ENCLOSURE 1

"In-Plane Behavior of Concrete Filled Steel (CFS) Elements" Presentation

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IN-PLANE BEHAVIOR OF CONCRETE FILLED STEEL (CFS) ELEMENTS

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- It is excerpted from the report:

- Varma, A.H., and Malushte, S. (2009). "In Plane Behavior, Analysis, and Design of Concrete Filled Steel Wall Panels." *Bowen Laboratory Research Report No. 2009 – 03*, School of Civil Engineering, Purdue University, West Lafayette, IN 27906, *under review by sponsor*.

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RESULTS FROM LEVEL 2 ANALYSIS

- The level 2 analysis of the structure will result in force and deformation demands on the components.
- In the spirit of LRFD, the level 2 analysis will be elastic that accounts for the effects of: (a) concrete cracking, (b) slip between the steel plates and concrete infill, and (c) connection flexibility.
- The results from the analysis will include for the cylindrical shield building the following demands:
  - In plane Force demands (Sx, Sy, and Sxy)
  - Moment (Mx, My, and Mxy)
  - Out of Plane shear (Vxz and Vyz)

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IN PLANE BEHAVIOR

- The effects of these demands on the behavior of the steel-concrete composite (SC) structure was investigated
- To begin, lets focus on the behavior for in-plane forces Sx, Sy, and Sxy.
- These are forces per unit length typically in kips/ft
- Just to give a sense of quantity
  - The pure tension strength of the AP1000 design = 0.75 x 2 x 12 x 50 ksi = 900 kips/ft
  - The pure compression strength (squash load) = 0.75 x 2 x 12 x 50 ksi + 34.5 x 12 x 6 ksi = 3384 kips/ft

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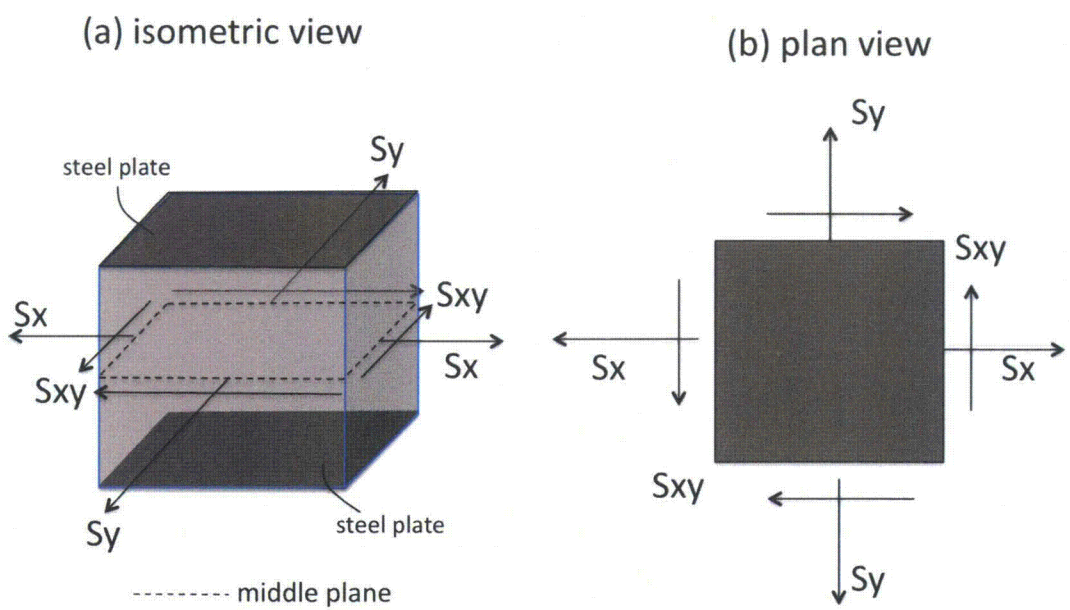
In Plane Behavior

- Consider a concrete filled steel (CFS) element taken from the SC structure
- It is subjected to in-plane forces Sx, Sy, and Sxy per unit length (in.).



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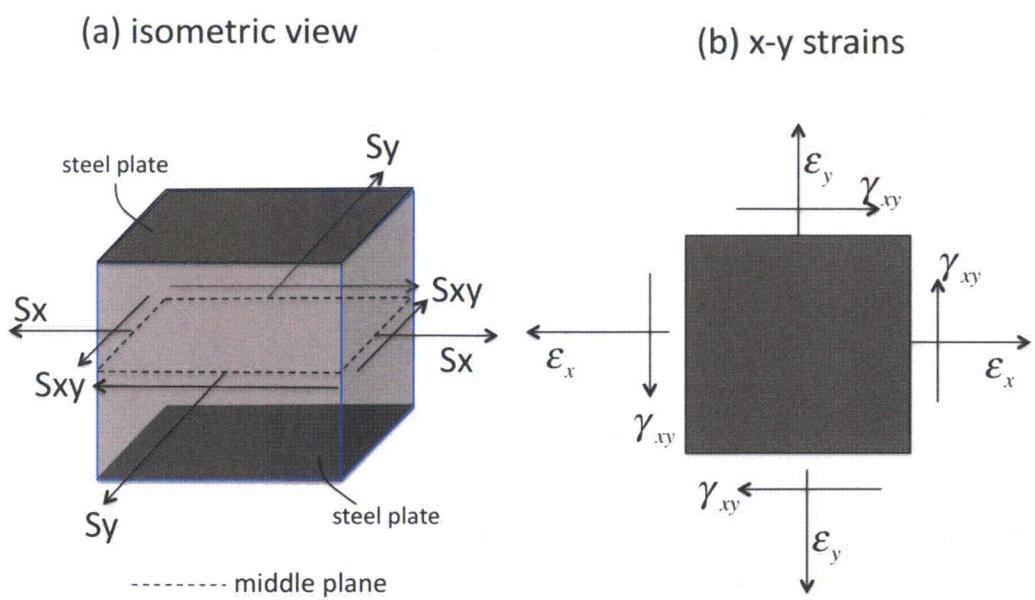
In Plane Behavior

- These in-plane forces will cause some deformations in the element. These deformations can be denoted by the strain terms: ε_x, ε_y, γ_{xy}



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ONE ASSUMPTION

- Let us assume that the strains ε_x, ε_y, γ_{xy} are compatible between the steel plate and the concrete infill.
- We will revisit this assumption later.
- The AP 1000 design will have to demonstrate that the shear stud spacing (8.5in.) is adequate to achieve this as an engineering approximation for the load cases.

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TWO FACTS

- If θ_p is the principal direction
- Stress Transformation relates the principal stresses to the x-y stresses,

$$[T]_\sigma = \frac{1}{2}\begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & 2\sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -2\sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

- Strain Transformation relates the principal strains to the x-y strains

$$[T]_\varepsilon = \frac{1}{2}\begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & \sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -\sin(2\theta_p) \\ -2\sin(2\theta_p) & 2\sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

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PRINCIPAL IN-PLANE FORCES SP1 AND SP2

- The difference between the two transformations is due to the definition of engineering shear strain γ_{xy} which is twice the shear strain ε_{xy}

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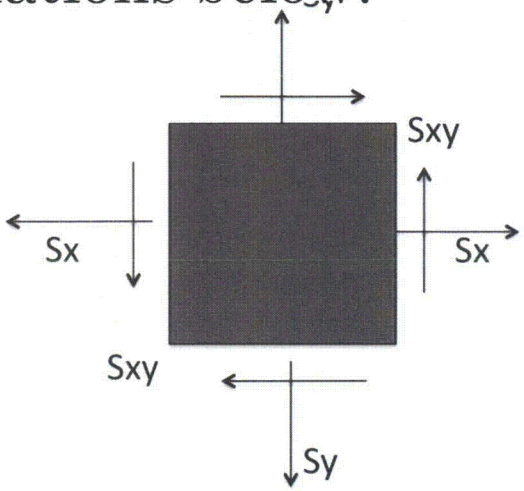
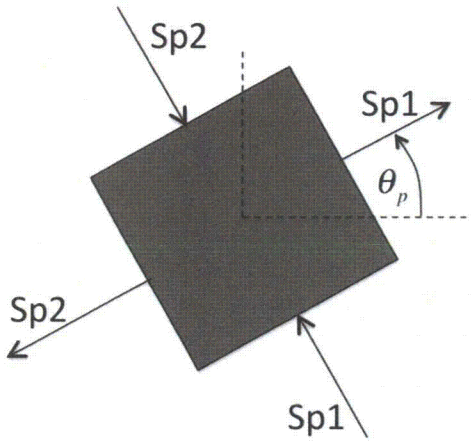
PRINCIPAL IN-PLANE FORCES (SP1, SP2)

- The in-plane forces (Sx, Sy, and Sxy) can be used to compute the principal direction (θ_p) and principal membrane forces S_{p1} and S_{p2} per unit length using Equations below:





$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y}$$

$$S_{p1} = \frac{(1+\cos(2\theta_p)}{2} S_x + \frac{(1-\cos(2\theta_p)}{2} S_y + \sin(2\theta_p)\, S_{xy}$$

$$S_{p2} = \frac{(1-\cos(2\theta_p)}{2} S_x + \frac{(1+\cos(2\theta_p)}{2} S_y - \sin(2\theta_p)\, S_{xy}$$

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CONCRETE CRACKING

- Before concrete cracking, the principal forces are resisted by the composite section (steel and concrete)
- Concrete cracking occurs after the principal membrane force (S_{p1} or S_{p2}) exceeds $S_{ct.}$

$$S_{ct} = \left(\frac{4\sqrt{f_c'}}{1000} - \varepsilon_{sh}E_c\right)\left(T_c + \frac{E_s}{E_c}2t_s\right) \quad kips/in.$$

- S_{ct} can be reduced to account for the effects of concrete shrinkage tensile strain (ε_{sh})
- This point is variable because the tensile strength of concrete can be highly variable and shrinkage effects are not easy to characterize.
- Cracking can occur due to one or both principal forces. The cracks will be oriented in the plane perpendicular to the principal force that exceeds S_{ct}.

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POST-CRACKING BEHAVIOR

- Post-cracking the concrete will offer very little stiffness and stress capacity in the principal direction perpendicular to the plane of cracking.
- However, it will have stiffness and strength in the principal direction parallel to the plane of cracking. This can be referred as cracked orthotropic behavior
- Japanese researchers recommend that the stiffness in the direction parallel to the plane of cracking can be assumed to be 70% of the uncracked stiffness (E_c).
- Lets assume the stiffness and strength perpendicular to the direction of cracking are assumed to be zero, which is a conservative assumption.

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PRINCIPAL STRAINS

- Assume that the principal strains (ε_{p1} and ε_{p2}) are in the same direction as the principal forces (S_{p1} and S_{p2}).
- They are related to the x-y strain (ε_x, ε_{y-}, and γ_{xy}) by the strain transformation rules. In these equations θ_p is the principal direction computed earlier.

$$\begin{Bmatrix} \varepsilon_{p1} \\ \varepsilon_{p2} \\ 0 \end{Bmatrix} = [T]_\varepsilon \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad [T]_\varepsilon = \frac{1}{2}\begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & \sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -\sin(2\theta_p) \\ -2\sin(2\theta_p) & 2\sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

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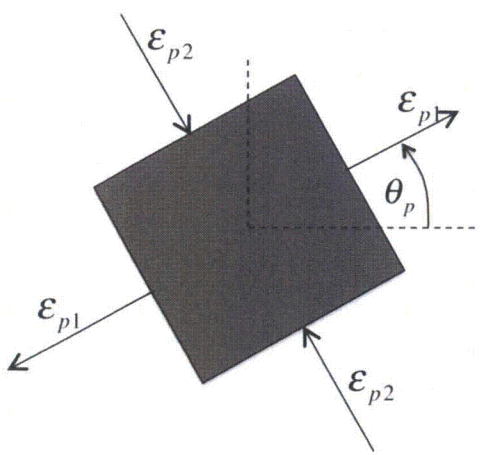
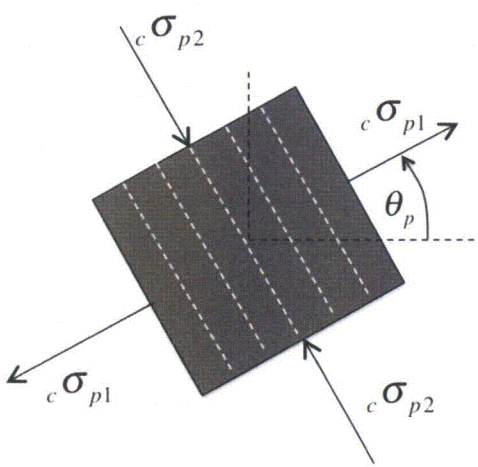
CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- The concrete infill will have principal stresses (${}_c\sigma_{p1}$ and ${}_c\sigma_{p2}$) corresponding to the principal strains (ε_{p1} and ε_{p2}). The principal stresses will be related to the strains via the reduced elastic modulus (E_c')

(a) principal strains



(b) principal stresses



$$\begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} = E_c' \begin{bmatrix} 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 \text{ or } 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{p1} \\ \varepsilon_{p2} \\ 0 \end{Bmatrix}$$

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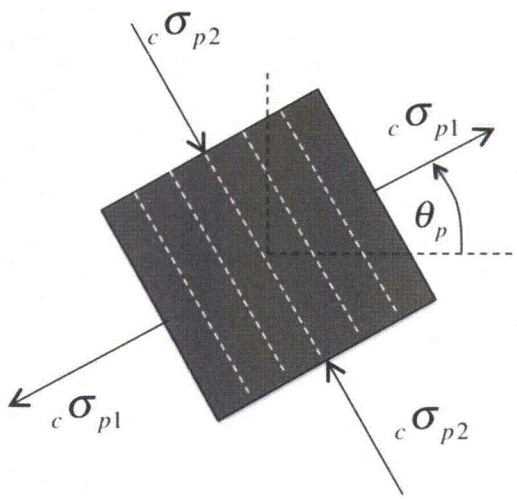
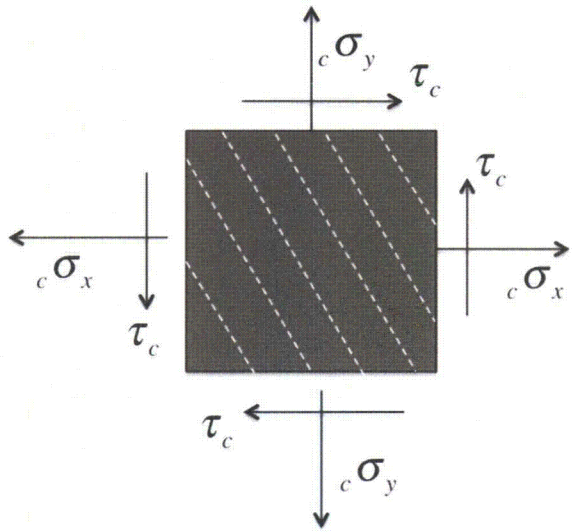
CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- The concrete principal stresses (${}_c\sigma_{p1}$ and ${}_c\sigma_{p2}$) can be transformed back to stresses in the x-y directions ${}_c\sigma_x$, ${}_c\sigma_y$, and ${}_c\tau$ as shown





$$\begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} = [T]_\sigma^{-1} \begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} \qquad [T]_\sigma = \frac{1}{2}\begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & 2\sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -2\sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

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CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- Collecting all the terms that we have developed so far we get the stress-strain relationship for cracked concrete:

$$\begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} = [K]_c \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$[K]_c = [T]_\sigma^{-1} \begin{bmatrix} 0 \; or \; E_c' & 0 & 0 \\ 0 & 0 \; or \; E_c' & 0 \\ 0 & 0 & 0 \end{bmatrix} [T]_\varepsilon$$

$$[K]_c = \frac{E_c'}{4} \begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & 2\sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -2\sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}^{-1} \begin{bmatrix} 0 \; or \; 1 & 0 & 0 \\ 0 & 0 \; or \; 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & \sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -\sin(2\theta_p) \\ -2\sin(2\theta_p) & 2\sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

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STEEL PLATE STRESS-STRAIN RELATIONSHIP

- The steel plates will have plane stress behavior, and the corresponding elastic isotropic behavior can be used to relate the steel plate membrane stresses (${}_s\sigma_x$, ${}_s\sigma_y$, and ${}_s\tau$) to the composite section strains (ε_x, ε_y, and Υ_{xy})

$$\begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix} = [K]_s \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad [K]_s = \frac{E_s}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

