20th International Conference on Structural Mechanics in Reactor Technology (SMiRT 20)
Espoo, Finland, August 9-14, 2009
SMiRT 20-Division ??, Paper ??

Reliability Analysis of Slope Stability at Nuclear Power Plant Site

Weijun Wang[a] & Carl Costantino[b]

[a]*Sr. Geotechnical Engineer, US Nuclear Regulatory Commission, Mail Sop T-7E18, Washington, DC 20555-0001, e-mail: Weijun.Wang@nrc.gov*
[b] *Professor Emeritus, 4 Rockingham Road, Spring Valley, New York, NY 10977e-mail: carl@cjcasso.com*

Keywords: Slope Stability, Uncertainty and variability, Probability, Reliability index,

1 ABSTRACT

It is well known that great uncertainty and variability exist in subsurface materials regarding layer uniformity and soil/rock engineering properties, as well as seismic loadings to be considered. Those uncertainties and variability not only affect the stability of the subsurface materials and foundations, but also affect the stability of the slopes that are usually present at the site, especially when seismic loadings are involved. This study focuses on the impact of the uncertainties of soil and seismic loads on the slope stability analysis and how to determine the reliability of the analysis results in terms of factor of safety. In this study, uncertainties involved in the slope stability analysis were first identified, then a practical engineering procedure is proposed to apply probability analysis method that takes those uncertainties into consideration and uses reliability index as the reliability measurement of the factor of safety of slope stability. After conducting sensitivity study, the influences of input parameters on slope stability analysis results were identified. Finally, the proposed procedure was applied to slope analyses for a slope on a U.S nuclear power plant to present an example. The analysis results showed that (1) the seismic loadings and soil shear strength parameters affect the slope stability analysis results more than other parameters involved in the analysis; (2) the proposed reliability analysis method can give a good indication of the degree of reliability of the factor of safety of slope stability; and (3) the degree of uncertainty of engineering properties of subsurface material and seismic loading conditions affect the reliability of slope stability analysis. Based on the results of this study, it is concluded that the uncertainties of site subsurface materials and site specific seismic loadings should be considered in the site stability analyses and their impacts on the analyses results need to be studied. The proposed reliability slope stability analysis method can be used as supplement to conventional deterministic analysis methods to gain more confidence to the analysis results.

2 INTRODUCTION

In nuclear power plant design and construction, engineers have to face and deal with many uncertainties, not only in risk evaluation and analysis, but also in site geologic, seismic and geotechnical engineering properties evaluations. In geotechnical engineering, the uncertainties and variability are generally related to the subsurface strata uniformity and soil/rock engineering properties, as well as seismic loadings to be considered at a specific site. Those uncertainties and variability not only affect evaluation of the stability of the subsurface materials and foundations, but also affect the stability analyses of slopes, dam and embankment at the site, especially when seismic loadings are involved.

To deal with uncertainties and variability, geotechnical engineers conventionally use factor of safety and conservatively chosen parameters in calculations and analyses. But how reliable the evaluations and analysis results often is everyone's guess. Because the probabilistic method is a powerful tool to deal with uncertainties and variation, this method has been recognized and applied in civil engineering field since early 70's, as described by Benjamin and Cornell (1970) and Harr (1987), among others. There are quite a few studies related to probability and reliability analysis of slope, dam and embankment stability. For examples, Wolff and Wang (1993) presented their study on reliability analysis of dams and locks, and later the U.S. Army Corps of Engineers (1997, 1999) introduced the probability and reliability methods for use in

Note: This paper was prepared, in part, by an employee of the United States Nuclear Regulatory Commission on his or her own time apart from his or her regular duties. NRC has neither approved nor disapproved its technical content.

