Comments on the Buckling Assessment of the Oyster Creek Drywell Shell with Emphasis on the Determination of Capacity Reduction Factors

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October 2, 2009

Summary:

The approach taken in assessing the integrity of the drywell shell against buckling is sound and in accord with the best practices. However, in reviewing the various studies, it appears to this reviewer that the approach set out in ASME Code Case 2286-1 accounting for transverse stress in the capacity reduction factors for spherical shells should be reviewed with eye to modification in the future. The emphasis of this report will focus on what we believe are possible shortcomings and inconsistencies of Code Case 2286-1. In spite of the concerns that will be spelled out about Code Case 2286-1, the capacity reduction factors generated by the alternative approach suggested here are in close agreement with those used in the assessment. Moreover, it will be argued that the alternative approach provides conservative results for the effect of transverse stress on the capacity reduction factor for the baseline case of large spherical shell segments. Thus, it is not our opinion that any possible shortcomings of Code Case 2286-1 should invalidate the conclusions of the buckling assessments. This report begins with a brief introduction to the use of the capacity reduction factor to account for the effect of imperfections on bucking of thin spherical shells with emphasis on the stabilizing role of transverse stress. It then reviews several prescriptions for reduction factors for spherical shells highlighting what this reviewer believes are the shortcomings of Code Case 2286-1. Finally, capacity reduction factors from the two approaches as applied to the spherical portion of the drywell shell are presented adding confidence to the conclusions of the drywell assessments reported.

Introductory Comments

The work summarized in the “Structural Evaluation of the Oyster Creek Drywell Summary Report” and in the earlier slides presented by AmerGen (all references are given at the end of this report) appears to be of high quality and in accord with current best practices, as far as the expertise of this reviewer enables him to judge. The area to which I have been asked direct my attention concerns the buckling assessment of the drywell shell prompted by corrosion with consequent thinning in certain sections of the shell. An essential step in the buckling assessment is the use of a capacity reduction factor (CRF) to account for the fact that the buckling of thin shells is highly sensitive to geometric imperfections in the shell. Use of the reduction factor is necessary since the numerical methods that are used to compute the buckling load do not account for the initial imperfections. The most recent assessments of integrity of the drywell against buckling make use of capacity reduction factors for cylindrical and spherical shells specified by ASME Code Case 2286-1. In addition, the method for applying the capacity reduction factor in the recent assessments follows the procedures apparently dictated by this Code. As far as I have been able to ascertain by my study of the several reports, this approach has been followed correctly and consistently. This statement also applies to the procedures used to assess the effect of thinning on buckling in areas affected by corrosion.

I have two concerns related to ASME Code Case 2286-1 that I believe should be addressed in the future. The first concerns the prescription of the capacity reduction factor (CRF) as it depends on the transverse stress as well as the baseline CRF for large spherical shell segments subject to equi-biaxial compression and uniaxial compression. The second concern is the procedure for choosing the transverse stress in applying the reduction factor. The issues identified here underlying these concerns constitute a mix of conservative and unconservative aspects which appear to cancel each other out in the current assessments. As noted in the summary, this reviewer regards the issues raised here as ones that should be addressed in the longer term independent of the present assessment and without prejudicing confidence in the current assessment given the comfortable margins of safety that emerged in the several studies. This will be made clear in the last section of the report.

In view of my concerns related to the prescription of the capacity reduction factor for spherical shells and to the manner in which it is applied, this report begins by laying out several results for the dependence of the reduction factor on transverse stress, including that used in the latest assessments and an accurate result for a specific initial imperfection. These are the baseline reduction factors for elastic buckling of large spherical shell segments with radius to thickness ratios exceeding 600. Then the report spells out two different ways in which the transverse stress can be accounted for in applying the reduction factor. One method follows the procedure adopted in the drywell shell studies as apparently specified by ASME Code Case 2286-1. The other, proposed here, is believed to be more consistent with both the underlying numerical bucking analysis and with the way transverse stresses develop for the most prevalent class of loadings in which all membrane stress components increase proportionally with load.

As general background, it can be noted that capacity reduction factors have been widely used in the design of thin shell structures with most of the developments stemming from the large body of experimental data on cylindrical shells assembled by NASA around 1960. In the aerospace industry it is common to refer to the capacity reduction factors as "knockdown factors". In addition to my own past research on the imperfection-sensitivity of shell buckling, I was involved as a consultant in the 1970's in the design of spherical tanks of LNG ships against buckling where issues similar to those in the drywell assessment arose. Like the drywell shell, the huge spherical aluminum tanks fell into the category of very thin shells with radius to thickness exceeding 600. The tank was supported at its equator by a cylindrical shell "skirt" rising from the ship hull. The critical condition with respect to buckling occurred under partial fill conditions when the segment of the spherical shell below the equator had a compressive circumferential membrane stress and a tensile meridional stress that was roughly equal in magnitude. As a consultant, I was asked to help assign an appropriate knockdown factor. As in the present assessment, the factor $\alpha \cong 0.2$, based largely on NASA's cylindrical shell data, was recognized to be overly conservative due to the stabilizing influence of the tensile transverse membrane stress component. At the time there was not much else to go by other than Yao's (1963) data on spherical shell segments which had a similar ratio of the pre-buckling membrane stresses. Based on this limited data and several theoretical imperfection-sensitivity studies, together with what we hoped to be good judgment, a reduction factor, $\alpha \approx 0.4$, was proposed. (I am unable to recall the precise value.)

