2 and 3). Such reduced-scale test facilities have been economically and reliably used to generate data for the assessment of safety system performance and to benchmark safety evaluation computer codes. However, scaling these integral test facilities can be challenging because they often consist of a complex interconnection of dynamical systems.

The traditional approach for the study of dynamical systems uses phase portraits. This well-known technique provides valuable insights into the stability, classification, and topological equivalence of system behaviors (References 8 and 9). [[

]]<sup>3(a)-(c)</sup> The objective of this report is to present a geometric approach for scaling single and coupled dynamical systems using the theory of process space-time geometry.

The following sections provide a brief overview of the key features of the theory of process space-time geometry needed to develop a scaling methodology based on [[

11 <sup>3(a)-(c)</sup>

#### 2.1 Integral Balance Equation

Let a dynamical system be defined as a finite control volume containing a conserved quantity such as mass, momentum, or energy and acted upon by internal and external agents of change. Furthermore, let a conserved process be defined as the sequential transition of the state of the system and the transition sequence governed by an integral balance law constrained by the system's initial state and boundary conditions. The integral balance law for a dynamical system is given by

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			[[	]] <sup>3(a)-(c)</sup>	[1]
In this instant	equation, [[ t. It is defined a	]] <sup>3(a)-(c)</sup> is the frac s follows:	tional amount of th	e integrated conserved	d quantity at a given
			Π	]] <sup>3(a)-(r</sup>	<sup>c)</sup> [2]
Here to volume maxim	the local and i e is denoted b num) value of th	nstantaneous amount y [[ ]] <sup>3(a)-(c)</sup> . I ne integrated conserved	of a conserved qu n this equation, [[ d quantity for the pu	uantity spatially distribu ]] <sup>3(a)-(c)</sup> is a time- rocess considered. Furt ]] <sup>3(a)-(c)</sup>	uted within a control independent (ideally hermore, [[
The pa	arameter [[	]] <sup>3(a)-(c)</sup> takes into acc	count all of the [[		
	]] <sup>3(a)-(c)</sup> It is d	efined as			
			Ξ	]] <sup>3(a)-(c)</sup>	[3]
[[	]] <sup>3(a)-(c)</sup>				
<b>2.1.1</b> [[	[[	]] <sup>3(a)</sup>	)-(C)		
					[4]
					[5]
				]] <sup>3(a).</sup>	(c) [6]

2.1.3 [[	[[	]] <sup>3(a)-(c)</sup>	]] <sup>3(a)-(c)</sup>			
						[8]
					]] <sup>3(a)-(c)</sup>	[9a, b, c, d]
<b>2.1.4</b> [[	[[			]] <sup>3(a)-(c)</sup>		
						[10]

[7]

[[	

2.1.2 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[[

[[				
			]] <sup>3(a)-(c)</sup>	[11]
<b>2.1.6</b>	[[	]] <sup>3(a)-(c)</sup>		
			]] <sup>3(a)-(c)</sup>	[12]
<b>2.1.7</b> [[	[[	]] <sup>3(a)-(c)</sup>		
				[13]
			[1	4a-d]

]]<sup>3(a)-(c)</sup>

2.1.5 [[

[20]

[19]

[[

2.1.8 [[

# ]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[18]

[17]

[16]

[[ [15]



#### 2.2 Dynamical System Scaling Analysis for a Single Balance Equation

Figure 2-1 presents a flow diagram that outlines the DSS analysis for a process governed by a single balance equation. Prior to conducting the analysis, it is essential to have a good understanding of the prototype process to be scaled. This includes obtaining the operating conditions and the physical dimensions of the full-scale prototype. [[

## ]]<sup>3(a)-(c)</sup>

The details of the DSS method are described in the sections that follow. After the description, the DSS method is applied to a practical engineering problem for purposes of illustration.

Figure 2-1. Overview of dynamical system scaling (DSS) analysis for a system described by a single balance equation.