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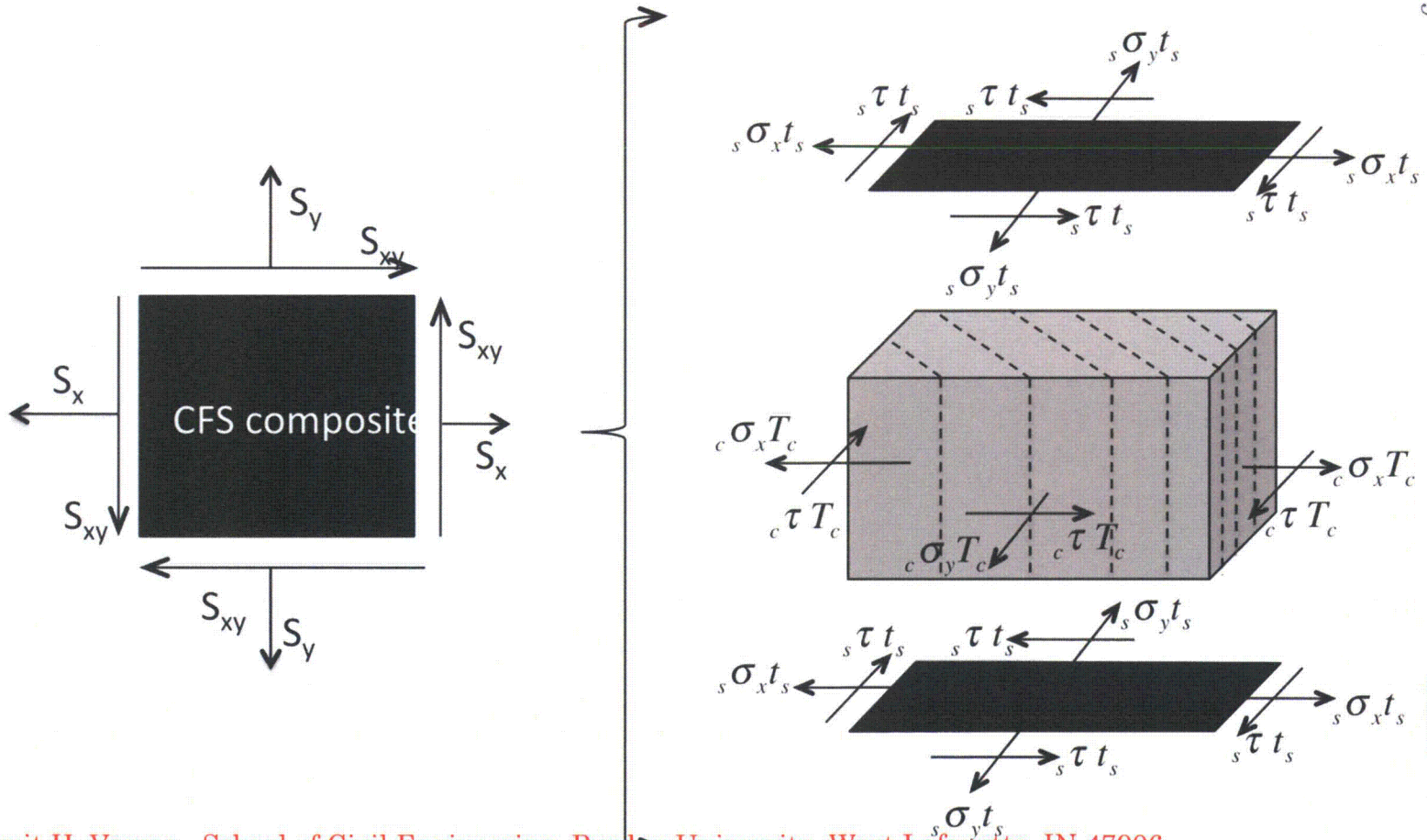
Force Equilibrium of composite section

- Static force equilibrium diagram for the CFS composite panel / element subjected to membrane forces (S_x, S_y, and S_{xy}) per unit length. (in.)



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FORCE EQUILIBRIUM OF COMPOSITE SECTION

$$\begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix} \times 2t_s + \begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} \times T_c$$

$$\begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} = 2t_s [K]_s \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + T_c [K]_c \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \left[2t_s [K]_s + T_c [K]_c \right]^{-1} \begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix}$$

- For the applied S_x, S_y, and S_{xy}, solve the equation above to get ε_x, ε_y, and γ_{xy}.

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FORCE EQUILIBRIUM OF COMPOSITE SECTION

- After obtaining the strains, calculate the steel plates stresses and concrete stresses using the following equations:

$$\begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix} = [K]_s \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \longrightarrow \begin{Bmatrix} {}_s\sigma_{p1} \\ {}_s\sigma_{p2} \\ 0 \end{Bmatrix} = [T]_\sigma \bullet \begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix}$$

$$\begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} = E_c' \begin{bmatrix} 0\ or\ 1 & 0 & 0 \\ 0 & 0\ or\ 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} [T]_\varepsilon \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

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DESIGN CHECKS AND LOAD RATIOS

- Establish yielding of the steel plates. Calculate the Von Mises stress caused by the loading

$$\sigma_{VM} = \sqrt{{}_s\sigma_{p1}^2 + {}_s\sigma_{p2}^2 - {}_s\sigma_{p1} \bullet {}_s\sigma_{p2}}$$

- The load ratio to cause yielding = F_y / σ_{VM}

- Since the concrete was assumed to remain elastic, check that the minimum principal stress (compressive) is still in the elastic range, i.e.,

$${}_c\sigma_{p2,p1} \leq 0.70 f_c'$$

- This entire process can be automated easily to determine the behavior of CFS panels subjected to a variety of in-plane membrane force combinations

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$ts := 0.3875$

$T := 30$

$Es := 29000$

$\nu := 0.3$

$fc := 6000$ $\quad Fy := 50$

$Ec := 57 \cdot 6000^{0.5} \rightarrow 4415.2010146764552491$

Material Properties Section

$Unitload := ts$

$Sxy := Unitload$

$Sx := 0.0000000001$

$Sy := 0$

Applied Loads

Fixed crack orientation

$$a := \left(2 \cdot \frac{Sxy}{Sx - Sy}\right) \rightarrow 7.75e9$$

$$\theta := \frac{1}{2}\,\mathrm{atan}(a) \rightarrow 0.78539816333293218058$$

$$Sxp1 := \frac{(Sx + Sy)}{2} + \sqrt{\left[\frac{(Sx - Sy)}{2}\right]^2 + Sxy^2} \rightarrow 0.38750000005$$

$$Sxp2 := \frac{(Sx + Sy)}{2} - \sqrt{\left[\frac{(Sx - Sy)}{2}\right]^2 + Sxy^2} \rightarrow -0.38749999995$$

$$a := \begin{cases} 0 & \text{if } Sxp1 > 0 \\ 1 & \text{otherwise} \end{cases} \qquad b := \begin{cases} 0 & \text{if } Sxp2 > 0 \\ 1 & \text{otherwise} \end{cases}$$

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$$Crackconcrete := \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Concrete contribution in principal directions

$$Strains := Totstiffness^{-1} \cdot \begin{pmatrix} Sx \\ Sy \\ Sxy \end{pmatrix} \rightarrow \begin{pmatrix} 0.00001341364243137941753194 \\ 0.00001341364242623966630965 \\ 0.00003983307197549749994407 \end{pmatrix}$$

Strains from Force Equilibrium

$$Steelstress := \frac{Steelstiff}{ts} \cdot Strains \rightarrow \begin{pmatrix} 0.55570804353658044540778778 8 \\ 0.55570804342192445660285701 8 \\ 0.44429195664977980706234491 2 \end{pmatrix}$$

Steel Stresses

$$Steeltrans := Stresstrans \cdot Steelstress \rightarrow \begin{pmatrix} 1.0000000001290322580715101 4 \\ 0.11141608682947264393913466 61 \\ 6.9814726368186078034258734 8e-21 \end{pmatrix}$$

Steel Principal Stresses

$$Concretestress := \frac{Concretestiffness}{\frac{T}{2}} \cdot Strains \rightarrow \begin{pmatrix} -0.014355791118028328172903568 5 \\ -0.014355791121733048462137057 1 \\ 0.014355791119880688317363427 7 \end{pmatrix}$$

Concrete Stresses

$$Concprinstress := Stresstrans \cdot Concretestress \rightarrow \begin{pmatrix} -3.2717216691711313940852610208e-23 \\ -0.028711582239761376635007908383 \\ -1.8311088432742040945524135676e-23 \end{pmatrix}$$

Concrete Principal Stresses

$$Steeleffstress := \sqrt{(Steeltrans_0)^2 + (Steeltrans_1)^2 - Steeltrans_0 \cdot Steeltrans_1} \rightarrow 0.94920885890230090630320192014$$

Calculating the von-mises stress

$$Loadratio := \frac{Fy}{Steeleffstress} \rightarrow 52.675446010714427324283345597 8$$

Load ratio to failure by yielding

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Pure Shear Behavior

Sxy – based on theory

PURE SHEAR BEHAVIOR

- For pure in-plane shear loading, the applied membrane forces S_x and S_y will be equal to zero, and the in-plane shear loading will be equal to S_{xy}.
- The principal direction (θ_p) =$\tan^{-1}$(inf.) = 45°
- The principal forces will be equal to S_{xy} in magnitude; S_{p1} will be in tensile and S_{p2} will be compressive.
- Concrete cracking will occur when the applied in-plane shear (S_{xy}) exceeds the tensile cracking strength (S_{ct}) of the composite CFS section

$$S_{ct} = \left(\frac{4\sqrt{f_c'}}{1000} - \varepsilon_{sh}E_c\right)\left(T_c + \frac{E_s}{E_c}2t_s\right) \times 12 \quad kips/ft$$

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PURE SHEAR BEHAVIOR

- The in-plane shear stiffness before the concrete cracking limit state can be calculated as:

$$K_{xy}^{cr} = G_s A_s + G_c A_c = \frac{E_s t_s}{2(1+\nu_s)} + \frac{E_c T_c}{2(1+\nu_c)} \text{ kips/in}$$

- The concrete cracking planes will be perpendicular to direction of the principal stress S_{p1}, i.e., at 135°.
- The principal direction θ_p will be equal to 45°. Substituting this value in the equation for $[K]_c$

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PURE SHEAR BEHAVIOR

$$[K]_c = \frac{E'_c}{4}\begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & 2\sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -2\sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}^{-1} \begin{bmatrix} 0\ or\ 1 & 0 & 0 \\ 0 & 0\ or\ 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1+\cos(2\theta_p) & 1-\cos(2\theta_p) & \sin(2\theta_p) \\ 1-\cos(2\theta_p) & 1+\cos(2\theta_p) & -\sin(2\theta_p) \\ -2\sin(2\theta_p) & 2\sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

- Substituting $\theta_p = 45^o$ The concrete $[K]_c$ becomes

$$[K]_c = \frac{E'_c}{4}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \qquad \begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} = [K]_c \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

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PURE SHEAR BEHAVIOR

- Force Equilibrium for Pure Shear

$$\begin{Bmatrix} 0 \\ 0 \\ S_{xy} \end{Bmatrix} = \left\{ \frac{2E_s t_s}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} + \frac{E_c' T_c}{4} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \right\} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

- Solve simultaneous equations to get the strains and calculate stresses etc. for pure shear case

$$\boxed{{}_s\sigma_x = {}_s\sigma_y = \frac{S_{xy}}{2t_s}\frac{K_\beta}{K_\alpha + K_\beta}}$$

$$\boxed{{}_s\tau = \frac{S_{xy}}{2t_s}\frac{K_\alpha}{K_\alpha + K_\beta}}$$

$$\boxed{{}_c\sigma_x = {}_c\sigma_y = -{}_c\tau = \frac{S_{xy}}{T_c}\frac{-K_\beta}{K_\alpha + K_\beta}}$$

$$K_\alpha = G_s 2t_s \qquad K_\beta = \frac{1}{\frac{4}{E_c' T_c} + \frac{2(1-\nu)}{E_s 2t_s}}$$

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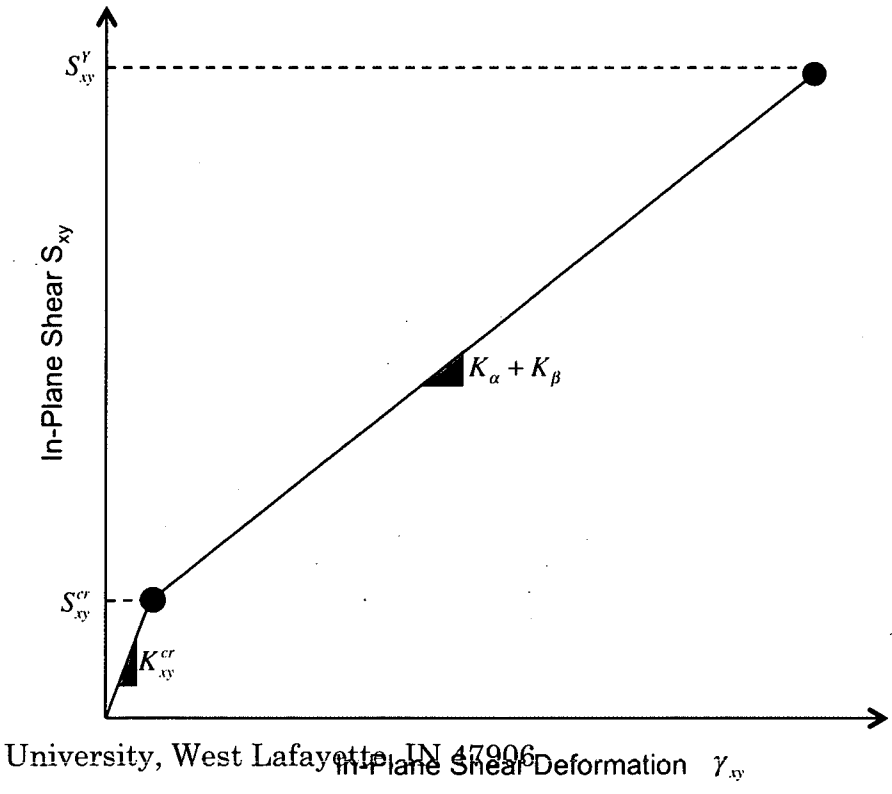
PURE SHEAR BEHAVIOR

- Use the stresses to calculate von-mises stress and establish the yielding of the steel plate.
- The yield load corresponds to:

$$S_{xy}^{Y} = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}}\left(2t_s F_y\right)$$

- The shear stiffness post-cracking corresponds to;

$$S_{xy} = (K_\alpha + K_\beta)\,\gamma_{xy}$$



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PURE SHEAR BEHAVIOR – PARAMETRIC STUDY

Effect of Concrete Thickness on In-plane Shear Strength and Stresses Assumed 0.5 in. steel plate thickness with Fy = 50 ksi							
Concrete T_c (in.)	**K_α (kips-ft/ft)**	**K_β (kips-ft/ft)**	$\frac{S_{nv}}{A_s F_y}$	$\frac{{}_s\sigma_x}{F_y} = \frac{{}_s\sigma_y}{F_y}$	$\frac{{}_s\tau}{F_y}$	$\frac{{}_c\sigma_x}{f'_c} = \frac{{}_c\sigma_y}{f'_c} = \frac{-\tau_c}{f'_c}$	$\frac{{}_c\sigma_p}{f'_c}$
18	133846	99852	0.93	0.40	0.53	0.24	0.48
20	133846	106206	0.94	0.42	0.52	0.23	0.45
22	133846	112040	0.95	0.44	0.52	0.21	0.43
24	133846	117414	0.97	0.45	0.52	0.20	0.41
26	133846	122381	0.98	0.47	0.51	0.19	0.39
28	133846	126986	0.99	0.48	0.51	0.19	0.37
30	133846	131266	1.00	0.49	0.50	0.18	0.36
32	133846	135256	1.00	0.50	0.50	0.17	0.34
34	133846	138983	1.01	0.51	0.50	0.16	0.33
36	133846	142472	1.02	0.52	0.49	0.16	0.32

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COMPARISON WITH EXPERIMENTAL RESULTS

◎ Japanese Tests by Ozaki et al. (2004)

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Study on steel plate reinforced concrete panels subjected to cyclic in-plane shear

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Received 19 February 2002; received in revised form 8 June 2003; accepted 10 June 2003

COMPARISON WITH EXPERIMENTAL RESULTS

12/23/09

o Japanese tests of Ozaki et al. (2004)

Table 2
Test specimen (research program I)

Specimens	Surface steel plate (t) (mm)	Headed stud bolt			Nodal force (MPa)	Partitioning web
		Pitch in welding (B) (mm)	Diameters (mm)	B/t		
S2-00NN	2.3	70	4	30	0.0	
S2-15NN					1.47	
S2-30NN					2.94	
S3-00NN	3.2	100	5	31	0.0	
S3-15NN					1.47	
S3-30NN					2.94	-
S3-00PS					0.0	Studs were welded
S3-00PN						Without studs
S4-00NN	4.5	135	9	30		-

Table 4
Experimental results

Specimen	Steel		Concrete	Elastic shear modulus	Post-cracking shear modulus	Cracking strength		Yield strength		Maximum strength	
	Yield stress, Young's modulus (MPa)	$A_w \times A_p$ (cm^2)	Compressive strength, tangential stiffness (MPa)	G_c ($\times 10^3$ MPa)	G_y ($\times 10^3$ MPa)	Q_c (kN)	γ_c ($\times 10^{-3}$)	Q_y (kN)	γ_y ($\times 10^{-3}$)	Q_u (kN)	γ_u ($\times 10^{-3}$)
S2-00NN	340 (1.97×10^5)	53.5 (17.1)	42.2 (2.72×10^4)	12.4	4.16	293	0.115	2290 (−2110)	2.50 (−1.99)	2960 (−2780)	9.41 (−6.12)
S2-15NN			41.6 (2.77×10^4)	13.2	4.14	433	0.133	2330 (−2290)	2.71 (−2.21)	3110 (−2930)	10.00 (−6.02)
S2-30NN			42.0 (2.79×10^4)	16.4	3.69	542	0.168	2490 (−2570)	3.01 (−2.41)	3110 (−3200)	10.48 (−6.03)
S3-00NN	351 (1.99×10^5)	75.4 (16.9)	41.9 (2.71×10^4)	12.9	4.88	311	0.134	3070 (−3070)	3.01 (−2.00)	3610 (−3430)	6.05 (−6.03)
S3-15NN			41.6 (2.67×10^4)	13.1	4.29	384	0.141	3130 (−3120)	2.99 (−3.01)	3760 (−3330)	[illegible] (−6.01)
S3-30NN			40.1 (2.70×10^4)	11.9	4.67	385	0.186	3170 (−3080)	2.80 (−2.96)	3730 (−3550)	5.57 (−5.63)
S3-00PS		75.4 (25.4)	41.9 (2.71×10^4)	13.1	5.81	350	0.141	2680 (−2640)	1.93 (−1.97)	3580 (−3220)	10.87 (−5.98)
S3-00PN			39.9 (2.72×10^4)	16.4	4.92	271	0.113	2350 (−2390)	2.01 (−2.03)	3510 (−3060)	17.00 (−6.02)
S4-00NN	346 (2.07×10^5)	104.9 (16.7)	42.8 (2.76×10^4)	16.4	8.22	349	0.103	3510 (−3560)	2.01 (−2.00)	4100 (−3790)	5.67 (−4.00)