geotechnical engineering. Vanmarcke (1977), Christian et al (1994) and others also presented either in detailed discussion or summary of applications of probabilistic concept to analysis of the stability of slope and embankments using so-called first-order, second-moment method. Duncan (2000) discussed the use of reliability concept in geotechnical engineering and gave some detailed examples on how to apply this concept in retaining wall stability, slope stability and settlement analyses. Although it have not been accepted as a formal method in geotechnical engineering practice for various reasons that include the variability involved in geotechnical engineering has not fully characterized, especially the correlations between all variables; how to use it in engineering design has not been standardized; and lack of confidence on use of this method in practice among geotechnical engineers, there is a general agreement that probabilistic and reliability methods should be applied in general geotechnical engineering practices as supplement to and enhance the conventional deterministic method.

3 RELIABILITY EVALUATION METHODS IN SLOPE STABILITY ANALYSIS

In engineering, reliability is generally defined as the ability of a system or component to perform its required functions under stated conditions for a specified period of time. In geotechnical engineering practices using reliability concept is a good way to account for uncertainties and variability resulting from spatial variation of subsurface materials –soil and rock, variation in soil/rock property determination, field and laboratory test results, dada analysis, analysis model errors, and others.

For slope stability analysis, uncertainties also exist in other variables, or parameters, such as geometry of slope, loading conditions, pore water pressure distribution and others. Among the variables, some of them are time dependent, such as the shear strength of soils, pore water pressure distribution, as well as dynamic loads, including earthquake induced seismic loading. In engineering practices, factor of safety (FOS) is used to account for the uncertainties, but reliability can provides a measurement on degree of confidence on the design and analysis results. In reliability analysis, some basic statistic and reliability will be used and the procedure of using reliability concept in slope stability analysis is needed and will be discussed as the follows.

3.1 Basic definition of variables

For a parameter, or variable *X* with *n* data points x_i, $i=1, 2, \ldots n$, it is defined that the mean, or expected value *E[X]* is

$$E[X] = \frac{1}{n}\sum_{1}^{n} x_i \qquad (1)$$

The variance *Var[X]* is

$$Var[X] = \frac{1}{(n-1)}\sum_{1}^{n} \left(x_i - E[X]\right)^2 \qquad (2)$$

The standard deviation σ[X] is

$$\sigma[X] = \sqrt{Var[X]} \qquad (3)$$

The coefficient of variation V(X) is

$$V(X) = \frac{\sigma[X]}{E[X]} \qquad (4)$$

For two variables, X and Y, the covariance of X and Y, Cov [X,Y] is defined as

$$Cov(X,Y) = \frac{1}{(n-1)}\sum_{1}^{n} \left(x_i - E[X]\right)\cdot\left(y_i - E[Y]\right) \qquad (5)$$

If two variables are unrelated, then *Cov(X,Y)* = 0.

The coefficient of correlation ρ_{XY}, a non-dimensional measure of the correlation, is defined as

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \tag{6}$$

3.2 Reliability index

Reliability index, β, is a very useful probability measurement and its definition is described as the follows.

In engineering reliability analysis, the safety margin, *SM*, is a commonly used variable as it provides a measurement of how the capacity (resistance) of a designed structure or component compares with demand (load). The *SM* is defined as

$$SM = C - D \tag{7}$$

where *C* is capacity and *D* is demand.

The mean and standard deviation of *SM* are

$$E[SM] = E[C] - E[D] \tag{8}$$

$$\sigma_{SM} = \sqrt{\sigma_C^2 + \sigma_D^2 - 2\rho_{c,D}\sigma_C\sigma_D} \tag{9}$$

where

E[C] and *E[D]* are mean of capacity and demand, respectively;

σ_C and σ_D are standard deviation of capacity and demand, respectively; and

$\rho_{C,D}$ is coefficient of correlation of capacity and demand.