#### 2.2.1 Normalized Balance Equation and Local Phenomena

[[

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[[

## 2.2.2 Dynamical System Coordinate Scaling

]]<sup>3(a)-(c)</sup>

[25]

[26]

[27]

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[[

[24]

[29]

[28]

[[

2.2.3 [[

Figure 2-2. [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[36]

[35]

[34]

[[

2.2.5 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[33]

[32]

]]<sup>3(a)-(c)</sup> 2.2.4 [[

]]<sup>3(a)-(c)</sup>

[31]

[[

[[

]] <sup>3(a)-(c)</sup> [30]

[[

#### Similarity Criteria and Scaling Approaches 2.3

[[

]]<sup>3(a)-(c)</sup>

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[39]

[40]

[38]

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[[

[[	]] <sup>3(a)-(c)</sup>	[37]
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 Table 2-1.
 Scaling methods and similarity criteria

[[

[[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[41]

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[[

2.3.1 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

[42] [43]

[[

				]]	3(a)-(c)
Figure 2-3.	Π	]] <sup>3(a)-(c)</sup>			
ננ					
(c)					]] <sup>3(a)-</sup>
[[			]] <sup>3(a)-(c)</sup>		
Table 2-2.	[[			]] <sup>3(a)-(c</sup>	)
ננ					

[[

Table 2-3. [[

[[

[[

Figure 2-4. [[ ]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

[[

2.3.2 [[

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

## 2.4 Application of DSS to a Fluid Mixing-Heating Process in [[ ]]<sup>3(a)-(c)</sup>

Consider the system shown in Figure 2-5. A fluid is contained inside a tank that is insulated and pressurized. It houses a heating plate at its base and a steam heating coil at its interior. The rotation of the mixer keeps the fluid at a uniform temperature and enhances heat transfer from the plate and the coil. The fluid temperature can be raised from its initial conditions to a final equilibrium temperature set by the steam coil and the heater plate.



Figure 2-5. Fluid mixing and heating tank

[[

[44]

]]<sup>3(a)-(c)</sup> [49]

[[

[48]

]]<sup>3(a)-(c)</sup>

[47]

[46]

[56]

2.4.1 Application of the [[

]]<sup>3(a)-(c)</sup> Scaling Method

]]<sup>3(a)-(c)</sup> [55]

[54]

[53]

[52]

[51]

[50]

[60]

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[59]

[58]

[[

[57]

Table 2-4. Comparison of prototype and model parameters

[[

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

- Figure 2-6. Solution curves for the fluid temperature ratio versus reference time for the prototype and reduced scale model
- 2.4.2 Analysis of Heating-Mixing Process Geometry
- [[

[61]

[62]

[63]

[64]

]]<sup>3(a)-(c)</sup>

[69]

[68]

]]<sup>3(a)-(c)</sup>

[[

[[

Table 2-5. ]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

[67]

[66]

[[

]]<sup>3(a)-(c)</sup> [65]

[[

						[71]
					]] <sup>3(a)-(c)</sup>	[72]
[[						
		]] <sup>3(a)-(c)</sup>		<b></b> 3(a)-(c)		
[[	ll			]] <sup>5(a)-(b)</sup>		[73]
						[74]
						[75]
						[76]
			]] <sup>3(a)-(c)</sup>			[77]

[[

[70]

]]<sup>3(a)-(c)</sup>

Figure 2-7. Fluid heating and mixing process. [[

[[

]]<sup>3(a)-(c)</sup>

[[

[[

]]<sup>3(a)-(c)</sup>

[[

2.4.3 [[

Figure 2-8. [[

[[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

[[

]]<sup>3(a)-(c)</sup>

[78]

Figure 2-9. Fluid mixing and heating process. [[

[[

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[79]

]]<sup>3(a)-(c)</sup>

[80]

]]<sup>3(a)-(c)</sup>

[81]

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[85]

[84]

[83]

[82]
Figure 2-10. [[

]]<sup>3(a)-(c)</sup>

[[

[[

Figure 2-11. [[

]]<sup>3(a)-(c)</sup>

# 3.0 Scaling Analysis and Design Framework for an Integral Effects Test Facility

The general objective of a scaling analysis for an integral effects test facility is to obtain the reduced-scale dimensions and operating conditions for a test facility capable of simulating the important flow and heat transfer behaviors. To develop a properly scaled test facility, the following specific objectives must be met for each operational mode of interest.

- The thermal hydraulic processes that should be modeled have been identified.
- The similarity criteria that should be preserved between the test facility and the full-scale prototype have been obtained.
- The priorities for preserving the similarity criteria have been established.
- Specifications for the test facility design or modifications have been provided.
- Biases due to scaling distortions have been quantified.
- The critical attributes of the test facility that must be preserved to meet QA requirements have been identified.

