APPENDIX A
SEISMIC CORE-DAMAGE FREQUENCY ESTIMATES

A.1 Elementary Estimates

An equation for seismic core-damage frequency (SCDF) can be developed from the site-specific seismic hazard curve and the plant-level fragility, which are defined in Table 2:

Table 2. Definitions and Properties of Seismic Hazard and Plant-Level Fragility

Common Name	Symbol	Definition	Properties
Seismic hazard curve	$H(a)$	The annual frequency at which the site earthquake-induced vibratory ground motion exceeds a given value, *a*.	• $H(a) \geq 0$ for $a \geq 0$ • $H(a)$ is not defined for $a < 0$ • $H(a)$ is a monotonic decreasing function: $H(a_1) \geq H(a_2)$ for $a_1 \leq a_2$)
Plant-level fragility	$P_{CD}(a)$	The probability of core damage as a function of the site earthquake-induced vibratory ground motion, *a*.	• $0 \leq P_{CD}(a) \leq 1$ for $a \geq 0$ • $P_{CD}(a)$ is not defined for $a < 0$ • $P_{CD}(a)$ is a monotonic increasing function: $P_{CD}(a_1) \leq P_{CD}(a_2)$ for $a_1 \leq a_2$)

In nuclear power plant seismic risk assessment, the site earthquake-induced vibratory ground motion is usually expressed in terms of the peak ground acceleration (PGA). Other characterizations of the vibratory ground motion may also be used (e.g., individual spectral accelerations, the average spectral acceleration over a select band of spectral frequencies). An estimate of the SCDF using a single ground motion characterization is called an “elementary SCDF estimate.”

The SCDF resulting from earthquakes that cause site ground motions in the interval $[a_L, a_U]$ can be found by partitioning the interval $[a_L, a_U]$ into *n* subintervals $[a_{i-1}, a_i]$, each of which is tagged with a distinguishing point a^*_i such that $a_{i-1} \leq a^*_i \leq a_i$. Let $\Delta a_i = a_i - a_{i-1}$ be the width of each subinterval. Adding up the contributions from each subinterval (a Riemann sum) provides an estimate of the SCDF over the interval $[a_L, a_U]$:

$$SCDF(a_L, a_U) \approx \sum_{i=1}^{n} \left[H(a_i) - H(a_i + \Delta a_i)\right] P_{CD}(a_i^*) \quad \text{(A-1)}$$

The difference $H(a_i) - H(a_i + \Delta a_i)$ is the frequency of earthquakes that cause site ground motions in the interval [ai, ai + Δa_i]. This frequency is analogous to the frequency of an initiating event defined in an internal event probabilistic risk assessment (PRA) (i.e., each subinterval can be interpreted as a unique seismic initiating event). Thus, each term in the Riemann sum represents the core-damage frequency due to its associated seismic initiating event, and the Riemann sum itself approximates the total seismic core-damage frequency resulting from earthquakes that cause site ground motions in the interval $[a_L, a_U]$.

Multiplying and dividing each term in the Riemann sum by Δa_i and rearranging gives:

$$SCDF(a_L,a_U) \approx \sum_{i=1}^{n} P_{CD}(a_i^*)\left[-\frac{H(a_i+\Delta a_i)-H(a_i)}{\Delta a_i}\right]\Delta a_i \qquad \text{(A-2)}$$

As the number of subintervals, n, increases, the width, Δa_i, of each subinterval decreases. In the limit:

$$\lim_{\Delta a_i \to 0} -\frac{H(a_i+\Delta a_i)-H(a_i)}{\Delta a_i} = -\frac{dH(a_i)}{da} \qquad \text{(A-3)}$$

In the limit, the Riemann sum converges to a definite integral:

$$SCDF(a_L,a_U) = \int_{a_L}^{a_U} P_{CD}(a)\left[-\frac{dH(a)}{da}\right]da \qquad \text{(A-4)}$$

The total SCDF is obtained by setting $a_L = 0$ and $a_U = \infty$:

$$SCDF = \int_{0}^{\infty} P_{CD}(a)\left[-\frac{dH(a)}{da}\right]da \qquad \text{(A-5)}$$

A.2 Solution of the Elementary Equation

Given functional forms for *H(a)* and $P_{CD}(a)$, it is straightforward to numerically integrate Equation (A-5). However, probabilistic seismic hazard analysis typically presents *H(a)* in tabular form, leading to the need to interpolate and extrapolate to determine the value of *H(a)* for any arbitrary value of *a*. Double logarithmic interpolation and extrapolation (i.e., ln *H(a)* is a linear function of ln *a*) is a commonly used approach and has been adopted in this analysis. Note that linear interpolation and extrapolation on log-log axes implies power-law behavior.