If C and D are not related, although is it unlikely in engineering practices, equation (9) then reduced to

$$\sigma_{SM} = \sqrt{\sigma_C^2 + \sigma_D^2} \tag{10}$$

Reliability index is a dimensionless quantitative value that represents the distance from expected condition to a limit state, or edge of a failure region. The reliability index is measured in unit of standard deviation of performance function and only depends on the statistical properties of variables involved not depend on the expression of the performance function. The reliability index β_{SM} is defined as

$$\beta_{SM} = \frac{E[SM]}{\sigma_{SM}} \tag{11}$$

The illustration of reliability index for normally distributed safety margin SM is presented in Figure 1.

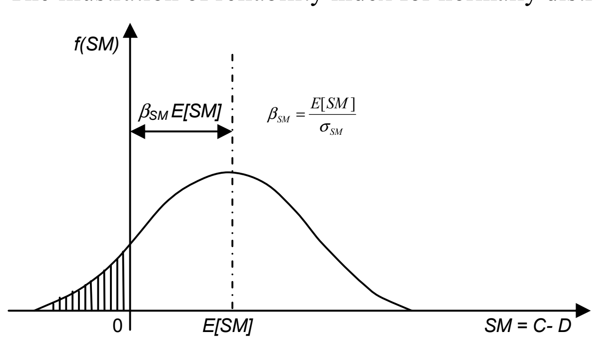

Figure 1. Reliability index of safety margin

In engineering practice, factor of safety (*FOS*) is another commonly used safety indicator and can be defined as the ratio of the *C* and *D*

$$FOS = C/D \tag{12}$$

For practical purpose, we can treat the *FOS* as a new variable and the reliability index of *FOS*, β_{FOS}, is defined as (for normal distribution)

$$\beta_{FOS} = \frac{E[FOS]-1}{\sigma_{FOS}} \tag{13}$$

If the FOS has lognormal distribution, then β_{FOS} becomes

$$\beta_{FOS} = \frac{\ln\left(\frac{E[FOS]}{\sqrt{1+V_{FOS}^2}}\right)}{\sqrt{\ln(1+V_{FOS}^2)}} \approx \frac{\ln(E[FOS])}{V_{FOS}} \tag{14}$$

Note that only when V_{FOS} is much smaller then 1, then the approximation of the β_{FOS} can be used.

3.3 Reliability index calculation

There are many methods available to determine reliability index, from vigorous integration method, if all variables statistic properties, including probability distributions are know, to approximate methods, such as Taylor's series method, point estimate method (PEM), and Monte Carlo simulation method, and those methods are often used in practices. Taylor's series method is based on expanding the performance function about its mean (expected value) according to Taylor's series theory, and expressing the performance function's mean and variance in terms of the expected values and variances of individual variables involved. The PEM is based on discretizing the variable probability distribution from its whole region into tow (or more) points along its own dimension with weighted probability concentration values, and then calculate the performance function's mean and variance by summarizing the points over all dimensions. The Monte Carlo simulation method, which first generates random variables according to their probability distributions, then simulates the performance function using those random variable points and the mean and variance of the performance function are calculated from the simulation outputs. Since the methods mentioned above are well established, therefore no detailed description and related mathematic expressions will be given in this paper.

4 APPLICATION OF RELIABILITY EVALUATION OF SLOPE STABILITY

4.1 Procedure of using reliability evaluation in slope stability analysis

To apply the reliability concept in slope stability, we should follow some basic steps:

1. Identify all variables (parameters) involved. Variables in slope stability analysis usually include slope geometry related parameters, soil properties, water table/phreatic surface and loading conditions.
2. Determine the uncertainties of the variables. Ideally, we would like to know the probability distribution of each variables and the correlation distribution among the variables, but very often, we only have limited data and do not know the exact probability distribution of the variables. Since according to central limit theorem, the sum of a sufficiently large number of independent random variables, each with finite mean and variance, will be approximately normally, we can reasonably assume the variables have either normal or lognormal distribution in engineering practice. Therefore all we need to know are some basic statistical properties, such as mean value, standard deviation and

coefficient of correlation. In practice, we should try our best to collect as many data as possible and use equations (1) to (6) to determine the statistic properties of the variables. When lack of information on some variable properties, such as standard deviation of variables, we can use some typical values given by many literatures, or use other practical methods, such as "three-sigma rule" (Dai and Wang (1992)) to have a rough estimate.

