Analyzing Uncertainty Using Monte Carlo Simulation

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What Is Monte Carlo Simulation?

- A technique that approximates the probability of certain outcomes by performing multiple trial runs, called simulations, randomly drawing values from a pre-defined distribution.
- Monte Carlo has become more popular as computers have become more powerful, particularly in fields that specialize in analyzing financial markets.

When to Use Monte Carlo

- When there is a variable/variables with a known range of values but an uncertain value for any particular time or event.
- Examples of such variables: interest rate, stock prices, weather conditions, long-term costs.

GAO's Use of Monte Carlo

General topic	GAO report
Capital program costs	• *GAO Cost Estimating and Assessment Guide: Best Practices for Developing and Managing Capital Program Costs.* GAO-09-3SP (supersedes GAO-07-1134SP). March 2009. • *Nuclear Waste Management: Key Attributes, Challenges, and Costs for the Yucca Mountain Repository and Two Potential Alternatives*. GAO-10-48. Nov. 2009. • *VA Construction: VA Is Working to Improve Initial Project Cost Estimates, but Should Analyze Cost and Schedule Risks*. GAO-10-189. Dec. 2009.
Trust fund sufficiency	• *Compact of Free Association: Palau's Use of and Accountability for U.S. Assistance and Prospects for Economic Self-Sufficiency*. GAO-08-732. June 2008. • *Compacts of Free Association: Trust Funds for Micronesia and the Marshall Islands May Not Provide Sustainable Income*. GAO-07-513. June 2007.

Other U.S. Agencies' Use of Monte Carlo

Congressional Budget Office (CBO)	**Social Security projection:** CBO's Long-Term Actuarial Model (LTAM) allows a more comprehensive analysis of uncertainty than does the Social Security Administration's (SSA) scenario-based approach. Although LTAM does not always operate at the same level of detail as SSA's model, it was written to follow the same methodology, and tests show that its estimates are consistent with those produced by SSA. LTAM can be run repeatedly under a range of assumptions about future events to obtain a full distribution of possible outcomes.
Department of the Treasury	**Bank stress testing:** Charge-off rates for different loan types are modeled as dependent on a set of economic and financial variables. To deal with nonstationary in the variables, the empirical Bayesian approach was employed. The estimation was carried out by running 10,000 Markov Chain Monte Carlo simulation.

Sample Questions Monte Carlo Can Help Answer

- What is the likelihood that any dedicated fund will be sufficient in the future?
- How will additional funding contribution requirements improve the likelihood?
- How will the required fund accumulation schedule affect the likelihood?

Monte Carlo's Limitations for Decision Makers

- Replication of capital market's actual behavior may not be accurate: recent historical period may not provide a realistic representation of potential future variations.
- Other unknown factors cannot truly be accounted for.
- Results can be sensitive to equation specifications, degrees of interdependence among variables, and the historical periods used for the estimates.
- Simulations provide possible scenarios to inform decision makers but do not make decisions.

Types of Monte Carlo Simulations

Type	What it does	The good	The bad
Non-parametric	Uses historical data (like drawing all past returns from a hat)	Simple	Uses only historical data, assumes future will resemble the past
Parametric	Based on means, standard deviations, and correlations of the random variables	Richer sets of results	May not find a distribution that truly resembles potential outcome distribution
Economic modeling	Models movements of the yield curve through time, then layers on various equity and fixed-income risk premium to derive returns	Most realistic	Most complicated

Steps of Monte Carlo Simulation

1. Start with a set of assumptions, such as the estimated means, standard deviations, and correlations for a set of variables, such as investment returns.
2. Generate random drawings based on the assumptions (like drawing numbers from a hat).
3. Calculate and display the range of possible outcomes.

Sample Monte Carlo Simulation

- **Background**: The U.S. compact with the Republic of Palau, a Pacific island nation, required the United States to set up a trust fund with an initial investment of $66 million in 1995 and an additional $4 million in 1997. The compact stated that the trust fund's objective was to produce an average annual distribution of $15 million for 35 years starting in 2010.
- **Question**: What is the likelihood that the Palau trust fund will be able to provide the desired level of income after U.S. direct assistance stops?

Where is Palau?

