

Westinghouse Non-Proprietary Class 3

WCAP-17203-NP  
Revision 0

June 2010

# Fast Transient and ATWS Methodology



**Westinghouse**

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WESTINGHOUSE NON-PROPRIETARY CLASS 3

**WCAP-17203-NP**  
**Revision 0**

## **Fast Transient and ATWS Methodology**

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**June 2010**

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**LIST OF ACRONYMS**

1D	One Dimensional
3D	Three Dimensional
ABWR	Advanced Boiling Water Reactor
ADS	Automatic Depressurization System
AOO	Anticipated Operational Occurrences
APRM	Average Power Range Monitor
ARI	Alternate Rod Insertion
ASME	American Society of Mechanical Engineers
ATWS	Anticipated Transients Without Scram
ATWS-RPT	Anticipated Transient Without Scram Recirculation Pump Trip
BISON	Westinghouse transient analysis code with a one dimensional kinetics model
BPV	Bypass Valve
BWR	Boiling Water Reactor
CCA	Code Capability Assessment
CFR	Code of Federal Regulations
CPR	Critical Power Ratio
CSAU	Code Scaling, Applicability, and Uncertainty
DCD	Design Control Document
DUA	Data Uncertainty Assessment
ECCS	Emergency Core Cooling System
EOC	End of Fuel Cycle
EM	Evaluation Model
FD	Feedwater Flow Decrease
FMCRD	Fine Motion Control Rod Drive
FI	Feedwater flow Increase
FINT	Fuel Assembly Peaking Factor
ICPR	Initial Steady-State Critical Power Ratio
IET	Integral Effect Test
LBLOCA	Large Break Loss of Coolant Accident
LOCA	Loss of Coolant Accident
LHGR	Linear Heat Generation Rate
LRNBP	Load-Rejection No Bypass
MC	Monte Carlo
MCPR	Minimum Critical Power Ratio
MSIVC	Main Steam Isolation Valve Closure
NRC	US Nuclear Regulatory Commission
OLMCPR	Operating Limit Minimum Critical Power Ratio
PCT	Peak Cladding Temperature
PD	Pressure Decrease
PDF	Probabilistic Density Function
PI	Pressure Increase
PIRT	Phenomena Identification and Ranking Table
RCPB	Reactor Coolant Pressure Boundary
RD	Recirculation Flow Decrease

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RI	Recirculation Flow Increase
RPV	Reactor Pressure Vessel
SAFDL	Specified Acceptable Fuel Design Limits
SAR	Safety Analysis Report
SET	Separate Effect Test
SLCS	Standby Liquid Control System
SLMCPR	Safety Limit Minimum Critical Power Ratio
SRP	US NRC Standard Review Plan (NUREG-0800)
SRV	Safety Relief Valve
TCV	Turbine Control Valve
TD	Feedwater Temperature Decrease
TI	Feedwater Temperature Increase
TMOL	Thermal Mechanical Operating Limit
TSV	Turbine Stop Valve
TTMOL	Transient Thermal Mechanical Operating Limit

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# **1 INTRODUCTION**

## **1.1 BACKGROUND AND REPORT OBJECTIVE**

The current Westinghouse NRC licensed methodology for fast transient analyses is described in CENDP-300-P-A, "Reference Safety Report for Boiling Water Reactor Reload Fuel" (Reference 1). While this methodology is applicable for the analysis of fast and slow transients for BWR/2 through BWR/6 reload analysis, it does not address all the transients required for first core applications. The objective of this topical report is to establish a methodology for analyzing both limiting and non-limiting fast transients in initial and reload cores for currently operating BWRs and new ABWRs as required by initial Safety Analysis Report (SAR) work. The methodology enclosed in this topical report applies to covers fast transients as the current methodology for slow transients documented in Reference 1 is still valid.

This topical report describes the methodology of the evaluation model, as recommended in NRC guidance (i.e. Reference 2), for non-limiting and limiting Anticipated Operational Occurrences (AOOs) including Anticipated Transients Without Scram (ATWS) and Infrequent Events. An AOO, as defined in 10CFR50 Appendix A, is a condition of normal operation that is expected to occur one or more times during the life of the nuclear power unit (Reference 3). Because ATWS events are considered AOOs that are followed by a failure of the scram protection system, they are addressed separately in the methodology (Reference 4). Whether an event is defined as an AOO, infrequent event or postulated accident is defined in plant specific documentation and is therefore not included in this methodology. Instead licensing basis documents for each plant will define categorization of each condition. The categorization of each event will determine which event acceptance criteria are evaluated.

The methodology described in this topical report is code independent and is applicable to both 1D and 3D transient analysis codes. As such, the complete evaluation model will utilize this document, a code dependent topical and other supporting licensing basis documents.

## **1.2 OVERVIEW OF THE TOPICAL REPORT**

Figure 1-1 illustrates the basic outline of this topical report.

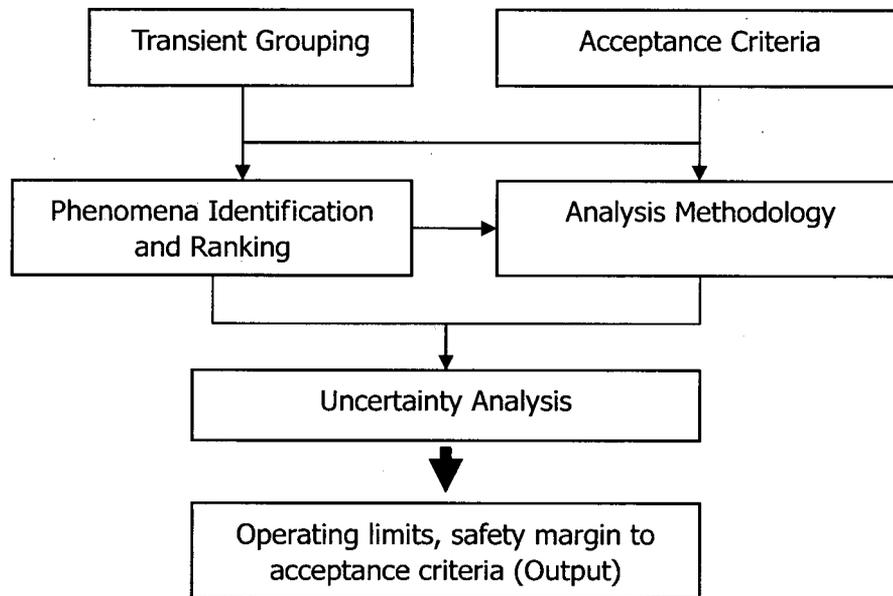


Figure 1-1 Overview of Topical Report

This topical report describes the methodology portion of the evaluation model, as defined in NRC guidance, for fast transients analyzed using a 1D or 3D dynamic transient analysis computer code. This methodology covers the fast transients defined in the Standard Review Plan (NUREG-0800, Reference 4) in order to meet the requirements of the General Design Criteria (GDC). Both limiting and non-limiting fast transients, including ATWS are included. For the limiting fast transients, the methodology is consistent with Reference 1.

The transients grouped in Chapter 15 of the SRP are re-grouped in an equivalent manner in this methodology. Using this transient grouping and the acceptance criteria for AOOs including ATWS events as outlined in Chapter 4 and Chapter 15.8 of the SRP, a Phenomena Identification and Ranking Table (PIRT) is created (References 5, 7, and 8). Transient grouping and acceptance criteria serves as input to the PIRT, which in turn serves as input into the analysis methodology. This PIRT table defines the phenomena which have to be addressed when evaluating the operating limits and safety margins to acceptance criteria. The validity of the PIRT contained herein has been verified by Westinghouse methodology experts. Ranking of the phenomena is on a High/Medium/Low scale, based on its influence on the figures-of-merit (defined in the Acceptance Criteria Section). The ranking is performed for each transient group. High ranked phenomena, together with Acceptance Criteria, Transient Grouping and Plant Specification are the complete input to Analysis Methodology and Uncertainty Analysis Sections.

Using the PIRT table as input, the analysis methodology describes the evaluation process for determining the operating limits and/or the safety margins to acceptance criteria.

Once operating limits and safety margins to acceptance criteria are determined for AOOs, uncertainty analysis is conducted to evaluate the impact of uncertainties and biases on these limits in order to account for the uncertainty in the best-estimate result. These uncertainties and biases result from input and modeling parameters and are evaluated using a Monte Carlo based method.

### **1.3 SUMMARY AND CONCLUSIONS**

The methodology discussed in this topical report is applicable to fast transient AOOs including ATWS and Infrequent Events that can occur in ABWRs as well as BWR/2 through BWR/6 designs. It serves to update the current fast transient and ATWS methodology to make it applicable to first cores and license a new Monte Carlo based uncertainty analysis.

This methodology ensures that AOOs and Infrequent Events meet the acceptance criteria related to the GDCs contained in 10CFR50 Appendix A by using NRC approved transient analysis codes and applying adequate uncertainties and biases using a Monte Carlo Method.

Westinghouse recognizes that this methodology does not fully meet the evaluation model requirements outlined in NRC guidance. Instead, this topical report serves only as a generically applicable methodology portion of a plant specific evaluation model. The final evaluation model for a plant will reference this document as well as other licensing documents, including a code specific topical report.

### **1.4 SCOPE OF REVIEW**

Westinghouse requests NRC review and approval of:

- The ranking designations in the PIRT enclosed
- Analysis Methodology for evaluating fast transients
- Monte Carlo based uncertainty analysis methods

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## **2 TRANSIENT GROUPING AND PLANT SPECIFICATION**

### **2.1 PLANT SPECIFICATION**

This methodology is applicable to Advanced Boiling Water Reactors (ABWRs) and BWR/2 through BWR/6 designs.

### **2.2 TRANSIENT SCENARIO SELECTION AND CATEGORIZATION ACCORDING TO EVENT TYPE**

The fast transients are divided in groups to facilitate the identification and ranking of phenomena influencing the figures-of-merit. The categorization of events according to types used for this topical report is equivalent to the grouping used in Chapter 15 of the SRP, but allows for one transient group to include more than one SRP Chapter 15 event type.

To facilitate the methodology development, the AOO events categorized in NRC guidance are grouped into the following four categories:

- Pressure increase/decrease (PI/PD)
- Reactor coolant flow increase/decrease (RI/RD)
- Feedwater flow increase/decrease (FI/FD)
- Reactor coolant temperature increase/decrease (TI/TD)

The group of events is defined by its phenomenological effect on the plant. In this way the potentially important phenomena that govern the transient response in each group are easily identified. The events included in each group are shown in Table 2-1 with a cross reference to the SRP section that describes the event. Main sections in the SRP may envelope various specific transients. For example, Section 15.1.3 "Increase in steam flow" include both pressure regulator and safety/relief valve malfunctions.

Table 2-1 Transient event categorization

Event category	Abbr.	SRP event
Pressure increase/decrease	PI/PD	15.1.2 Increase in feedwater flow 15.1.3 Increase in steam flow 15.1.4/15.6.1 Inadvertent opening of a safety/relief valve 15.2.1 Loss of external load 15.2.2 Turbine trip 15.2.3 Loss of condenser vacuum 15.2.4 Closure of main steam isolation valve 15.2.5 Steam pressure regulator failure (closed) 15.2.6 Loss of nonemergency AC power to station auxiliaries
Reactor coolant flow increase/decrease	RI/RD	15.3.1 Loss of forced reactor coolant flow including trip of pump motor and flow controller malfunctions 15.3.3 Reactor coolant pump rotor seizure 15.3.4 Reactor coolant pump shaft break 15.4.4 Startup of an inactive recirculation pump at an incorrect temperature 15.4.5 Flow controller malfunction causing an increase in core flow rate
Feedwater flow increase/decrease	FI/FD	15.2.7 Loss of normal feedwater flow
Reactor coolant temperature increase/decrease	TI/TD	15.1.1 Decrease in feedwater temperature 15.5.1 Inadvertent startup of ECCS

Based on the frequency of initiating events, the categorization of transients occurs on a plant design specific basis. Therefore variations may occur. For example, the transient "Trip of all recirculation pumps" may be categorized as a postulated accident in the ABWR while it is identified as an AOO in BWR/2-6 plants. These variations are the result of plant design features.

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Specific transients and their appropriate frequency of occurrence classification are established in the plant licensing bases and documented in the SAR or design control document (DCD). Whether an event is classified as an AOO or a postulated accident does not affect the phenomena identification and ranking tables.

AOOs and infrequent events capture the phenomena classified in the PIRT that occur during the initiating event of an ATWS prior to reactor scram. Phenomena that result after an initiating event and after the failure of scram are considered under the ATWS column in the PIRT.

ATWS events evaluated are mitigated by the following manual and automatic shutdown scenarios:

- Reactor shutdown by Alternate Control-Rod Insertion (ARI), as stated in 10CFR50.62
- Reactor shutdown by Fine-Motion Control Rod Drive (FMCRD) run-in<sup>1</sup>
- Reactor shutdown by activation of Standby Liquid Control System (SLCS)

The first two ATWS scenarios do not introduce any new challenging phenomena that are not already considered in the third ATWS scenario, or in the AOO initiating events.

