The Effect of Gaps on the Response of Spent Fuel Transportation Package Closure Lid Bolts During a Drop Impact

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ABSTRACT

During an impact event, gaps between the various components of a spent fuel transportation cask may create secondary impacts that result in higher dynamic loads than would have occurred if the gaps had not been present. A condition of particular interest is the gap that may exist between the cask internal contents (fuel assemblies, canister, etc,) and cask closure lid, and the effect this gap may have on amplifying the response of the closure lid bolts during an impact.

Through the use of a simple dynamic model this paper investigates the effect of a secondary impact due to a gap between the cask internals and the cask closure lid on the response of the closure lid during a 30 foot end drop. The response results are presented in terms of the Dynamic Load Factor (DLF) for the closure lid. Response is shown to be a nonlinear function of the impact limiter deceleration, gap size and closure lid diameter, thickness and inertial properties. These results provide valuable insights into the parameters that affect response and show the conditions under which gaps of sufficient size may significantly influence response.

INTRODUCTION

When spent fuel is shipped in a transportation cask, gaps exist between the cask closure lid (the containment boundary) and the cask internal contents (fuel assemblies, fuel basket, etc.). If a transportation accident was to occur these gaps may result in a secondary impact of the cask contents onto the lid that could significantly increase the response of the lid above the values that would have occurred if the gap had not been present. The regulations in 10 CFR Part 71.73(c) (1) for the 30-foot drop requires the cask to be dropped "in a position for which maximum damage is expected." Some might suggest that "position" means "orientation," and that the inclusion of gaps in the hypothetical accident condition (HAC) is not within the intent of the provisions of Part 71.73. There are, however, strong arguments to suggest otherwise. 10 CFR Part 71.74 "Accident conditions for air transport of plutonium" in 71.74(a)(1) states the following:

> "A package must be physically tested to the following conditions… Impact at a velocity of not less than 129 m/sec (422 ft/sec) at a right angle onto a flat essentially unyielding, horizontal surface in the orientation (e.g., side, end, corner) expected to result in maximum damage…"

In Part 71.73 the word "orientation" is not used, instead the word "position" is used. Therefore, the word "position" must be interpreted more broadly than simply "orientation." In this broader context the word position refers to both the spatial orientation of the cask as well as the geometric position of the cask and its contents relative to one another. Thus, to meet this broader intent, gaps, whose size is sufficient to significantly influence

the dynamic response of the closure lid bolts, must be incorporated in tests and finite element analyses of spent fuel transportation packages.

Reviewing the history of the development of radioactive material transport regulation (Pope 2004), Messenger and Fairbairn "proposed that, for the practical purposes of approval of design of Type B packagings, …the postulated 'maximum credible accident' shall be regarded as comprising: (i) Head-on collision at 30 mph (or 30 ft free fall, which is equivalent) with a rigid structure…" The intent, therefore, is to simulate a head-on collision at 30 mph. In such a transportation accident gaps will exist. The inclusion of gaps in tests and analysis would not be an issue if it were not for the fact that the secondary impact of package contents onto the closure lid can be potentially significant.

PROBLEM DESCRIPTION

A vertical section through a typical cylindrical spent fuel transportation package subjected to a top-down end drop is shown in Figure 1. The transportation package consists of the following basic components: (1) the cask overpack, (2) closure lid, (3) closure bolts, (4) cask internal contents (including fuel assemblies) and (5) impact limiter. The gap between the internal contents and closure lid is also shown in the figure.

For the case considered herein, the package is dropped from 30 feet in a vertical position with the top end down and impacts an unyielding surface at a velocity, V_o = 44 ft/sec. To maintain the gap between the contents and the lid during the drop in an actual physical test, the contents would be suspended on a weak tether to create the gap size that would be present during an actual transportation impact event.