A Brief Overview of Capacity Reduction Factors and Their Application for Spherical Shells Subject to Unequal Pre-buckling Membrane Stresses

a) The transverse stress dependence of α

First let me define the capacity reduction factor, α, as I understand it for large spherical shell segments with radius, R, to thickness, t, falling in the range $R/t \geq 600$. For most of the drywell shell buckling assessments, plasticity does not come into play and thus it will not be considered in this report—the plasticity reduction factor, η, will be taken to be unity. Throughout this report the following notation will be used: σ_1 is the pre-buckling membrane stress in the meridional direction and σ_2 is the corresponding stress in the circumferential direction. Both are taken to be *positive in compression*. The ratio of the pre-buckling membrane stresses at any load level is

$$r = \frac{\sigma_2}{\sigma_1} \tag{1}$$

Young's modulus is E and Poisson's ratio is ν.

In all cases considered here, $\sigma_1 \geq \sigma_2$ such that compression in the 1-direction drives the buckling. Denote the elastic theoretical buckling stress by $(\sigma_1)_C$. For example, $(\sigma_1)_C$ has been obtained in the 3-dimentional drywell study from an FEM eigenvalue analysis for the lowest buckling eigenvalue λ_C without accounting for initial imperfections. With respect to one of the concerns raised in this report, it is important to distinguish between two classes of loadings: (A) Those that produce membranes stress that increase in proportion with fixed ratio, r, as the load increases, such as gravity loading on the drywell or the loading of the spherical shell segments tested by Yao (1963). (B) Those in which the transverse membrane stress is fixed while the primary compressive membrane stress increases with the load, such as an axially compressed cylindrical shell under internal pressure in which the pressure is fixed and the axial load is increased. For want of better terminology, the first class will be referred to as *proportional transverse stress* and the latter class will be called *fixed transverse stress*. Of course, there are loadings which combine these two types of loadings, however the discussion in this report will treat them individually. In a theoretical buckling calculation for the first class of loadings, all of the membrane stress components are jointly scaled by λ. This appears to be the case for the two main loading cases considered in the buckling analysis of the drywell shell in which λ is identified with the acceleration of gravity, g. For the second class, the theoretical buckling calculation should fix the transverse stress (associated with a fixed secondary load parameter) at the appropriate level and scale stresses associated with the primary load by λ. The relevance of these points will emerge later in discussion of how the reduction factor should be evaluated in terms of the transverse stress.

The capacity reduction factor, α, relates the buckling stress accounting for imperfections to the theoretical buckling stress by

$$(\sigma_1)_{Buckling} = \alpha\,(\sigma_1)_C \tag{2}$$

Based on a large body of experimental data for buckling of cylindrical shells under uniaxial compression assembled by NASA there is general agreement that $\alpha = 0.207$ applies to sufficiently large cylindrical shell segments (i.e. for sufficiently large $M = \ell / \sqrt{Rt}$ where ℓ is the minimum length between reinforcements such as stiffeners or rings, as discussed in CC-2286-1). One concern this reviewer has concerning CC-2286-1 is that the code stipulates that $\alpha = 0.207$ should also apply to large spherical shell segments with $R/t \geq 600$ *under uniaxial compression* ($r = 0$). We believe this is not correct (and, in fact, is overly conservative) as discussed in the next paragraph. As noted below, we believe $\alpha = 0.207$ should apply to the spherical shell under *equi-biaxial compression* ($r = 1$). CC-2286-1 assumes $\alpha = 0.6 \times 0.207 = 0.124$ for large spherical shell segments under equal biaxial compression with $R/t \geq 600$.

If there is experimental evidence for taking $\alpha = 0.124$ for thin *spherical shells* under equi-biaxial compression and/or $\alpha = 0.207$ for uniaxial compression, this reviewer is not aware of it. If such experimental data exists then these values should indeed be regarded as being firmly grounded. However, absent such experimental data, the understanding of cylindrical and spherical shell buckling would argue for similar values of the CRF, α, for the cylindrical shell under uniaxial axial compression and the spherical shell under equi-biaxial compression. Both these loadings (and *only these loadings*) on the respective shell structures give rise to many simultaneous buckling modes associated with the lowest buckling eigenvalue. It is the existence of the multiple modes and the way they couple together that produces the catastrophic buckling behavior and the extreme imperfection-sensitivity of these structures. The background for these assertions is the extensive work of W.T. Koiter. In this reviewer's experience, this reasoning has led practioners in the field to conclude that the factor $\alpha = 0.207$ (at least approximately) should apply to large spherical shell segments under equi-biaxial compression *not* under uniaxial compression. Uniaxial compression of a spherical segment does not give rise to multiple simultaneous bucking modes and it should not be as imperfection-sensitive. However, we emphasize again that ultimately the CRF is dictated by experimental data, and if there exists data on spherical shell bucking that would suggest that $\alpha = 0.124$ is a better choice for equi-biaxial compression than $\alpha = 0.207$ then that choice should prevail.

The issue addressed in the reminder of this sub-section is the dependence of α on $r = \sigma_2 / \sigma_1$ for large elastic spherical shell segments. In presenting the results from CC 2286-1 we will use the recommendation of the Code that $\alpha = 0.207$ for uniaxial compression ($r = 0$). For the alternative approach suggested we take $\alpha = 0.207$ for equi-biaxial compression ($r = 1$) for the reasons stated above.