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<sup>1</sup> If this feature is included in the design.

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### 3 ACCEPTANCE CRITERIA

#### 3.1 AOO ACCEPTANCE CRITERIA

The following acceptance criteria for AOOs have been defined to meet the requirements related to the GDC for nuclear power plants specified in Appendix A to 10CFR50. Limits set forth in the SRP are used to verify acceptance of the analysis:

1. **Radioactive effluents** – The limits for radioactive effluents are those contained in 10CFR20 and 10CFR100.
2. **Specified Acceptable Fuel Design Limits** – SAFDLs, which are addressed in GDC 10, are divided into limits on Minimum Critical Power Ratio (MCPR), Linear Heat Generating Rate (LHGR) for clad strain and fuel centerline temperature, and peak fuel enthalpy.

The MCPR safety limit is used as an event acceptance limit to protect the fuel cladding from overheating as documented in SRP Section 4.4. The specific value for this limit is core and fuel design dependent and is identified for each plant cycle application.

The overpower LHGR limit protects the fuel cladding from exceeding 1% plastic strain and the fuel pellet from centerline melting as required by SRP Section 4.2. The specific value for this limit is fuel design dependent.

The fuel enthalpy limit is consistent with SRP Section 4.2.

3. **Peak Reactor Vessel Pressure** – The ASME Code Section III (Reference 6) upset limit of 110% of the Reactor Pressure Vessel (RPV) design pressure is used as peak pressure limit for AOO as recommended in SRP Section 4.2.
4. **Suppression Pool Temperature** – The suppression pool temperature is used as an acceptance limit for AOOs to assure that the suppression pool is available to function as a heat sink for events involving operation of the safety relief valves. The heat capacity temperature limit identified in the plant specific Emergency Operating Procedures is used for this limit.

##### 3.1.1 ATWS acceptance criteria

The BWR should meet the following acceptance criteria to demonstrate plant compliance with the requirements of 10CFR50.62. Criteria described in the SRP are used to verify that acceptable limits have been met during ATWS analysis:

1. **Fuel Integrity** – As stated in SRP Section 4.2, the long-term core cooling capability is assured by meeting the cladding temperature and oxidation criteria of 10CFR50.46 (i.e., peak cladding temperature not exceeding 1204°C (2200°F), and the local oxidation of the cladding not exceeding 17% of the total cladding thickness).

2. **Primary System** - The RPV integrity is assured by limiting the maximum primary stress within the Reactor Coolant Pressure Boundary (RCPB) to the emergency limits as defined in the ASME Code, Section III. RPV integrity is required by GDC 15 of 10CFR50 Appendix A.
3. **Containment Integrity** - The long-term containment capability is assured by limiting the maximum containment pressure to the design pressure of the containment structure and the suppression pool temperature to the wetwell design temperature in order to ensure compliance with GDC 31.
4. **Long-Term Shutdown Cooling** - Subsequent to an ATWS event, the reactor shall be brought to a safe shutdown condition, and be cooled down and maintained in a cold shutdown condition as required by GDC 35.

### 3.2 FIGURES-OF-MERIT

Figures-of-merit, as defined in SRP Section 15.0.2 are quantitative standards of acceptance that are used to define acceptable answers for safety analysis. In this methodology, figures-of-merit are the evaluation criteria used to judge the relative importance of each phenomenon. For the transients being considered in this topical report, the figures-of-merit are derived from the regulatory requirements corresponding to the acceptance criteria discussed earlier. The figures-of-merit for this methodology are summarized in Table 3-1.

Table 3-1 Figures-of-merit for AOO and ATWS

Transient class	Figures-of-merit
AOO	Minimum critical power ratio (MCPR) Reactor vessel pressure (RVP) Linear heat generation rate (LHGR)
ATWS	Cladding temperature Reactor vessel pressure (RVP) Mass and energy release to containment

For AOOs, the figures-of-merit are taken to be MCPR, RVP and LHGR. For phenomena identification purposes there is no need for evaluation against fuel enthalpy because, for the transients of interest, the fuel enthalpy limit is met if the MCPR safety limit is met. Thus, a fuel enthalpy analysis will not introduce any new phenomena that are not already covered by the MCPR figure-of-merit.

Hence, MCPR, LHGR and RVP are the relevant figures-of-merit to capture all important phenomena associated with AOO acceptance criteria.

Containment integrity and radiological consequences are not included in the scope of the current methodology for fast transients. Phenomena of importance for containment response or the radiological consequences are evaluated separately in containment analysis that is beyond the scope of this topical.

The figures-of-merit for ATWS are the cladding temperature, the RVP, and the mass and energy released to the containment. For phenomena identification, the cladding temperature is an acceptable figure-of-merit for the fuel integrity acceptance criteria defined in Section 3.1.1 (i.e., cladding temperature and oxidation criterion). The energy and mass release to the containment was defined as a figure-of-merit for the containment integrity acceptance criteria. The transient code calculates mass and energy released to containment during an ATWS. This information is passed on to a containment analysis code to ensure containment acceptance criteria are met.

## 4 PHENOMENOLOGICAL DESCRIPTION

Because each transient scenario is plant specific, a general description of the transient groups this methodology pertains to is provided in the subsequent sections. Plant specific scenarios will be described in other licensing documents such as the SAR, DCD or License Amendments.

### 4.1 PRESSURE INCREASE/DECREASE

The pressure increase/decrease group covers transients that generate a pressure increase transient or a pressure decrease transient.

Since the phenomena involved in pressure increase and pressure decrease are very similar, these transients are included in the same group. However, the transient scenario is different and therefore they are described in two different sections below.

#### 4.1.1 Pressure Increase

Transients that will generate pressure increase are summarized in Table 4-1:

Table 4-1 Pressure Increase Transients

SRP section	Description
15.1.2	Increase in feedwater flow
15.2.1	Loss of external load
15.2.2	Turbine trip
15.2.3	Loss of condenser vacuum
15.2.4	Closure of main steam isolation valves
15.2.5	Steam pressure regulator failure (closed)
15.2.6	Loss of non emergency AC power to station auxiliaries

A pressure increase transient is initiated by a valve position disturbance (closing) in the steam lines that causes a decrease in steam flow. The pressure in the reactor pressure vessel will after a very short time start to increase. This will cause a pressure increase wave to travel backwards through the steam lines that will reach the RPV. When the pressure starts to increase in the RPV the core inlet subcooling will increase and cause the boiling boundary in the core to move upwards, resulting in a core average void decrease. Due to negative void reactivity feedback the fission power will increase. The larger the pressure increase, the larger the subcooling increase, and the larger the power increase.

The core average void decrease will be interrupted by one or several of the following phenomena in the core:

- If no actions are taken by the plant safety systems, the fuel heat flux will increase with a time delay based on the fuel rod time constant. This increase in heat flux, resulting from an increase in heat generation in the fuel will eventually generate more boiling. The increase in heat flux interrupts the core average void decrease effect of the transient and causes the core void average to increase. This increase results in a reduction of fission power, eventually causing a fuel surface heat flux decrease causing an increased CPR.
- If the reactor scrams, power will be rapidly reduced starting from the core bottom due to the insertion of control rods. Due to the negative reactivity inserted into the core by the control rods, reactor fission power decreases while CPR starts to increase.
- If recirculation flow is rapidly reduced by a fast change in recirculation pump speed (e.g. in an ABWR) a recirculation pump head decrease will occur resulting of the inversion of the core void fraction decrease to a core average void fraction increase. Because the loss of pump head results in a reduced core flow, the boiling boundary will move closer to the core or bundle inlet where less energy is required to generate voids.

#### 4.1.2 Pressure Decrease

Table 4-2 lists the transients classified as pressure decrease transients.

Table 4-2 Pressure Decrease Transients

SRP classification	Description
15.1.3	Increase in steam flow
15.1.4/15.6.1	Inadvertent opening of a safety/relief valve

A pressure decrease transient results when steam flow is increased, an event usually caused when a valve in the steam line starts to increase its flow area or when one or several Safety or Relief Valves that inadvertently open.

In some cases, as in the BWR/3 Pressure Regulated Failure, the pressure decrease will lead to the MSIV closure and the transient becomes a pressure increase transient. When this occurs, the methodology for pressure increase is applied.

In all cases a pressure decrease occurs at the location where steam flow increases. Pressure decrease results in decreased core inlet subcooling. This decrease in core inlet subcooling causes the core boiling boundary to move downwards in the core. This fast change in boiling length generates a core average void increase and a fission power decrease. The larger the pressure decrease, the larger the subcooling decrease and the larger the power decrease.

The core average void increase will be interrupted by one or several of the following phenomena in the core.

- If no actions are taken by the plant safety systems, the heat flux from the fuel will start to decrease with a time delay depending on the fuel rod time constant. This decrease in heat flux (expressed as an increase in CPR) will eventually generate less boiling and will eventually result in a stabilized new steady void fraction because the core average void increase will cease.
- If reactor scram is initiated, power will be rapidly reduced starting from the core bottom due to the insertion of negative reactivity by the control rods. This reduces reactor fission power, causing the CPR to start to increase.
- If recirculation flow is rapidly reduced by a fast change in recirculation pump speed, a fast recirculation pump head decrease will occur, causing the core average void fraction to increase even further. This decrease in core flow moves the boiling boundary move closer to the core or bundle inlet (less energy is required to generate void).

## 4.2 REACTOR COOLANT FLOW INCREASE/DECREASE

This transient group covers transients that generate either a coolant flow increase or a coolant flow decrease.

Since the phenomena involved in reactor coolant flow increase and reactor coolant flow decrease are very similar, these transients are included in the same group. However, the transient scenario is different and therefore they are described in two different chapters below.

### 4.2.1 Reactor Coolant Flow Increase

Transients that generate reactor coolant flow increases are presented in Table 4-3:

Table 4-3 Reactor Coolant Flow Increase Transients

SRP section	Description
15.4.4	Start up of an inactive recirculation pump
15.4.5	Flow controller malfunction with increasing core flow

A coolant flow increase transient, usually resulting from start up of an inactive recirculation pump or an operator/controller error, causes the amount of water flowing into the reactor core to increase, resulting in a gradual power increase. This gradual power increase results from a decrease in the core average void.

The core average void decrease will be interrupted by one or several of the following phenomena in the core:

- If no action is taken by the core safety systems, the core average void will continue to decrease, causing power to increase. As power increases, the steam flow increased leading to an increased

pressure in the steam dome, thus resulting in a large steam line pressure drop. This event then becomes equivalent to a pressure increase scenario.

- If a turbine trip occurs, all turbine stop valves close, resulting in a sudden decrease in steam flow. This sudden decrease results in a pressure increase in the nuclear systems and a core void decrease to occur. This core void decrease results in an increase in power as the neutron flux increases. Eventually an over power situation will lead to a reactor scram.
- If reactor scram is initiated, power will be rapidly reduced starting from the core bottom and due to the negative reactivity inserted into the core by the control rods resulting in an increase in CPR.

#### 4.2.2 Reactor Coolant Flow Decrease

Transients generating reactor coolant flow decreases are specified in Table 4-4.

Table 4-4 Reactor Coolant Flow Decrease Transients

SRP section	Description
15.3.1	Loss of forced reactor coolant flow including trip of pump motor and flow controller malfunctions
15.3.3	Reactor coolant pump rotor seizure
15.3.4	Reactor coolant pump shaft break

Reactor coolant flow decrease transients, such as those caused by a trip of a pump or a flow controller malfunction results in a smaller amount of coolant entering the reactor core. This causes the core average void to increase, resulting in a reduction of power.

The effects this transient can have on the core are:

- Fuel specifications and limits being exceeded because of a mismatch caused by a rapid change in flow. If flow is decreased rapidly, heat cannot be transferred out of the fuel to the coolant.
- A large decrease in CPR for plants such as the ABWR that have internal recirculation pumps while only a small decrease in CPR for BWRs with large recirculation pump inertia.

---

### 4.3 FEEDWATER FLOW INCREASE/DECREASE

#### 4.3.1 Feedwater Flow Increase

SRP Section 15.1.2 discusses transients that will generate feedwater increases.

A feedwater flow increase transient, usually resulting from an operator/controller error, causes the amount of water flowing into the reactor core to increase, which causes excess heat removal. This excess heat removal causes the moderator void and temperature to decrease, reducing the CPR in the core.

The core average void decrease will be interrupted by one or several of the following phenomena in the core:

- If no action is taken by the core safety systems, the core average void will continue to decrease, causing power to increase. As power increases, the steam flow increases leading to an increased pressure in the steam dome. CPR is also reduced as a result of the increase in power.
- If a turbine trip occurs, all turbine safety valves close, resulting in a sudden decrease in steam flow. This sudden decrease results in a pressure increase in the nuclear systems and a core void decrease to occur. This core void decrease results in an increase in power as the neutron flux increases. Because of the pressure increase, this transient is then treated like a pressure increase transient, as discussed in Section 4.1.1.
- If reactor scram is initiated, power will be rapidly reduced starting from the core bottom and due to the negative reactivity inserted into the core by the control rods resulting in an increase in CPR.