It is assumed that the seismic hazard curve is defined by *n* points:

$$\{a_i, H_i \mid i = 1,2,\ldots,n\} \qquad \text{where } H_i = H(a_i) \text{ and } a_1 < a_2 < \ldots < a_n \qquad \text{(A-6)}$$

Using any two adjacent points, *i-1* and *i*, the seismic hazard curve can be approximated with a power law:

$$H_i(a) = K_{I,i}\, a^{-K_{H,i}} \qquad \textit{for } a_{i-1} \le a \le a_i \qquad \text{(A-7)}$$

where:

$$\left.\begin{aligned} K_{I,i} &= H_{i-1} - a_{i-1}^{K_{H,i}} \\ K_{H,i} &= \frac{\ln\left(\dfrac{H_i}{H_{i-1}}\right)}{\ln\left(\dfrac{a_i}{a_{i-1}}\right)} \end{aligned}\right\} \quad i = 2,3,\ldots,n \tag{A-8}$$

Seismic PRAs often provide either a tabulation or graph of $P_{CD}(a)$, which is reasonably approximated by a log-normal function over a wide range of ground motions (see Appendix C for a detailed discussion of plant-level fragility):

$$P_{CD}(a) = \Phi\left[\frac{\ln a - \mu}{\beta_C}\right] = \int_0^a \frac{1}{\sqrt{2\pi}\beta_C x} \exp\left[-\frac{(\ln x - \mu)^2}{2\beta_C^2}\right] dx \qquad \text{for } C_{50}, \beta_C > 0 \textit{ and } \mu = \ln C_{50} \tag{A-9}$$

where C_{50} denotes the median seismic capacity and β_C denotes composite logarithmic standard deviation. The derivative of $P_{CD}(a)$ with respect to *a* is:

$$\frac{dP_{CD}(a)}{da} = \frac{1}{\sqrt{2\pi}\beta_C a} \exp\left[-\frac{(\ln a - \mu)^2}{2\beta_C^2}\right] \tag{A-10}$$

As a result, Equation (A-5) could be numerically integrated by using Equations (A-7) and (A-9). However, numerical integration is not needed because Equation (A-5) has a closed-form solution under these assumptions.

Equation (A-5) can be broken into *n-1* integrals according to the points of the seismic hazard curve:

$$\begin{aligned} SCDF &= SCDF(0,a_2) + SCDF(a_2,a_3) + \cdots + SCDF(a_{n-1},\infty) \\ &= \int_0^{a_2} P_{CD}(a)\left[-\frac{dH_2(a)}{da}\right]da + \int_{a_2}^{a_3} P_{CD}(a)\left[-\frac{dH_3(a)}{da}\right]da + \ldots + \int_{a_{n-1}}^{\infty} P_{CD}(a)\left[-\frac{dH_n(a)}{da}\right]da \end{aligned} \tag{A-11}$$

Integrating Equation (A-4) by parts yields:

$$SCDF(a_i, a_{i-1}) = \int_{a_{i-1}}^{a_i} H_i(a)\left[\frac{dP_{CD}(a)}{da}\right]da - H_i(a)P_{CD}(a)\Big|_{a_{i-1}}^{a_i} \tag{A-12}$$

Substituting Equations (A-7) and (A-10) into the first term on the right-hand side of Equation (A-12):

$$I(a_{i-1},a_i) = \int_{a_{i-1}}^{a_i} H_i(a)\left[\frac{dP_{CD}(a)}{da}\right]da = \frac{K_{I,i}}{\sqrt{2\pi}\beta_C}\int_{a_1}^{a_2} \exp\left[-\frac{(\ln a-\mu)^2}{2\beta_C^2} - K_{H,i}\ln a\right]\frac{da}{a} \tag{A-13}$$

To further reduce Equation (A-13), it is convenient to make a change of variable in the integral:

$$x = \frac{\ln a - \mu}{\beta_C} \quad \Rightarrow \quad dx = \frac{da}{\beta_C a} \tag{A-14}$$

After making the change of variable, Equation (A-13) becomes:

$$I(a_{i-1},a_i) = \frac{K_{I,i}}{\sqrt{2\pi}}\exp(-K_{H,i}\mu)\int_{x_{i-1}}^{x_i} \exp\left(-\frac{x^2}{2} - K_{H,i}\beta_C x\right)dx \tag{A-15}$$