The capacity reduction factor used in the drywell assessments in the recent reports have been taken from ASME Code Case 2286-1. For sufficiently large elastic spherical shell segments with $R / t \geq 600$ (ACRS January 18, 2007; Miller Report June 15, 2006; Structural Evaluation of the Oyster Creek Drywell Summary Report, January 22, 2009) the reduction factor for transverse stresses that are zero or tensile is

$$\alpha = \frac{0.124}{0.6} + \frac{1.752}{3.24 + Et/(|\sigma_2| R)} \quad \text{for} \quad \sigma_2 \leq 0 \qquad (3)$$

Note that for σ_2=0, $\alpha = 0.124/0.6 = 0.207$. As Code Case 2286-1 is applied in the drywell shell studies, σ_2 in (3) is identified as the transverse stress component evaluated *at the loading condition*, as will be discussed in more detail in the next sub-section.[1] This

[1] Case Code 2286-1 as laid out in the cited reports at the end of this report and by Miller (2006), for example, does not state clearly how σ_2 should be identified. It appears to this reviewer that in all the drywell shell assessments σ_2 has been identified with the transverse stress at the applied load in (3), not the buckling load, whether the loading produces transverse stresses that are proportional or fixed.

is clearly correct for loadings that produce fixed transverse stresses. However, for loadings which produce proportional transverse stress, the transverse stress increases in direct proportion to the primary compressive stress according to $\sigma_2 = r\sigma_1$ independent of load. If σ_2 is identified as the transverse stress *at buckling*, such that $\sigma_2 = r\sigma_1$, then (3) becomes

$$\alpha = 0.207 + \frac{1.752}{3.24 + E(t/R)/(|r|\sigma_1)} = 0.207 + \frac{1.752}{3.24 + c/(|r|\sigma_1/\sigma_C)} \quad \text{for } r \le 0$$

In the last expression the theoretical elastic buckling stress for large, perfect spherical shell segments with $r \le 1$ has been introduced:

$$(\sigma_1)_C \equiv \sigma_C = \frac{Et}{cR}, \quad \text{with } c = \sqrt{3(1-\nu^2)} \tag{4}$$

Noting that $\sigma_1/\sigma_C = \alpha$, one obtains from (3) the equation relating α and r for $r \le 0$

$$\alpha = 0.207 + \frac{1.752}{3.24 + c/(|r|\alpha)}$$

which can be re-expressed as a quadratic equation for α:

$$3.24\alpha^2 - \left(2.422 - c/|r|\right)\alpha - 0.207c/|r| = 0, \qquad r \le 0 \tag{5}$$

This relation is plotted in Fig. 1 and it will be discussed following presentation of other results for the r-dependence of the reduction factor.

An accurate analysis of the effect of an imperfection in the shape of the axisymmetric buckling mode in a large segment of the sphere in regions away from its poles is reported in the Appendix. This analysis closely parallels the approach that Koiter (1963) employed in his famous analysis of the effect of an axisymmetric imperfection on the buckling stress of an elastic cylindrical shell in axial compression. By extending the analysis of the spherical shell to pre-buckling membrane stresses, σ_1 and $\sigma_2 = r\sigma_1$, that are unequal, one can compute the dependence of α on r for specific values of the imperfection amplitude. This dependence is plotted in Fig. 2 for four values of the imperfection amplitude. The lowest curve is relevant to shells having $R/t \ge 600$ because the imperfection amplitude has been chosen such that $\alpha = 0.207$ for the limit of equi-biaxial compression (r =1), as motivated earlier in this section. This curve has also been included in Fig. 1. An important feature of this result is that the approach guarantees that it provides an upper-bound to the buckling stress for the imperfection assumed. Table I lists three values of the capacity reduction factor of particular relevance.

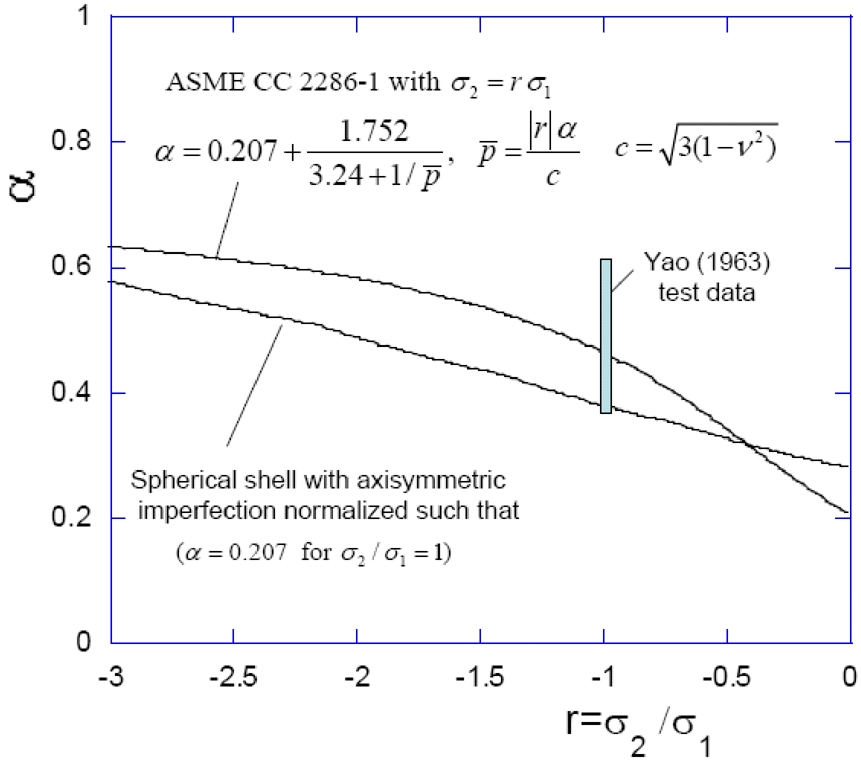

Fig. 1 Capacity reduction factor for large spherical shell segments as a function of $r = \sigma_2/\sigma_1$ as predicted by CC 2286-1 with σ_2 set as $r\sigma_1$ and $\nu = 0.3$, and as given by the results in the Appendix for a axisymmetric imperfection with amplitude chosen such that $\alpha = 0.207$ for equi-biaxial compression ($r = 1$). The experimental data of Yao (1963) is included.