#### 4.3.2 Feedwater Flow Decrease

Loss of normal feedwater flow, as covered in SRP 15.2.7, is a condition that results in a feedwater flow decrease transient.

During a loss of feedwater transient, the downcomer water level decreases, while the temperature increases to a value close to saturation. This causes the core inlet temperature to also increase. The decreasing water causes a reduction in the core inlet subcooling, increasing the core average void while decreasing the power.

The following core effects are possible:

- Decreased power will also reduce the fuel temperature. This fuel temperature decrease will slightly limit the power reduction due to positive Doppler feedback.
- At low water level signal actuation in the downcomer, one or more auxiliary feedwater system starts and a recirculation pump runback or trip as well as reactor scram may be initiated.

## 4.4 REACTOR COOLANT TEMPERATURE INCREASE/DECREASE

### 4.4.1 Reactor Coolant Temperature Increase

Transients that involve a loss of normal feed water flow, as discussed in SRP 15.2.7, will result in a reactor coolant temperature increase.

This group of transients phenomenological descriptions and core impacts are the same as for the transient group "Feedwater flow decrease" described in Section 4.3.2

### 4.4.2 Reactor Coolant Temperature Decrease

Transients that will generate reactor coolant temperature decrease are summarized in Table 4-5:

Table 4-5 Reactor Coolant Temperature Decrease Transients

SRP section	Description
15.1.1	Decrease in feedwater temperature
15.5.1	Inadvertent startup of ECCS

Transients of this nature, caused by a change in feedwater temperature or the inadvertent startup of the ECCS, initially lead to a decreased temperature in the downcomer. This temperature decrease will increase core subcooling leading to a reduction in the core average void. Power is increased, causing steam flow to increase. This transient will have the following effects on the reactor:

- As more steam is formed, the pressure in the steam dome will increase.
- A high APRM may cause the reactor to scram. Operator action may also result in a scram.

---

## **5 PHENOMENA IDENTIFICATION AND RANKING**

### **5.1 INTRODUCTION**

One of the basic steps in NRC guidance methodology is the development of a PIRT, which consists of identifying and ranking dominant phenomena during a transient with respect to their influence on the figures-of-merit. Upon completion of the PIRT, computer code selection occurs and the qualification basis of the model is evaluated to ensure the computational code is adequately qualified for the intended application. The PIRT is used to guide uncertainty analysis and/or in the model adequacy evaluation.

### **5.2 PURPOSE**

This chapter describes the PIRT developed for the evaluation of ABWR and BWR/2-6 designs infrequent events and AOOs including ATWS events. The PIRT considers phenomena as it impacts plant transient analysis.

Phenomena definitions are provided in Appendix A. While some phenomena are considered in other applications of a PIRT, others are not. Phenomena, not originally considered in previously approved PIRTs, such as those in Reference 5, 7, and 8 were developed based on the expert opinions and engineering judgment of a panel of methodology experts. The PIRT enclosed in this topical includes the identification of plant components according to their impact on the figures-of-merit. Components that have little or no bearing on the safety parameters of interest are not included in the PIRT.

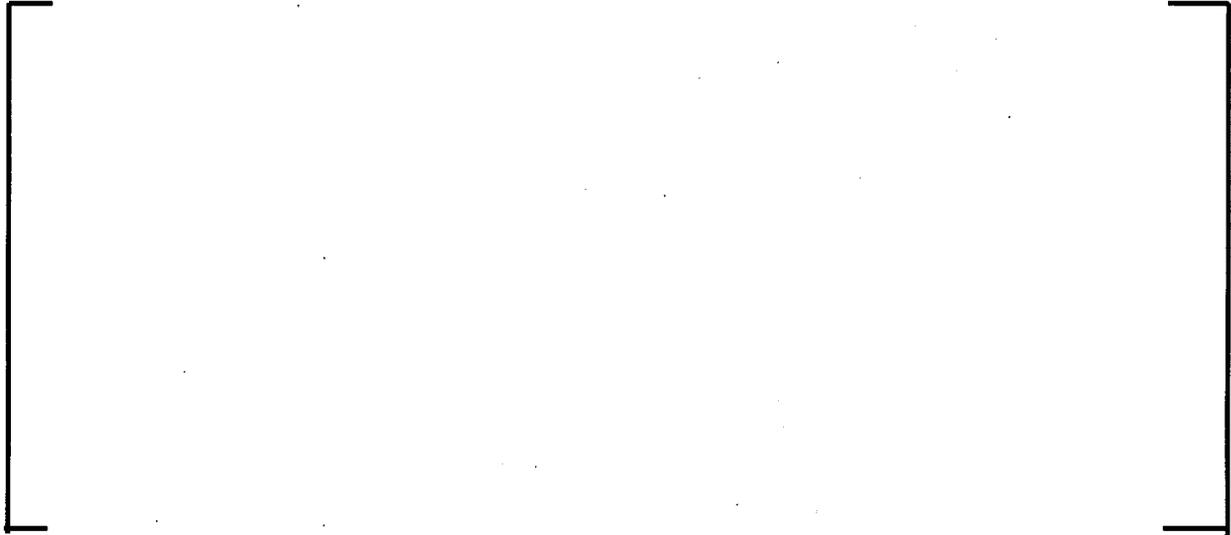
### **5.3 PHENOMENA IDENTIFICATION AND RANKING TABLE**

The PIRT includes phenomena that are related to the reactor kinetics and reactivity response, mass/energy release to the containment, and transient thermal analysis of the fuel rod.

Phenomena captured in the PIRT are limited to fast transient and ATWS scenarios in BWR plants. Because many phenomena occur in both AOOs and ATWS events, one PIRT table was developed. However, due to the different figures-of-merit used in AOO and ATWS analysis, rankings of some phenomena differ for the two event scenarios. Specific ATWS phenomena not already considered for AOOs are captured in the PIRT. Specific phenomena are identified in Table 5-1.

Table 5-1 Specific ATWS Phenomena/Components

a,c



The PIRT is divided into the following eight categories:

1. initial conditions,
2. transient power distribution,
3. steady state and transient cladding to coolant heat transfer and core spray heat transfer,
4. transient coolant conditions as a function of elevation and time,
5. fuel rod response,
6. multiple rod mechanical effects,
7. multiple rod thermal effects, and
8. plant component data.

Each phenomenon is assigned an “importance” grade corresponding to their influence on the figures-of-merit. Importance is assigned based on a High/Medium/Low scale. High (H) implies that the phenomenon has a dominant impact on the figures-of-merit. Medium (M) implies that the phenomenon has a moderate impact on the figures-of-merit. Low (L) implies that the phenomenon has a minimal or no impact on the figures-of-merit. Rankings are tabulated according to one of the four transient classes discussed in Section 2.2. Rationale for each ranking is provided for each phenomenon considered.

Table 5-2 PIRT for fast transients and ATWS

a,c









## 5.4 APPLICATION OF PIRT

The PIRT is used to determine the requirements for the physical model development, scalability, validation, and uncertainty and sensitivity studies. Explicit and accurate modeling of High (H) ranking phenomena is required because of the impact each has on the figures-of-merit. Because Medium (M) only moderately affect figures-of-merit, only a moderate degree of accuracy is required during the modeling phase of the analysis. Low (L) implies that the phenomenon has a minimal or no impact on the figures-of-merit. Rankings captured in this PIRT are used to guide the uncertainty analysis, as discussed in Section 7.

---

## 6 ANALYSIS METHODOLOGY

This section describes the analysis methodology for each transient group. The evaluation process consists of the following parts:

- Definition of limiting condition that drive the analysis based on plant operating conditions.
- Evaluation of core and fuel operating limits.
- Parameter selection process.
- Specification of input parameters to uncertainty analysis.

### 6.1 LIMITING PLANT STATES AND EVENTS

Each potentially limiting transient and ATWS event is evaluated for the limiting plant condition(s) throughout the plant operating domain. A single operating state or operating condition can conservatively bound all other possible states. Verification that this condition is bounding allows for the use of a bounding analysis during first core and reload safety analyses since performing an event analysis for limiting operating states and conditions conservatively bounds all other operating states and conditions.

### 6.2 FUEL AND CORE OPERATING LIMITS

Fuel and core operating limits consist of different limiting parameters that cannot be violated during operation of the plant, such as the Operating Limit Minimum Critical Power Ratio (OLMCPR) and LHGR limitations introduced by transient overpower.

Other operating limits consist of maximum or minimum allowed transient parameter values that are shown to not be violated during limiting transients, such as RPV pressure expressed as the percentage of the design RPV pressure.

The core operating limits are included in the Core Operating Limits Report and are established on a cycle-specific basis considering the contribution of both fast and slow transients. In the following sections the methodology for determining the operating limits for the fast transients is described.

#### 6.2.1 Operating Limit MCPR

##### Methodology

[

] <sup>a,c</sup>

[ ]<sup>a,c</sup>

[ ]<sup>a,c</sup>

For each analyzed AOO a transient change in CPR, ΔCPR, and a minimum transient CPR (CPRmin) is calculated using an NRC approved analysis code. [

] <sup>a,c</sup> Determination of transient specific full power MCPR uses the definition of CPR and the actual power, as shown in Equations 1 and 2.

$$[ ]^{\text{a,c}} \tag{1}$$

$$[ ]^{\text{a,c}} \tag{2}$$

[ ]<sup>a,c</sup>

The transient change is proportional to ICPR which allows for the calculation of OLMCPR(i) for a specific transient i by applying Equation 3:

$$[ ]^{\text{a,c}} \tag{3}$$

which can be rewritten as

$$[ ]^{\text{a,c}} \tag{4}$$

Discussion

[ ]<sup>a,c</sup>

$$[ \dots ]^{a,c} \quad (5)$$

**6.2.1.1 MCPR Operating Limit with Uncertainty**

Methodology

The following procedure is followed when evaluating the OLMCPR with uncertainty:

- [ ]<sup>a,c</sup> See Table B-1 for details.
- Transient calculation is performed, [ ]<sup>a,c</sup> is calculated according to Equation 4.
- OLMCPR(i) for transient i is then calculated by utilizing [ ]<sup>a,c</sup>

**6.2.2 LHGR Operating Limit**

Methodology

The plant LHGR operating limit is specified for each fuel type present in a given cycle. The plant LHGR operating limit is the most restrictive of:

1. [ ]<sup>a,c</sup>

\*The choice of the estimator grade depends on several factors and is decided by the type of analysis performed. It is a trade-of between the computational time and the risk of the overconservatism when evaluating the operating limits or the safety margins to acceptance criteria. With an increasing estimator grade, the number of calculations increases in the same time as the risk of overconservatism decreases.

[

] <sup>a,c</sup>

The LHGR change is determined by the same NRC approved evaluation model as used for transient CPR.

[

] <sup>a,c</sup>

[

] <sup>a,c</sup>

(6)

[

] <sup>a,c</sup>

### Discussion

[

] <sup>a,c</sup>

## **6.2.3 Peak Pressure**

The overpressurization protection analysis is a Special Event conservatively analyzed to address the adequacy of the plant's pressure relief system. Special Events are evaluated to probe a plant capability, as per current methodology.

### **6.2.3.1 Design Bases**

#### Basis

The plant overpressure protection system capability shall be confirmed adequate for the cycle specific reload. The specific plant licensing basis ASME code overpressure protection design limit, as identified in Reference 6, shall not be exceeded.

#### Discussion

Potentially limiting plant overpressurization events are analyzed to confirm that the reactor pressure limit is not exceeded. The maximum pressure acceptance limit shall be that limit established in the plant

licensing basis. For most BWRs, a conservative upset condition limit of 110% of design pressure is used in the code overpressure protection analysis, as stated in SRP Section 4.2.

### 6.2.3.2 Overpressurization Protection Methodology

#### Methodology

[ ]<sup>a,c</sup> This event is used to confirm the adequacy of the plant's pressure relief system prior to each reload cycle. The following evaluation procedure for this event is consistent with ASME evaluations:

- [ ]

] <sup>a,c</sup>

The overpressurization MSIV closure event is analyzed with the NRC approved dynamic analysis methods. The plant model developed for rapid pressurization events analysis is also used for calculating the ASME overpressurization event.

[ ]

] <sup>a,c</sup>

#### Discussion

The overpressurization MSIV closure event could be treated as an emergency condition consistent with the current version of the ASME code (Reference 6), with acceptable results compared to the ASME emergency condition limits (i.e., the reactor pressure acceptance limit of 120% of design pressure). However, the current approach [ ]

] <sup>a,c</sup>

---

[ ]<sup>a,c</sup> Because of the conservatism in this approach, and conservatism assumed in the event conditions, no other failures are assumed.

## 6.3 ANALYSIS CODES

All transient events can be grouped into fast and slow transients based on the dynamic characteristics of the transient. Fast transients are those events of relatively short duration such that the impact of the spatial and temporal dynamics on the system nuclear and thermal-hydraulics is important to the overall plant response. Slow transients are defined as those transients where the dynamic changes during the transient are sufficiently slow so that the assumption that steady state conditions are achieved at each time step is either realistic or conservative. Slow transient evaluation methodology is captured in Reference 1 and is therefore beyond the scope of this topical report.