Comparison with Experimental Results

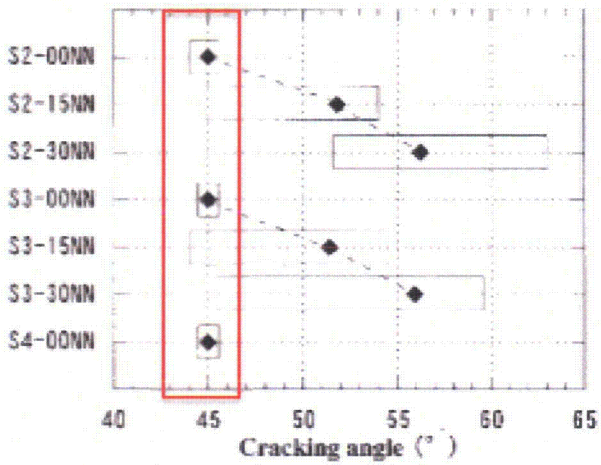
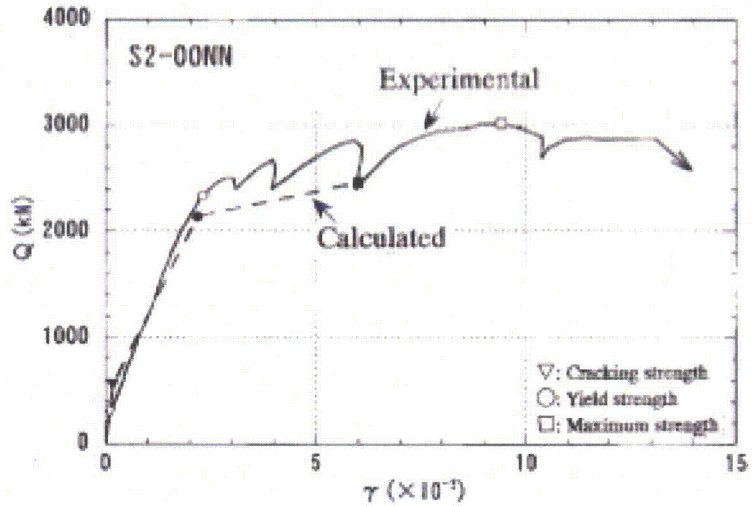
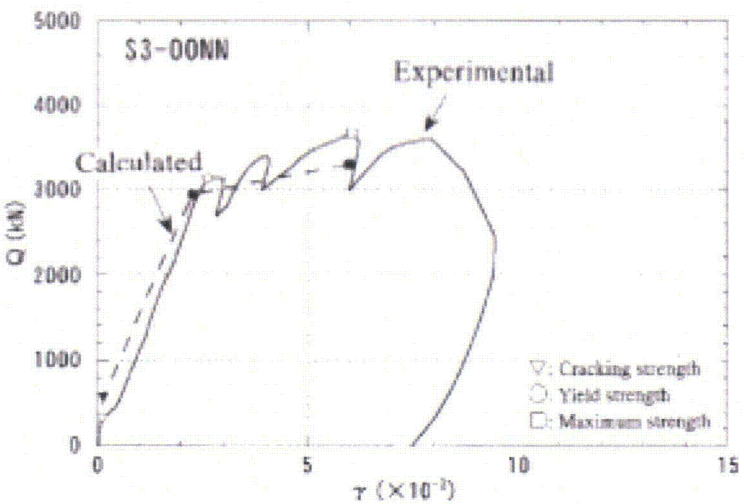
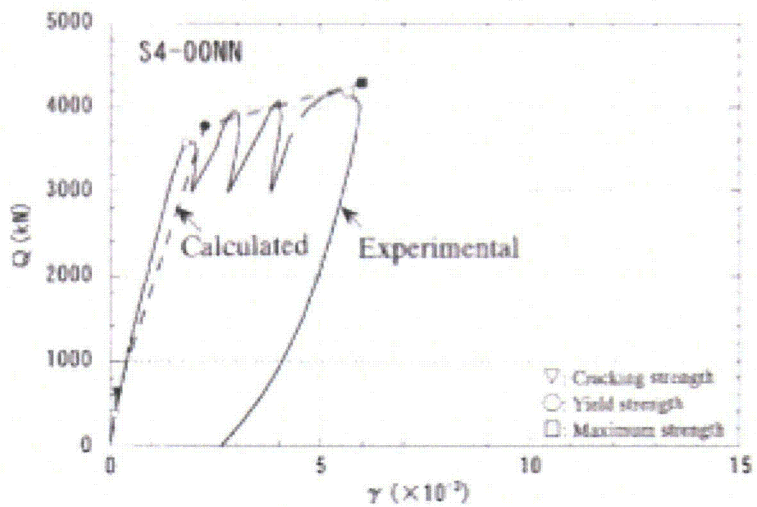


Fig. 7. Cracking angles.

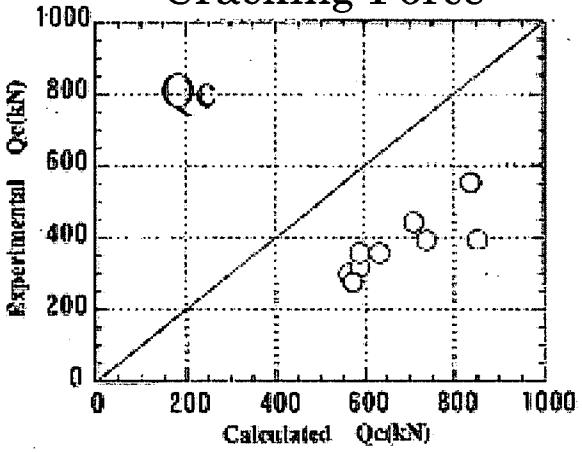




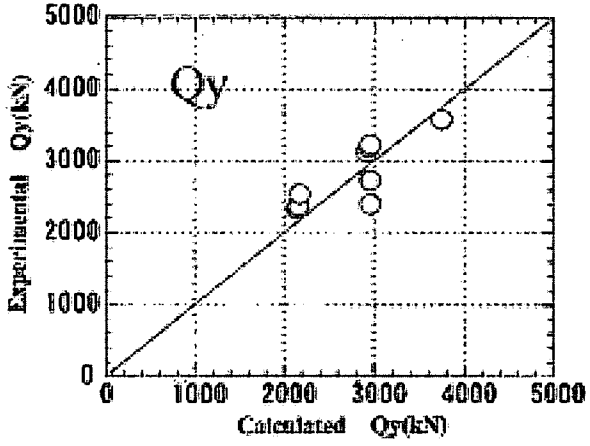


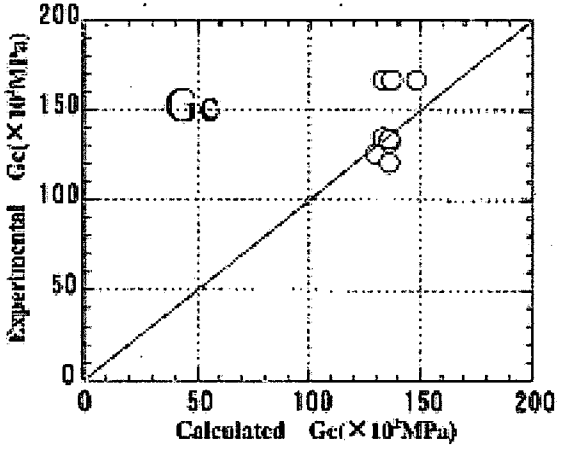
COMPARISON WITH EXPERIMENTAL RESULTS

Cracking Force

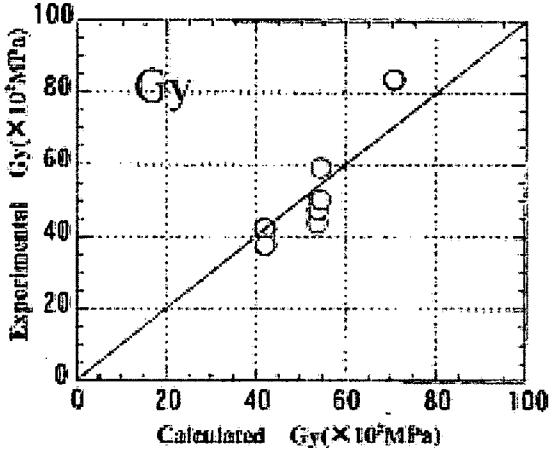


Yield Force





Pre-Cracking Stiffness



Post-Cracking Stiffness

COMPARISON WITH EXPERIMENTAL RESULTS

◎ Tests conducted by Lee, Choi, Hong , and Lee in Korea

The 5th International Symposium on Steel Structures
March 12-14, 2009, Seoul, Korea

In-Plane Shear Behavior of Composite Steel Concrete Walls

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Designation	Thickness (mm)	Diameter of Stud (φ-mm)	Spacing (mm)	Axial Force Ratio (%)	Axial Force (kN)
S30/400 F3.2 N00-1				0	0
S30/400 F3.2 N00-2	3.2	6-57	100	0	0
S30/400 F3.2 N08				8	705.6

Comparison with Experimental Results

- Setup used by these researchers tries to simulate a large panel zone in beam-to-column cruciform
- The setup worked ok. First specimen failed due to welding failure

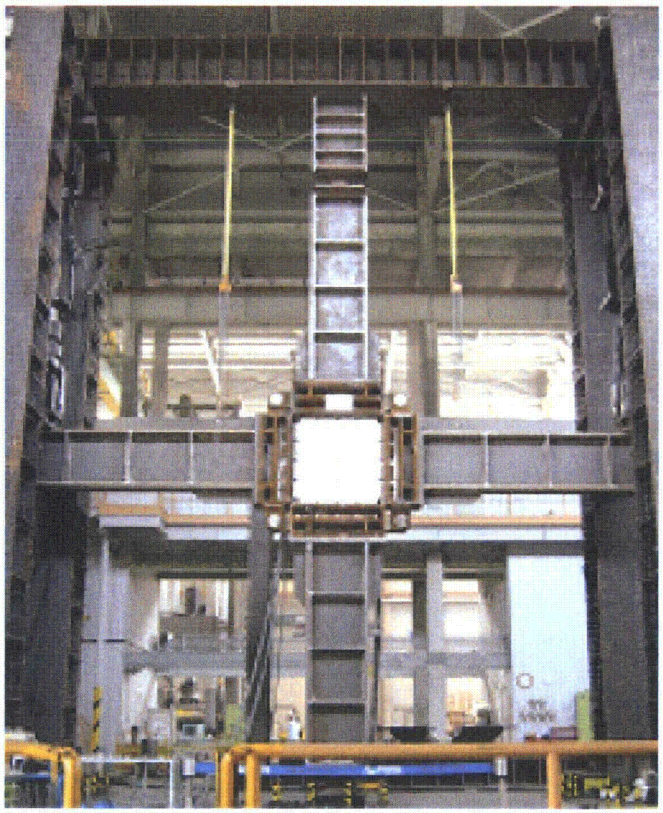

Figure 1. Loading Set-up for Pure Shear

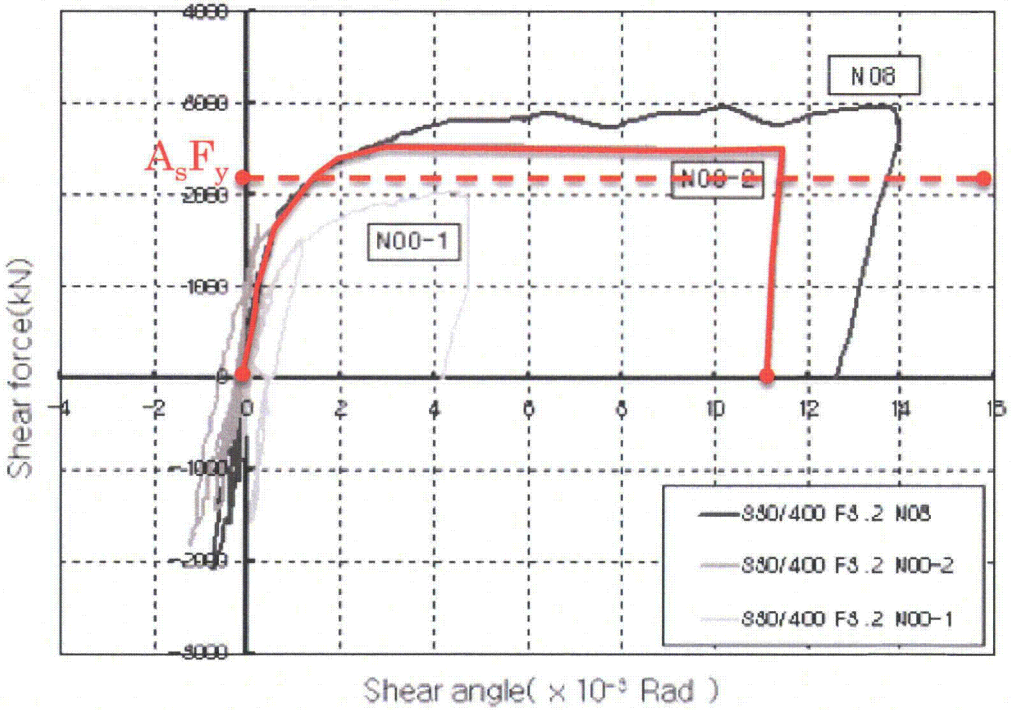

Figure 3. Shear Behavior of SC walls without stiffener

COMPARISON WITH EXPERIMENTAL RESULTS

Photograph showing 45 deg. cracks in concrete for the pure shear specimen

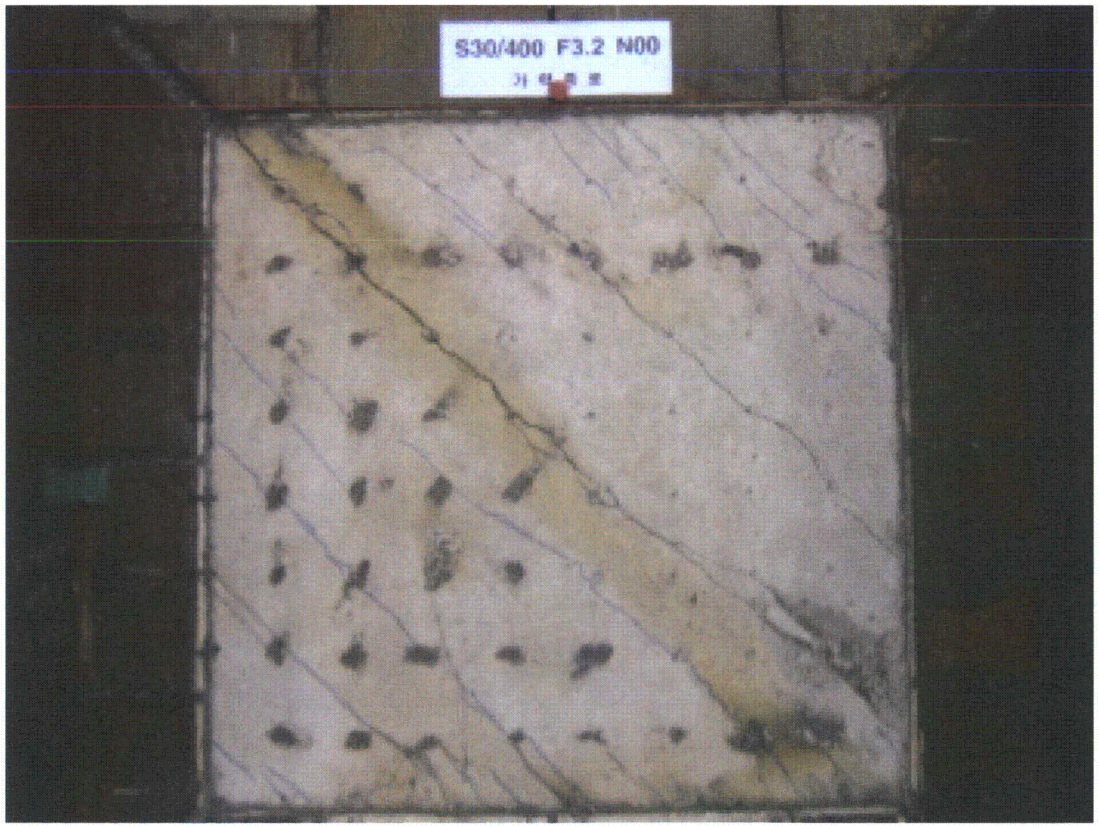

Figure 5. Diagonal Cracks in Concrete

Comparison with Experimental Results

Table 3. Strength of Non-Stiffened Steel Concrete panel

Specimen	Ultimate Shear (Q kN)	Max Shear Angle (rad ×10-3)	Yield Strength (kN)	Specification (kN)	Material Strength	
					steel (MPa)	conc. (MPa)
S30/400 F3.2 N00-1	2019	4.39	1601	2233	297	36
S30/400 F3.2 N00-2	2597	6.46	1601	2233	297	36
S30/400 F3.2 N08	2900	13.6	1601	2262	297	36