$r = \sigma_2/\sigma_1$	ASME CC 2286-1	Imperfection analysis
$r = 1$	$\alpha = 0.124$	$\alpha = 0.207$
$r = 0$	$\alpha = 0.207$	$\alpha = 0.281$
$r = -1$	$\alpha = 0.465$	$\alpha = 0.381$

Table I. Capacity reduction factor, α, for elastic buckling of large spherical shell segments with $R/t > 600$ based ASME Code Case 2286-1 as expressed in terms of r and based on the imperfection analysis in the Appendix.

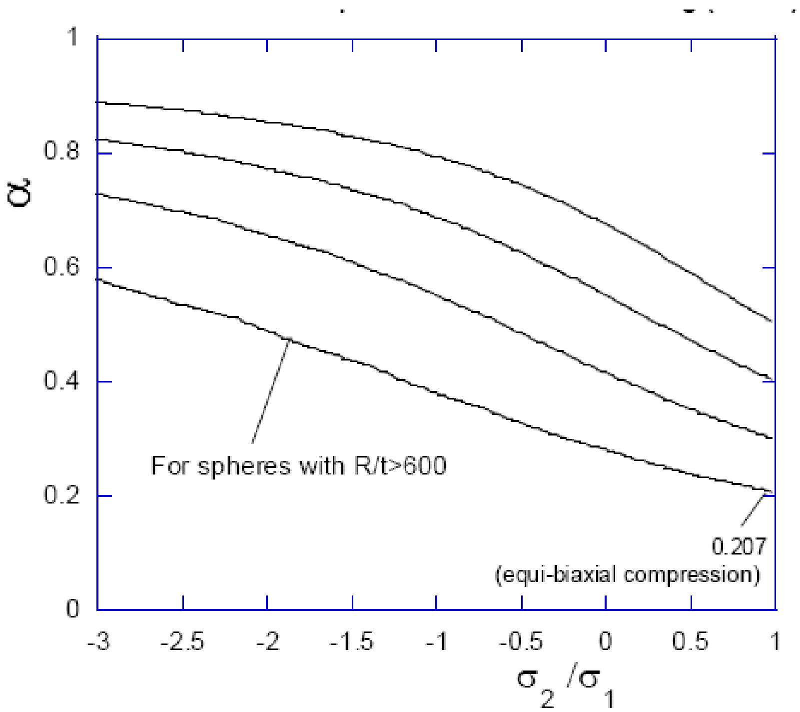

Fig. 2 Capacity reduction factor for large spherical shell segments as a function of $r = \sigma_2 / \sigma_1$ as predicted by the analysis in the Appendix for axisymmetric imperfections. The lowest curve has the imperfection set such that $\alpha = 0.207$ at $r = 1$ such that it is relevant to shells with $R/t \geq 600$.

Included in Fig. 1 is the range of experimental data for the large spherical shell segments tested by Yao (1963). These shells were tested in tension such that the circumferential stress was compressive with equal and opposite tensile stress in the meridional direction. In the present notation, with axes rotated 90 degrees, all of these test correspond to $r = -1$. The seven shells had R/t ranging from 455 to 1600 and the ratio of the experimental to the theoretical buckling loads ranged from 0.38 to 0.67. Two shells buckling at 0.38, including one shell with R/t =476.

Another plot of capacity reduction factors as a function of r is included here in Fig. 3. This plot is Slide #26 of the AmerGen presentation to the ACRS of February 2, 2007. The lower (red) curve is the same as that plotted in Fig. 1 for CC 2286-1. The data range of the Yao tests has been plotted incorrectly in Fig. 3 (c.f. Fig.1). Four of the seven shells buckled below the prescription of CC 2286-1.

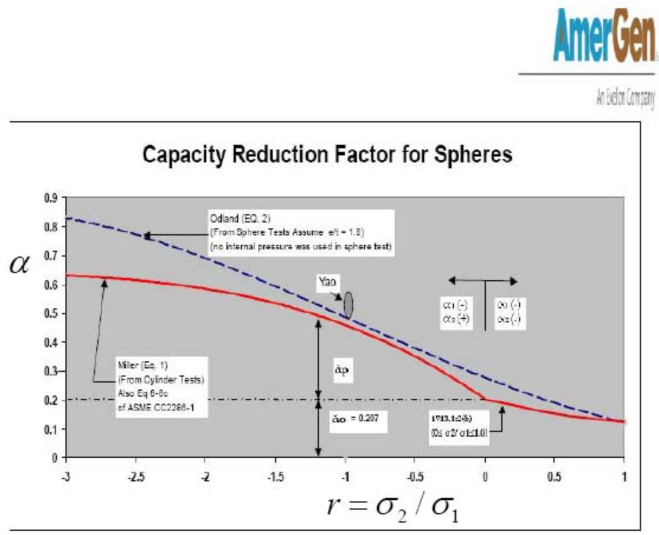

Fig. 3 Capacity reduction factors for spherical shells from slides presented to ACSR on February 2, 2007.