For fast transients a dynamic analysis code is required.

### 6.3.1 Fast Transients

#### *Methodology*

For fast transients, an NRC approved system dynamic computational code will be used to analyze 1D or 3D analysis simulations.

#### 6.3.1.1 1D Analysis

##### *Discussion*

Current Westinghouse methodology for analyzing fast transients consists of using a one-dimensional (1D) code. [

]<sup>a,c</sup>. This collapsing process is generic and applies to both Westinghouse and non-Westinghouse fuel designs.

Fuel rod performance and thermal-hydraulic data is simulated using thermal mechanical and thermal hydraulic modeling codes. Data obtained from these simulations can be integrated into the 1D transient analysis code.

[

]<sup>a,c</sup>

The fuel rod performance code uses radial heat conduction to determine the fuel rod time constant, which influences the peak and integrated core power and maximum heat flux in fast pressurization events. [

]<sup>a,c</sup>

---

An average channel core hydraulic model is derived from the three dimensional model used for the nuclear and thermal-hydraulic evaluations. [

]a,c

### **6.3.1.2 3D Analysis**

#### *Discussion*

For the analysis of fast transient using a 3D kinetics dynamic code, all nuclear and thermal-hydraulic data used to simulate the three-dimensional situation can be taken directly from a static core simulator. Thus, no 3D to 1D core collapse is required.

In addition, fuel rod performance is simulated with a built-in approved thermal mechanical model.

---

## 6.4 ANALYSIS METHODOLOGY

### 6.4.1 Pressure Increase/Decrease

#### 6.4.1.1 Analysis Code Requirements

Both pressure increase and pressure decrease transients have the same analysis code requirements. The requirements of the analysis code capability are based on the phenomenological descriptions used as the basis for the PIRT (Table 5-2). The data uncertainty assessment (discussed later in Section 7) may require that conservative input data is selected for the transient code analysis methodology. The requirements of the physical evaluation model, based on the phenomenological description, are the following:

- [

] a,c

To ensure that the condition captured in the PIRT is valid, confirmatory analysis will be performed to ensure that:

1. The limiting case that utilizes the PIRT captures unique and significant plant specific design features.

- 
2. The analysis of transient events include combinations of the transient categories defined herein if these conditions are more limiting than the conditions analyzed in the PIRT.

#### **6.4.1.2 Pressure Increase**

Transients belonging to this group are described in Table 4-1.

##### **6.4.1.2.1 Pressure Increase Transient Methodology**

###### Methodology

The analysis methodology is based on the PIRT (Table 5-2), the code capability assessment (Section 7.2) and the data uncertainty assessment (Table 7-1).

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] a,c

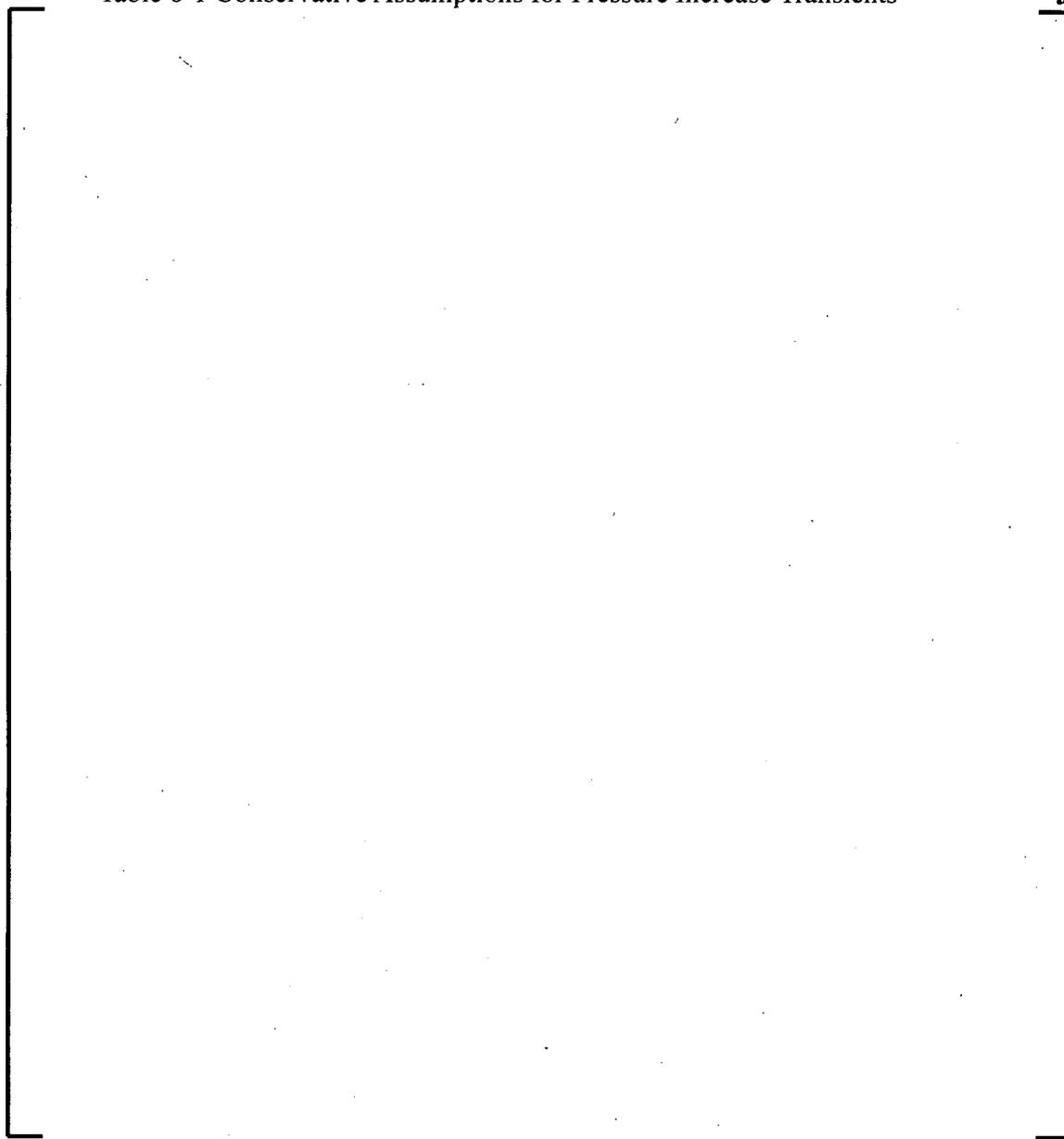
• [

] <sup>a,c</sup>

Table 6-1 summarizes the conservatisms that should be included in computer based evaluation models.

Table 6-1 Conservative Assumptions for Pressure Increase Transients

a,c



When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

Table 6-2 Conservative Assumptions for PCT calculations

] a,c

### 6.4.1.3 Pressure Decrease

Transients belonging to this group are described in Table 4-2.

#### 6.4.1.3.1 Pressure Decrease Transient Methodology

##### Methodology

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] a,c

- [
- 
- 

] <sup>a,c</sup>

Table 6-3 summarizes the conservative assumptions for pressure decrease transients.

Table 6-3 Conservative Assumptions for Pressure Decrease Transients

--

<sup>a,c</sup>

When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

---

## 6.4.2 Reactor Coolant Flow Increase/Decrease

### 6.4.2.1 Analysis Code Requirements

Both reactor coolant flow increase and decrease transients have the same analysis code requirements. The requirements of the analysis code capability are based on the phenomenological descriptions used as the basis for the PIRT (Table 5-2). The data uncertainty assessment (discussed later in Section 7) may require that conservative input data is selected for the transient code analysis methodology. The requirements of the physical evaluation model, based on the phenomenological description, are the following:

- [

] <sup>a,c</sup>

To ensure that the condition captured in the PIRT is valid, confirmatory analysis will be performed to ensure that:

1. The limiting case that utilizes the PIRT captures unique and significant plant specific design features.
2. The analysis of transient events include combinations of the transient categories defined herein if these conditions are more limiting than the conditions analyzed in the PIRT.

### 6.4.2.2 Reactor Coolant Flow Increase

Transients belonging to this group are described in Table 4-3.

#### 6.4.2.2.1 Analysis Code Requirements

In addition to the Analysis Code Requirements listed in Section 6.4.2.1, the following requirements also must be met for reactor coolant flow increase transients:

- [

] <sup>a,c</sup>

#### 6.4.2.2.2 Reactor Coolant Flow Increase Transient Methodology

The analysis methodology is based on the PIRT (Table 5-2), the code capability assessment (Section 7.2) and the data uncertainty assessment (Table 7-1).

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] <sup>a,c</sup>

Table 6-4 summarizes the conservative assumptions for modeling this group of transients.

Table 6-4 Conservative Assumptions for Reactor Coolant Flow Increase Transients

	<sup>a,c</sup>
--	----------------

---

When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

### 6.4.2.3 Reactor Coolant Flow Decrease

Transients belonging to this group are described in Table 4-4.

#### 6.4.2.3.1 Analysis Code Requirements

In addition to the Analysis Code Requirements described the following requirement must also be met for Reactor Coolant Flow Decrease transients:

- [

] <sup>a,c</sup>

#### 6.4.2.3.2 Reactor Coolant Flow Decrease Methodology

The analysis methodology is based on the PIRT (Table 5-2), the code capability assessment (Section 7.2) and the data uncertainty assessment (Table 7-1).

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] <sup>a,c</sup>

Table 6-5 summarizes the conservative assumptions used to accurately model Reactor Coolant Flow Decrease Transients.

Table 6-5 Conservative Assumptions for Reactor Coolant Flow Decrease Transients



a,c

When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

---

### 6.4.3 Feedwater Flow Increase/Decrease

#### 6.4.3.1 Analysis Code Requirements

Both feedwater flow increase and decrease transients have the same analysis code requirements. The requirements of the analysis code capability are based on the phenomenological descriptions used as the basis for the PIRT (Table 5-2). The data uncertainty assessment (discussed later in Section 7) may require that conservative input data is selected for the transient code analysis methodology. The requirements of the physical evaluation model, based on the phenomenological description, are the following:

- [

] <sup>a,c</sup>

To ensure that the condition captured in the PIRT is valid, confirmatory analysis will be performed to ensure that:

1. The limiting case that utilizes the PIRT captures unique and significant plant specific design features.
2. The analysis of transient events include combinations of the transient categories defined herein if these conditions are more limiting than the conditions analyzed in the PIRT.

#### 6.4.3.2 Feedwater Flow Increase

Transients that will generate feedwater flow increases are discussed in Section 15.1.2 of the SRP.

### 6.4.3.2.1 Feedwater Flow Increase Methodology

The analysis methodology is based on the PIRT (Table 5-2), the code capability assessment (Section 7.2) and the data uncertainty assessment (Table 7-1).

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

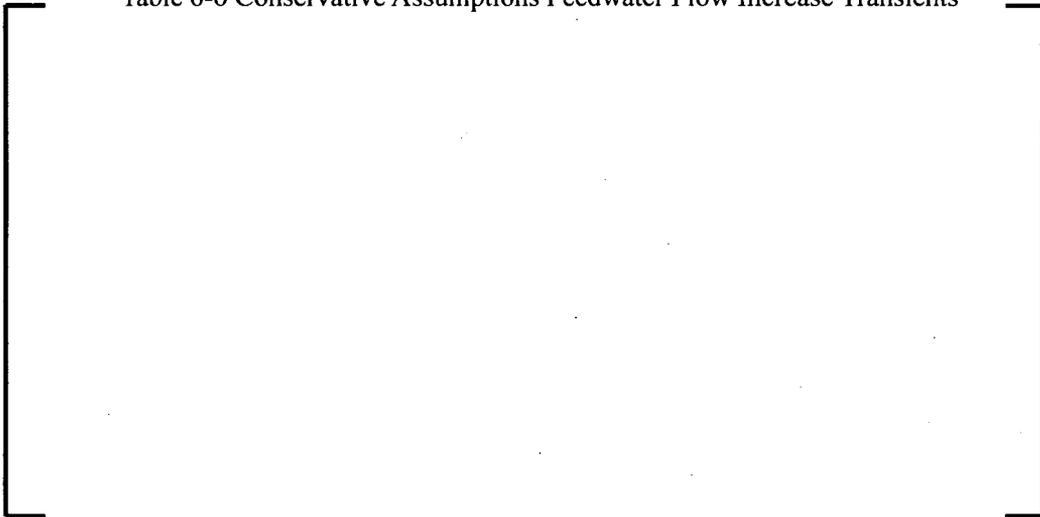
- [

] <sup>a,c</sup>

Table 6-6 summarizes these conservatisms.

Table 6-6 Conservative Assumptions Feedwater Flow Increase Transients

<sup>a,c</sup>



When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

### 6.4.3.3 Feedwater Flow Decrease

Transients that will generate feedwater flow are discussed in Section 15.2.7 of the SRP.