As Fy = 1200 mm x 3.2 mm x 2 x 297 N/mm^2 = 2280 kN

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IN PLANE BEHAVIOR

TENSION or COMPRESSION + IN PLANE SHEAR

IN-PLANE BEHAVIOR

- Used the Mathcad program to analyze the CFS element for different combinations of membrane force (tension or compression) and shear force.
- The limit state was always yielding of the steel plate, and the concrete stresses were limited to the elastic range (less than 0.70 f'_c)
- The last point on the compression + shear portion of the curve had concrete stress equal to 0.70 f'_c

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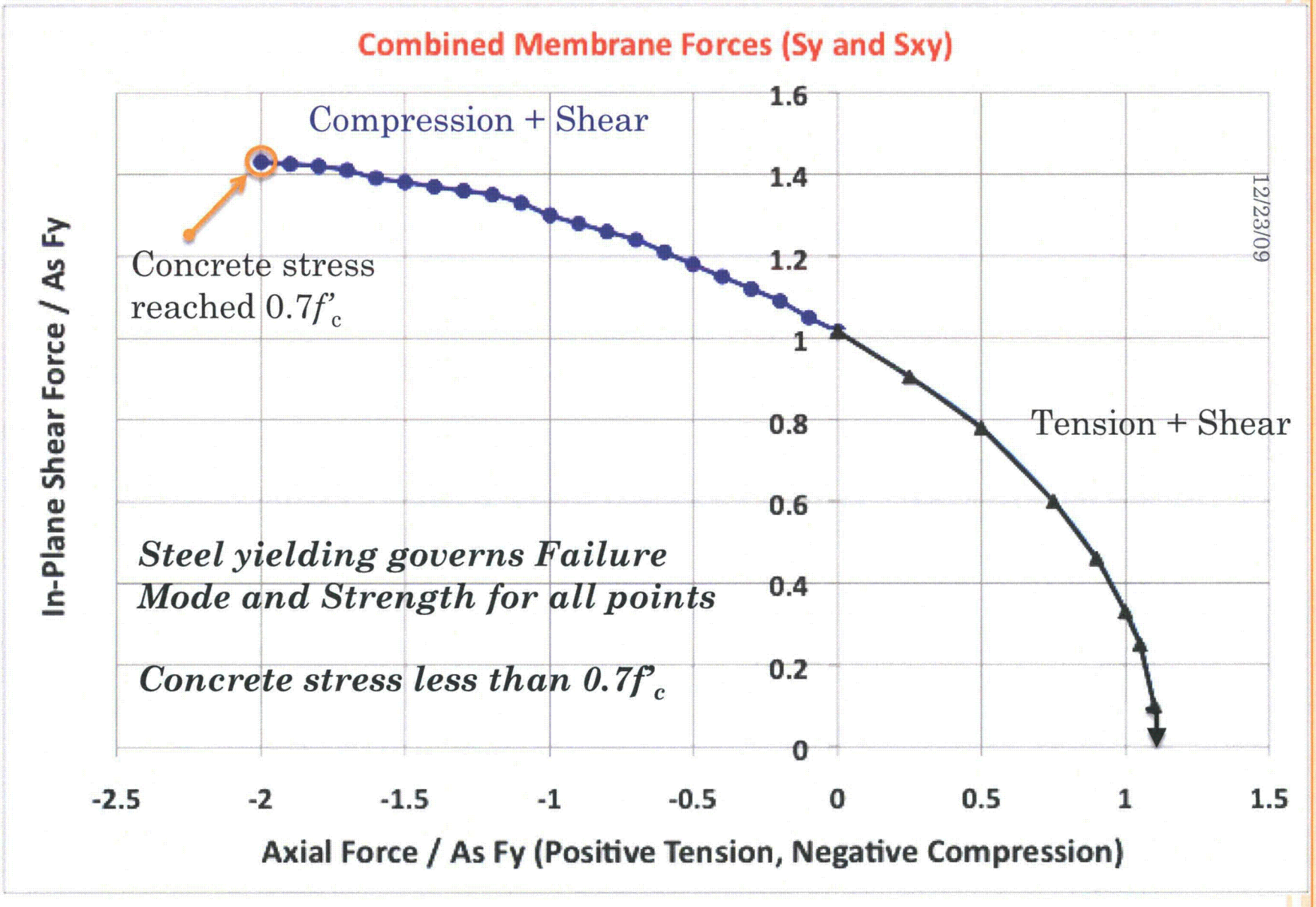

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IN-PLANE BEHAVIOR WITH SY AND SXY

- Combined effects of axial force (Sy) and in-plane force Sxy.
- Results obtained using Mathcad sheet.
  - Assumptions – Concrete remains elastic (checked by making sure that concrete principal stress is less than 0.7f'c).
  - Concrete has no tension strength
- Note the change in shear strength with axial tension and axial compression.
- Axial compression increases the in-plane shear strength.
- Need more sophisticated analysis to investigate the behavior of CFS panels elements to membrane forces.

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LIMITATIONS OF MATHCAD APPROACH

- The concrete material model was elastic with no compression yielding possible. This limitation becomes significant for cases with larger compression + shear.

- The mathcad program assumes zero tension stiffness and strength. This is conservative assumption but needs refinement

- Theory is not verified using independent finite element analyses

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In-Plane Behavior

Finite Element Models

NONLINEAR FINITE ELEMENT ANALYSIS OF CFS PANELS TO IN-PLANE FORCES

- Three different types of models
  - (1) Shell Solid (SSo) Model
  - (2) Shell Shell (SSh) Model
  - (3) Layered Composite (LCS) Shell Model

- Three different concrete material models
  - (1) Smeared cracking – linear Drucker-Prager – associated flow rule concrete model
  - (2) Concrete damaged plasticity model - smeared cracking – parabolic Drucker – Prager - non-associated flow rule – and damaged elasticity
  - (3) Linear elastic orthotropic model for cracked concrete

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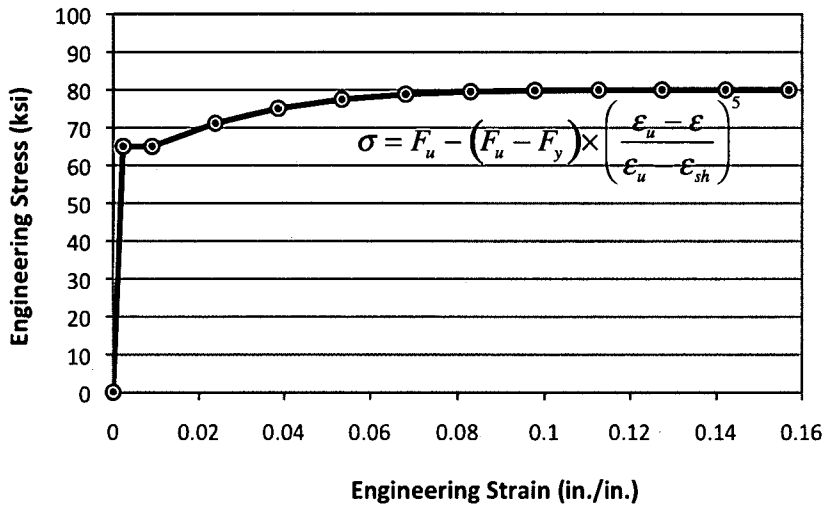
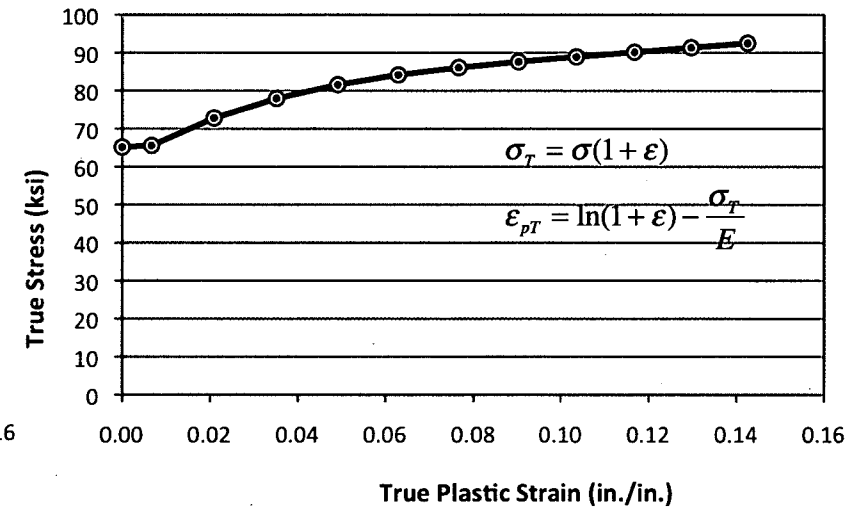
INPUT PARAMETERS

- Steel stress-strain curve





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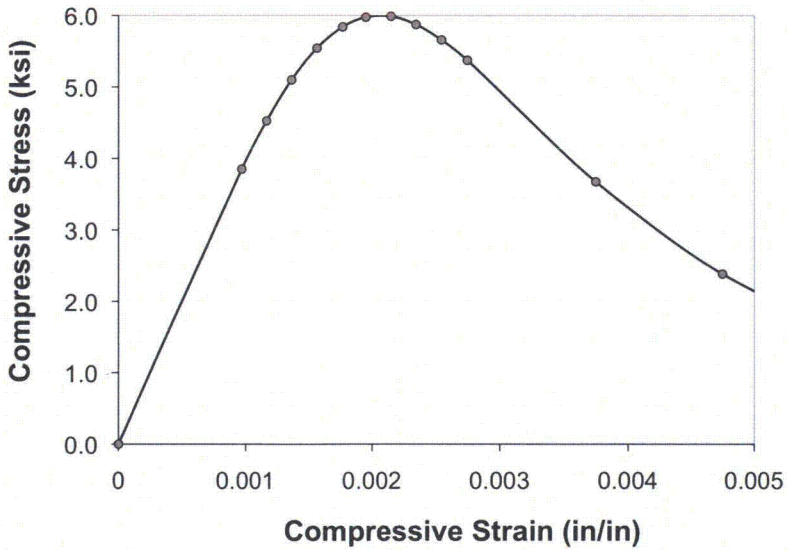
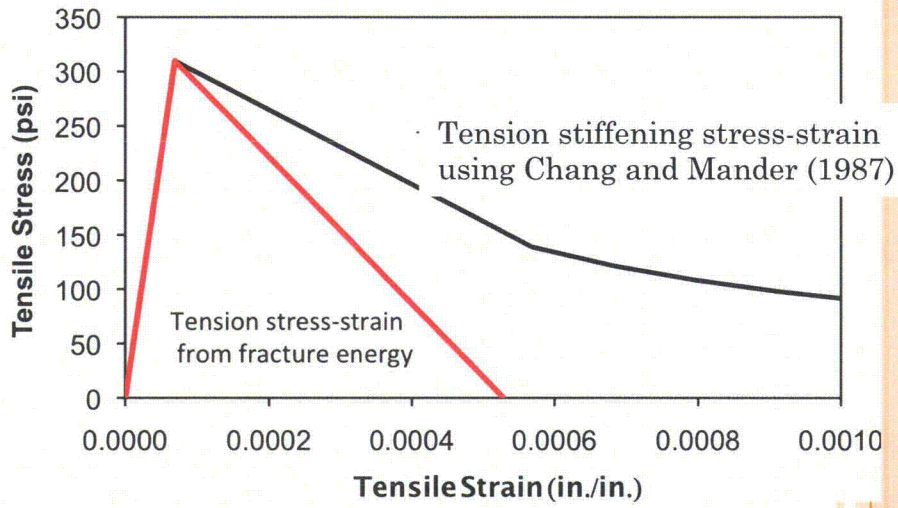
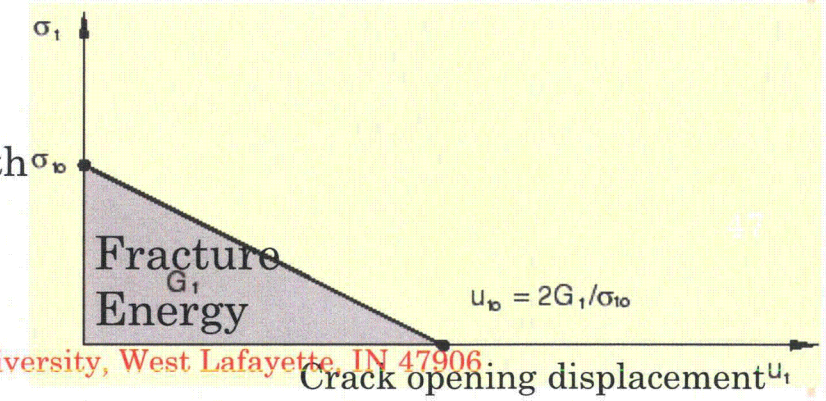
INPUT PARAMETERS

- Concrete compression and Tension behavior







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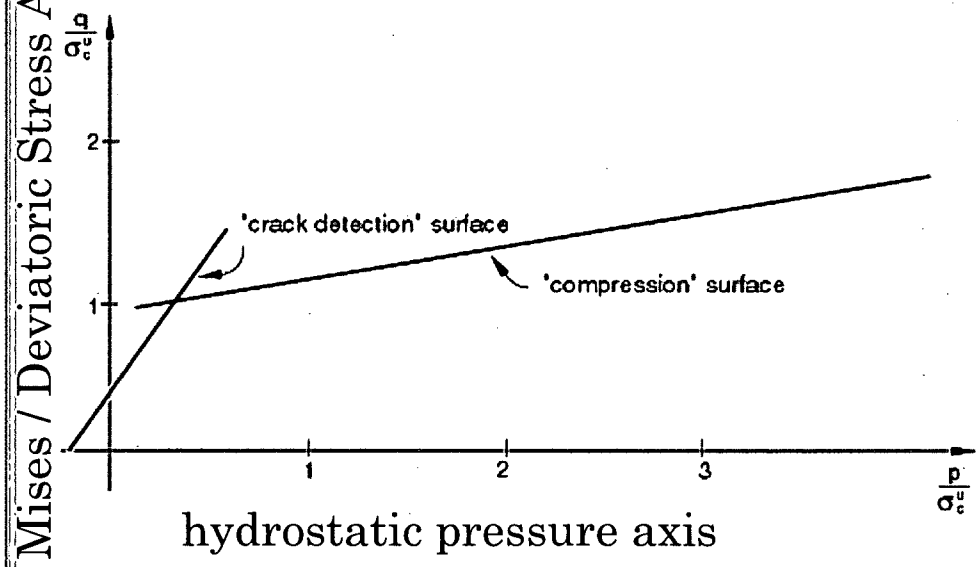
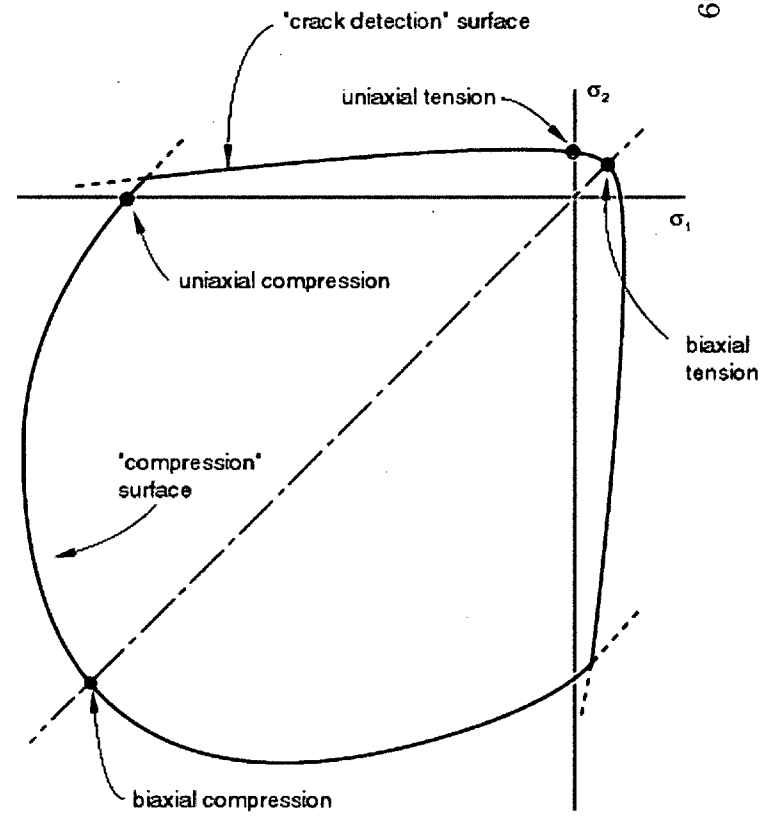
INPUT PARAMETERS

- Concrete Smeared Cracking Model

Failure Surfaces in p-q space





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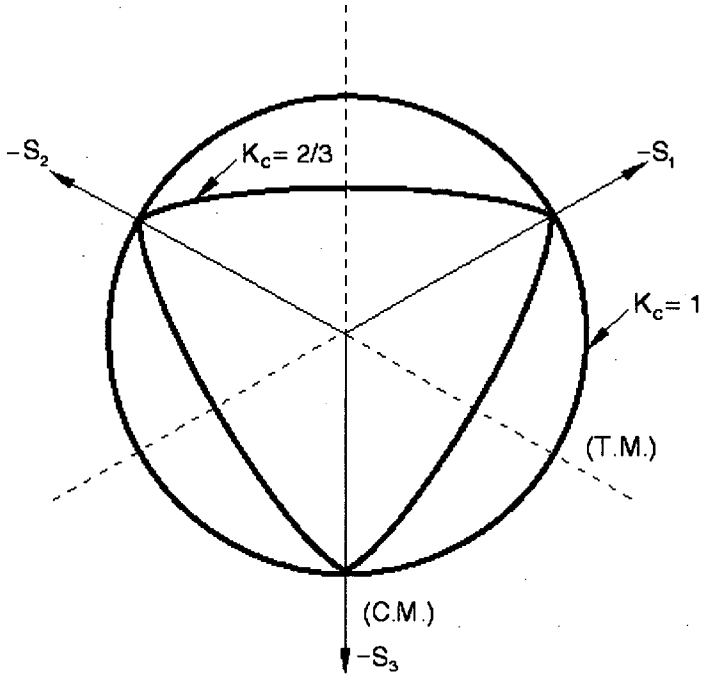
INPUT PARAMETERS

- Concrete Damaged Plasticity



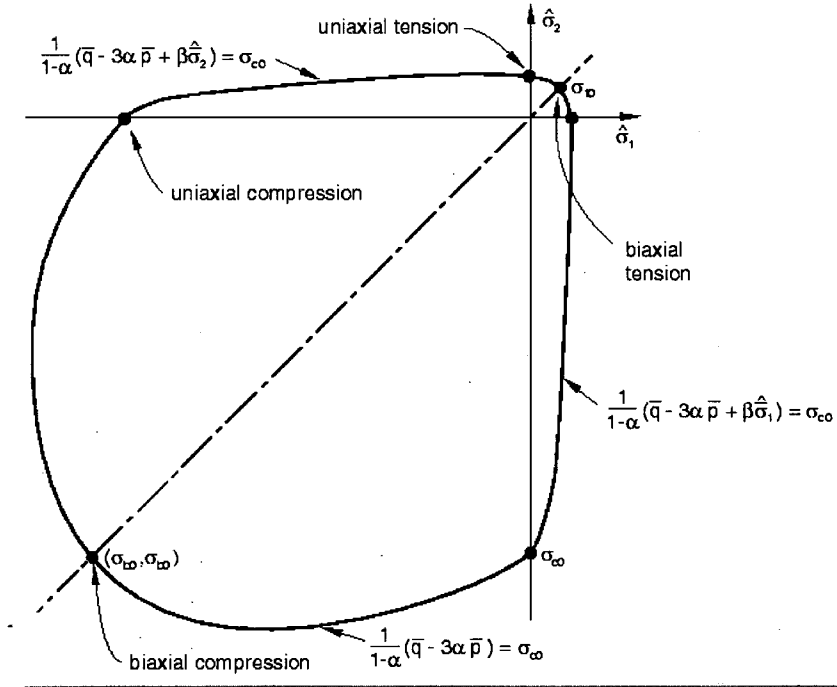


Figure 4.5.2–5 Yield surface in plane stress.