Methodology

Key inputs in simulation:

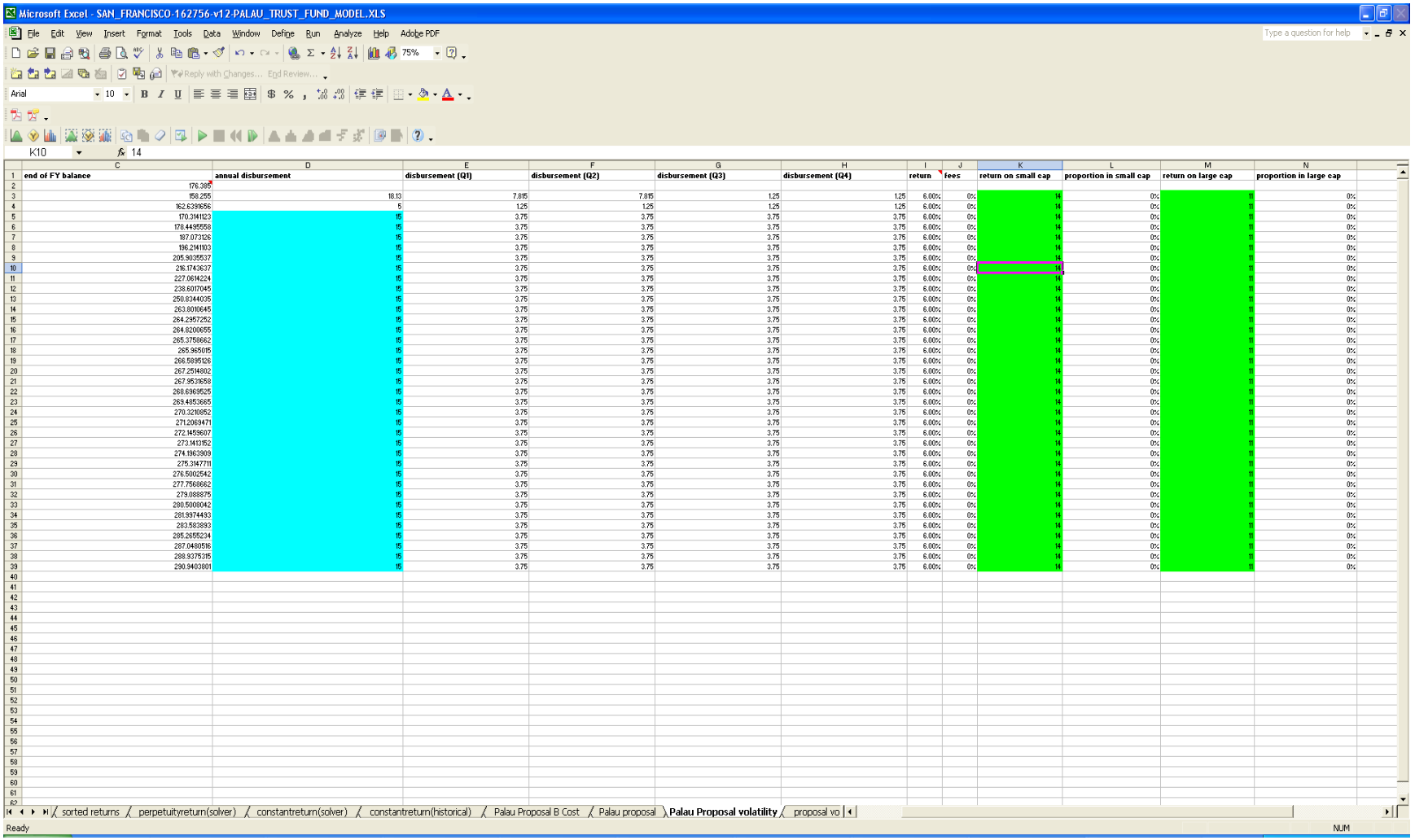
- the trust fund's balance at the beginning of the projection
- disbursement and inflation adjustment
- the trust fund balance projection equation
- distribution of returns for the Monte Carlo simulation.