#### 6.4.3.3.1 Analysis Code Requirements

In addition to the requirements specified in Section 6.4.3.1, the following requirements must be met by the computer evaluation model:

- [

] <sup>a,c</sup>

#### 6.4.3.3.2 Feedwater Flow Decrease Methodology

The analysis methodology is based on the PIRT (Table 5-2), the code capability assessment (Section 7.2) and the data uncertainty assessment (Table 7-1).

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] <sup>a,c</sup>

Table 6-7 summarizes the conservatisms used during Feedwater Flow Decrease transients.

Table 6-7 Conservative Assumptions for Feedwater Flow Decrease Transients

--	--

] <sup>a,c</sup>

When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

## **6.4.4 Reactor Coolant Temperature Increase/Decrease**

To ensure that the condition captured in the PIRT is valid, confirmatory analysis will be performed to ensure that:

1. The limiting case that utilizes the PIRT captures unique and significant plant specific design features.
2. The analysis of transient events include combinations of the transient categories defined herein if these conditions are more limiting than the conditions analyzed in the PIRT.

### **6.4.4.1 Reactor Coolant Temperature Increase**

Transients that will generate reactor coolant temperature increase are described in Section 15.2.7 of the SRP.

#### **6.4.4.1.1 Analysis Code Requirements**

Please refer the transient group "Feedwater Flow Decrease" requirements described above in Section 6.4.3.1 and 6.4.3.3.1.

#### **6.4.4.1.2 Reactor Coolant Temperature Increase Methodology**

Please refer the transient group "Feedwater Flow Decrease" methodology described above in Section 6.4.3.3.

### **6.4.4.2 Reactor Coolant Temperature Decrease**

Transients belonging to this group are described in Table 4-5.

#### **6.4.4.2.1 Analysis Code Requirements**

An event in this transient category may be so slow that it can be modeled as a slow transient. This methodology is described in the latest revision of Reference 1. The following requirements apply when modeling this event as a fast transient.

The requirements of the analysis code capability are based on the phenomenological descriptions used as the basis for the PIRT (Table 5-2). The data uncertainty assessment (discussed later in Section 7) may require that conservative input data is selected for the transient code analysis methodology. The requirements of the physical evaluation model, based on the phenomenological description, are the following:

- [

]<sup>a,c</sup>

[

] <sup>a,c</sup>

#### 6.4.4.2.2 Reactor Coolant Temperature Decrease Methodology

To ensure the computer models used in the analysis of transients are conservative, the following modeling techniques are used:

- [

] <sup>a,c</sup>

Table 6-8 summarizes the conservatisms used in the analysis of this transient.

Table 6-8 Conservative Assumptions for Reactor Coolant Temperature Decrease Transients



a,c

When the transient is evaluated against PCT the conservative assumptions according to Table 6-2 are used.

## 6.5 ANTICIPATED TRANSIENTS WITHOUT SCRAM

Because an ATWS event requires multiple failures to occur, it is considered beyond the plant design basis and is analyzed to demonstrate conformance to 10 CFR Part 50.62 (Reference 3).

By its definition, ATWS represents a spectrum of events due to the number of different potential event initiators. The spectrum of event initiators is generically evaluated to establish which ones are the most limiting. The most limiting initiators are usually caused by a rapid reduction in steam flow (rapid pressurization events) or events that can evolve to a rapid pressurization event during the course of the transient. These potentially limiting transients are analyzed as part of the plant design bases safety analysis.

The plant response during ATWS events is highly dependent on the event initiators. For rapid pressurization events, there is a rapid increase in the reactor coolant pressure boundary pressure, and core power. The pressure and power increase is limited by the reactor protection system, typically an automatic recirculation pump trip (ATWS-RPT) on high reactor pressure and operation of the safety/relief valves.

Reactor shutdown is accomplished by automatic or manual initiation depending on the plant design. One or several different systems may exist that can shut down the reactor.

- Alternate Rod Insertion (ARI)  
An alternate method of activating the hydraulic scram
- Fine Motion Control Rod Drive (FMCRD)  
Slow electromechanical insertion of the control rods
- Standby Liquid Control System (SLCS)  
Shutdown with boron injection into the vessel coolant

### 6.5.1 Methodology

#### 6.5.1.1 ARI or FMCRD

[

] <sup>a,c</sup>

#### 6.5.1.2 SLCS

[

] <sup>a,c</sup>

• [

]a,c

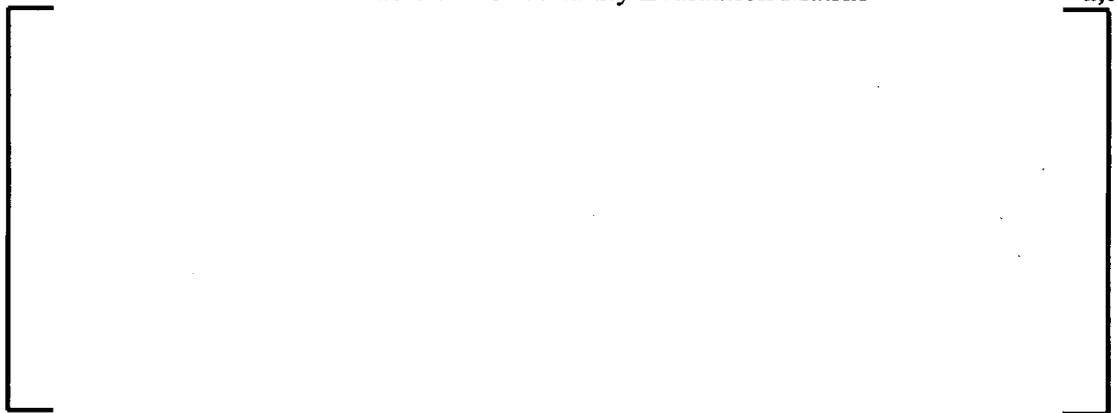
## 7 UNCERTAINTY ANALYSIS

The purpose of the uncertainty analysis is to predict a best estimate value accounting for uncertainties and biases of the relevant input and modeling parameters to ensure operating limits and safety margins meet the acceptance criteria. This section describes the process for defining input and model parameters that are subject to an uncertainty analysis and the process of uncertainty evaluation. A new method using 95% probability with a 95% confidence level is introduced for calculating these resulting conservative values.

### 7.1 SELECTION OF INPUT PARAMETERS TO UNCERTAINTY ANALYSIS

The selection of input parameters to the uncertainty analysis is based on the results from the PIRT table and the analysis methodology. Using the results of the PIRT, the CCA process, described in Section 7.2, ranks the code capability to simulate the high ranked phenomena. When the code capability is high for a certain phenomenon, the phenomenon input and modeling parameters are treated for biases and uncertainties. Table 7-1 comprises the details of how a phenomenon is treated based on the PIRT and CCA ranking.

Table 7-1 Uncertainty Evaluation Matrix



a,c

The Data Uncertainty Assessment (DUA) process (described in Section 7.3) links corresponding code-dependent input and model parameters for phenomena which are included in the uncertainty analysis. DUA also establishes the probabilistic distributions and/or bounding values for the Relevant Parameters used in the uncertainty evaluation. A situation may arise when the probabilistic distribution function for a certain parameter cannot be established. In that case, the parameter is treated conservatively.

The following sections contain the description of CCA and DUA on an overall process level. Neither CCA nor DUA is performed for any particular code in this topical report.

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## 7.2 CODE CAPABILITY ASSESSMENT

### 7.2.1 Introduction

The CCA evaluates the capability of the computer code for performing analyses of fast transients and ATWS events in BWRs. The PIRT presented in Section 5 identifies important phenomena that must be addressed in the analyses considered. The experimental database that supports the code models is defined and used to determine whether the identified relevant phenomena are treated with sufficient accuracy. Findings of the evaluation are summarized in a CCA matrix.

The transient methodology presented here is code-independent and therefore can be applied regardless of the computer code used. Because of this, a CCA is not presented here but will instead appear in a code-specific topical. In an attempt to provide a complete picture of the evaluation model, the CCA process described herein is done on an overall process level, defining the input to the CCA, identifying the steps taken in the process, and summarizing the results of a CCA effort.

### 7.2.2 Assessment Base

The existing experimental database is defined in order to determine the code accuracy, to assist the code ranking, and the establishment of uncertainties and relevancy of input parameters. As such, the database includes the following items:

- Separate effect tests (SETs) needed to develop and assess empirical correlations and other closure models
- Integral effect tests (IETs) to assess system interaction and global code capability
- Benchmark with other codes (optional)\*
- Plant transient data (if available)
- Simple test problems (or analytical solutions) to illustrate fundamental calculational device capability\*

The test database can include both separate effect tests that investigate empirical correlations and individual phenomena and integral effects tests that incorporate many or all of the important phenomena to assess system interactions and global code capability.

The result of the assessment base is the qualification records matrix. The qualification records matrix list contains the list of records (SETs, IETs, plant transient data, etc.) assigned to each high ranked phenomenon (defined in the PIRT).

---

\*Benchmarking with other codes and simple test problems are intended to complement and not be a substitute for SETs, IETs or plant transient data.

### 7.2.3 Review of Code Adequacy

The review of code adequacy focuses on the capabilities and performance of the computer code. The models are examined to see if all relevant phenomena (as defined in the PIRT) can be analyzed by verifying that a model with appropriate accuracy exists in the code for each of the phenomena identified in the PIRT. In these assessments, judgment of the code capability for predicting each important PIRT phenomenon is performed by an evaluation panel of subject matter experts. The results are then presented in a model assessment table.

Overall judgments on the qualification data are made by taking into account the quality of the test data, uncertainties, and knowledge of the test facility configurations and operations.

The model capability is stated according to a three-level scale: High/Medium/Low. This is related to how the phenomenon is calculated or used in determining the figures-of-merit.

- High (H) - the average bias and deviation, when comparing the code model results with experimental data, is low and that the code results follows the general trend of experimental data.
- Medium (M) - Either the low averaged bias is combined with high deviation, or the low deviation is combined with high average bias, when comparing the code results with experimental data.
- Low (L) - General trend in code results does not follow the experimental data and/or both the bias and deviation are high. Low ranking applies also when the phenomenon is not modelled by the code.

### 7.2.4 Example of Code Capability Assessment Matrix

Table 7-2 is a demonstrative example of a CCA Matrix.

Table 7-2 Code Capability Assessment Matrix - Example

Category	Phenomenon	Qualification data		Code Capability	Rationale
		Sufficiency	Relevancy		
Initial conditions	Gap size	H	H	H	WCAP-xxx
Multiple rod thermal effects	Rod-to-rod radiative heat transfer	-	-	L	Not modeled

### **7.3 DATA UNCERTAINTY ASSESSMENT**

The DUA process links corresponding code-dependent input and model parameters (so called Candidate Parameters) for pertinent phenomena to allow for further uncertainty and sensitivity analyses. Many of the Candidate Parameters have very small uncertainties themselves or an insignificant impact on the figures-of-merit. Such parameters are eliminated from further analyses while the remaining Relevant Parameters are subject to further uncertainty and sensitivity analyses. Probabilistic distribution functions and respective bounding values are then defined for each Relevant Parameter. These parameters are then an input to further uncertainty analysis. DUA consists of the following steps:

- Identification of Candidate Parameters
- Specification of Relevant Parameters, if applicable
- Establishment of probabilistic distributions and respective bounding values for the further uncertainty analysis

#### **7.3.1 Identification of Candidate Parameters**

Candidate Parameters are all code input and modeling parameters needed to simulate the high ranked phenomena identified in the PIRT. The list of candidate parameters is established at this phase, based on code documentation.

#### **7.3.2 Specification of Relevant Parameters**

Many of the Candidate Parameters have very small uncertainties themselves or an irrelevant impact on the figures-of-merit. These parameters may be therefore removed from further analysis following confirmation of their insignificance.

The rest of the parameters, having a significant influence on figures-of-merit, are considered the Relevant Parameters and serve as input to further analysis.

The specification of Relevant Parameters may be omitted. If that is the case, then all the candidate parameters are automatically considered as Relevant Parameters in further analysis.

#### **7.3.3 Establishment of Probabilistic Distributions and Respective Bounding Values for the Further Uncertainty Analysis**

The evaluation of the Relevant Parameters uncertainty intervals follows the guidelines defined in Code Scaling, Applicability, and Uncertainty (CSAU). According to CSAU there is no single method providing the uncertainty range or bias for all the Relevant Parameters. Instead, the Relevant Parameters are grouped and assessed by the following methods<sup>\*</sup>:

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<sup>\*</sup>Even though these methods are defined for the LBLOCA, Westinghouse finds them general enough to be applied in the case of any transient or accident analysis.

• [

] <sup>a,c</sup>

If a statement on the distribution of a certain Relevant Parameter cannot be made, but the data are considered sufficient to treat the Relevant Parameters statistically, the uniform probability distribution should be used in the further analysis. Uniform probability distribution represents the maximum ignorance about the distribution and leads to conservative uncertainty estimates.