Different similarity criteria will be obtained for the different modes of system operation. These criteria depend on the geometry of the components, the scaling level required to address the transport phenomena of interest, and the initial and boundary conditions for each particular mode of operation.

To assure that the scaling objectives are met in an organized and clearly traceable manner, a general design framework (GDF) is established. The model for this framework includes features drawn from the severe accident scaling methodology presented in NUREG/CR-5809 (Reference 18). A flow diagram for the GDF is presented in Figure 3-1. The following sections describe how the GDF can be applied to the design of an integral system test facility.

Figure 3-1. General design framework for an integral test facility

### 3.1 Experiment Objectives

The first task outlined by the GDF is to specify the experimental objectives. The experimental objectives define the types of tests that will be performed to address specific design or certification needs. These objectives determine the general modes of operation that should be simulated in the test facility. There are practical limits with regard to what can be studied in a single facility. The objective of a nuclear plant design certification test facility is to obtain qualified data to benchmark the computer codes and models that will be used to evaluate the safety of the plant design.

[[

#### 3.2 Phenomena Identification and Ranking Table

The second task outlined by the GDF is the development of a phenomena identification and ranking table (PIRT). A useful description of the PIRT process is found in Wilson and Boyack, 1998 (Reference 19). The role of the PIRT for a new design is twofold. First, if conducted early in the design process, the PIRT can serve to inform the design. This process can result in enhancements to safety system design particularly with regard to the timing and sequencing of safety systems. Second, the nature of scaling forbids exact similitude of all of the parameters of a reduced-scale test facility with those of a full-scale prototype. As a result, the design and operation of the test facility is based on simulating the thermal hydraulic processes most important to the system operational modes that will be explored. A valuable function of a PIRT is to identify the different phases of a scenario and the most important thermal hydraulic phenomena within those phases that should be simulated in the test facility. For example, many of the thermal hydraulic phenomena of importance to light water reactor (LWR) behavior during a loss-of-coolant accident (LOCA) have already been identified in existing PIRTs (Shaw, 1985) (Reference 20). Therefore, it is of value to conduct a literature review to determine what PIRT studies already exist for similar designs and transient conditions. Note that full-scale plant information is needed to develop a well-informed PIRT. In addition to the expert assessment, the scaling analysis results will also identify the dominant thermal hydraulic phenomena for a specific process. However, their importance to plant safety or operation will be determined by the PIRT.

#### 3.3 Hierarchical Levels for Similarity Criteria

The third step in the GDF is to define the hierarchical levels and operational modes for which the similarity criteria should be developed. This is determined by examining the phenomena being considered. For example, if the phenomenon being considered involves mass, momentum, or energy transport between materials such as water and solid particles, then the scaling analysis would be performed at the constituent level. If the phenomenon of interest involves mass, momentum, or energy transport between vapor and liquid, then the scaling analysis would be performed at the phase level. Thermal hydraulic phenomena involving integral reactor coolant system interactions, such as primary system depressurization or loop natural circulation, would be examined at the subsystem level. Thermal hydraulic phenomena, such as steam generator heat transfer, would be examined at the subsystem level. Specific interactions between the steam-liquid mixture and the stainless steel structure would be examined at the constituent level. Therefore, identifying the scaling level depends on the phenomenon being addressed.

For purposes of illustration, this section identifies the scaling level at which the similarity criteria should be developed for a NuScale SBLOCA. Based on the integral system test requirements, the following modes of plant operation will be examined in the test facility:

]]

Figure 3-2 presents the breakdown of the NuScale system into hierarchical levels. The reactor coolant system and the emergency core cooling system will be the primary focus of the test program.

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]]<sup>3(a)-(c)</sup>

Figure 3-2. NuScale system breakdown into hierarchical levels

#### 3.4 Application of the DSS Method to Integral System Test Facility Design

The fourth step in the GDF is to perform a scaling analysis for each of the hierarchical levels (e.g., systems and subsystems) and their modes of operation. The method used herein is the dynamical system scaling (DSS) method. The term "dynamical system" represents any system that undergoes transient behavior according to a set of governing equations. The DSS method has been described in Chapter 2.0 and provides [[

<sup>3(a)-(c)</sup> This method is particularly well suited for complex systems having interconnected components.