Completing the square in the exponential term inside the integral:

$$-\frac{x^2}{2} - K_{H,i}\beta_C x = -\frac{1}{2}\left[(x+K_{H,i}\beta_C)^2 - K_{H,i}^2\beta_C^2\right] \tag{A-16}$$

Substituting Equation (A-16) into Equation (A-15) yields:

$$\begin{aligned} I(a_{i-1},a_i) &= K_{I,i}\exp\left(-K_{H,i}\mu + \frac{1}{2}K_{H,i}^2\beta_C^2\right)\int_{x_{i-1}}^{x_i}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x+K_{H,i}\beta_C)^2}{2}\right]dx \\ &= K_{I,i}\exp\left(-K_{H,i}\mu + \frac{1}{2}K_{H,i}^2\beta_C^2\right)\left[\Phi\left(\frac{\ln a_i - \mu}{\beta_C} + K_{H,i}\beta_C\right) - \Phi\left(\frac{\ln a_{i-1} - \mu}{\beta_C} + K_{H,i}\beta_C\right)\right] \end{aligned} \tag{A-17}$$

The second term on the right-hand side of Equation (A-12) is readily determined as:

$$H(a)P_{CD}(a)\Big|_{a_{i-1}}^{a_i} = K_{I,i}a_1^{-K_{H,i}}\Phi\left(\frac{\ln a_i - \mu}{\beta_C}\right) - K_{I,i}a_2^{-K_{H,i}}\Phi\left(\frac{\ln a_{i-1} - \mu}{\beta_C}\right) \tag{A-18}$$

Combining Equations (A-17) and (A-18) gives the closed-form solution to Equation (A-12):

$$\begin{aligned} &SCDF(a_{i-1},a_i) \\ &= K_{I,i}\exp\left(-K_{H,i}\mu + \frac{1}{2}K_{H,i}^2\beta_C^2\right)\left[\Phi\left(\frac{\ln a_i - \mu}{\beta_C} + K_{H,i}\beta_C\right) - \Phi\left(\frac{\ln a_{i-1} - \mu}{\beta_C} + K_{H,i}\beta_C\right)\right] \\ &\quad + K_{I,i}a_{i-1}^{-K_{H,i}}\Phi\left(\frac{\ln a_{i-1} - \mu}{\beta_C}\right) - K_{I,i}a_i^{-K_{H,i}}\Phi\left(\frac{\ln a_i - \mu}{\beta_C}\right) \end{aligned} \tag{A-19}$$

The total seismic core-damage frequency, SCDF< can be determined by substituting Equation (A-19) into Equation (A-11):

$$
\begin{aligned}
SCDF &= K_{I,2} \exp\left(-K_{H,2}\mu + \frac{1}{2}K_{H,2}^2\beta_C^2\right)\Phi\left(\frac{\ln a_2 - \mu}{\beta_C} + K_{H,2}\beta_C\right) \\
&+ \sum_{i=3}^{n-1} K_{I,i} \exp\left(-K_{H,i}\mu + \frac{1}{2}K_{H,i}^2\beta_C^2\right)\left[\Phi\left(\frac{\ln a_i - \mu}{\beta_C} + K_{H,i}\beta_C\right) - \Phi\left(\frac{\ln a_{i-1} - \mu}{\beta_C} + K_{H,i}\beta_C\right)\right] \qquad \text{(A-20)} \\
&+ K_{I,n} \exp\left(-K_{H,n}\mu + \frac{1}{2}K_{H,n}^2\beta_C^2\right)\left[1 - \Phi\left(\frac{\ln a_n - \mu}{\beta_C} + K_{H,n}\beta_C\right)\right]
\end{aligned}
$$

A.3 Derived Estimates

As discussed in Section A.1, the site earthquake-induced vibratory ground motion is usually expressed in terms of the peak ground acceleration. Other characterizations of the ground motion could also be used in Equation (A-20). In the Safety/Risk Assessment of GI-199, elementary SCDF estimates were computed for the 10-Hz, 5-Hz, and 1-Hz spectral accelerations in addition to the peak ground acceleration.