### 7.3.4 Example of Data Uncertainty Assessment

Table 7-3 is a demonstrative example of a DUA Table

Table 7-3 Data Uncertainty Assessment Table - Example

Phenomenon or Plant component	Code Input/Modelling Parameter	Data Uncertainty					
		Minimum	Nominal	Mean Value	Standard Deviation	Distribution	Maximum
Recirculation Pump (Jet Pump)	Jet pump fluid inertia multiplier	0.9	1.0	N/A	N/A	Uniform	1.2
	Jet pump M ratio multiplier	N/A	1.0	1.0	0.035	Normal	N/A
	Jet pump N ratio multiplier	N/A	1.0	1.0	0.05	Normal	N/A

## 7.4 UNCERTAINTY ANALYSIS METHODOLOGY

This section contains the methodology for the determination of combined bias and uncertainty when evaluating operating limits or safety margins to acceptance criteria. The technical basis for this method is explained in Appendix B. Following sections describe the process of uncertainty evaluation, utilizing appropriate statistical uncertainty evaluation method, and define a final list of uncertainty input parameters.

### *Transient Uncertainty Evaluation Methodology*

The following steps describe the transient uncertainty evaluation process adopted by Westinghouse:

- **Define the tolerance limits** - probability level of 95% and confidence interval of 95% is used in the uncertainty evaluation for each operating limit or safety margin to acceptance criteria.
- **Specify the number of evaluated event acceptance criteria (output parameters)** – the number of parameters depends on the type analysis under consideration. The list of event acceptance criteria that are evaluated for each transient are defined throughout Section 6.
- **Decide on number of runs (based on previous steps)** – when the tolerance limits and the number of analysis output parameters is specified, the number of code runs is calculated from Equation 11 or 22 in Appendix B.
- **Define the input parameters in terms of probabilistic distribution** –Table 7-1 serves as a guide for determining the phenomena subject to uncertainty analysis (based on PIRT and CCA). The treatment of the input and modeling parameters for each phenomena falls into 3 basic categories, detailed in Section 7.4.1.
- **Create the run matrix** – the run matrix for the Monte-Carlo simulation is generated by sampling the probabilistic distributions of the input parameters n-times. The value for n is determined from two previous steps.
- **Perform the code run** – Using the run matrix parameters, the plant's response to a transient event is computed for each case and the event acceptance criteria under consideration are extracted from the output files.
- **Analyze results** – The results are tested for normality using normality tests as shown in Appendix B. If the data passes the normality test, then the 95<sup>th</sup> percentile, determined with 95% confidence is calculated according to the analysis of variance method captured by Equation 28 in Appendix B. In cases where computer results do not pass the normality test, the order statistics method is used instead to determine the 95<sup>th</sup> percentile, with 95% confidence. The results are tallied, ranking them from highest to lowest. Using the order statistics, the 95<sup>th</sup> percentile event acceptance criterion/criteria is/are determined with 95% confidence, by selecting the largest (in case of upper limit) or the lowest (in case of lower limit), from the obtained results. If the n<sup>th</sup> largest estimate is used instead, the number of code runs must be increased according to Equation 11 in Appendix B.

Discussion

In creating the run matrix, the random numbers are obtained by a generator adapted from Press (Reference 9). The period of the generator of  $2.3 \times 10^{18}$  is considered large enough. The algorithm generates the uniformly distributed values within a 0 to 1 interval. Sampling from the uniform distribution is therefore straightforward. Sampling of values from normal and log-normal distributions is performed by employing the rejection type approach.

When analyzing the results using the analysis of variance method, the constant  $Z_{95,95}$  depends on the sample size (number of code runs) and is estimated from standard statistical tables.

**7.4.1 Parameters Considered in Uncertainty Analysis**

Table 7-1 serves as a guide for determining the phenomena which are subject to uncertainty analysis (based on PIRT and CCA). The treatment of the input and modeling parameters for each phenomena falls into 3 basic categories:

- Nominal with uncertainty – [ ]<sup>a,c</sup>
  
- Nominal without uncertainty – [ ]<sup>a,c</sup>
  
- Conservative – [ ]<sup>a,c</sup>

---

\* All parameters related to the mentioned phenomena are called Candidate Parameters. Many of these parameters still have a negligible influence on the event acceptance criteria and these parameters are therefore eliminated from the further uncertainty study, based on a detailed sensitivity study. Those parameters are then treated as nominal without uncertainty. The rest is called Relevant Parameters and are those which are subject to the uncertainty analysis.

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## 8 DEMONSTRATION ANALYSIS

The implementation of the Westinghouse Fast Transient and ATWS methodology, described in this document, is demonstrated here for the Load Rejection without Bypass transient event. The description consists of the following sections:

- Specification of the transient group and power plant type
- Definition of the operating limits or safety margins to acceptance criteria under evaluation
- PIRT creation
- Selection of computational tool (analysis methodology)
- CCA
- DUA
- Analysis of the nominal case
- Analysis including uncertainty evaluation

### 8.1 TRANSIENT GROUP AND POWER PLANT TYPE SPECIFICATION

The generator load rejection without bypass event is the postulated complete loss of electrical load to the turbine generator coupled with the assumed failure of the turbine bypass system. Fast closure of the turbine control valves is initiated whenever electrical grid disturbances occur which result in significant loss of load on the generator. The turbine control valves are required to close rapidly to prevent overspeed of the turbine generator rotor due to the loss of load. The rapid closure of the turbine control valves causes a sudden reduction of steam flow which results in a nuclear system pressure increase. The generator load rejection without bypass belongs therefore to the pressure increase transient class. See Section 4.1.1 for the phenomenological description of this transient group.

Plant type used for the demonstration analysis is the ABWR design.

### 8.2 OPERATING LIMITS AND SAFETY MARGINS TO ACCEPTANCE CRITERIA

The transient is evaluated as an AOO in the plant design basis and therefore is evaluated to determine the plant operating limits. The example considered evaluates only one parameter, the OLMCPR. Methodology for calculating OLMCPR is described in detail in Section 6.2.1.

### 8.3 PHENOMENA IDENTIFICATION AND RANKING

Table 8-1 contains the list of the phenomena which significantly influences the OLMCPR figures-of-merit. These phenomena are treated further in the uncertainty analysis.

Table 8-1 Load Rejection without Bypass Demonstration Example – High Ranked Phenomena

a,c

#### **8.4 SELECTION OF COMPUTATIONAL TOOLS**

Analysis code requirements for the pressure increase transient are defined in Section 6.4.1.1 under Analysis Methodology. Westinghouse 1D kinetic analysis code BISON is selected for the demonstration analysis. The three-dimensional nodal simulator POLCA7 is used to establish cross section dependence on coolant density, fuel temperature, and control rod fraction for the preparation of appropriate polynomial forms for the one-dimensional BISON model.

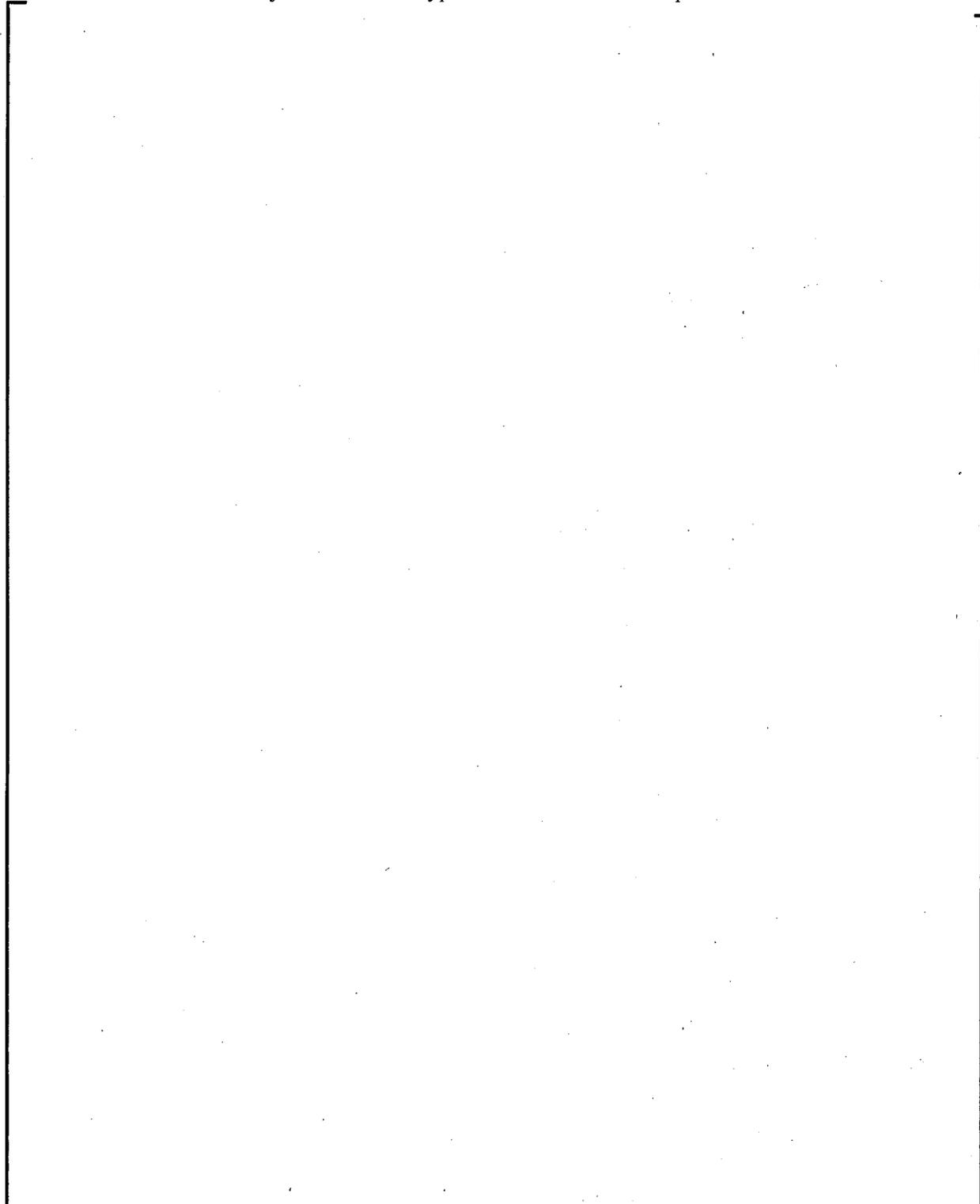
#### **8.5 CODE CAPABILITY ASSESSMENT**

The CCA process provides a statement on the 1D code's capability to simulate the high ranked phenomena listed in Table 8-1. An example CCA Matrix for BISON (in the form of Table 7-2) which is needed as input to the uncertainty analysis is shown in Table 8-2.

Table 8-2 is presented only for demonstrational purposes. It is given without reference to the rationale for the code ranking and will not be used in licensing applications.

Table 8-2 Load Rejection without Bypass Demonstration Example – BISON CCA Matrix

a,c





## 8.6 DATA UNCERTAINTY ASSESSMENT

As described in Section 0 of this topical report, the data uncertainty assessment process serves two purposes. The first one is to link the high ranked phenomena with the corresponding relevant input and modeling parameters and the second is to define the probabilistic distribution functions, respective bounding values for each relevant input and modeling parameter. These then become the uncertainty analysis input parameters.

Table 8-3 (DUA Table) comprises the list of relevant input and model parameters together with their distributions and bounding values. Probabilistic data are not defined for all of the relevant parameters. Several phenomena are treated conservatively instead. Rationale for the conservatism in these phenomena is provided in Section 8.6.1 and is based on the analysis methodology described above (Section 6.4.1.2).

Table 8-3 is presented for demonstrational purposes only. The values of the uncertainty parameters provided here will not be used in licensing applications without reference to the model validation reports from which the data are obtained.

Table 8-3 Load Rejection without Bypass Demonstration Example - DUA Table

a,c

1

a,c



---

## 8.6.1 Parameters Treated Conservatively

### 8.6.1.1 Fuel Cycle Design

As explained in Section 6.2.1 (Analysis Methodology), the analysis is conservatively performed at the end of fuel cycle (EOC) where the OLMCPR has the largest value. EOC is selected because the control rod insertion time is largest at this time in life. Also at EOC, the axial power profile shape is “top-peaked” which prolongs the influence of control rod insertion.

### 8.6.1.2 Coolant conditions – Including pressure, temperature, quality, void fraction, and mass flow rates

Conservative operating point (see Section 8.7.1) is chosen for the analysis.

### 8.6.1.3 Critical Heat Flux, Dryout

NRC approved CPR correlation demonstrated to be conservative is used in the analysis.

### 8.6.1.4 Rod-to-spacer grid thermal hydraulic interaction

Rod-to-spacer grid thermal hydraulic interaction is accounted for in CPR correlation.