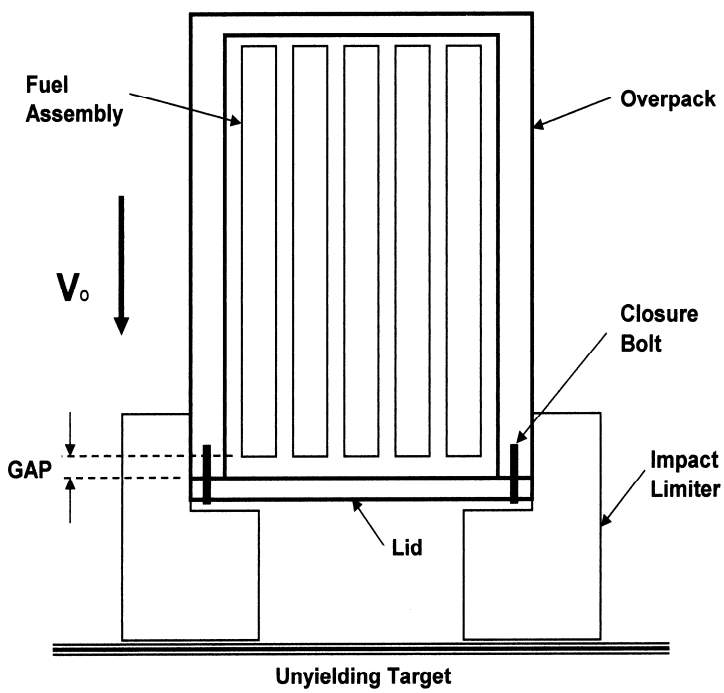

Figure 1: Vertical section through a typical cylindrical spent fuel transportation package subjected to a top-down end drop

Upon impact with the unyielding target it is assumed that the impact limiter applies a constant crush force to the cask overpack and outer annulus region of the lid (i.e., it is assumed that no crush forces are applied to the central region of the lid.). As the cask overpack decelerates and slows down the gap closes and the cask contents catches up to the closure lid causing a secondary impact that briefly reduces the deceleration of the overpack. This brief reduction in overpack deceleration is conservatively neglected, and the deceleration of the overpack is assumed to remain constant during the entire event. In addition, it is assumed that the impact between the contents and lid is fully plastic with no rebound. As such, once the contents mass impacts the lid it is subjected to the same deceleration as the cask overpack (i.e., the lid). It is further assumed that the closure lid and bolts remain elastic during the impact. This is consistent with the Standard Review Plan (NUREG-1617), which recommends that the closure lid bolts and closure lid system within the region of the lid bolts remain elastic in order to demonstrate leak-tightness by finite element analysis.

METHODOLOGY

Time and Relative Velocity at Gap Closure

The cask overpack deceleration resulting from the constant force applied to the overpack by the impact limiter is

$$a = -\alpha g \tag{1}$$

where, α, is the g magnitude of the deceleration. Integrating the above expression with respect to time, we obtain the cask overpack (and lid) velocity, v, at any time, t, after initial impact with the unyielding target as

$$v = V_o - \alpha g t \tag{2}$$

where V_o is the cask impact velocity. Finally, integrating the velocity with respect to time, we obtain the cask overpack (and lid) displacement, x, at any time after initial impact as

$$x = V_o t - \frac{1}{2}\alpha g t^2 \tag{3}$$

Letting x = 0 be the location of the lid inner surface at the time of primary impact (t = 0), Figure 2 plots the displacement of the lid and contents over time, where the displacement of the contents is given by the expression

$$x = V_o t - \Delta \tag{4}$$

and, Δ, is the gap between the contents and lid. [Note that the positive x direction is actually a downward displacement.] Equating the lid displacement to the contents displacement, one obtains the time, t_i, at which gap closure (secondary impact) occurs as

$$t_i^{\,2} = \frac{2\Delta}{\alpha g} \tag{5}$$

The relative velocity between the contents and lid, v_i, at the time the contents impacts the lid is obtained by subtracting the lid velocity (Equation 2) from the contents velocity, V_o