A "derived SCDF estimate" is an estimate of the seismic core-damage frequency that is developed from the four elementary SCDF estimates. Let:

$SCDF_{pga}$ = SCDF estimate obtained by using the PGA-based seismic hazard and plant-level fragility curves
$SCDF_{10}$ = SCDF estimate obtained by using the 10-Hz seismic hazard and plant-level fragility curves
$SCDF_{5}$ = SCDF estimate obtained by using the 5-Hz seismic hazard and plant-level fragility curves
$SCDF_{1}$ = SCDF estimate obtained by using the 1-Hz seismic hazard and plant-level fragility curves

Three derived SCDF estimates are defined as follows:

$$SCDF_{\max} = \max\left(SCDF_{pga}, SCDF_{10}, SCDF_{5}, SCDF_{1}\right) \quad \text{(A-21)}$$

$$SCDFavg = \frac{1}{4}SCDF_{pga} + \frac{1}{4}SCDF_{10} + \frac{1}{4}SCDF_{5} + \frac{1}{4}SCDF_{1} \quad \text{(A-22)}$$

$$SCDF_{IPEEE} = \frac{1}{7}SCDF_{pga} + \frac{2}{7}SCDF_{10} + \frac{2}{7}SCDF_{5} + \frac{2}{7}SCDF_{1} \quad \text{(A-23)}$$

Equation (A-23) is termed the "IPPEE weighted average SCDF" because the weights were obtained from Appendix A of NUREG-1407.

A.4 Weakest Link Model

In general, the four elementary SCDF estimates are unique (i.e., they yield different values of the SCDF). This effect happens because the seismic hazard curves at various spectral frequencies are not parallel; rather, they have different slopes. The derived estimates represent an attempt to reconcile the different elementary SCDF estimates but cannot be developed from fundamental principles. Specifically, no technical basis exists for assigning the weights.

An alternative to the log-normal function used to characterize the plant-level fragility, $P_{CD}(a)$, can be developed by considering the relationship between the spectral shape used to determine the PGA-based plant-level fragility curve and the uniform hazard spectrum at a given annual exceedance frequency, h. The uniform hazard spectrum at a given annual exceedance frequency, h, may be determined from the following four-step process.

- Step 1. Select a set of spectral frequencies from a range of spectral frequencies that govern system, structure, and component fragilities. In the Safety/Risk Assessment, four spectral frequencies were selected: 1 Hz, 5, Hz, 10 Hz, and PGA.

- Step 2. Obtain the spectral seismic hazard curves that correspond to the selected spectral frequencies, for example, H1(a1), H5(a5, H10(a10), and Hpga(apga).

- Step 3. For the given annual exceedance frequency, h, find the corresponding spectral accelerations by inverting each spectral hazard curve (see Figure A-1):

$$h = H_1(a_1) = H_5(a_5) = H_{10}(a_{10}) = H_{pga}(a_{pga})$$
$$\rightarrow a_1 = H_1^{-1}(h), a_5 = H_5^{-1}(h), a_{10} = H_{10}^{-1}(h), \text{and } a_{pga} = H_{pga}^{-1}(h) \quad \text{(A-24)}$$

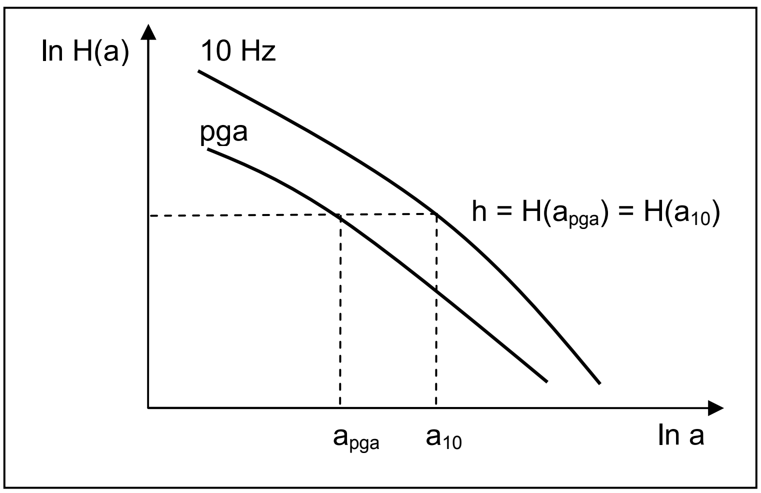

Figure A-1. Construction of a Uniform Hazard Spectrum.

- Step 4. Plot the corresponding spectral accelerations against their spectral frequencies (see Figure A-2).