The spread in Fig. 1 between the capacity reduction factor based on ASME Code Case 2286-1 and that based the buckling analysis reported in the Appendix for $r < -0.5$ may be a cause for concern. As noted earlier, this reviewer is not familiar with the deliberations that led to the development of the ASME Code Case 2286-1, nor am I aware of a significant new body of experimental results (more recent, say, than those of Yao (1963)) for spherical shell segments that could be used to motivate the capacity reduction factor dependence on σ_2 specified by (3) in the Code. The pivotal role of $\alpha = 0.207$ for the uniaxial case, $\sigma_2 = 0$, in (3) would seem to require validation by experiments. (I have not seen the experimental data generated by the Miller tests referred to in some of the correspondence.) The Yao data shown in Fig. 1 (and incorrectly plotted in Fig. 3) indicates that the Code prescription is somewhat unconservative when $r = -1$ ($\alpha = 0.465$ compared with 0.38 for two of the shells).

The r-dependence from the imperfection analysis in the Appendix and plotted in Fig. 1 is almost certainly conservative for the following reason. That analysis is based on an imperfection that varies only with x_1. Such imperfections are less influenced by the transverse stress, σ_2, than non-axisymmertic imperfections that have variations in both directions. It is not possible to carry out an analysis for the r-dependence of α for non-axisymmetric imperfections without resorting to a full nonlinear FEM computation. Such an analysis would almost certainly produce larger values of α for $r = 0$ and $r = -1$ than those listed in Table I for the axisymmetric imperfection. Nevertheless, it remains true that shells with imperfection shapes exist that would buckle below predictions based on the capacity reduction factor specified by ASME Code Case 2286-1. This point is driven home in Fig. 4 where trends are presented on the effect of a positive transverse stress on the buckling of *cylindrical shells* loaded by an axial load in combination with an internal pressure. The lowest theoretical curve follows very closely the prediction based on the same type of Koiter-analysis presented in the Appendix for the effect of axisymmetric imperfections on cylindrical shells. The conclusion to be drawn from the experimental data in Fig. 4 is that trends in the capacity reduction factor with the transverse stress as predicted under the assumption of an axisymmetric imperfection may not be overly conservative because shells dominated by axisymmetric imperfections do exist. This conclusion is further reinforced by the fact that two of the seven shells tested by Yao (1963) coincide almost exactly with the prediction for axisymmetric imperfections with $r = -1$, i.e. $\alpha = 0.38$.

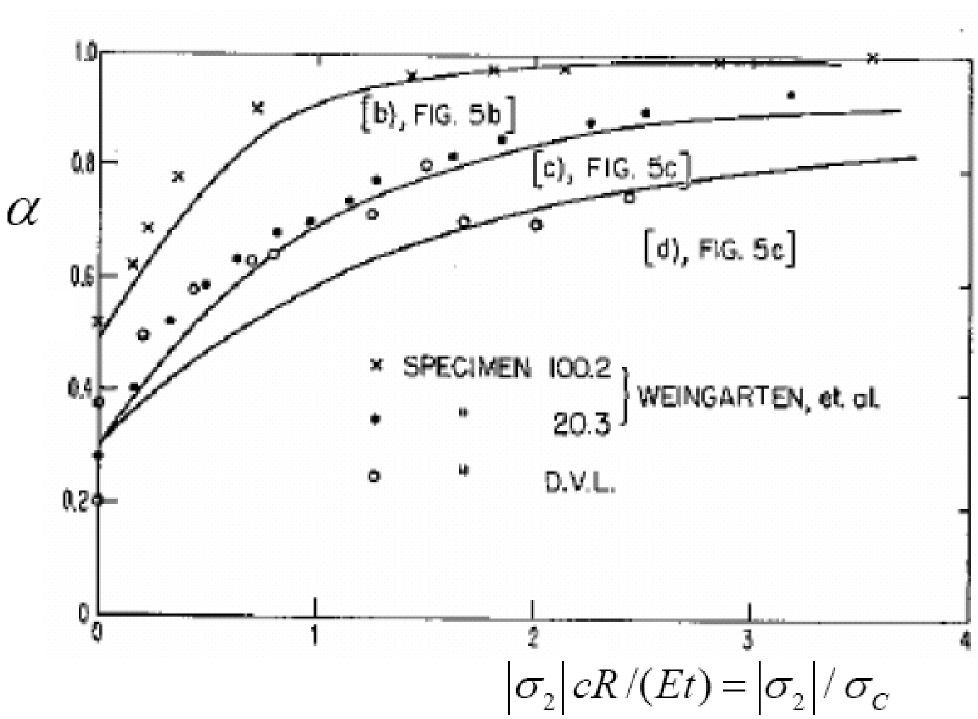

Fig. 4 The effect of both axisymmetric imperfections (lowest solid curve) and combinations of axisymmetric and non-axisymmetric imperfections (upper two solid curves) on the buckling of cylindrical shells under axial compression subject to internal pressure. The circumferential transverse stress, σ_2, is tensile. This is Fig. 7 taken from Hutchinson (1965), and the experimental points are data cited in that reference. Note that these shells do not have radius to thickness exceeding 600 and therefore the capacity reduction factor lies above 0.207 when $\sigma_2 = 0$ for most of these shells. The transverse stress stabilizes shells dominated by non-axisymmetric imperfections more effectively than those dominated by axisymmetric imperfections. In the latter case, the Koiter analysis (essentially the lower curve) accurately captures the increase of the capacity reduction ratio with increasing transverse stress.

b) The procedure for determining the capacity reduction factor

(i) The procedure as applied in the assessments of the drywell shell based on AMSE Code Case 2286-1

As noted in the previous subsection, the method used to determine α using CC 2286-1 that appears to have been followed consistently in the drywell studies is as follows, illustrated here for large elastic spherical shell segments. Evaluate α using equation (3) based on the transverse membrane stress σ_2 computed at *the applied load* for the case in question. With $(\sigma_1)_C$ as the theoretical stress computed with no imperfections taken into account, the actual buckling stress accounting for imperfections is given by (2), i.e., $(\sigma_1)_{Buckling} = \alpha(\sigma_1)_C$, assuming no further reduction due to plasticity. The maximum allowable stress is reduced by the factor of safety, FS, to

$$(\sigma_1)_{Allowable} = \alpha(\sigma_1)_C / FS \ .$$

For loading cases producing *a fixed transverse membrane stress*, this reviewer believes that this procedure is correct because this *is* the transverse stress at buckling. However, for loading cases producing *proportional transverse membrane stress*, this reviewer believes that the appropriate choice of σ_2 should be its value *at buckling*. This is the second of the two concerns this reviewer has about the procedure of CC 2286-1.