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NONLINEAR FINITE ELEMENT ANALYSIS

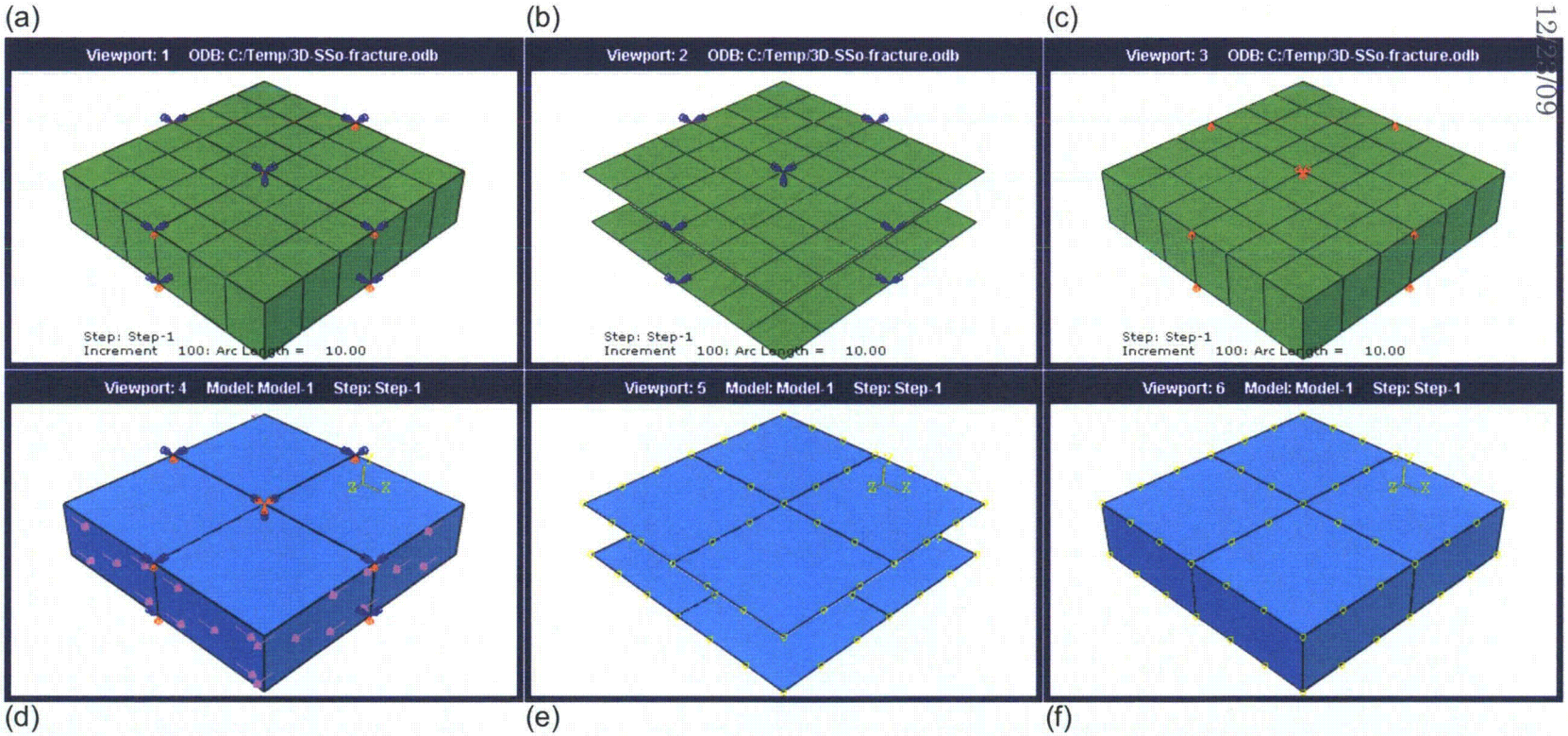

Figure 5-7. Details of SSo model:
(a) Model with mesh and boundary conditions.
(b) Steel plates with shell (S4R) elements, fully tied to concrete elements
(c) Concrete infill modeled using solid (C3D8R) elements
(d) Loading applied as uniformly distributed shear traction on the concrete element surfaces
(e) Steel plates tied to concrete
(f) Concrete surfaces tied to steel plates

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NONLINEAR FINITE ELEMENT ANALYSIS

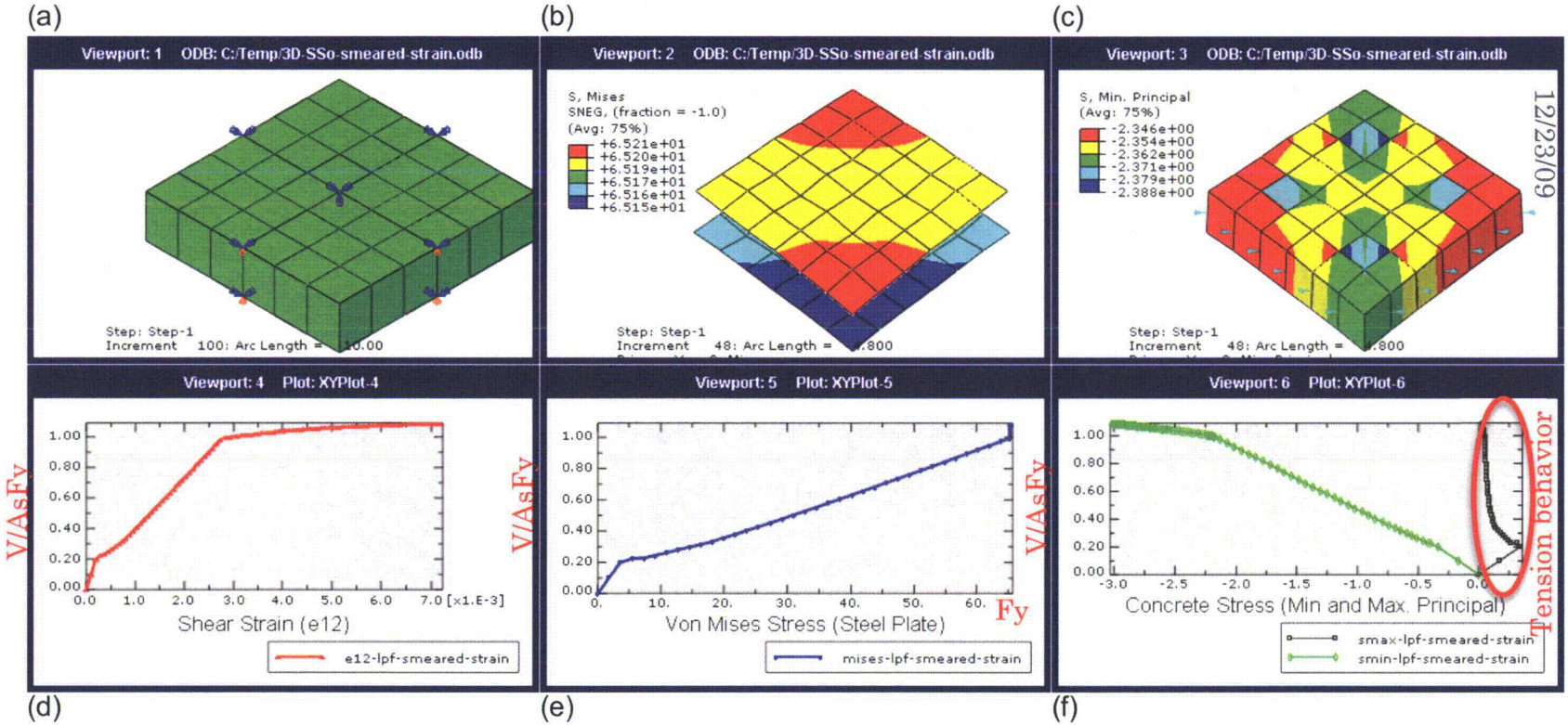

Figure 5-8. Results from SSo model with smeared concrete cracking model (tension stiffening stress strain curve):
(a) Model with boundary conditions,
(b) Von Mises stresses in steel plate when shear force ratio (S_{xy}/A_sF_y)=1.0,
(c) Minimum principal stress in the concrete when shear force ratio=1.0,
(d) Shear force ratio (S_{xy}/A_sF_y) vs. shear strain response,
(e) Shear force ratio (S_{xy}/A_sF_y) vs. Von Mises stress in steel plate, and

(f) Shear force ratio vs. minimum and maximum principal stress in concrete.

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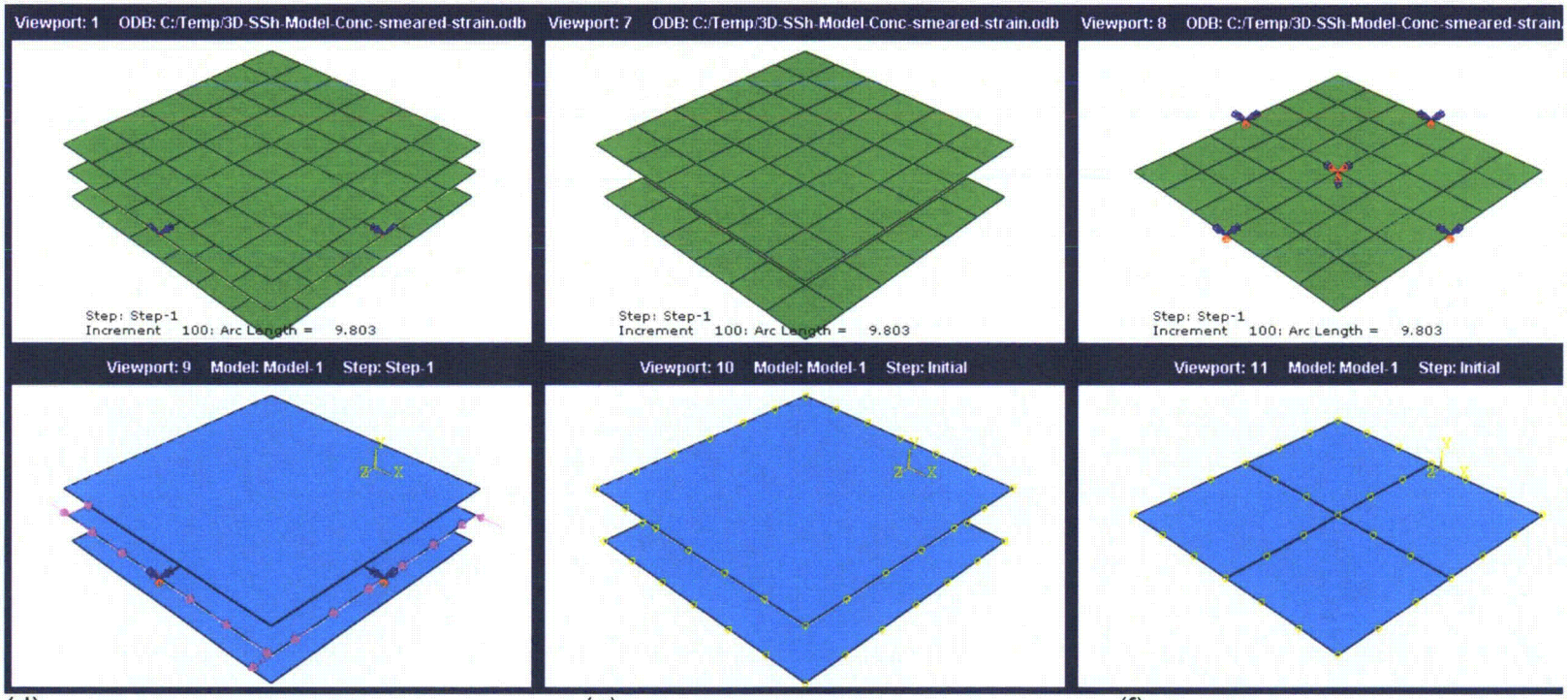
NONLINEAR ANALYSIS – SSH MODEL



(d) (e) (f)

Figure 5-11. Details of SSh model:
(a) Model with mesh and boundary conditions.
(b) Steel plates with shell (S4R) elements, fully tied to concrete elements
(c) Concrete infill modeled using shell (S4R) elements, boundary conditions included
(d) Loading applied as uniformly distributed shear traction on the concrete shell element edges
(e) Steel plates tied to concrete
(f) Concrete surfaces tied to steel plates

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NONLINEAR ANALYSIS – SSH MODEL

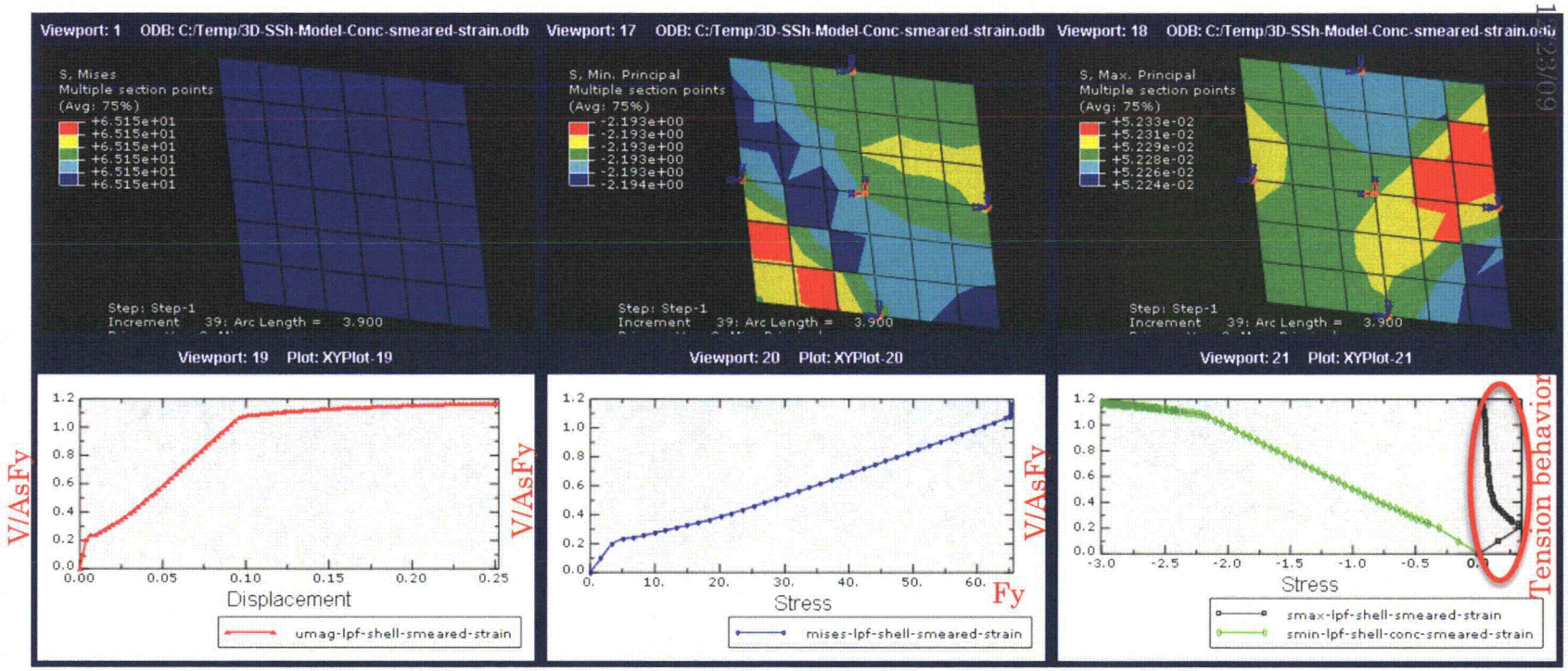

Figure 5-13. Results from SSh model with concrete smeared cracking model (tension stiffening stress-strain curve):
(a) Von Mises stresses in steel plate when shear force ratio (S_{xy}/A_sF_y)=1.0,
(b) Minimum principal stress in the concrete when shear force ratio=1.0,
(c) Maximum principal stress in the concrete when shear force ratio=1.0
(d) Shear force ratio (S_{xy}/A_sF_y) vs. shear strain response,
(e) Shear force ratio (S_{xy}/A_sF_y) vs. Von Mises stress in steel plate, and
(f) Shear force ratio vs. minimum and maximum principal stress in concrete.