3. Choose slope stability analysis model. There are a few models available for slope stability analysis, such as Ordinary, Bishop, Janbu, Spencer, Morgenstern-Price and General Limit Equilibrium (GLE) methods. Since al method has its own advantages and limitations, either a best suitable method, or more than one method should be used in slope stability analysis.
4. Calculate the mean/expected value and standard deviation of slope stability analysis results, in term of factor of safety, by using any well established and available method as mentioned before.
5. Determine reliability index of FOS, or degree of confidence of slope stability analysis using equation (13).

During the analysis, not all parameters need to be treated as random variable if uncertainty and variability for those parameters is very small. For example, if the geometry of the slope is carefully measured, then we can treat the dimension of the slope as constant. It will also reduce the calculation effort if we know which variables have minor influence to the slope stability analysis result, and therefore to treat those variables as constant in our calculation as well.

It should be pointed out that in engineering slope stability analysis, the time dependency property of variables, such as soil permeability distribution, strain softening of soil shear strength, and slope progressive failure are usually not considered. Also, to simplify the analysis, 3-D effect and spatial correlation of variables are not considered although these factors can be important in some special cases.

4.2 Example of reliability evaluation of slope stability analysis

To illustrate how to perform reliability evaluation of slope stability analysis using the proposed procedure, a slope at a proposed U.S. nuclear power plant site is used as an example, and the analysis procedure and results are presented as the follows.

4.2.1 Slope description

One of the slopes at proposed new reactor site is a 10.7 m high, with 3 horizontal to 1vertical slope, and about 120 m long new-cut slope. The materials in the slope consist mostly of Zone IIA saprolites, a further stage of weathering beyond weathered rock with mainly silty clays and clays, the underlying Zone IIB saprolites are generally very dense silty sands on weathered rock. The thickness of Zone IIA is about 10.7 m and Zone IIB is about 3 m. The ground water level is about 8.5 m below the top of slope. The geometry of the slope is illustrated in Figure 2.

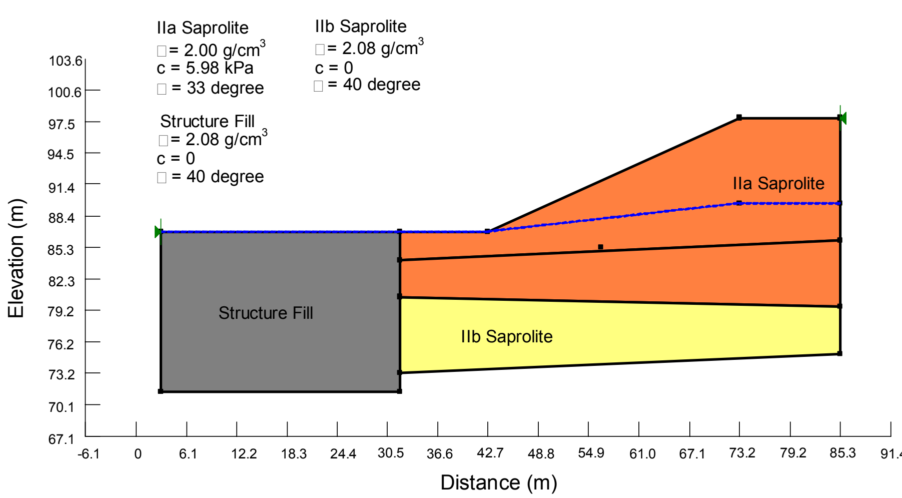

Figure 2. Cross section, material properties of a slope used in slope stability analysis