## 3.5 Define Component Coupling, Balance Equations, and Local Phenomena

The DSS method requires identifying the interconnections among components within the system or subsystem being analyzed to establish the transient transport behavior of conserved quantities. The objective is to define the time-dependent coupling among the set of dynamical components at a specific hierarchical level. Figure 3-3 illustrates this concept for a three-component system interacting with the environment.

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]]<sup>3(a)-(c)</sup>

Figure 3-3. Example of component coupling at a specific hierarchical level

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[86a-c]

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]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

[87a-c]

The subsequent integral scaling analyses require that each term of the component balance equations be specified. This is done in tabular form to make the analysis traceable and auditable. Table 3-1 provides an example of the types of balance equations, local phenomena, and the corresponding models for the three component system shown in Figure 3-3.

Table 3-1.	Sample	list	of	component	balance	equations,	phenomena,	and	models	for	the	three-
	compone	ent s	syst	em								

Component	Integral Balance Equations	Local Phenomena	Models and References
Component #1	Mass Balance	Break Flow Rate	Choked flow model
	Energy Balance	Core power Break flow energy	LWR decay heat standard
	Metal Structure Energy Balance Equation	Wall Heat Transfer	Free convection heat transfer correlations for vertical flat plates
Component #2	•	•	•
	•	•	•
Component #3	•	•	•
•	•	•	•
	•	•	•

### 3.5.1 Integral Scaling Analysis

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[88]

[89a-c]

# 3.5.1.1 Component Level Scaling Analysis

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]]<sup>3(a)-(c)</sup>

# 3.5.1.2 System Level Scaling Analysis

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[93]

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3.5.2 [[

[[

3.5.2.1 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup> [99]

[98]

[97]

[96]

The temporal displacement rate for the aggregate is given by

NP-TR-1010-867-NP

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[94]

[95]

]]<sup>3(a)-(c)</sup>

[109]

[108]

[107]

[[

3.5.2.2 [[

	[[	I	[100]
		]] <sup>3(a)-(c)</sup>	
	[[		[101]
		I	[102]
			[103]
		I	[104]
			[105]
			[400]
			[106]
]] <sup>3(a)-(c)</sup>			

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[110]

[111]

[112]

[113]

]]<sup>3(a)-(c)</sup>

#### 3.5.3 Scale Ratios

The next step in the DSS analysis is to select the physical dimensions and operating conditions for the test facility such that the similarity criteria obtained for the coordinate transformation method selected, are satisfied. This requires detailed information about the prototype geometry and operation. The result of this portion of the analysis is to produce a set of scale ratios. This includes height, length, area, volume, flow, power, time, and fluid property scale ratios.

### 3.5.4 Identifying Scale Distortions

Because it is impossible to identically satisfy all of the similarity criteria simultaneously, it is necessary to assess the scaling distortions that arise as a result of the selection of a particular set of geometric dimensions and operating conditions for the test facility. The objective of the distortion analysis is to minimize overall scaling distortions while faithfully simulating the most important phenomena identified by the PIRT. The distortion analysis serves to identify the dominant terms in the balance equations. Preserving the dominant transport terms on a scaled basis preserves the dominant phenomena. Hence, the similarity criteria representing the "weaker" transport terms need not be preserved identically since their influence on the overall process being modeled would not be significant.

The DSS method introduces a new approach to understanding time-dependent scale distortions. Because every process [[

]]<sup>3(a)-(c)</sup>

### 3.5.5 Component Design Specifications and Prioritization

Having completed the scaling analysis for a specific set of components (i.e., hierarchical level) and a specific SBLOCA period or operational mode, the same scaling process is repeated for all of the SBLOCA periods and operational modes for each hierarchical level. Because the scaling ratio requirements for one set of operating conditions could partially conflict with the scaling ratio requirements for another set of

operating conditions, the final step in the scaling analysis is to prioritize the test facility design specifications.

#### 3.6 Developing Test Facility Design Specifications

The fifth step of the GDF is to document all of the test facility design and operation specifications. All of the essential geometric features and operating parameters that must be controlled to assure an accurate simulation of the important thermal hydraulic phenomena are identified and designated as critical attributes for use in the quality assurance plan.