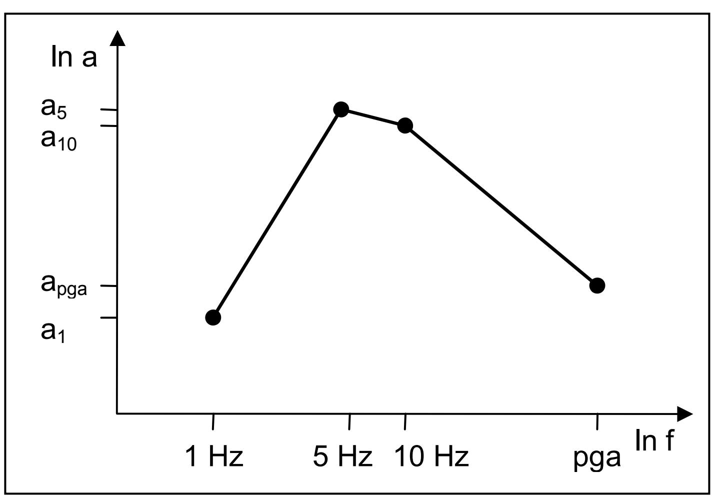

Figure A-2. A Uniform Hazard Spectrum.

In general, the shape of the normalized uniform hazard spectrum (UHS) depends on annual exceedance frequency (it is typical to normalize response spectra with respect to the peak ground acceleration). In addition, the shape of the normalized UHS is different, in general, than the shape of the response spectrum used to develop the PGA-based plant-level fragility curve. (Even if the 10^{-4} per year UHS is used to develop the PGA-based plant-level fragility curve, it will have a different normalized shape than, for example, the normalized 10^{-5} UHS.)

The fragility of an SSC is usually developed at that SSC's natural (fundamental) frequency. This is done because the presence of the natural frequency causes the SSC to resonate, which greatly increases the forces and moments impressed on the SSC. In general, the natural frequency for one SSC will be different than the natural frequency for another SSC, depending on the SSC's design. In a seismic PRA, these differing natural frequencies are converted to peak ground acceleration by using a review-level response spectrum. The key parameter in this process is the spectral ratio. The spectral ratio for a given spectral frequency, m_f, is defined as the ratio of the spectral ordinate on the review-level response spectrum corresponding to that spectral frequency to the spectral ordinate on the review-level response spectrum for the peak ground acceleration. For example, suppose that the natural frequency of an SSC is 10 Hz and that analysis indicates the SSC will fail when the 10-Hz spectral acceleration exceeds 1.5 g. If the 10-Hz spectral ratio, m_{10}, is 2, then the SCC is assumed to fail when the peak ground acceleration reaches 1.5 g / 2 = 0.75 g.

In a seismic PRA, seismically induced failure is not a threshold effect; rather, various probabilities of failure exist depending on the acceleration caused by an earthquake. These failure probabilities are described by the fragility curve, which can be interpreted using random variables and load-strength interference theory. Let C be a random variable that represents the PGA-based seismic capacity. Then, an SSC will fail if C is less than the acceleration caused by an earthquake, and the probability of failure as a function of the peak ground acceleration (the fragility) can be expressed as:

$$\Pr\{failure\} = \Pr\{C \le a\} \qquad \text{(A-25)}$$

Considering how fragilities are developed in a seismic PRA, it is reasonable to assume that the seismic capacity at an arbitrary spectral frequency, C_f, would be the product of the spectral ratio for that spectral frequency and the PGA-based seismic capacity:

$$\Pr\{failure\} = \Pr\{C_f \le a_f\} = \Pr\{m_f C \le a_f\} \qquad \text{(A-26)}$$

Figure A-3 illustrates the relationships among the 1-hz, 5-Hz, and 10-Hz spectral fragilities and the PGA-based fragility for an arbitrary SSC. For example, suppose that an earthquake causes a peak ground acceleration equal to the HCLPF, then the probability of failure is 0.01. If the 10-Hz spectral ratio, m_{10}, is 2, then the occurrence of an earthquake that causes a 10-Hz spectral acceleration of twice the HCLPF also implies a failure probability of 0.01.