(ii) An alternative procedure for loadings that produce proportional membrane stresses

To motivate the alternative proposal for determining the influence of the transverse stress on the capacity reduction factor, consider, as an example, the tests conducted by Yao (1963) on spherical shell segments. The segments were loaded under axial tension so that the roles σ_1 and σ_2 are interchanged in the present notation. The important point is that the ratio of these membrane stresses is *fixed* as the load is increased and corresponds to $r \cong -1$. This fact is recognized by the data points of Yao plotted (incorrectly) in the AmerGen slide in Fig. 3. Thus, $\sigma_2 = r\sigma_1$, when buckling occurs in the test. A second motivation for the alternative proposal is that the standard procedure for computing the theoretical buckling stress, $(\sigma_1)_C$, scales *all* of the membrane stress components by a single eigen-load parameter, λ, unless there are two independent loads, one of which increases until buckling occurs and the other is stationary. In other words, for load cases producing proportional transverse stress, the ratio of the membrane stresses is *fixed* according to $\sigma_2 = r\sigma_1$ in computing the theoretical buckling stress. The fixed proportionality of the membrane stresses in such an analysis is a natural consequence of the fact that they depend linearly on the load parameter in the pre-buckling state. In the numerical buckling calculations for this class of loads, it would not be permissible to fix the transverse stress.

From the above it follows that for loadings producing proportional transverse stress, a logical way to determine the dependence of the capacity reduction factor on the transverse stress is to evaluate α in terms of $r = \sigma_2 / \sigma_1$, independent of the applied load, ensuring that σ_2 *is* the transverse membrane stress at buckling (or at any other load). With this approach, α is read directly off a curve such as one of those in Figs. 1-3 or evaluated from a formula for α in terms of r. Once α is identified, the remaining steps for determining the allowable stress are identical to those outlined above in (i).

(iii) Comments on the two approaches

As indicated above, this reviewer believes that the capacity reduction factor should be evaluated using the transverse membrane stress *at buckling* in all cases. For loading cases that produce *fixed transverse stress,* the fixed transverse stress *is* the value at buckling and thus there is no issue. However, for loading cases which produce *proportional transverse stress*, the procedure used in the dry well assessment uses the transverse stress at the applied load and *not* at buckling. The rationale for evaluating the transverse stress at buckling will be further motivated below.

One could argue that the relevant value of the transverse stress is the value at the applied load and therefore this value should be used in determining α. However, if this option is invoked, the buckling experiments used to establish α should also be devised so that the transverse stress at buckling is that set by the applied load (as opposed to the buckling load). This is not how most bucking tests are conducted. Most buckling tests have a single load parameter such that all components of the membrane stress increase proportionally. An example of an exception would be a cylindrical shell subject to a fixed internal pressure and an increasing axial load. However, loading cases relevant to most of those for the spherical portion of drywell shell produce proportional membrane stresses. The Yao tests exemplify this class of loading. Moreover, if the factor of safety reflects an uncertainty of applied load, for example, such that an accidental overload might be twice the design applied load, then for this class of loading *all* the membrane stresses will be increased by a factor of two. Here, again, it makes sense to adopt $\sigma_2 = r\sigma_1$ in the evaluation of α.

It is worth emphasizing again that the procedure for evaluating α using all the pre-buckling membrane stress components *at the buckling load* encompasses both classes of loadings, *fixed transverse stress* and *proportional transverse stress*. As noted earlier, for both classes of loadings, this procedure is consistent with the way in which the pre-buckling membrane stresses vary, both in buckling tests and in the finite element computation of theoretical buckling stresses. As noted above, the procedure which evaluates α using the transverse stress at the applied load is not consistent with either of these two legs of buckling assessment when the actual loading produces a transverse stress that increases proportionally with the load.

The two procedures are identical when $\sigma_2 = 0$. They are also identical when the factor of safety is unity, $FS = 1$, because then the applied stresses, including the transverse stress, are the stresses at buckling. However, for cases with $FS > 1$ with nonzero transverse membrane stress the approaches differ. For loading cases producing proportional transverse stress, a procedure evaluating α at the transverse stress of the applied load, as has been done in the drywell shell assessments, will underestimate α if $\sigma_2 < 0$ and overestimate α if $\sigma_2 > 0$ compared to the procedure that uses the transverse stress at buckling. (This follows because the magnitude of transverse buckling stress at the applied load is never greater than its magnitude at buckling and because α is a decreasing function of r, as seen in Figs. 1-3.) Since many of the loading cases involved in the drywell shell assessment have $\sigma_2 < 0$, it follows that this aspect of the procedure employed in the assessment is conservative compared to the proposed alternative. This observation must be tempered by the fact that the capacity reduction factor employed in these assessments based on CC-2286-1 in Fig. 3 or in (3) may not be conservative when $r < -0.5$.

Finally, it can be mentioned that employing the capacity reduction factor, α, using the procedure favored by this reviewer for loads generating proportional transverse stress is no more difficult to apply than the procedure employed in the drywell shell assessments. One simply identifies $r = \sigma_2 / \sigma_1$ from the pre-buckling calculation and then obtains α from a formula or a curve as in Fig.1. Cases that involve combinations of the two classes of loading are slightly more complicated but can be readily addressed.