4.2.2 Input parameters and variable properties

In this slope stability analysis, we treat the geometry of the slope as constant based on field measurement. Also, since the maximum possible ground water level will be used in the analysis, this parameter is also treated as a constant. The soil properties, such as unit weight, γ, effective internal friction angle, ϕ, and cohesion, *c*, are based on field investigation data. The seismic loading was determined from site seismic response analysis in terms of maximum ground motion accelerations, a_h and a_v, denoted as horizontal and vertical seismic induced ground acceleration, respectively. The variable properties used in slope stability analysis are listed in Table 1. Note that the properties for structure fill are assumed based on the type of soil to be used. The variation of ground motion accelerations is unknown although the maximum seismic ground motion acceleration is result of probabilistic analysis, and its variation may have a wide range as one can imagine.

Table 1. Slope stability analysis input variable properties

	Variable	Mean Values	Coefficient of variation V (%)
Zone IIA Saprolite	*Unite weight* γ, g/cm^3 (pcf)	2.00 (125)	5
	Effective internal friction angle ϕ, degree	33	7
	Cohesion c, kPa (psf)	5.98 (125)	10
Zone IIB Saprolite	*Unite weight* γ, g/cm^3 (pcf)	2.08 (130)	5
	Effective internal friction angle ϕ, degree	40	7
	Cohesion c, kPa (psf)	0	
Structure Fill	*Unite weight* γ, g/cm^3 (pcf)	2.08 (130)	3*
	Effective internal friction angle ϕ, degree	40	5*
	Cohesion c, kPa (psf)	0	
Seismic Loading	Horizontal acceleration a_h, g	0.50**	Unknown
	Vertical acceleration a_v, g	0.25**	Unknown

* Assumed value.

** One-half of the peak ground acceleration value is used in seismic slope stability analysis.

4.2.3 Sensitivity study

As mentioned previously, there are quite a few slope stability analysis methods available. For the purpose of this study, we choose to use commercially available computer software GeoSlope to perform slope stability analysis for it containing various analysis models, and the ability of conduct probabilistic analysis.

Following the proposed procedure, sensitivity study was conducted to identify which variables have more influence on the slope stability. First, slope stability analysis was performed using mean values for all input parameters to determine the factor of safety of the slope stability, which is the standard deterministic analysis. Then series slope stability analyses were performed in such a way that in analysis runs, only one input parameter changed varying over certain range while other parameters unchanged at their mean value. Finally the analyses results were examined to see how the changes of each input variable affect the FOS. The deterministic method calculation results using different models are listed in Table 2 and some of the sensitivity study results are presented in Figure 3 to Figure 7.

Table 2. Factor of safety value comparison using different model

Model	Factor of Safety
Ordinary	1.114
Bishop	1.122
Janbu	1.049

General Limit Equilibrium (GEL)	1.128
Spencer	1.128
Morgenstern-Price	1.128

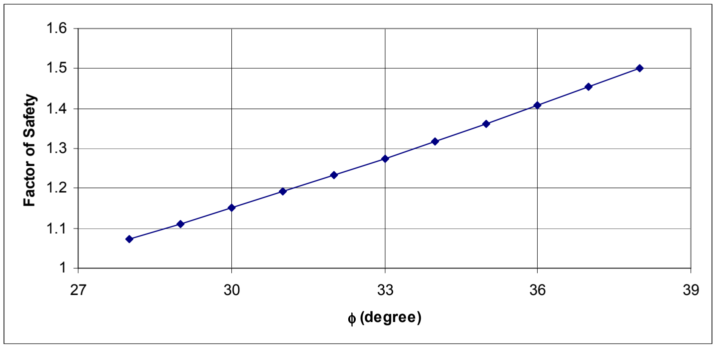

Figure 3. Effect of changing internal friction angle ϕ of zone IIa saprolite soil on factor of safety