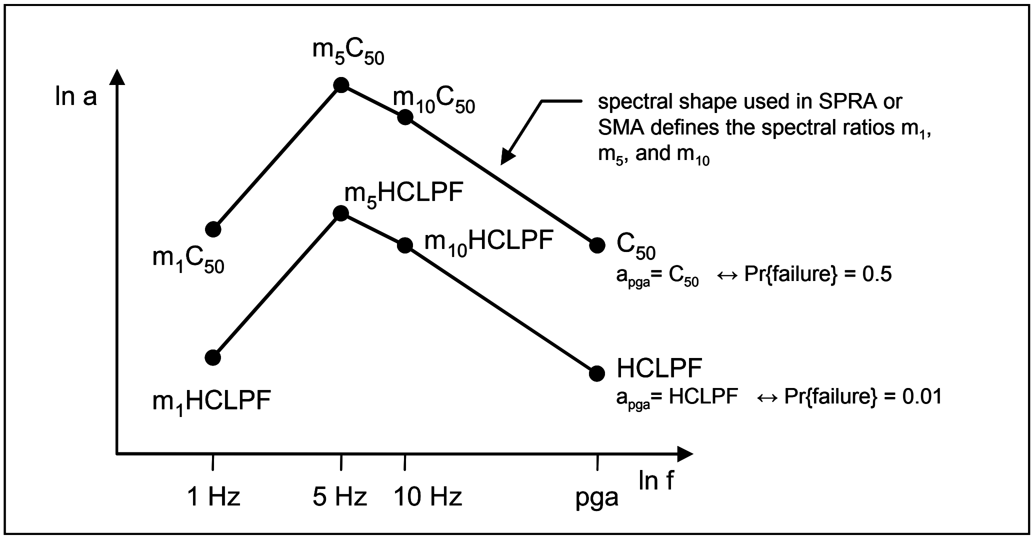

Figure A-3. The Definition of Spectral Fragility.

To define the probability of failure based on a UHS and the definition of spectral fragility just discussed, consider Figure A-4 that shows the UHS superimposed onto the review-level response spectrum:

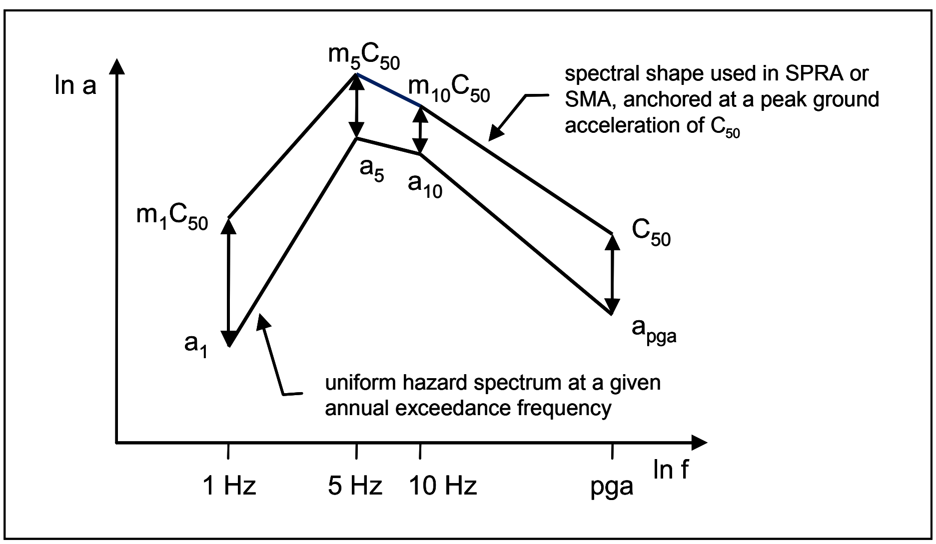

Figure A-4. Development of the Weakest Link Model.

Figure A-4 indicates that the probability of failure depends on “how close” the UHS is to the spectral fragility curve. Specifically:

$$
\begin{aligned}
\Pr\{failure\} &= 1-\Pr\left[C > a \text{ and } C_1 > a_1 \text{ and } C_5 > a_5 \text{ and } C_{10} > a_{10}\right] \\
&= 1-\Pr\left[C > a \text{ and } m_1 C > a_1 \text{ and } m_5 C > a_5 \text{ and } m_{10} C > a_{10}\right] \\
&= 1-\Pr\left[C > a \text{ and } C > \frac{a_1}{m_1} \text{ and } C > \frac{a_5}{m_5} \text{ and } C > \frac{a_{10}}{m_{10}}\right] \qquad \text{(A-27)} \\
&= 1-\Pr\left[C > \max\left(a, \frac{a_1}{m_1}, \frac{a_5}{m_5}, \frac{a_{10}}{m_{10}}\right)\right]
\end{aligned}
$$

The relationship described by Equation (A-27) is termed the “weakest link” model in this assessment.