$$v_i = -\alpha g t \tag{6}$$

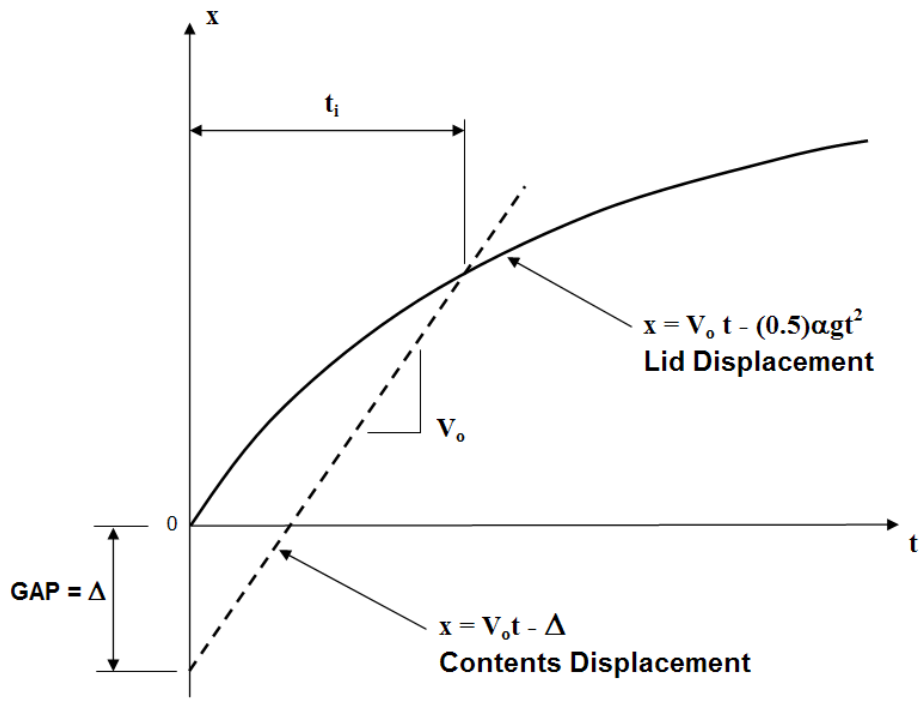

Figure 2: Plot of the displacement of the lid and contents over time

Development of a Simple Dynamic Model

The objective of the paper is to develop a simple dynamic model to estimate the influence of gaps and various other parameters on closure lid response during a drop impact event. This can most easily be accomplished by creating an equivalent single degree of freedom idealization of the problem. Such an approach is a well established technique for the approximate solution of dynamic problems in the areas of impact and impulse loading (Biggs 1964).

A simple representation of the problem, prior to gap closure, is shown in Figure 3. In the figure the uniformly distributed mass of the contents, m, is still moving down with a velocity, V_o. The simply supported lid is moving down with an ever decreasing velocity, v, which is equal to the impact velocity, V_o, minus the reduction in velocity due to cask overpack (lid) deceleration given by Equation (2).

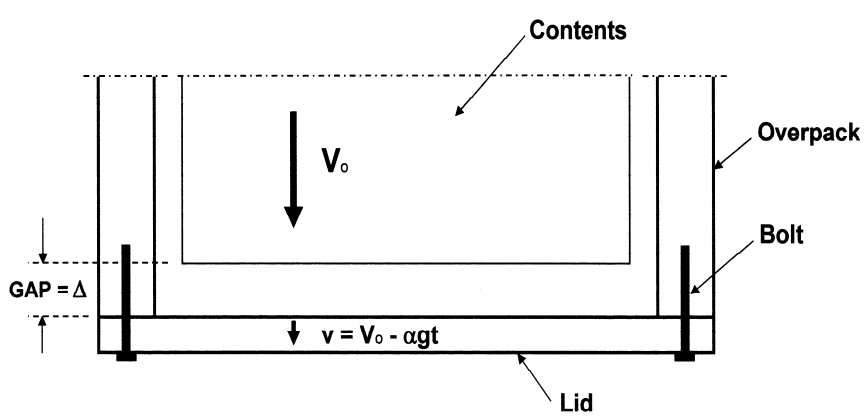

Figure 3: A simple representation of the problem, prior to gap closure.

A general equivalent single degree of freedom system representation is shown in Figure 4, where, K_e, is the equivalent spring stiffness (in general non-linear), M_e, is the total equivalent mass and, F_e, is the equivalent impulse force, which in this case is zero since no impulsive forces are applied to the lid.