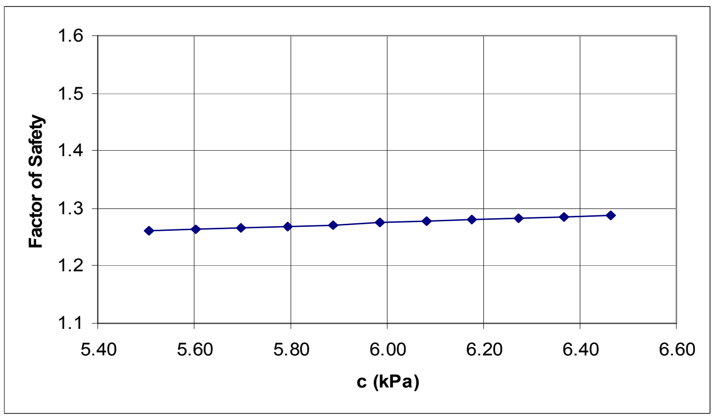

Figure 4. Effect of changing cohesion *c* of zone IIa saprolite soil on factor of safety

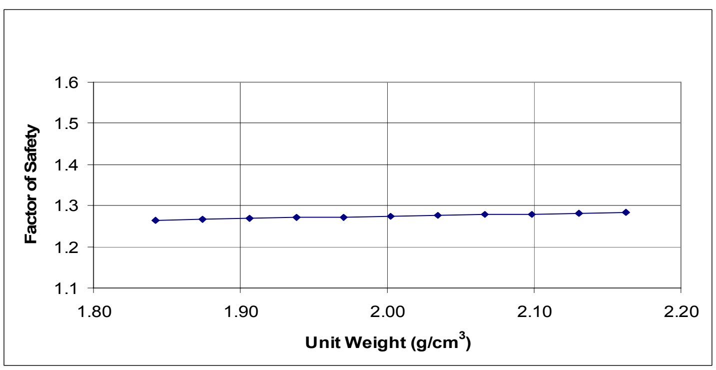

Figure 5. Effect of changing unit weight of zone IIa saprolite soil on factor of safety

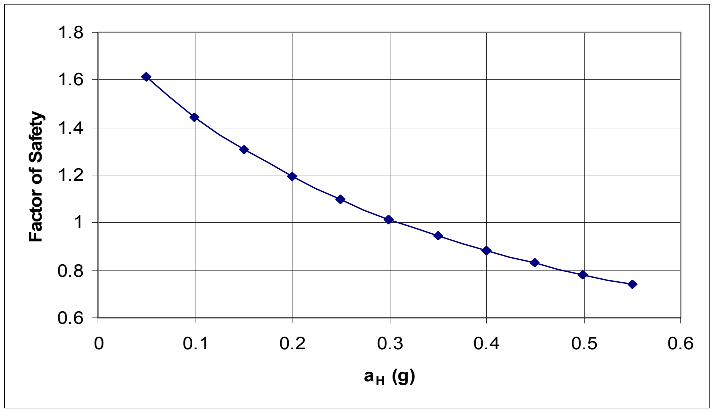

Figure 6. Effect of changing Horizontal Seismic Load on factor of safety

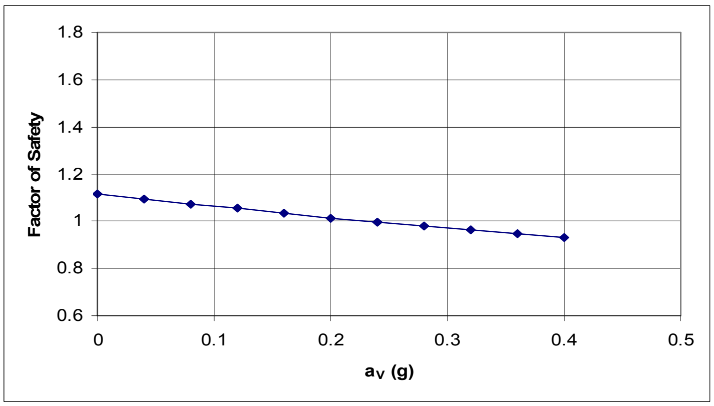

Figure 7. Effect of changing Vertical Seismic Load on factor of safety

The deterministic method calculations yielded very similar results – about 7 percent different among all methods, which indicated that for the given slope and loading conditions, there is not much difference between the factor of safety with respect to moment equilibrium, and to horizontal force equilibrium.