### 8.6.1.5 Recirculation Pumps

As specified in Section 6.4.1.2.1 of Analysis Methodology, the recirculation pump model is conservative, [

] <sup>a,c</sup>

### 8.6.1.6 Scram Speed/Timing

In the demonstration example, the control rod insertion is given in the scram tables. SCRAM2 model is not used and therefore the code capability to simulate the scram is low and analysis shall use a conservative value. As specified in Section 6.4.1.2.1 of Analysis Methodology, [

] <sup>a,c</sup>

### 8.6.1.7 Safety Relief Valves

In order to model the SRV conservatively, [

] <sup>a,c</sup>

### 8.6.1.8 Turbine Valves

In order to generate a faster void decrease in the core, [

] <sup>a,c</sup>

### 8.6.1.9 Steam Lines

According to the analysis methodology, the steam line is modeled in a conservative way with respect to CPR such that [

] <sup>a,c</sup>

## 8.7 ANALYSIS OF NOMINAL CASE

### 8.7.1 Nominal case - input

This analysis example represents an ABWR preliminary equilibrium reactor core, obtained after 5 initial cycles loaded with [ ]<sup>a,c</sup> fuel assemblies. Core operating parameters are given in Table 8-4. Analysis is performed at EOC.

Table 8-4 Demonstration analysis – Core operating parameters

	a,b,c

Nominal case calculations evaluate the best estimate value of transient MCPR without consideration of uncertainty arising from the input and modelling parameters. The analysis of the nominal case is presented here as a baseline so that the value of calculated MCPR could be compared to the final OLMCPR value which contains the evaluation of uncertainty.

All input and modelling parameters used in the calculation are set to their nominal values. The exceptions are noted below:

- [ ]

] <sup>a,c</sup>

### 8.7.2 Nominal case – results

Table 8-5 shows the predicted sequence for the generator load rejection without bypass transient event. Table 8-6 shows the important results from the analysis event including the evaluation of ICPR, CPRmin, and resulting OLMCPR(i).

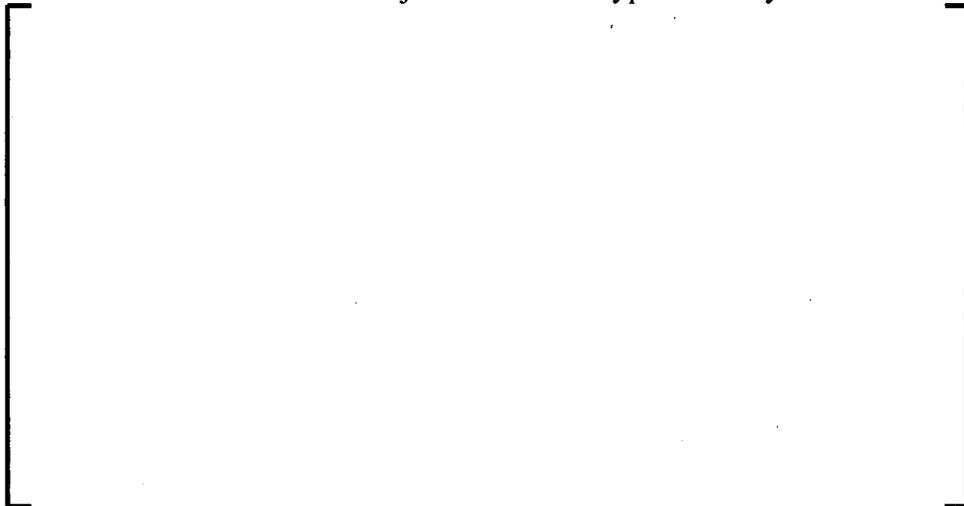
Table 8-5 Generator load rejection without bypass – predicted sequence

a,c



Table 8-6 Generator load rejection without bypass – analysis results

a,c



---

## **8.8 ANALYSIS INCLUDING THE UNCERTAINTY EVALUATION**

Uncertainty evaluation methodology, as described in Section 7.4 is applied here to account for the uncertainty in input and modeling parameters when calculating the best estimate value of OLMCPR for generator load rejection without bypass transient event. Upper probability bound for OLMCPR at 95% is estimated with 95% confidence.

As the analysis requires the evaluation for one parameter at the limits of 95/95 , the minimum amount of code runs is 59 assuming that the OLMCPR is not normally distributed and the uncertainty is calculated by the order statistics method (see Appendix B for details).

### **8.8.1 Development of Run Matrix**

The individual uncertainty contributors, as defined in Table 8-3, are sampled according to their probabilistic distribution functions. Run matrix of 59 components is created. Relevant code input and modeling parameters which are not treated stochastically are set to their conservative value as defined by the analysis methodology (same as in the analysis of nominal case).

### **8.8.2 Uncertainty Evaluation Results and Determination of the 95/95 OLMCPR Limit**

The ICPR(i,j) and CPRmin(i,j) values are extracted for each of 59 code runs and OLMCPR(i,j) is calculated according to Equation 4. An Anderson-Darling normality test is performed and Anderson-Darling statistics  $A^*$  is calculated to be larger than 0.751 . Hypothesis of normality is therefore rejected for a 5% level test and order statistics method is used instead to calculate the OLMCPR(i) for the generator load rejection without bypass transient event as the maximum value of 59 OLMCPR(i,j). In this case the maximum value is OLMCPR(i)= 1.354 . This value may be compared to OLMCPR(i)= 1.332 calculated without the uncertainty consideration.

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## 9 REFERENCES

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APPENDIX A:  
PHENOMENA DEFINITIONS FOR AOO AND ATWS

## **ABSTRACT**

This appendix summarizes the definitions of all phenomena used in PIRT creation process.

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## ACRONYMS

BPV	Bypass Valve
BWR	Boiling Water Reactor
ECCS	Emergency Core Cooling System
ID	Inner Diameter
OD	Outer Diameter
SLCS	Standby Liquid Control System
TCV	Turbine Control Valve
TSV	Turbine Safety Valve

## APPENDIX A: PHENOMENA DEFINITIONS FOR AOO AND ATWS

Table A-1 Phenomena definitions for AOO and ATWS

ID	Category	Phenomenon	Definition
A1	A. Initial conditions	Gap size	Distance between pellet outside and inside clad diameters.
A2		Gas pressure	Pressure of the gas in the fuel rod gap.
A3		Gas composition	Composition of the gas in the rod (mole fractions of the fill and fission gas components).
A4		Pellet and cladding dimensions	Characteristic physical dimensions, as a function of burnup.
A5		Fuel cycle design	Fuel burnup and loading pattern, control rod pattern, nuclide concentrations (xenon, heavy nuclides and fission products).
A6		Cladding oxidation (ID & OD)	The amount of prior zirconium oxide on both the inside and outside cladding surfaces.
A7		Coolant conditions Including pressure, temperature, quality, void fraction, and mass flow rates.	Thermal-hydraulic conditions in the core including pressure, temperature, quality, void fraction, and mass flow rate.
A8		Rod free volume	The plenum and other free volumes within the fuel rod occupied by the gas.
A9		Gas communication (full)	The ability of the gas in the free volume to move axially within the fuel rods, thereby providing uniform gas pressure.
A10		Gadolinium distribution (conductivity effect)	The spatial distribution of gadolinium within the core, which affects the thermal conductivity of the fuel rods.
A11		Initial stored energy-fuel	The total energy content of the fuel rods at initial power conditions.

ID	Category	Phenomenon	Definition
A12		Initial stored energy-structures	The total energy content of structures within the vessel at initial power conditions.
A13		Initial core pressure drop (grids)	The initial axially varying pressure within the core.
A14		Pellet radial power distribution	The radial distribution of the power produced in the fuel rods.
A15		Fuel assembly axial power distribution	The magnitude and axial distribution of the power produced in the fuel assembly.
A16		Fuel assembly peaking factors	A fuel assembly's power compared to the core average (radial peaking factor).
A17		Pin peaking factors	Pin power distribution within an assembly.
A18		Coolant flow distribution in the core.	Distribution of coolant mass flow in the active channels, bypass channels, and internal water channels.
B1	B. Transient power distribution	Moderator feedback	Reactivity feedback from moderator density and density changes in active channels. These changes are a result of direct deposition to the coolant and heat transfer from the cladding.
B2		Decay heat power	The power produced due to decay reactions of actinides and fission products.
B3		Fuel temperature feedback	Reactivity feedback from fuel temperature changes. This effect results from the heating of the fuel and associated neutronic effects, in particular the Doppler effect, and heat transfer from the fuel rod cladding.
B4		Delayed neutron fraction	The fraction of fission neutrons that are not emitted instantaneously designated beta ( $\beta$ ).
B5		Fractional energy deposition in moderator and structures	The fraction of total fission and decay energy that is deposited directly in the coolant and the structures.
B6		Control rod reactivity	Control rod/SCRAM worth.

ID	Category	Phenomenon	Definition
B7		Heat-transfer through box wall to inter-assembly bypass	Self-explained.
B8		Active channel-to-inner channel and water rod heat transfer	Active channel is the fuel bundle channel (water in contact with the fuel rods in each subbundle). Inner channel is any internal assembly bypass (such as the water cross or water rods).
B9		Boron reactivity feedback	Reactivity feedback due to presence of soluble boron in the core coolant. Effect of Standby Liquid Control System (SLCS) injection of boron to shutdown the reactor in ATWS calculations.
B10		Boron transport	Transport and mixing process of soluble boron within the reactor vessel.
B11		Boron settling	Thermal stratification of injected boron solution in the lower plenum under natural flow conditions. This is an issue for BWRs injecting boron into the lower plenum.
C1	C. Steady-state and transient cladding to coolant heat transfer and core spray heat transfer	Single phase convection	Heat transfer from fuel outer surface to adjacent single-phase liquid.
C2		Subcooled boiling, nucleate boiling, bulk boiling, and forced convection vaporization	Heat transfer to adjacent liquid resulting in the formation of vapor at nucleation sites on the cladding surface or in the bulk liquid.
C3		Critical heat flux/dryout	The heat flux that causes vaporization sufficient to prevent liquid from arriving at the heated surface.
C4		Steam cooling/transition boiling	Heat transfer from the cladding outer surface to single-phase gas.
C5		Radiation heat transfer to coolant	Radiative thermal energy transport to the surrounding vapor/liquid environment.

ID	Category	Phenomenon	Definition
C6		Rewet	Heat transfer occurring from liquid contact with the cladding surface after dryout; occurs when the surface temperature has decreased to the minimum film boiling point.
C7		Rod-to-spacer grid thermal-hydraulic interaction	The enhanced convective heat transfer effects downstream of the spacer grids due to mixing and flow redistribution for single- or two-phase flows.
D1	D. Transient coolant conditions as a function of elevation and time	Temperature	Temperatures of the gas and liquid phases of coolant flowing along the fuel rod.
D2		Flow rate/directions (CCFL)	Flow rate and direction of gas and liquid phases flowing along the fuel rod (including crossflow and counter current flow limiting effects).
D3		Quality	The mass flow fraction of steam (gas) in the two-phase mixture flowing along the fuel rod.
D4		Void fraction	The volume fraction of steam (gas) in the two-phase mixture.
D5		Pressure	The absolute total pressure in the coolant channel along the rod.
D6		Partial vapor pressure	The partial steam pressure in the coolant channel along the rod.
E1	E. Fuel rod response	Plastic deformation of cladding (thinning, ballooning and burst)	Irreversible changes in cladding dimensions caused by pressure differentials or mechanical loadings at high temperatures. If cladding burst occurs, the final plastic deformation at the burst location is characterized by the burst strain.

ID	Category	Phenomenon	Definition
E2		Heat resistances in fuel, gap and cladding	The resistances offered by the fuel, gap, and cladding to the flow of thermal energy from regions of high temperature to regions of lower temperature. The resistance is dependent upon path length and thermal conductivity, which change with burnup and other processes, e.g., the buildup of oxide on the cladding surfaces.
E3		Fuel temperature	Fuel temperature distribution, as used to determine fuel temperature feedback, pellet properties, radial thermal expansion, gas temperatures.
E4		Metal-water reaction heat addition	The additional heat generated in the cladding due to metal-water reactions.
E5		Cladding oxidation magnitude (ID/OD)	Thickness of oxide layers on inner and outer surfaces of cladding.
E6		Cladding temperature	The cladding temperature as used in determining cladding properties, cladding-to-coolant heat transfer, and radial thermal expansion.
E7		Time dependent gap-size heat transfer	The gap size is a result of plastic, thermal, and elastic deformation. The heat transfer across the gap is a function of gap size, conductance of the gas mixture, and the temperatures of the pellet outside diameter and cladding inside diameter (radiative heat transfer).
E8		Thermal and mechanical properties of pellet and cladding	The thermal and mechanical properties of the pellet and cladding, e.g., heat capacity, conductivity, yield stress, and creep, are needed to calculate the temperature and deformation response of the fuel rod.
F1	F. Multiple rod mechanical effects	Rod-to-rod mechanical interactions	Interaction between two or more rods, including guide tubes, water rods, and channels. Occurs when one or all rods are deformed due to swelling or bowing, including mechanical contact and conduction heat transfer. Such that the rods are in physical contact.
F2		Rod bow between spacer grids	Bowing of a fuel rod due to axially constrained thermal expansion.