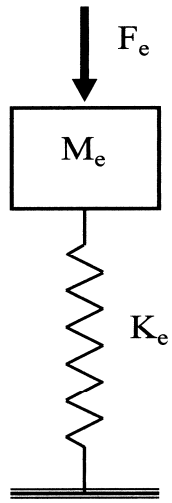

Figure 4: A general equivalent single degree of freedom system

The equivalent mass is the modal mass, which for a continuous uniform mass distribution, m, is given by the expression

$$M_e = \int m\phi(x)^2 dx \qquad (7)$$

where $\phi(x)$ is the assumed-shape function on which the equivalent system is based. For a simply supported circular plate, the unit displacement shape function is

$$\phi(r) = \frac{(R^2 - r^2)}{R^2} \qquad (8)$$

where, R, is the plate radius and, r, is the radial coordinate measured from the center. In cylindrical coordinates the equivalent mass becomes

$$M_e = \iint m\phi(r)^2 r dr d\theta \qquad (9)$$

where, m is the mass per unit area. Substituting Equation (8) into Equation (9) and integrating, the equivalent mass becomes

$$M_e = \frac{m\pi R^2}{3} \qquad (10)$$

which is exactly one third of the total mass, $M = m \pi R^2$. Since both the contents mass and lid mass are uniformly distributed, the equivalent mass of the contents and lid can both be calculated from Equation (10). In addition, since both the contents and lid are undergoing the same deceleration (after gap closure), the total system equivalent mass, M_e, is then the sum of the contents and lid equivalent masses.

The equivalent stiffness is computed for the equivalent spring such that the spring deflection is the same as that of the reference point on the structure for which the shape function is assumed. For the closure lid, the

equivalent stiffness is determined from the maximum displacement of a simply supported circular plate, which is (Timoshenko 1959)

$$z_{\max} = \frac{q(5+\nu)R^4}{64(1+\nu)D} \tag{11}$$

where, q, is the uniform load per unit area, ν, is Poisson's ratio and, D, is the plate flexural rigidity. Equating the maximum plate displacement from Equation (11) caused by a load per unit area equal to, mg, to the displacement of the equivalent system acted upon by a load equal to, $M_e g$, (i.e., $\delta = M_e g/K_e$) one obtains

$$K_e = \frac{64\pi(1+\nu)D}{3(5+\nu)R^2} \tag{12}$$

The equivalent mass together with the equivalent stiffness determine the systems fundamental frequency.

SOLUTION

As mentioned previously, the lid closure system is assumed to remain elastic during the impact event. This allows the equivalent single degree of freedom model of the lid closure system to be treated as a free vibration problem with initial conditions. For this case, the vibratory motion of the mass, M_e, about the static equilibrium position is given by the equation (Timoshenko 1955)

$$x = x_o \cos(\omega t) + \left(\frac{v_o}{\omega}\right)\sin(\omega t) \tag{13}$$

where x_o is the initial displacement of the mass from the static equilibrium position, x = 0, v_o is the initial velocity of the mass at x_o, and ω is the natural frequency of the equivalent system.

The equivalent spring mass system is shown in Figure 5, where M_{ec} and M_{el} are the equivalent masses of the contents and lid respectively and constitute the total equivalent mass of the system, M_e.

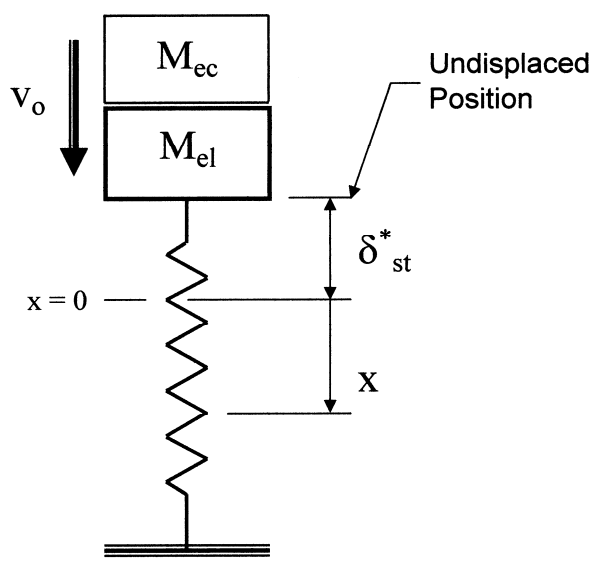

Figure 5: The equivalent spring mass system

The initial displacement, as measured from the static equilibrium position, is

$$x_o = \delta_{st}^* = \alpha\delta_{st} \tag{14}$$

where δ_{st}^* is the static displacement of the mass, M_e, acted upon by a constant deceleration, αg, and, δ_{st} is the static displacement of the total mass, M_e, in a normal gravitational field, g.