The sensitivity study results clearly show that among major input parameters, soil shear strength parameters, especially the internal friction angle of soil, and the horizontal component of seismic loading have greater impact on the slope stability. The analysis results showed that if the internal friction angle decreases from 33 degree to 30 degree, the FS will linearly reduce from 1.27 to 1.15; if the horizontal component of seismic load increase from 0.2g to 0.3g, then the FS will decrease from 1.19 to 1.0. The results indicate that (1) in order to get more realistic stability analysis results, we should pay more attention on determination of soil shear strength property so that the values are closer to field condition and with less variation; and (2) caution should be exercised with the seismic loading used in stability analysis, any change of the loading values should be justified.

4.2.4 Reliability analysis of slope stability

In reliability analysis, the variables with their statistic properties are listed in Table 1 and normal distributions for those variables are assumed with certain value ranges based on either field investigation data or engineering judgement.

The Monte Carlo simulation method was chosen in reliability analysis because this method is a build-in function in the GeoSlope program and can be easily used to perform probabilistic analysis. Since the variability of seismic loading is unknown, several standard deviation values were used in the analyses to evaluate its impact on the reliability index of the FOS of slope stability. For Monte Carlo simulation, 2000 simulation runs were performed for each individual analysis to ensure sufficient factor of safety date to be generated. The reliability analysis results are listed in Table 3.

Table 3. Reliability analysis results of slope stability analysis

Coefficient of variation of ground motion acceleration (%)	Mean of factor of safety	Standard deviation of factor of safety	Reliability index	Probability of failure (%)
10	1.131	0.068	1.924	0.05
20	1.141	0.112	1.263	6.15
30	1.147	0.154	0.954	17.30
40	1.154	0.189	0.815	21.30
50	1.152	0.235	0.647	28.65
100	1.115	0.317	0.363	43.50

The reliability analysis results show that (1) the variation of variables affects the expect value of factor of safety of slope stability but not significantly; (2) variation of variables affects the reliability of factor of safety more significantly. As the coefficient of variation of seismic ground motion acceleration increases from 10% to 100%, the reliability index decreases from 1.924 to 0.363; or the slope failure probability increases from 0.05% to 43.5%. Therefore, reducing variability of input parameters, the reliability of slope stability will increase; and (3) even assume there is 16% of chance (V =1.0) that the ground motion acceleration will be double or more from its mean value, the probability of slope failure is still under 50%.

5 SUMMARY AND CONCLUSIONS

This study provides a simple and practical reliability analysis method for slope stability analysis. This study and example slope stability analysis results showed that:

1. Since uncertainties and variation exist in soil properties and their parameters determination processes, no absolute single value involved in slope stability analysis can truly represent actual site condition. Therefore commonly used analysis methods generally can only provide an estimate with certain degree of confidence. In current engineering practice, requirements of FOS greater than unity, and use of conservative estimates of soil property parameters in analysis when using deterministic method are necessary because they actually take those uncertainties and variation into consideration. The reliability analysis method is a good supplement to the conventional deterministic method for providing confidence measure of the analysis results.

2. The sensitivity study results show that among major input parameters, soil shear strength parameters, especially the internal friction angle of soil, and the horizontal component of seismic loading have greater impact on the slope stability. In order to get more realistic stability analysis results, we should pay more attention on determination of soil shear strength property during site investigation so that the related parameter values are closer to field condition and with less variation.