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NONLINEAR ANALYSIS – LCS MODEL

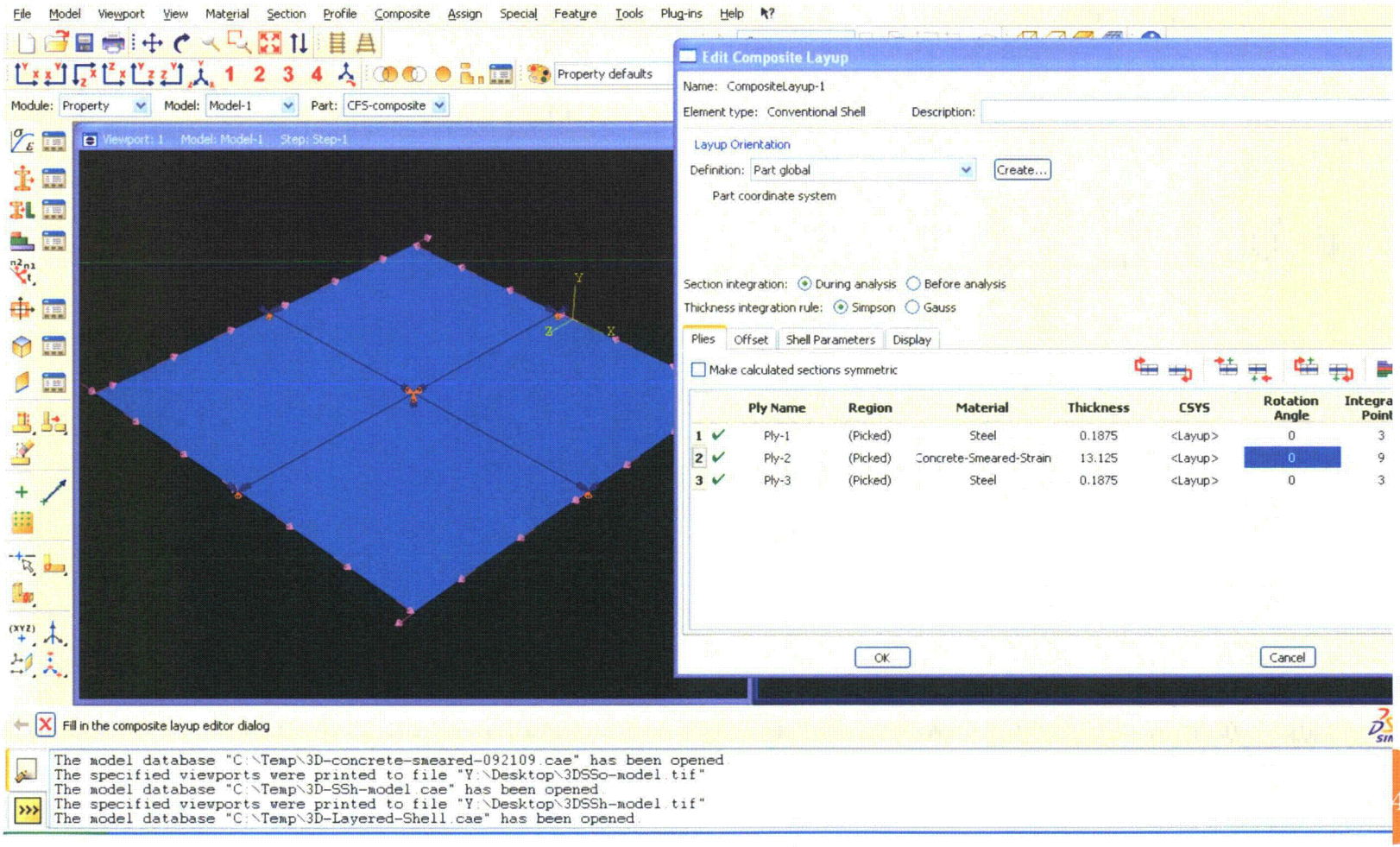

Figure 5-15. Layered Composite Shell (LCS) model with ply details, boundary conditions, and shell edge loading.

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NONLINEAR ANALYSIS – LCS MODELS TENSION STIFFENING AND FRACTURE ENERGY

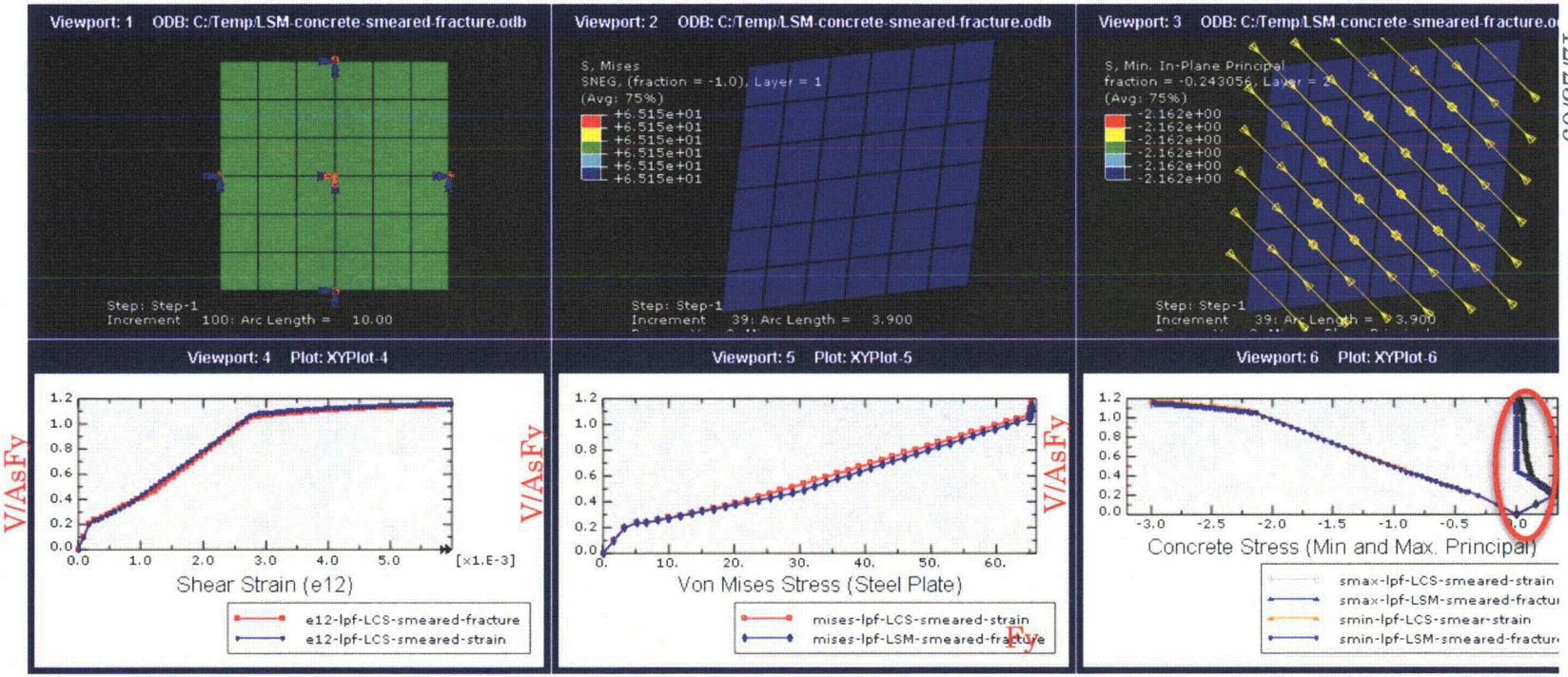

Figure 5-16. Results from LCS model with concrete smeared cracking model (tension stiffening and fracture energy):
(a) Model with boundary conditions
(b) Deformed shape and Von Mises stresses in steel plate when shear force ratio (S_{xy}/A_sF_y)=1.0
(c) Minimum principal stress in the concrete when shear force ratio=1.0
(d) Shear force ratio (S_{xy}/A_sF_y) vs. shear strain response
(e) Shear force ratio (S_{xy}/A_sF_y) vs. Von Mises stress in steel plate

(f) Shear force ratio vs. minimum and maximum principal stress in concrete.

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NONLINEAR ANALYSIS – LCS MODEL WITH ORTHOTROPIC ELASTIC MODEL

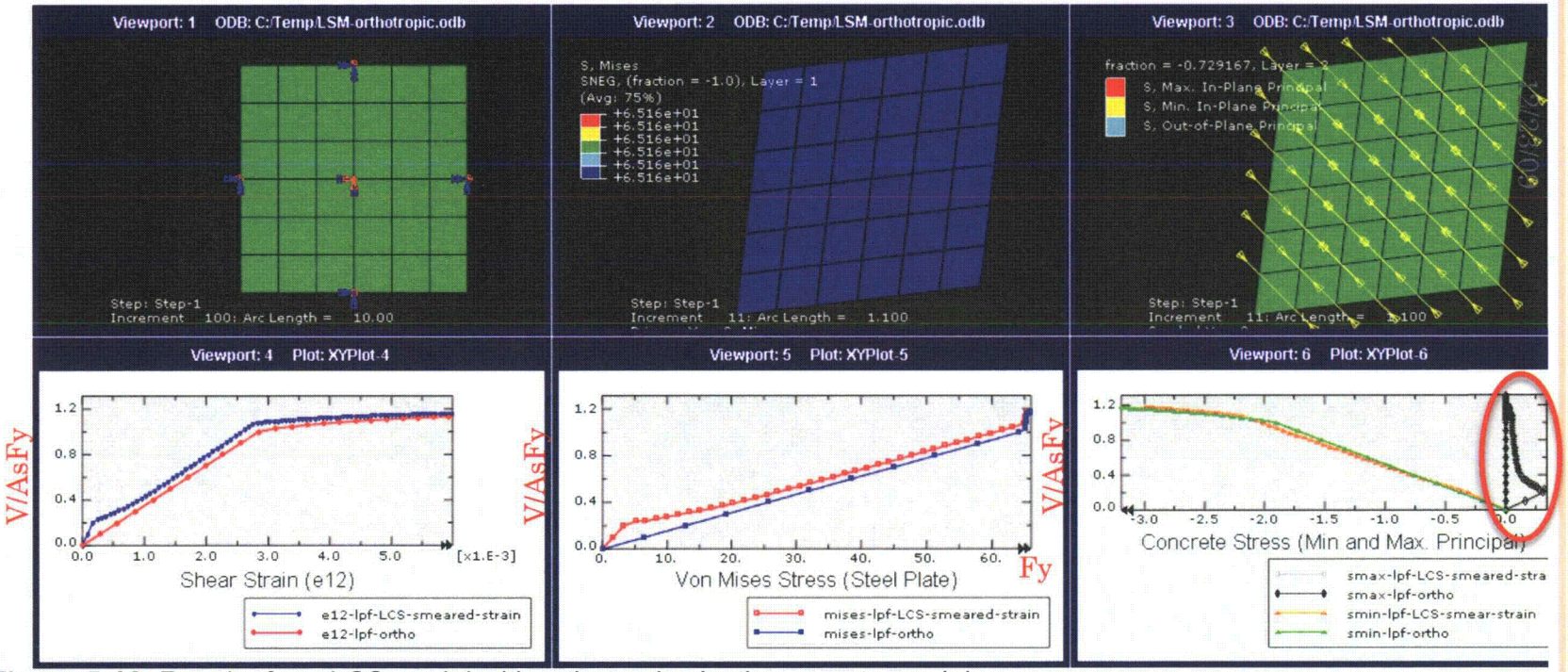

Figure 5-18. Results from LCS model with orthotropic elastic concrete model:
(a) Model with boundary conditions
(b) Deformed shape and Von Mises stresses in steel plate when shear force ratio (S_{xy}/A_sF_y)=1.0
(c) Minimum principal stress in the concrete when shear force ratio=1.0
(d) Shear force ratio (S_{xy}/A_sF_y) vs. shear strain response
(e) Shear force ratio (S_{xy}/A_sF_y) vs. Von Mises stress in steel plate

(f) Shear force ratio vs. minimum and maximum principal stress in concrete.

Tension behavior

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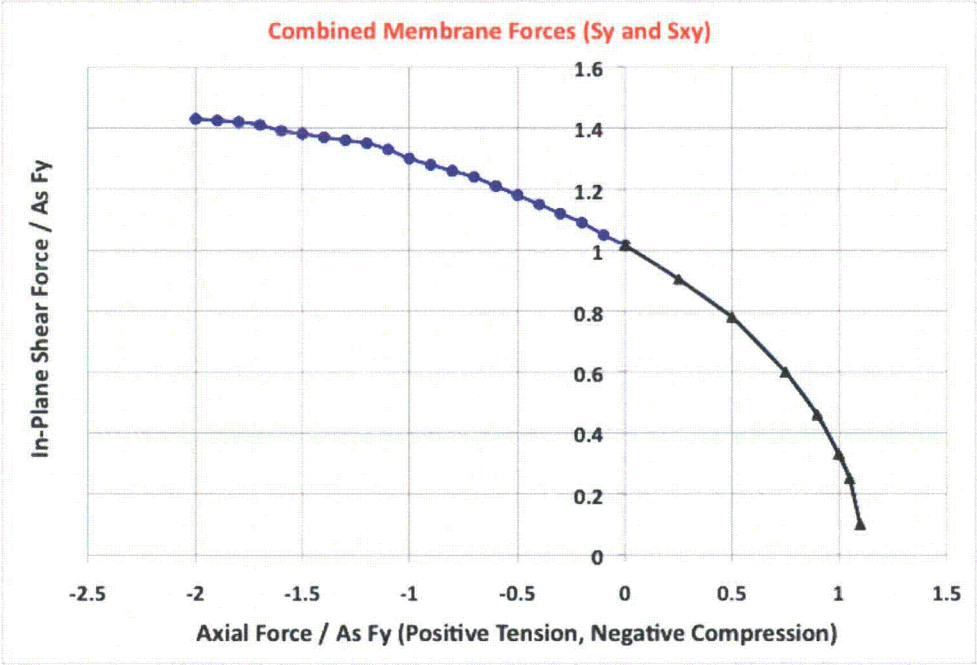
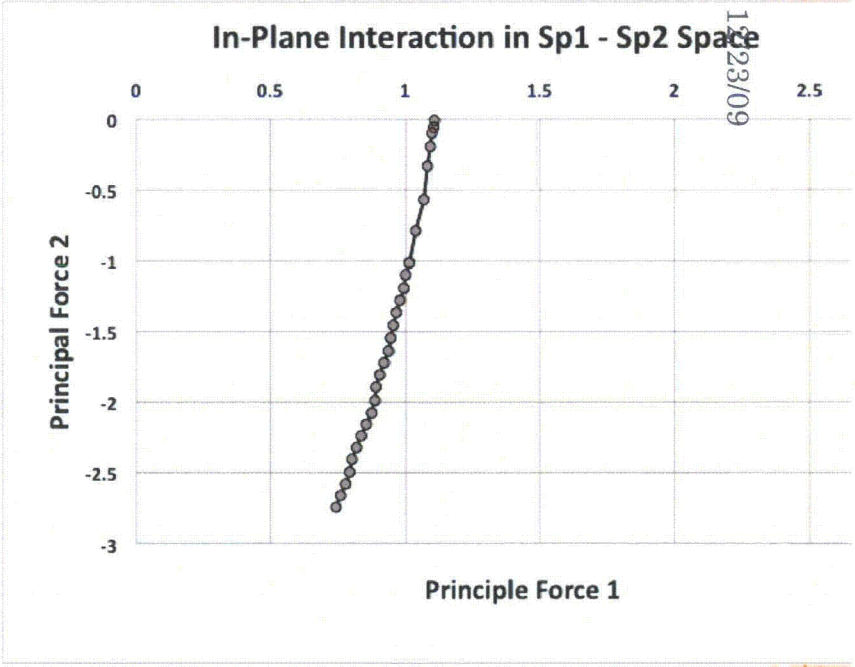
Nonlinear Analysis using Analytical Model with Orthotropic Cracked Concrete





$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y}$$

$$S_{p1} = \frac{(1+\cos(2\theta_p))}{2} S_x + \frac{(1-\cos(2\theta_p))}{2} S_y + \sin(2\theta_p)\, S_{xy}$$

$$S_{p2} = \frac{(1-\cos(2\theta_p))}{2} S_x + \frac{(1+\cos(2\theta_p))}{2} S_y - \sin(2\theta_p)\, S_{xy}$$