The initial velocity, v_o, of the mass, M_e, is the resultant velocity of masses M_{ec} and M_{el} after the fully plastic impact of the contents with the lid, and is given by the equation (Den Hartog 1948)

$$v_o = v_i \frac{M_{ec}}{M_{ec} + M_{el}} \tag{15}$$

where v_i is the relative impact velocity between the contents and lid as given by Equation (6).

The natural frequency, ω, of the equivalent single degree of freedom system is calculated from the equation

$$\omega^2 = \frac{K_e}{M_e} = \frac{\frac{M_e g}{\delta_{st}}}{M_e} = \frac{g}{\delta_{st}} \tag{16}$$

The maximum amplitude of the displacement response as measured from the static equilibrium position, x = 0, can be calculated from Equation (13) and is

$$x_{\max} = \sqrt{x_o^{\,2} + \left(\frac{v_o}{\omega}\right)^2} \tag{17}$$

Substituting Equations (14), (15) and (16) into Equation (17) and using the results from Equations (5) and (6) one obtains the maximum response amplitude:

$$x_{\max} = \alpha\delta_{st}\sqrt{1 + \left(\frac{2\Delta}{\alpha\delta_{st}}\right)\left(\frac{M_{ec}}{M_{ec} + M_{el}}\right)^2} \tag{18}$$

The total displacement of the lid from its undeformed position equals the static displacement of the lid with the attached contents acted upon by the cask deceleration, αg, plus the maximum amplitude of the free vibration response due to the secondary impact of the contents, x_{max} , thus

$$\text{Total Displacement} = \alpha\,\delta_{st} + x_{max} \tag{19}$$

Dividing the total displacement in Equation (19) by the static displacement, $\alpha\,\delta_{st}$, one obtains the Dynamic Load Factor, DLF, for the response of the closure lid due to the presence of a gap between the package contents and the closure lid as

$$DLF = 1 + \sqrt{1 + \left(\frac{2\Delta}{\alpha\delta_{st}}\right)\left(\frac{M_{ec}}{M_{ec} + M_{el}}\right)^2} \qquad (20)$$

which is the desired result. An interesting observation is the fact that if the lid mass is neglected and $\alpha = 1$, Equation (20) reduces to the same result one would obtain for a mass being dropped from a height, Δ, onto an elastic spring (Housner 1959).

RESULTS

To illustrate the result presented in Equation (20) a generic spent fuel transportation package and drop scenario is selected. Assume that when the package is dropped from 30 feet and strikes an unyielding target the impact limiter applies a constant crush force to the package that decelerates the package at a constant rate of 50g's. Let the package itself have the following properties:

Lid:	Steel Material: E = 28,000,000 psi, ν = 0.3, Density = 495 lbs/ft^3
	Radius = 34 inches; Thickness = 2, 4 and 8 inches
Contents:	Total weight = 40,000 lbs
Gap Size :	Gap = 0 to 2.5 inches

Lid Response

In Figure 6 the Dynamic Load Factor (DLF) in Equation (20) is plotted as a function of gap size for three lid thicknesses. The results show the influence of gap size and lid stiffness on the displacement response of the lid. To put the results in context, a DLF = 1.0 represents the static displacement of the lid loaded by the contents under the influence of a gravity field of αg. A DLF = 2.0 represents the total displacement of the lid and contents due to the instantaneous application of a gravity field of αg, and is the maximum response of the lid and contents for the case of a zero gap.