#### 3.7 Post Test Scaling Assessment

The sixth and final step of the GDF is to perform a post-test scaling assessment. This requires plotting the test data and/or evaluation model calculations [[

]]<sup>3(a)-(c)</sup> Several types of post-test scaling analyses can be performed depending on the use of the test data. A key application of separate effect testing SET and integral effect testing IET data is to benchmark evaluation models used for safety assessment. The NRC has issued Regulatory Guide 1.203 to provide general guidance for an evaluation model assessment and development program. The following chapter describes the application of the DSS methodology to the applicable EMDAP requirements.

# 4.0 Application of the DSS Methodology to EMDAP

The DSS methodology can be used to address several key objectives of the elements of the Evaluation Model Development and Assessment Process (EMDAP) described in U.S. Nuclear Regulatory Commission (NRC) Regulatory Guide 1.203. In particular, the DSS methodology can be used for the following aspects of the EMDAP for integral effect tests (IET), separate effect tests (SET), and Evaluation Model (EM) assessments:

#### EMDAP Element 2, Development Assessment Base

- Step 6 Perform scaling analysis and identify similarity criteria.
- Step 8(a) Evaluate effects of IET distortions.
- Step 8(b) SET scale-up capability

#### EMDAP Element 4 – Assess Evaluation Model Adequacy

- Step 15 Assess scalability of models.
- Step 19 Assess scalability of integrated calculations and data for distortions.

A detailed discussion of each of the EMDAP steps listed above is described in Regulatory Guide 1.203. The objective of these different scaling assessments is to determine the effect of scale on the overall uncertainty of the evaluation model calculations. Specifically, scale distortions in the IET and SET facilities used to benchmark the evaluation model and the correlations within the evaluation model need to be quantified and included in the uncertainties of the code results. IAEA Safety Report Series #52 provides a useful summary of uncertainty evaluation methods for best estimate safety analysis computer codes (Reference 21).

Figure 4.1 is a flow chart that describes how the DSS Methodology can be used to address the EMDAP steps listed above. The chart is divided into two categories: test facility scaling analyses and data/evaluation model scaling analyses.

#### 4.1 Test Facility Scaling Analyses

The objective of Step 6 of the EMDAP is to demonstrate that the IET or SET facility properly simulates the important phenomena identified to occur in the prototype for a given process. Application of the DSS methodology to the design of an IET facility has already been described in the previous chapter. The same approach can be applied to an SET facility. The approach is summarized by the flow diagram for the General Design Framework presented in Figure 3-1 and fully discussed in Chapter 3.

EMDAP Step 8a is aimed at identifying and correcting, if possible, scaling distortions in the IET facility. This is also important for SET facilities. A novel part of the DSS methodology is the geometric approach to assessing scale distortions. For an IET or SET facility undergoing design, the approach requires providing analytical or numerical solutions to the normalized balance equations for both the test facility and the prototype for a specified transient, (e.g., SBLOCA). [[

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### Figure 4-1. Application of the DSS Methodology to applicable EMDAP steps

### 4.2 Data and EM Scaling Analyses

The DSS methodology can also be applied to scaling assessments of *measured data* obtained from IET or SET facilities. This could be a comparison of counterpart tests performed in multiple IET or SET facilities or a comparison of a process initiated at a range of initial conditions in a single IET or SET facility. As such, the DSS methodology can be used to address EMDAP Steps 8a and 8b.

The following sections present the method of converting measured data obtained from an SET facility, an IET facility, or an evaluation model [[ ]]<sup>3(a)-(c)</sup>

#### 4.2.1 Graph of Data in Engineering Units

Transient data obtained from an IET or SET facility is typically graphed as [[

]]<sup>3(a)-(c)</sup>. Assuming that the fluid mixing-heating test facility examined in Chapter 2 was constructed and operated to obtain measurements of fluid temperature versus time. Figure 4.2 provides an example of a standard plot of the data curve that would be expected as expressed in engineering units.



Figure 4-2. Example of measured temperature curve obtained from a fluid mixing-heating test facility

#### 4.2.2 Graph of Data as a Normalized Conserved Quantity

For this example of temperature vs. time, it is recognized that the conserved quantity is the energy within the control volume. Let the normalized conserved quantity,  $\beta$ , be defined [[

[[

4.2.3 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[114]

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Figure 4-3. [[

]]<sup>3(a)-(c)</sup>

[[

4.2.4 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

Figure 4-4. [[

]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

Figure 4-5. [[

]]<sup>3(a)-(c)</sup>

#### 4.3 IET/SET/EM Scaling Distortion Acceptance Criteria

The DSS methodology provides a means of calculating the scaling distortions between processes in counterpart IET facilities, between processes in counterpart SET facilities, between EM predictions and data generated by counterpart IET facilities, and between correlations and the data generated by counterpart SET facilities.