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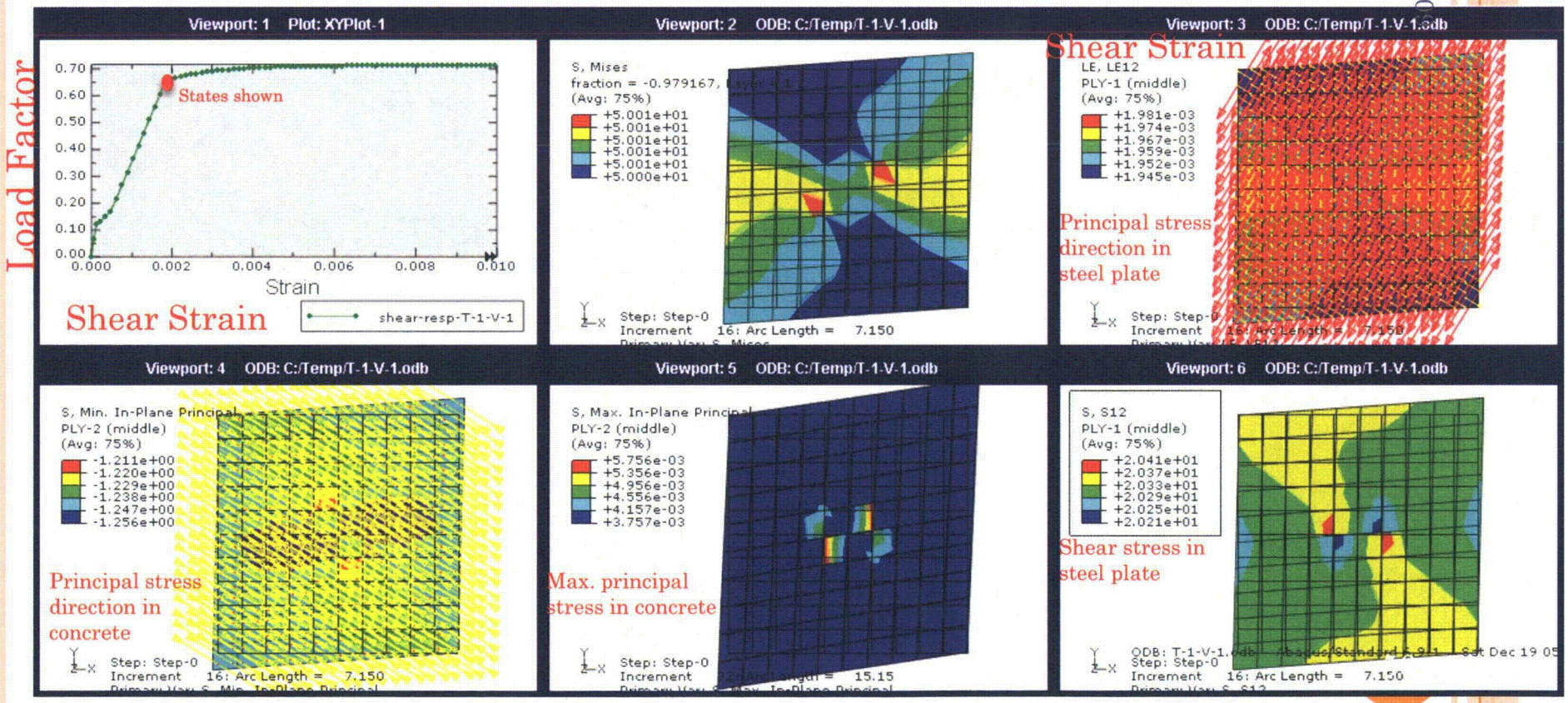
FEM Analysis for Tension + Shear

Applied Tension = Shear = As Fy: Analysis using modified Riks method



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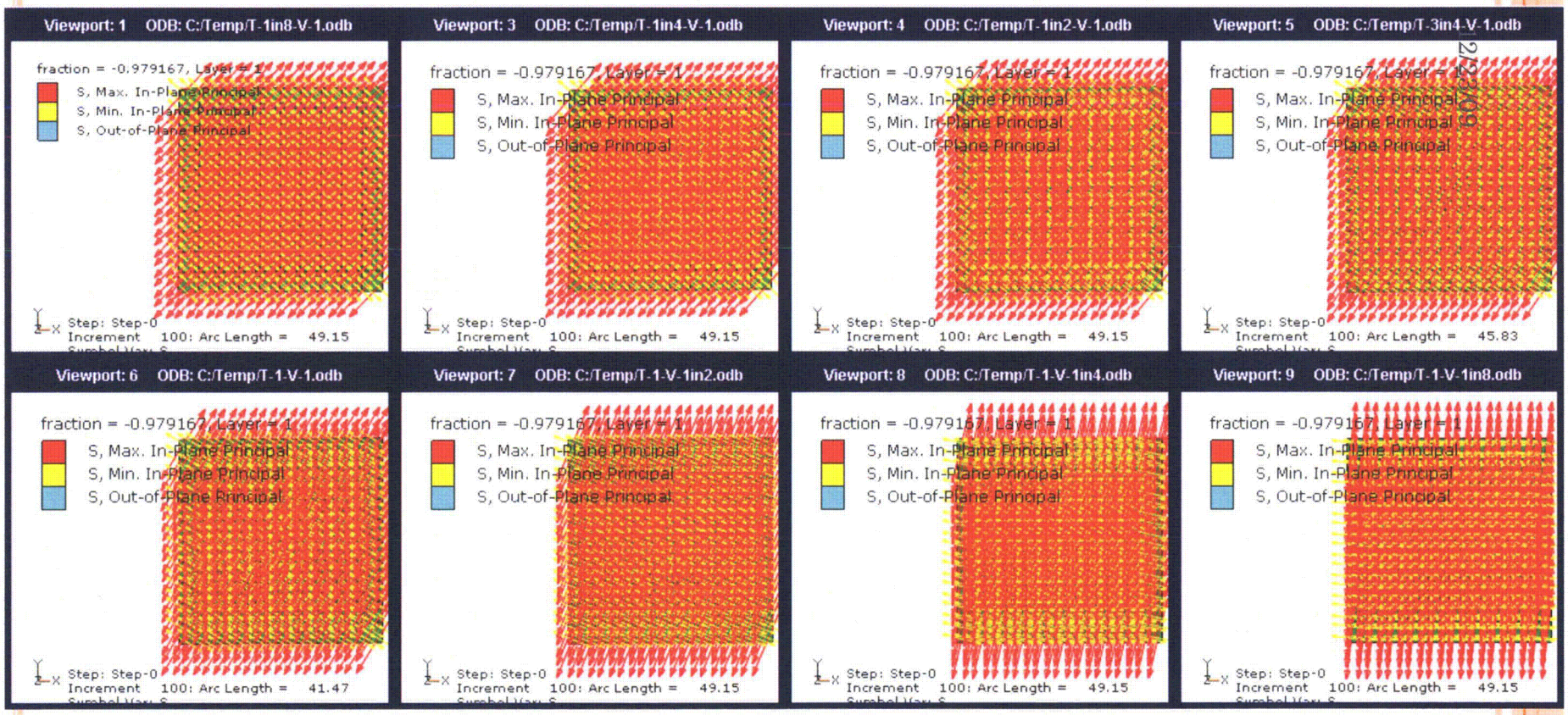
FEM ANALYSIS FOR TENSION + SHEAR
CRACK ORIENTATION FOR CASES (PROPORTIONAL LOADING)

T/V=1/8 T/V=0.75 T/V=0.5 T/V=0.75



T=V T=2V T=4V T=8V

As shown, the crack orientation changes slowly with increasing T.
As T/V changes from 0 to 1, there is little change in crack orientation.
Crack orientation changes more rapidly as the T/V ratio increases from 1 to inf.

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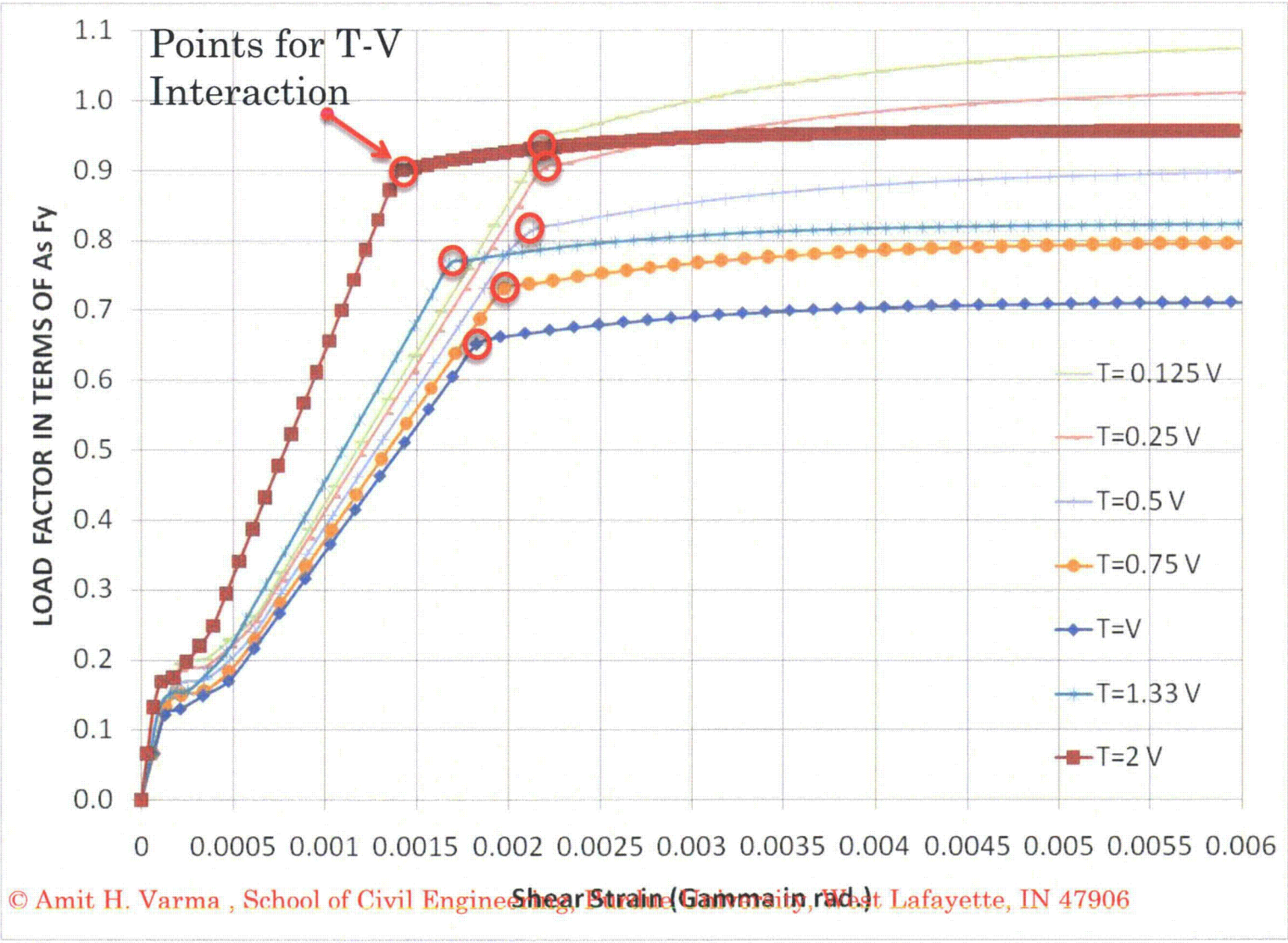
Results from Finite Element Analysis Tension + Shear (Proportional Loading)



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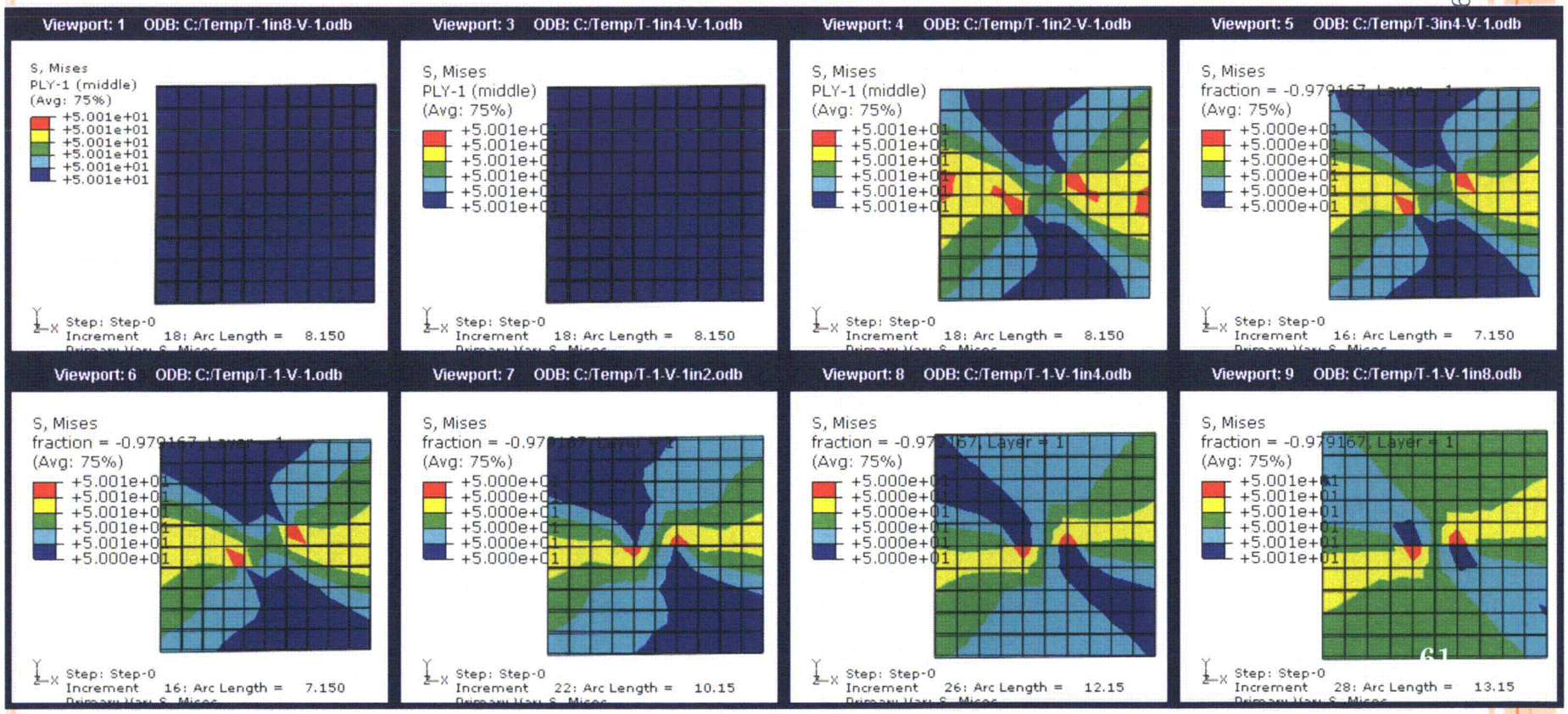
RESULTS FROM TENSION + SHEAR ANALYSIS

Von Mises stress in steel plates corresponding to the points of yielding i.e,. turning points in the shear force-strain responses on previous slide

T/V=1/8 T/V=0.75 T/V=0.5 T/V=0.75



T=V T=2V T=4V T=8V

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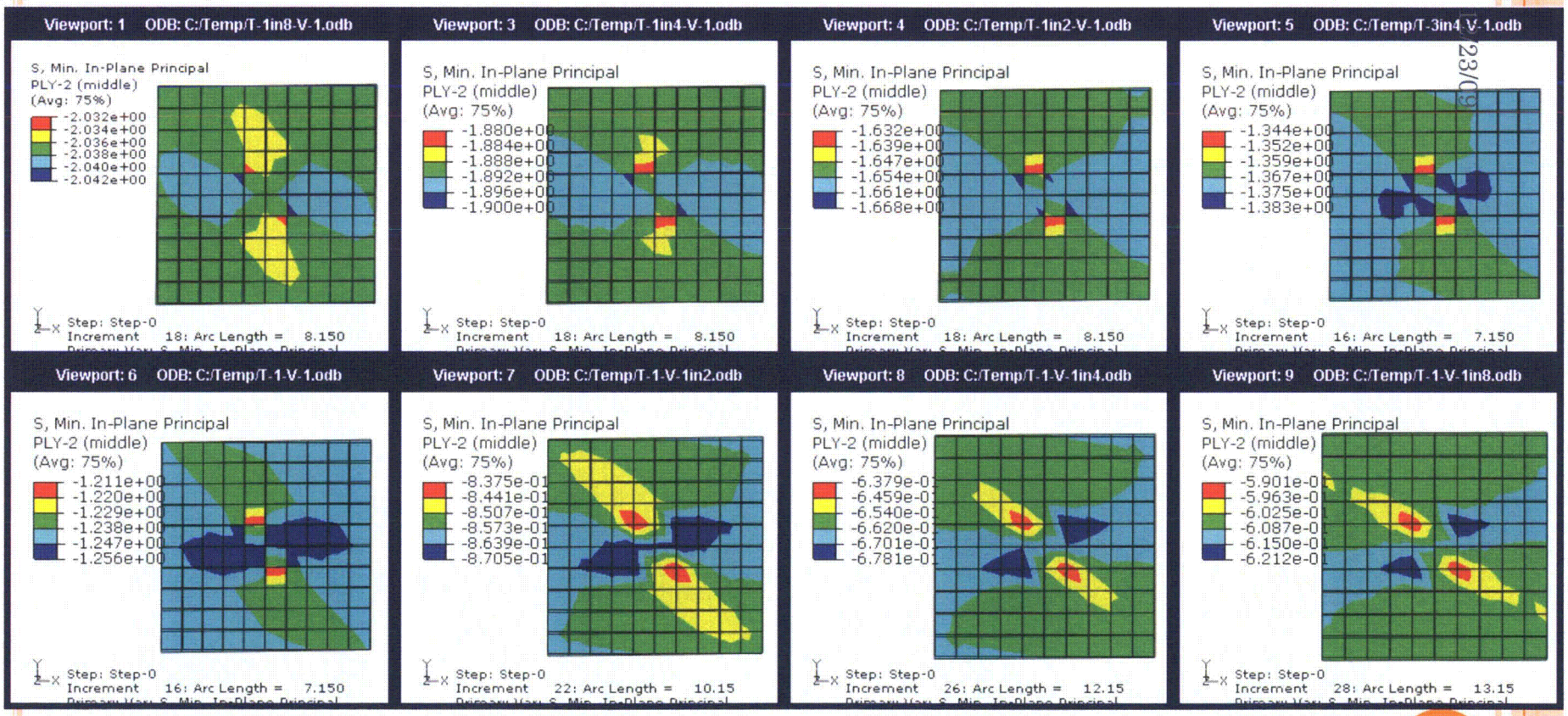
RESULTS FROM TENSION + SHEAR ANALYSIS

Minimum principal stress in concrete at the onset of yielding

T/V=1/8 T/V=0.75 T/V=0.5 T/V=0.75



T=V T=2V T=4V T=8V

Concrete contribution (min. princ. stress) decreases as the T/V increases.