Cross-Correlation and Serial Correlation of Historical Annual Returns

	Small company	Large company	U.S. treasury bills
Small company	1		
Large company	0.66	1	
Treasury bills	-0.01	0.05	1
Serial correlation	-0.003	0.03	0.81

Source: GAO calculation.

Screen Shot 1



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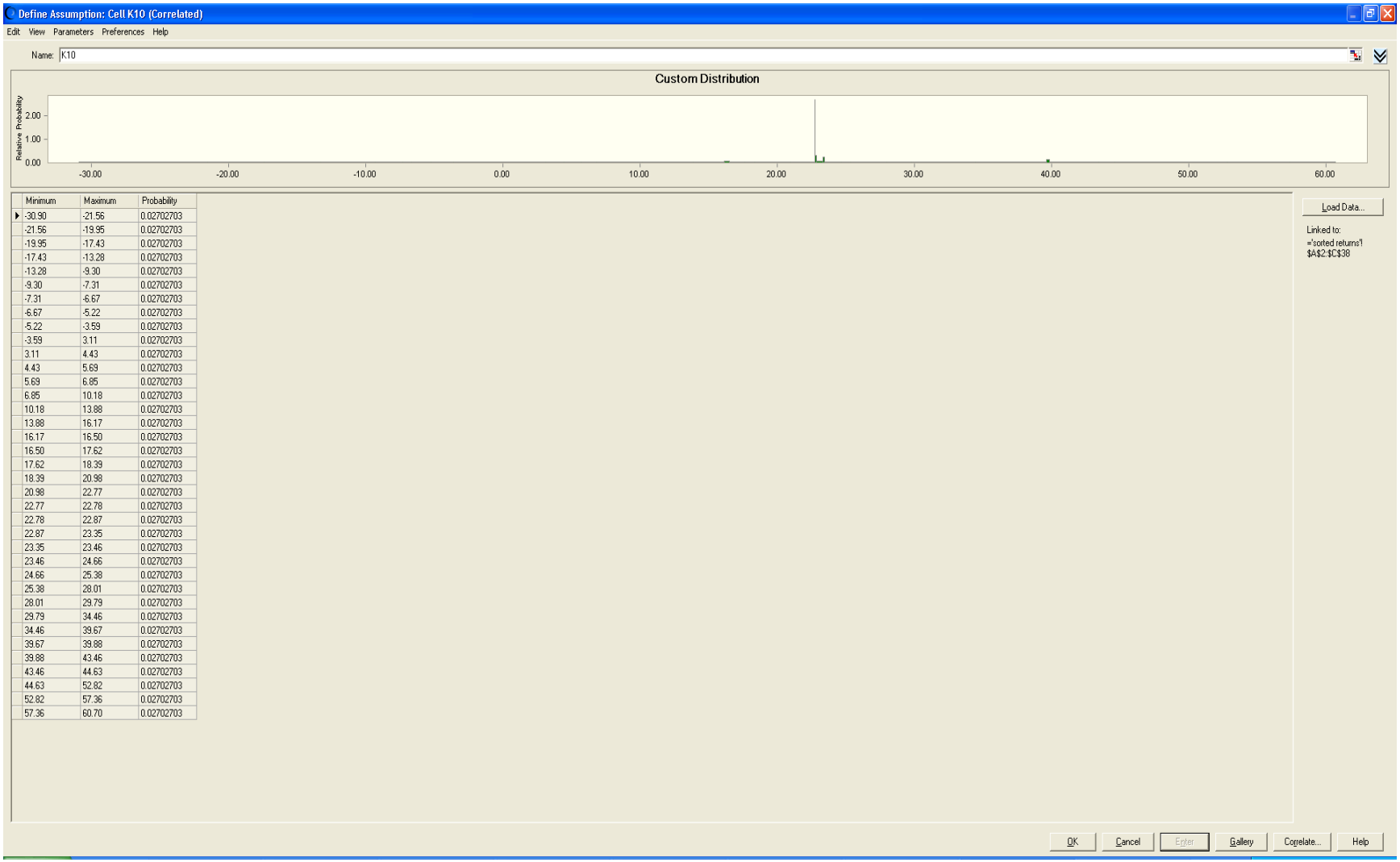