Figure 4-1 presents three scale distortion acceptance criteria. The first two criteria address the adequacy of the IET/SET facilities from the standpoint of producing test results that faithfully simulate the important phenomena of the full-scale prototype. The last criterion addresses the adequacy of the test results for use in determining the effect of scale on the overall uncertainty of the evaluation model calculations.

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[117]

[116]

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# Appendix A. [[

]]<sup>3(a)-(c)</sup>

#### A.1 Abstract

This report introduces the theory of [[

]]<sup>3(a)-(c)</sup>

#### A.2 Introduction

Let a system be defined as a finite control volume containing a conserved quantity such as mass, momentum, or energy and acted upon by internal and external agents of change. Furthermore, let a conserved process be defined as the sequential transition of the state of the system; the transition sequence governed by an integral system balance law constrained by the system's initial state and boundary conditions. These definitions, whether stated explicitly or implied by application, have served as the backbone of engineering analyses for centuries. However, their interpretation and implementation suffer from some fundamental weaknesses. [[

]]<sup>3(a)-(c)</sup>

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Figure A-1. [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

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#### Integral Balance Equation A.3

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[A1]

[A2]

[A3]

]]<sup>3(a)-(c)</sup>

[A4]

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[A5]

[[

A.3.1 [[

[A6] [A7] [A8] [A9] ]<sup>3(a)-(c)</sup>

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A.3.2 [[

[[

]]<sup>3(a)-(c)</sup>

Figure A-2. [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

[A11]

Figure A-3. [[ ]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

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A.3.3 [[

]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

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[A13]

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Figure A-4. [[

]]<sup>3(a)-(c)</sup>

[A15] [A16] ]] <sup>3(a)-(c)</sup> [A17] [( [A18a, b, c, d] ]] <sup>3(a)-(c)</sup> ]] <sup>3(a)-(c)</sup> []				[A14]
[A16] ] <sup>3(a)-(c)</sup> [A17] [( [A18a, b, c, d] ] <sup>3(a)-(c)</sup> [ [ ]				[A15]
[[ [A17] [[ [A18a, b, c, d] ]] <sup>3(a)-(c)</sup> [[ ]				[A16]
[A18a, b, c, d] ] <sup>3(a)-(c)</sup> [[	[[		]] <sup>3(a)-(c)</sup>	[A17]
]] <sup>3(a)-(c)</sup> [[				[A18a, b, c, d]
A.3.5 [[ ]] <sup>3(a)-(c)</sup> [[				]] <sup>3(a)-(c)</sup>
[[	A.3.5 [[	]] <sup>3(a)-(c)</sup>		
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			<b>11</b> 3(a)-(c)	[0.40]

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A.3.4 [[



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[21d]

[21c]

[21b]

[A21a]

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A.3.7 [[ ]]<sup>3(a)-(c)</sup>

]]<sup>3(a)-(c)</sup>

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A.3.6 [[

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[A29]

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]]<sup>3(a)-(c)</sup>

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A.3.8 [[

]]<sup>3(a)-(c)</sup>

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3(a)-(c)		

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A.3.9 [[

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A.3.10 [[

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]] <sup>3(a)-(c)</sup>	[A50]

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3.11 [[	]] <sup>3(a)-(c)</sup>	]] <sup>3(a)-(c)</sup>	

]]<sup>3(a)-(c)</sup>

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[A52a-c]

[A51a, b]

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[A57a-d]

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]]<sup>3(a)-(c)</sup>

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A.3.13 [[

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]]<sup>3(a)-(c)</sup>

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A.3.12 [[

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]]<sup>3(a)-(c)</sup>

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]]<sup>3(a)-(c)</sup>

Table A-1. [[

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]]<sup>3(a)-(c)</sup>

## A.4 Conclusions

An axiomatic formulation of the theory of process space-time geometry has been presented as outlined in Figure A-1. [[

]]<sup>3(a)-(c)</sup>

## Rev. 1

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