Concrete contribution is highest for the pure shear case.

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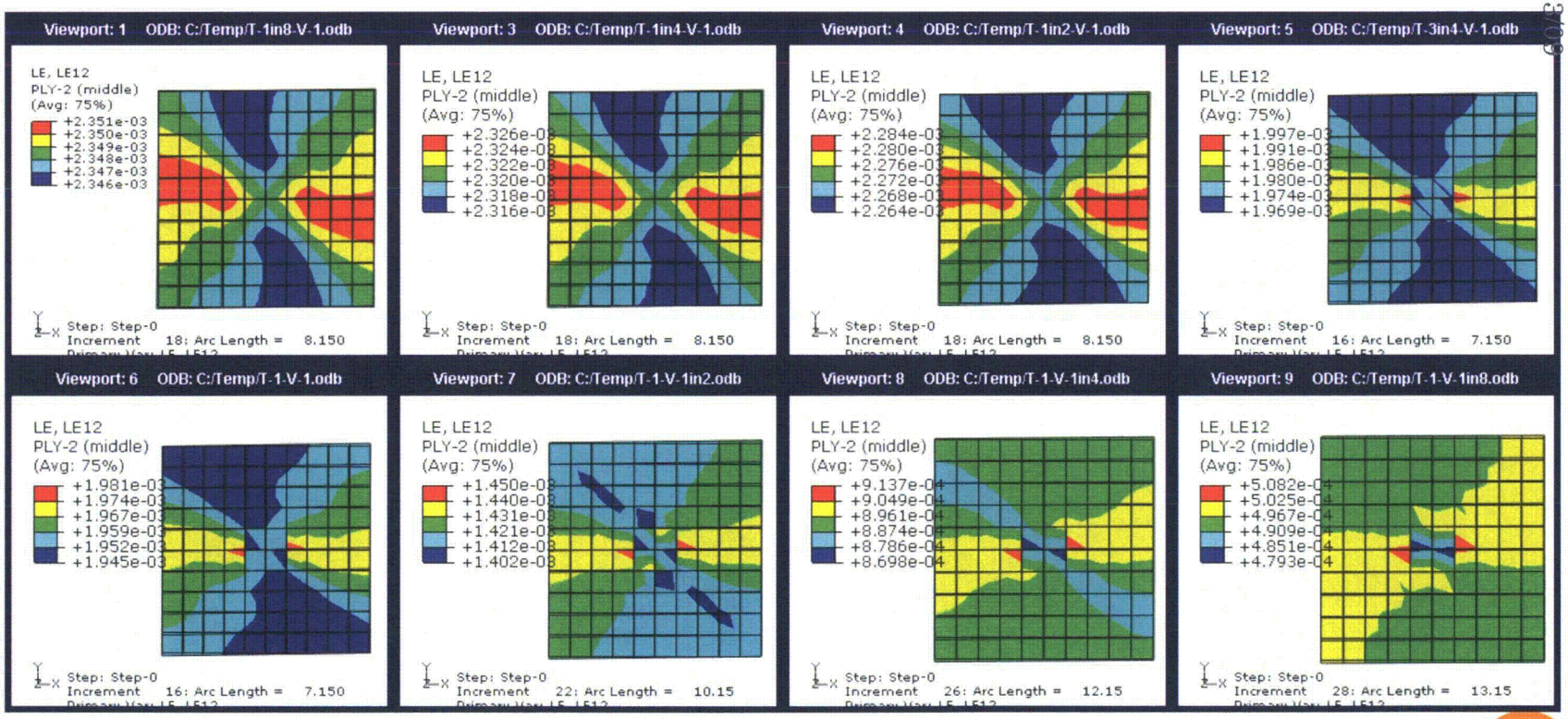
Results from Tension + Shear Analysis

Shear strain at onset of yielding decreases as the T/V increases.
Largest shear strain at yielding occurs for the pure shear case.

T/V=1/8 T/V=0.75 T/V=0.5 T/V=0.75



T=V T=2V T=4V T=8V

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SUMMARY FROM FEM ANALYSIS RESULTS

- The steel plates govern the behavior even more for tension + shear loading cases.
- The cracking direction does not change until there is significant tension (T/V greater than or equal to 1)
- The concrete contribution to the shear behavior decreases with increasing tension. It is maximum for the pure shear case (as compared to tension + shear).

- Compression + shear results in significant increase in the in-plane shear strength of the composite design.

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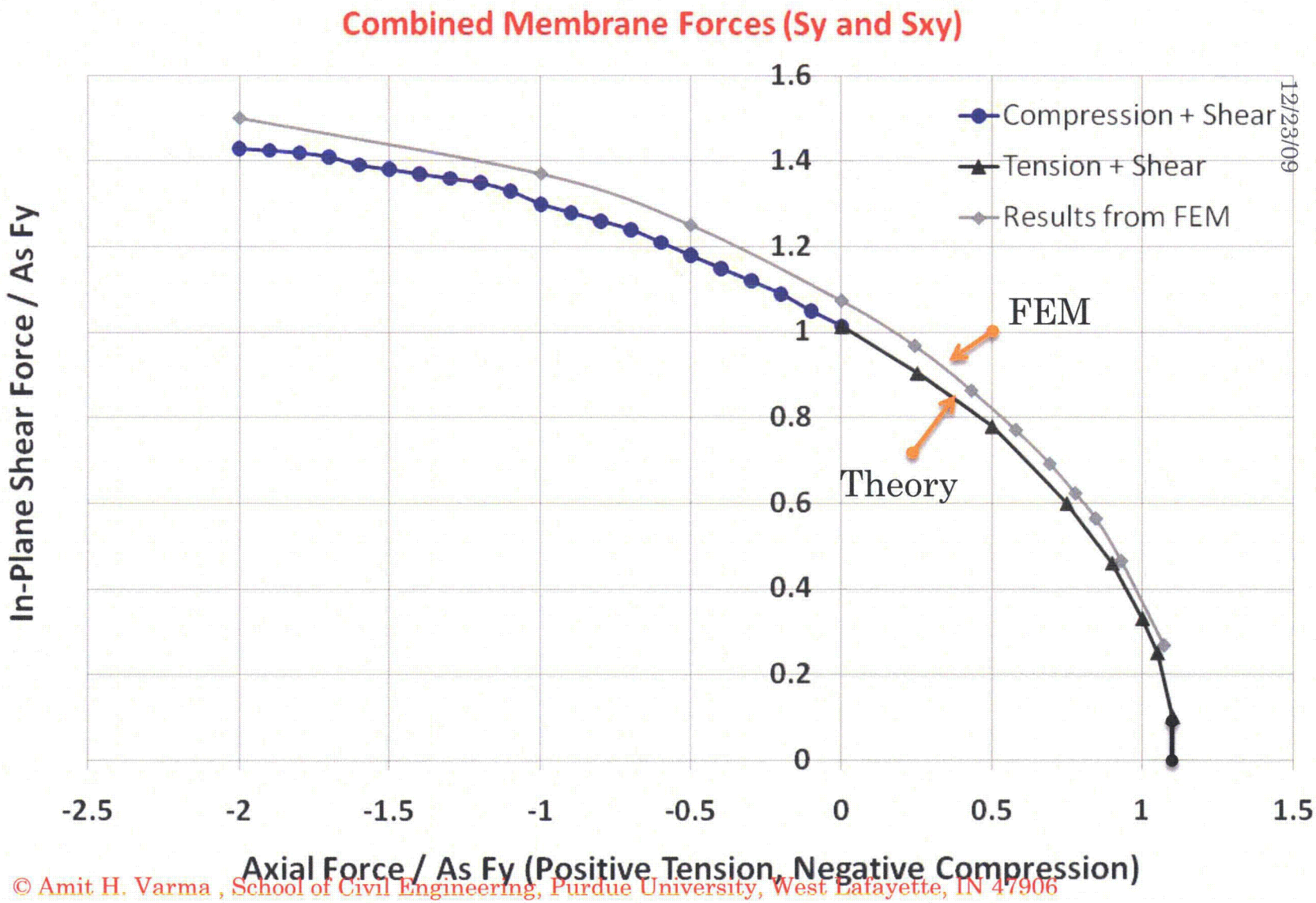
Analysis for In-Plane Force



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RESULTS IN PRINCIPAL FORCE SPACE

- Take the LCS model with smeared cracking concrete model and fracture energy.
- Analyze for combinations of axial and shear force
- Take the results for Sy and Sxy, and convert into or calculate Sp1 and Sp2.

$$S_{p1,p2} = \frac{S_x + S_y}{2} \pm \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + \left(S_{xy}\right)^2}$$

- The principal forces were plotted from the results.

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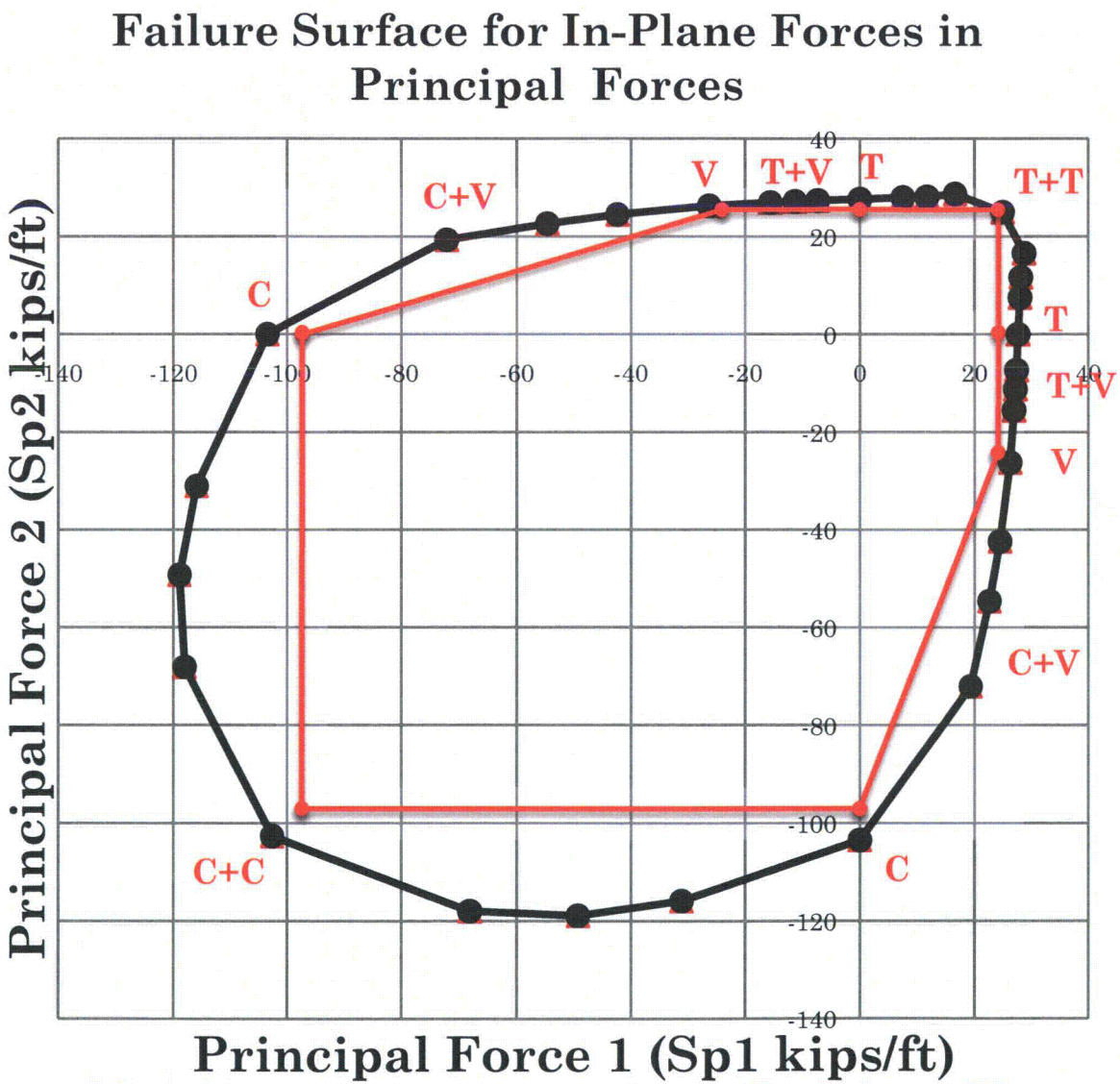
FAILURE SURFACE IN SP1-SP2 SPACE



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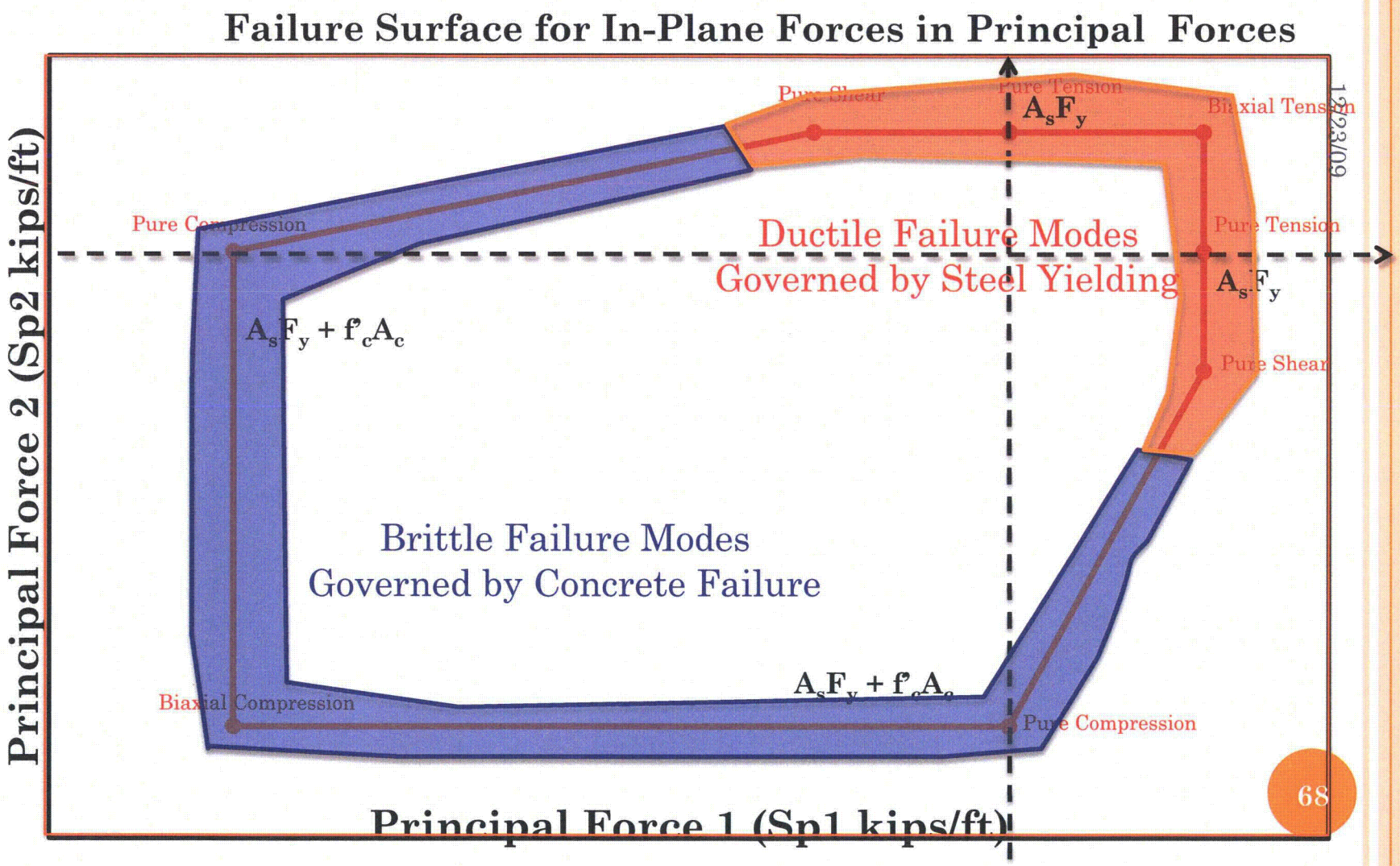
Capacity Surface in Principal Force Space



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DESIGN CAPACITIES

- The phi-factors should be representative of the failure mode (brittle and ductile)
- Creep and shrinkage effects can be included by using F_R (residual stress).
- The compression capacity can be reduced to include the effects of creep, shrinkage, local buckling (if any), and locked in stresses
- For example, As (Fy-Fr) + β f'c Ac

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