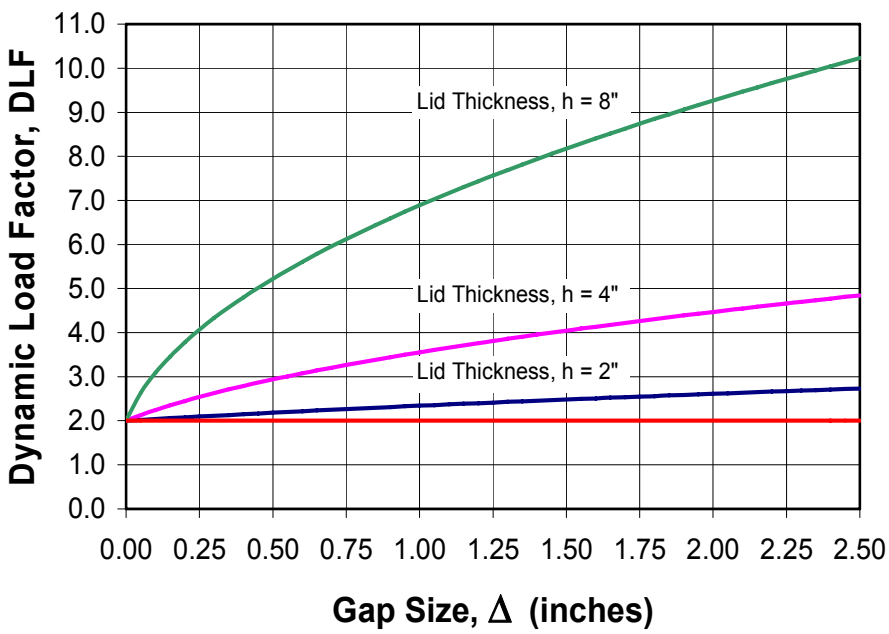

Figure 6: Dynamic Load Factor plotted as a function of gap size for three lid thicknesses

Another factor that also influences response is the mass of the lid, the effect of which is already captured in Equation (20). For an 8" thick lid with a gap greater than 1 inch, for example, neglecting the mass of the lid increases the response by almost 30%, which illustrates the importance of incorporating the inertial effects of the lid in the model.

Support Reactions (Bolt Loads)

For the case of a closure lid loaded impulsively, as in Figure 4, the dynamic reactions of the real structure would have no direct counterpart in the equivalent one-degree system, since the reaction (spring force) of the equivalent system is not the same as the real reaction. This is because the equivalent system was deliberately selected so as to have the same dynamic deflection as the real structure. For the case in Figure 4, the resultant reaction would be obtained by considering the dynamic equilibrium of the system where the inertia force of the mass, M_e, resists (opposes) the impulsive force, F_e. However, this is not the situation in our case.

In our case, as represented in Figure 5, it is the inertia force of the lid and contents that are directly causing the displacement and loading of the spring. The lid and contents inertia force does not resist the motion, it causes the motion. Therefore, the displacement response, DLF, shown in Figure 6 is directly proportional to the support reactions (lid bolt forces).

CONCLUSION

A simple dynamic model has been developed for estimating the effect of a gap between a spent fuel transportation package's contents and closure lid on the response of the lid during a drop impact event. The resulting displacement response of the lid is expressed as the Dynamic Load Factor given by Equation (20) and plotted in Figure 6. The results show the importance of considering the effect of gaps on the drop impact response of transportation packages and can provide a valuable tool to aid designers in developing preliminary package designs.

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SYMBOLS

α	The g magnitude of the cask deceleration
Δ	Gap between the contents and closure lid
δ_{st}^*	Static displacement of the total equivalent mass, M_e, acted upon by a constant deceleration, αg
δ_{st}	Static displacement of the total equivalent mass, M_e, in a normal gravitational field, g
DLF	Dynamic Load Factor for the displacement response of the lid due to the presence of a gap between the package contents and closure lid
D	Plate flexural rigidity, $D = \dfrac{Eh^3}{12(1-\upsilon^2)}$
E	Elastic Modulus
$\phi(x)$	Assumed-shape function on which the equivalent system is based
g	Gravitational constant
h	Closure lid thickness
K_e	Equivalent spring stiffness (in general non-linear)

m	Mass per unit area
M_e	Total equivalent mass
M_{ec}	Equivalent mass of the contents
M_{el}	Equivalent mass of the lid
ν	Poisson's ratio
q	Uniform load per unit area
r	Radial coordinate measured from the center of the lid.
R	Closure lid radius
t_i	Time at which gap closure occurs
v_i	Relative velocity between the contents and lid at the time the contents impacts the lid
v_o	The initial velocity of the equivalent mass, M_e, at x_o, after the fully plastic impact of the contents with the lid
V_o	Cask impact velocity, 44 ft/sec
ω	Natural frequency of the equivalent system
x_o	Initial displacement of the equivalent mass from the static equilibrium position, $x = 0$
x_{max}	Maximum amplitude of the displacement response of the equivalent system as measured from the static equilibrium position, $x = 0$
x	Displacement coordinate as measured from the static equilibrium position

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