Comparisons of the Capacity Reduction Factors from the Two Approaches as Applied to the Spherical Sections of the Drywell Shell

Tables 8-5 and 8-6 in the SIA report present the applied stresses and the modified capacity reduction factor, $\alpha \equiv \alpha_m$, for the two most relevant loading cases of drywell shell, refueling and flooding, respectively. Results are presented for upper, middle and lower sections the spherical shell. These results are listed here in Table II. Also included is the r-value based on the ratio of the applied membrane stresses listed in the SIA tables and the value of α predicted based on the approach proposed here in conjunction with the lower curve in Fig. 1 based on the axisymmetric imperfection analysis.

The two sets of predictions are quite close and in all cases the values used in the assessment of the Oyster Creek drywell shell are slightly more conservative than those of the present approach except for the upper section in the refueling case. In that one case, the spherical segment is smaller than a fully "large section" and the α -value in Table 8-5 has been justifiably increased in accord to the procedure based on the value of M .

Spherical sections	Upper section	Middle section	Lower section
α SIA Table 8-5	0.436	0.338	0.328
r SIA Table 8-5	-0.58	-0.96	-0.93
α From Appendix	0.35	0.37	0.36
α SIA Table 8-6	0.533	0.492	0.502
r SIA Table 8-6	-2.5	-3.4	-2.15
α From Appendix	0.54	0.60	0.51

Table II. Comparisons between the capacity reduction factors from the SIA 2009 report and the approach suggested by the present report in conjunction with the results from the analysis in the Appendix and plotted in Fig. 1.

Tables 8-7 and 8-8 of the SIA report present similar results for the various bays of the sand bed region of the drywell shell. For the refueling case, the modified capacity reduction factor, α , ranges from 0.30 to 0.34. For all of the bays, $r \cong -1$, implying that $\alpha = 0.38$ based on the present approach, again falling just above the values used in the assessment. For the flooding case, Table 8-8 gives α in the range from 0.48 to 0.55. For all bays, $r \cong -2$, such that Fig. 1 gives $\alpha = 0.50$.

In conclusion, the procedure of ASME Case Code 2286-1 gives very similar predictions for the capacity reduction factor for large elastic spherical shell segments to the procedure advanced here, in spite of several significant differences between them. It appears to this reviewer that the mix of conservative and unconservative aspects in the CC-2286-1 approach combine to yield sound results in the applications relevant to the drywell shell. Nevertheless, for the reasons detailed in this report, this reviewer feels strongly that the approach laid out in CC-2286-1 should be carefully reviewed.

Appendix: Capacity Reduction Factor α for Buckling of Large Spherical Shell Segments with Axisymmetric Imperfections

Consider an elastic spherical shell of radius, R, thickness, t, Young's modulus, E, and Poisson's ratio, ν, that is governed by the nonlinear Donnell-Mushtari-Vlasov (DMV) shell equations. Denote the in-plane pre-buckling membrane stresses of the perfect shell by σ_1 and σ_2 defined to be positive in compression with $\sigma_1 \geq \sigma_2$ ($\sigma_{12} = 0$). These are aligned with middle surface coordinates, x_1 and x_2, where, consistent with the loadings of the spherical sections of the drywell shell, x_1 is in the direction of the meridian and x_2 in the circumferential direction. The spherical segment is considered to be large compared to the wavelengths of the imperfection and buckling modes but still relatively shallow. The analysis which follows is very similar to that given by Koiter (1963) in his solution for the upper-bound to the buckling of a cylindrical shell subject to axial compression with an axisymmetric imperfection. The corresponding analysis of the large spherical shell segment subject to equi-biaxial compression was given by Hutchinson (1967), and the present results reduce to this case in the limit $\sigma_1 = \sigma_2$. The objective of the present analysis is to obtain explicit results for the dependence of the capacity reduction factor on the biaxial stressing ratio, $r = \sigma_2 / \sigma_1$. Only the essential details of the analysis will be included in this Appendix; the full details are similar to those given in the above two references.

The initial imperfection is

$$\bar{w} = \bar{\xi} t \cos\left(q x_1 / R\right)$$

with $\bar{\xi}$ as the normalized amplitude of the imperfection and $q^4 = 12(1-\nu^2)(R/t)^2$. The imperfection is in the shape of the classical axisymmetric buckling mode in a shallow section of the sphere (Hutchinson, 1967). The nonlinear DMV equations for the imperfect shell admit an exact axisymmetric solution which written in terms of the normal displacement and the Airy stress function is

$$W_A = -\frac{(1-\nu) p R^2}{E t^2} + \bar{\xi} t \frac{\bar{\sigma}_1}{1-\bar{\sigma}_1} \cos(\frac{q x_1}{R})$$

$$F_A = -\frac{1}{2}\left(\sigma_1 x_2^{\,2} + \sigma_2 x_1^{\,2}\right) - \bar{\xi} t \frac{E R t}{q^2} \frac{\bar{\sigma}_1}{1-\bar{\sigma}_1} \cos(\frac{q x_1}{R})$$

Here, the pressure, p, normal to shell equilibrates pre-bucking membrane stresses according to $p = (\sigma_1 + \sigma_2)t / R$. Additionally, $\bar{\sigma}_1 \equiv \sigma_1 / \sigma_C$ and $\bar{\sigma}_2 \equiv \sigma_2 / \sigma_C$ with

$$\sigma_C = \frac{Et}{\sqrt{3(1-\nu^2)}R}$$

as the classical buckling stress of the perfect shell. The classical stress applies to all large, perfect spherical shell segments as long as $r \le 1$.