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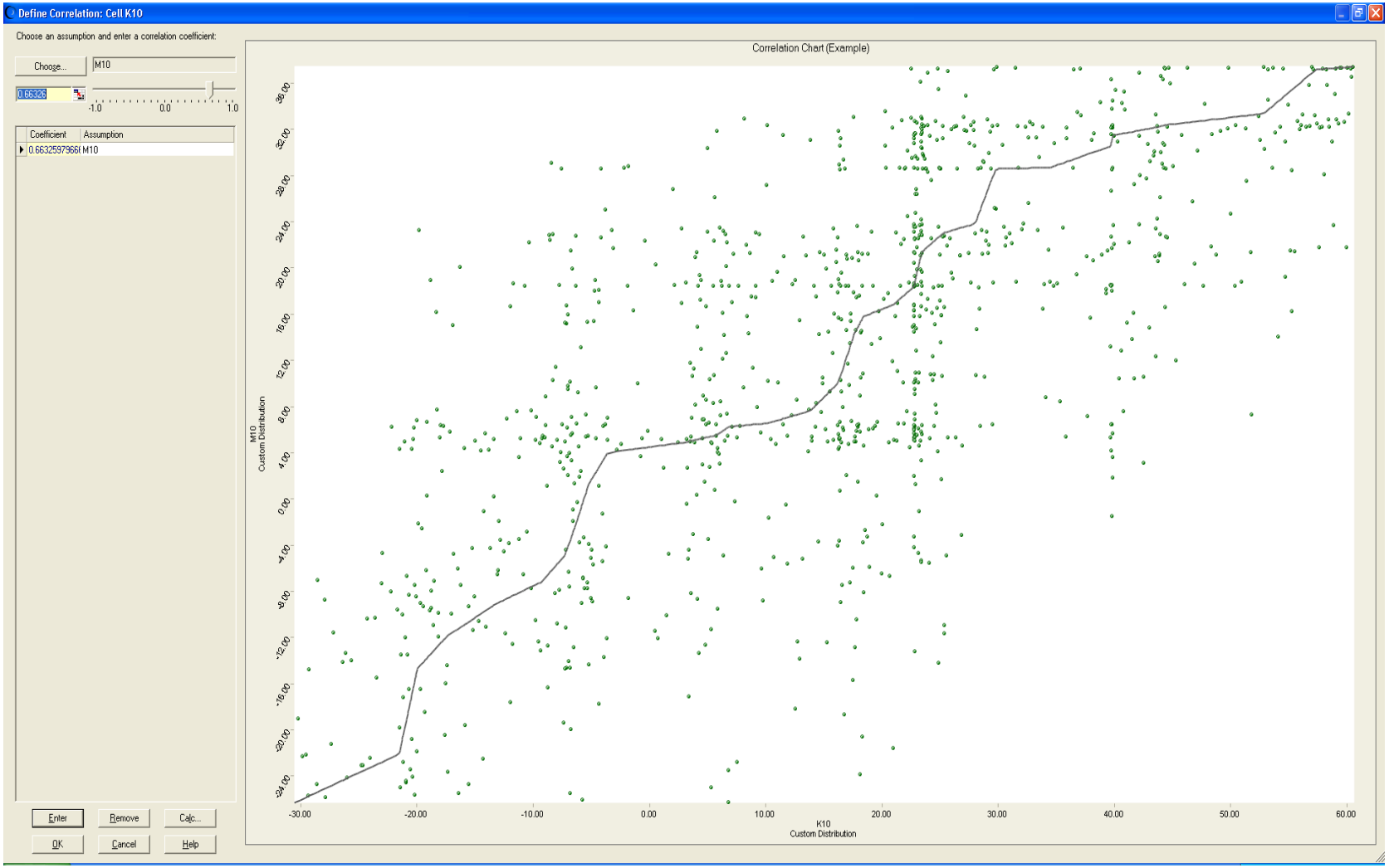

What GAO found

- Palau will be able to withdraw $15 million per year from its trust fund for the planned 35 years—from 2010 through 2044—if the fund earns a compounded annual return of at least 8.1.
- Forecasts of future returns are subject to considerable uncertainty.
- Market volatility could lead to the trust fund's depletion after 2016.

Probability of shortfall in Palau trust fund