ID	Category	Phenomenon	Definition
G1	G. Multiple rod thermal effects	Rod-to-rod radiative heat transfer	Thermal radiation heat transfer between fuel rods.
G2		Rod-to-channel box radiative heat transfer	Thermal radiation heat transfer between a fuel rod and the channel box in a BWR.
G3		Rod-to-spacer grid local heat transfer	Heat transfer between a fuel rod and a spacer grid due to thermal radiation and conduction heat transfer.
G4		Rod-to-water rod radiative heat transfer	Thermal radiation heat transfer between a fuel rod and a water rod (BWR).
G5		Rod-to-inner channel radiative heat transfer	Thermal radiation heat transfer between a fuel rod and the inner channel box (BWR).
H1	H. Plant component/system data	Recirculation pumps	Recirculation pump characteristics (inertia, flywheels, coast-down characteristics).
H2		Steam separators	Separator pressure drop and carry-under.
H3		Steam dryers	Steam dryers pressure drop and carry-over.
H4		Scram speed/timing	Control rods hydraulic insertion speed/time.
H5		Upper plenum	Geometric and hydraulic data for upper plenum.
H6		Lower plenum	Geometric and hydraulic data for lower plenum, coolant mixing effects.
H7		Downcomer	Geometric and hydraulic data for downcomer.
H8		Steamlines	Geometric and hydraulic data for steamlines.
H9		Safety Relief Valves	Capacity and characteristics of safety relief valves.
H10		Emergency Core Cooling Systems (ECCS)	Capacity and temperature of ECCS systems.
H11		Turbine valves (TCV,TSV,BPV)	Capacity and characteristics of turbine valves.

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ID	Category	Phenomenon	Definition
H12		Condenser tank	Volume and temperature of condenser hotwell.
H13		Feedwater system	Volumes and temperature of feedwater system.
H14		Boron injection system	Injection capacity, boron concentration, temperature.
H15		Level control system	Level controller characteristics.
H16		Pressure control system	Pressure controller characteristics.

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APPENDIX B:  
UNCERTAINTY EVALUATION METHODS

## **ABSTRACT**

This appendix summarizes the theoretical basis for the statistical method used in the uncertainty evaluation. The methodology used by Westinghouse is based on two methods described below, namely the Analysis of Variance Method and the Order Statistics Method.

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## ACRONYMS

AOO	Anticipated Operational Occurrences
ATWS	Anticipated Transients Without Scram
MCPR	Minimum Critical Power Ratio
OLMCPR	Operating Limit Minimum Critical Power Ratio

## APPENDIX B: UNCERTAINTY EVALUATION METHODS

### B.1 TECHNICAL BASIS – SINGLE PARAMETER UNCERTAINTY EVALUATION BY ORDER STATISTICS METHOD

The order statistics method was first derived by Wilks (Reference 1). The derivation presented below follows the later formulation, adopted by Guba, Makai, and Pal (Reference 2). The problem of estimating the tolerance limit, derived below, applies for the single event acceptance criterion. If several parameters are evaluated simultaneously, the generalized Equation 22 applies.

Let's assume that the output parameter  $y$  (event acceptance criterion) is a stochastic variable described by the probabilistic density function  $g(y)$ . Nothing is known about this function except that it is continuous. If  $N$  code runs are performed with fluctuating input parameters (sampled from their distribution functions), then a sample  $\{y_1, y_2, \dots, y_N\}$  of the event acceptance criterion,  $y$ , is obtained.

Tolerance limits  $L=L(y_1, y_2, \dots, y_N)$  and  $U=U(y_1, y_2, \dots, y_N)$  can be defined such that:

$$P\left(\int_L^U g(y) dy > \beta\right) = \gamma \quad (7)$$

Here,  $\gamma$  represents the probability that  $\beta$  fraction of the random output variable  $y$  population falls within the tolerance limits  $L$  and  $U$ .

If we now arrange the stochastic sample  $y_1, y_2, \dots, y_N$  in increasing order such that\*:

$$y(1) = \min_{1 \leq k \leq N} y_k \quad \text{and} \quad y(N) = \max_{1 \leq k \leq N} y_k \quad (8)$$

and define:

$$y(0) = -\infty \quad \text{and} \quad y(N+1) = +\infty \quad (9)$$

Guba (Reference 2) demonstrated that for some positive  $\beta < 1$  and  $\gamma < 1$ , two functions  $L=L(y_1, y_2, \dots, y_N)$  and  $U=U(y_1, y_2, \dots, y_N)$  can be constructed such that the probability  $\gamma$  that:

$$\int_L^U g(y) dy > \beta \quad (10)$$

is determined from:

\* Assumption is made that the probability of equal values of  $y$  occurring is neglected, because the probability density function  $g(y)$  has been assumed to be continuous.

$$\gamma = 1 - I(\beta, s-r, N-s+r+1) = \sum_{j=0}^{s-r-1} \binom{N}{j} \beta^j (1-\beta)^{N-j} \quad (11)$$

where:

$$I(\beta, j, k) = \int_0^{\beta} \frac{x^{j-1} (1-x)^{k-1}}{B(j, k)} dx \quad (12)$$

$$B(j, k) = \frac{(j-1)!(k-1)!}{(j+k-2)!} \quad (13)$$

$$0 \leq r \leq s \leq N \quad \text{and} \quad L = y(r), U = y(s) \quad (14)$$

Equation 11 is used to determine the necessary number of code runs in order to estimate the tolerance limit with probability  $\beta$  on a confidence interval  $\gamma$  using  $s^{\text{th}}$ , resp.  $r^{\text{th}}$  estimator.

If for example the smallest and the largest of  $N$  calculations are used, as an estimate ( $r=1$  and  $s=N$ ), then Equation 11 simplifies to (two sided tolerance limit):

$$\gamma = 1 - \beta^N - N(1-\beta)\beta^{N-1} \quad (15)$$

And if only the upper tolerance limit has to be estimated, as is the case for all event acceptance criteria for AOO and ATWS, then Equation 11 simplifies to (assuming  $r=0$  and  $s=N$ ):

$$\gamma = 1 - \beta^N \quad (16)$$

If  $\beta=0.95$  and  $\gamma=0.95$ , the number of calculations estimated by Equation 16 is  $N=59$ . The statistical interpretation of this result is that when there is a random sample of size  $N=59$  observed, then there is a  $\gamma=95\%$  probability that 95% of the population for the considered output parameter (event acceptance criterion) is below the maximum of the 59 samples.

A single one-sided tolerance limit may also be estimated by other estimator than the largest from the sample set. Let's say we would like to estimate the event acceptance criterion by the second largest value from the  $N$ -sample set. Assume that the upper tolerance limit ( $\beta=95\%$ ) is estimated by the second largest estimator from the sample set. 95% confidence in the result is required. Then the necessary number of run is calculated from Equation 17 (derived by assuming  $r=0$  and  $s=N-1$  in Equation 11):

$$\gamma = 1 - \beta^N - N(1-\beta)\beta^{N-1} \quad (17)$$

$N=93$  is obtained by solving Equation 17 for  $\gamma=0.95$  and  $\beta=0.95$ .

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The required number of code runs needed to estimate the upper/lower tolerance limit of 95% with 95% confidence by  $k^{\text{th}}$  largest/smallest estimator is shown in Table B-1.

Table B-1 Number of code runs for different estimators and single parameter

Estimator Grade	Number of Runs
1	59
2	93
3	124
4	153
5	181
6	208

## B.2 TECHNICAL BASIS –MULTIPLE PARAMETER UNCERTAINTY EVALUATION BY ORDER STATISTICS METHOD

The derivation of non-parametric statistic method, as provided in previous chapter assumes a single uncertainty analysis output parameter. A situation may arise, requiring the simultaneous evaluation of several output parameters, such as operating limit MCPR and reactor vessel pressure.

Guba (Reference 2) therefore provides an extension of his formulation to multiple variables for the case when the dependency of output variables is represented by an unknown joint density distribution function. Following is the derivation according to Guba (Reference 2), utilized as the uncertainty analysis of several output parameters by Westinghouse.

Let's assume the output comprising p output variables ( $y_1, y_2, \dots, y_p$ ) and let  $g(y_1, y_2, \dots, y_p)$  to be their joint probabilistic distribution function and let Y to be defined as:

$$Y = \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1N} \\ y_{21} & y_{22} & \dots & y_{2N} \\ \dots & \dots & \dots & \dots \\ y_{p1} & y_{p2} & \dots & y_{pN} \end{pmatrix} \quad (18)$$

where each element of matrix Y is the result of n<sup>th</sup> code run ( $1 < n < N$ ) for p<sup>th</sup> output parameter ( $1 < p < P$ ). Let  $g(y_1, y_2, \dots, y_p)$  to be unknown and continuous. Then, in the case of  $P > 1$  dependent output variables, it is possible to construct p-pairs of random intervals  $[L_j, U_j]$ ,  $j=1, \dots, P$  such that the probability of the following inequality:

$$\int_{L_1}^{U_1} \dots \int_{L_p}^{U_p} g(y_1, \dots, y_p) dy_1 \dots dy_p > \beta \quad (19)$$

is independent of  $g(y_1, y_2, \dots, y_p)$  and is given by:

$$\begin{aligned} P \left\{ \int_{L_1}^{U_1} \dots \int_{L_p}^{U_p} g(y_1, \dots, y_p) dy_1 \dots dy_p > \beta \right\} &= \gamma \\ &= 1 - I(\beta, s_p - r_p, N - s_p + r_p + 1). \end{aligned} \quad (20)$$

Function  $I(\cdot)$  is given by Equations 12 and 13 and

$$s_p \leq s_{p-1} - r_{p-1} - 1 \leq s_1 - \sum_{j=1}^{p-1} (r_j + 1) \quad \text{and} \quad r_p \geq r_{p-1} \geq \dots \geq r_1 \quad (21)$$

If  $r_1 = r_2 = \dots = r_p = 0$  and  $s_p = N - p + 1$ , then the solution of the following equation defines the number of code runs needed to obtain  $\beta$  probability at  $\gamma$  confidence interval, for one-sided probability interval and p-parameters evaluated simultaneously:

$$\gamma = 1 - I(\beta, N - p + 1, p + 2) = \sum_{j=0}^{N-p} \binom{N}{j} \beta^j (1 - \beta)^{N-j} \quad (22)$$

The solution of Equation 22 for  $\beta=0.95$  and  $\gamma=0.95$  for 1-3 parameters evaluated simultaneously is shown in Table B-2:

Table B-2 Number of code runs for simultaneous evaluation of 1-3 event acceptance criteria

Number of Parameters	Number of Runs
1	59
2	93
3	124

### B.3 TECHNICAL BASIS –UNCERTAINTY EVALUATION BY ANALYSIS OF VARIANCE METHOD

Analysis of Variance uncertainty evaluation method is based on the assumption that the output parameter (event acceptance criterion) is distributed normally, described by finite mean value ( $\mu$ ) and finite standard deviation ( $\sigma$ ).

Ideally, the infinite number of code runs is performed (normal distribution is continuous) and the mean value is calculated according to Equation 23:

$$\mu = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n X_i \quad (23)$$

Similarly the standard deviation:

$$\sigma = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \mu)^2} \quad (24)$$

where  $X_i$  are the random values from distribution  $X$ .

In practical situations, the finite number of code runs ( $N$ ) is performed and the sample mean is calculated instead as:

$$\bar{\mu} = \frac{1}{N} \sum_{i=1}^N X_i \quad (25)$$

and analogically the sample standard deviation  $s_n$ :

$$s_n = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \bar{\mu})^2} \quad (26)$$

Theoretically, if  $\mu$  and  $\sigma$  are known, then the upper estimate of 95<sup>th</sup> probability level for the parameter under investigation ( $X_{95,100}$ ) could be calculated with 100% confidence by the following formula:

$$X_{95,100} = \mu + 1.645\sigma \quad (27)$$

In a practical situation, both  $\mu$  and  $\sigma$  are unknown and are estimated by the sample mean and sample standard deviation from a vector of  $N$  realizations of random parameter  $X$ . In that case, a 95<sup>th</sup> probability level of the parameter under investigation is estimated with 95% confidence by Equation 28:

$$X_{95,95} = \bar{\mu} + z_{95,95} s_n \quad (28)$$

Here,  $X_{95,95}$  is the 95/95 estimate of the parameter under investigation and  $z_{95,95} > 1.645$  is a *factor for one-sided normal tolerance limit*. This factor depends on the desired probability level (95%), confidence interval (95%) and the number of parameter realizations (code runs N). A factor for one-sided normal tolerance limit can be found from statistical tables and for 95% probability, 95% confidence and  $N=59$ ,  $z_{95,95}=2.024$ .

If for example the OLMCPR0(i) value has to be estimated from the sample of 59 runs, on a 95% probability level, with 95% confidence, then the sample mean is calculated according to Equation 25, sample standard deviation according to Equation 26,  $z_{95,95}=2.024$ , and the OLMCPR0(i) according to Equation 28. The assumption is taken that the code run samples are coming from a normal distribution. This assumption has to be verified using standard normality tests such as Anderson-Darling test.

## REFERENCES

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2. Guba, A., Makai, M., and Pal, L., "Statistical Aspects of Best Estimate Method-I", Reliability Engineering and System Safety, 80, pp. 217-232, 2003