Following Koiter's (1963) approach for cylindrical shells, we analyze the bifurcation problem for buckling from the axisymmetric state into a non-axisymmetic mode. The bifurcation is generally unstable giving rise to a dynamic collapse. An important feature of this approach is that it is carried out in such a way that the result is not only highly accurate but also provides a rigorous upper-bound to the bifurcation stress for the particular imperfection. The reader is referred to Koiter's paper for the physical motivation involving essential aspects of nonlinear mode coupling underlying the steps that follow.

The solution in the buckled state is written as

$$W = W_A + w(x_1, x_2) \ \ \& \ \ F = F_A + f(x_1, x_2)$$

A non-axisymmetric buckling deflection of the form

$$w(x_1, x_2) = At \sin\left(\frac{1}{2}\frac{qx_1}{R}\right)\sin\left(\gamma \frac{qx_1}{R}\right)$$

is assumed where γ sets the wavelength in the circumferential direction and is to be determined in the solution process. The next step is to note that the nonlinear DMV compatibility equation can be solved *exactly* for f in terms of w:

$$f = AEt^3\left[b_1 \sin\left(\frac{1}{2}\frac{qx_1}{R}\right) + b_2 \sin\left(\frac{3}{2}\frac{qx_1}{R}\right)\right]\sin\left(\gamma \frac{qx_2}{R}\right)$$

where

$$b_1 = -\frac{1}{2c}\left(\frac{1}{4}+\gamma^2\right)^{-1} + \frac{\gamma^2}{2}\left(\frac{1}{4}+\gamma^2\right)^{-2}\frac{\bar{\xi}}{1-\bar{\sigma}_1},\ \ b_2 = -\frac{\gamma^2}{2}\left(\frac{9}{4}+\gamma^2\right)^{-2}\frac{\bar{\xi}}{1-\bar{\sigma}_1}$$

with $c = \sqrt{3(1-\nu^2)}$. Because only terms linear in A will be required, the quadratic terms in A are not shown.

The final step in the analysis is to evaluate the potential energy difference of the shell in buckled state from that in the axisymmetric state. The eigenvalue problem for the bifurcation problem only requires the quadratic terms in w and f in the potential energy change. Koiter's notation for this term is $P_2(w, f)$, and the expression for it is

$$P_2(w, f) = \frac{1}{2}\int_S \Big\{ \; D\left[(1-\nu)w_{,\alpha\beta}w_{,\alpha\beta} + \nu w_{,\gamma\gamma}{}^2\right] + \frac{1}{Et}\left[(1+\nu) f_{,\alpha\beta} f_{,\alpha\beta} - \nu f_{,\gamma\gamma}{}^2\right]$$
$$-\sigma_1 t\, w_{,1}{}^2 + \left(-\sigma_2 t + \frac{\bar{\xi} Et^2}{R}\frac{\bar{\sigma}_1}{1-\bar{\sigma}_1}\cos\left(\frac{qx_1}{R}\right)\right) w_{,2}{}^2 \; \Big\} dS$$

The potential energy change can be evaluated in closed form:

$$P_2 = \frac{Et^3SA^2}{4R^2}\left\{\left(\frac{1}{4}+\gamma^2\right)^2 + 4\left(cb_1\left(\frac{1}{4}+\gamma^2\right)\right)^2 + 4\left(cb_2\left(\frac{9}{4}+\gamma^2\right)\right)^2 - \left(\frac{1}{2}+2\gamma^2 r\right)\bar{\sigma}_1 - c\bar{\xi}\gamma^2\frac{\bar{\sigma}_1}{1-\bar{\sigma}_1}\right\}$$

where S is the area of the spherical segment and

$$r = \frac{\sigma_2}{\sigma_1}$$

The only approximation in the above calculation occurs in the final step where terms such as $\int_S \cos(qx_1/R)dS$ are neglected compared to S. The neglected terms are of order $1/q$ relative to those retained.

For prescribed $\bar{\xi}$ and r with specified, γ, the eigenvalue for bifurcation from the axisymmetric state, $\bar{\sigma}_1$, is given by $P_2 = 0$. For prescribed $\bar{\xi}$ and r, the lowest buckling stress is obtained by minimizing this eigenvalue with respect to γ. Note that the normalized lowest buckling stress, $\bar{\sigma}_1 = \sigma_1/\sigma_C$, is precisely the desired capacity reduction factor, α. The fact that the result so obtained is an upper-bound to the reduction factor follows because the field used to evaluate P_2 is kinematically admissible due to fact that f is obtained exactly in terms of w. Inspection of P_2 shows that $c\bar{\xi}$ appears in combination with no other dependence on ν. Thus, the plots of the reduction factor as a function of r introduced in this report do not depend on Poisson's ratio.

The capacity reduction factor, $\alpha = \bar{\sigma}_1$, is plotted for specified $\bar{\xi}$ and r in Figs. 1 and 2. The lowest curve in Fig. 2 is relevant to the drywell sphere since the imperfection is set such that $\alpha = 0.207$ for equi-biaxial compression ($r = 1$): $c\bar{\xi} = 1.396$ or $\bar{\xi} = 0.845$

for $\nu = 0.3$. The upper three curves correspond to $c\bar{\xi} = 0.824$, $c\bar{\xi} = 0.512$ and $c\bar{\xi} = 0.316$, respectively. The results for $r = 1$ coincide with those presented in Fig. 5 of Hutchinson (1967). Analogous results for *cylindrical shells* subject to a combination of axial load and internal pressure have been presented by Hutchinson (1965).

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