3. Degree of uncertainties of parameters involved in the slope stability analysis affects the reliability of factor of safety. Generally, the smaller the variation is, the higher the confidence level we can get. As illustrated in this study, for the same mean values of seismic loading, when decreasing the degree of uncertainty, or reduces the value of coefficient of variation from 1.0 to 0.1, the confidence level, or probability of FOS greater than 1.0 increases from about 56% to 99%. Therefore, it is desirable to reduce the variability of all parameters involved in slope stability analysis, which can be achieved by conduct sufficient field and laboratory tests, improving quality of all procedures involved in parameter determination, such as sample collection, storage and transportation, laboratory testing, data collection and analysis.

4. Since the seismic loading used in stability analyses are based on the probabilistic analysis results of the site-specific maximum ground acceleration, which involves the uncertainties of the earthquake occurrence frequencies for different magnitudes from different sources, the

uncertainties of site subsurface materials, as well as the uncertainties of the analysis method itself, therefore this parameter is a probabilistic variable. Because this parameter has great influence on slope stability analysis result, the probabilistic property of seismic ground motion acceleration needs to be better understood and adequate values should be used in analysis. Caution should be exercised with the seismic loading used in stability analysis; any change of the loading values should be justified.

5. The proposed reliability method can be used in slope stability analysis in engineering practice, which can provide more confidence to the analysis results together with the conventional deterministic analysis. The reliability analysis concept should be used to other geotechnical engineering analysis so that the uncertainties can be better accounted for and engineers can have more confidence on their design and analyses.

Acknowledgements. *The authors would like to express their great appreciation to Dr. Nilesh C. Choshi for his encouragement and technical advices for this study.*

REFERENCES

Benjamin, J. R., and Cornell, C. A. 1970. Probability, statistics, and decision for civil engineers, McGraw-Hill, New York.

Christian. J. T. , Ladd, C. C., and Baecher, G. B. 1994. Relaibility applied to slope stability analysis. Jounal of geotechnical engineering, ASCE. Vol. 120:12. P 2180-2207.

Dai, S-H., and Wang, M-O. 1992. Reliability analysis in engineering applications. Van Nostrand Reinhold, New York.

Duncan, J. M. 2000. Factors of safety and reliability in geotechnical engineeing. Jounal of geotechnical and geoenvironmental engineering, ASCE. Vol. 126:4. P 307-316.

Harr, M. E. 1987. Reliability-based design in civil engineering. McGraw-Hill, New York.

Oka, Y. And Wu, T.H. 1990. System reliability of slope stability. Jounal of geotechnical engineering, ASCE. Vol. 116:8. P 1185-1189.

U.S. Army Corps of Engineers 1997. Engineering and design introduction to probalility and reliability methods for use in geotechnical engineering. Engineering technical letter No. 1110-2-547, 30 September 1997.

U.S. Army Corps of Engineers 1999. Risk-based analysis in geotechnical engineering for support of planning studies. Engineering technical letter No. 1110-2-556, 28 May 1999.

Vanmarcke, E.H. 1977. Reliability of earth slopes. Jounal of geotechnical engineering, ASCE. Vol. 103:11. P 1227-1246.

Wolff, T. F., and Wang, W. 1993. Reliability analysis of navigation structures in geotechnical practice in dam rehabilitation. Proceedings of the specialty conference sponsored by the Geotechnical Engineering Division, ASCE, Raleigh, North Carolina, April 25-28, 1993, ASCE Geotechnical Specialty Publication No. 35, P 159-173.

Wolff, T. F. 1994. Evaluation the reliability of existing levees. Report prepared for U.S. Army Engineer Waterways Experiment Station, Geotechnical Laboratory, Vicksburg, MS, September 1994. This is also Appendix B to U.S. Army Corps of Engineers ETL 1110-2-556, Risk-Based Analysis in Geotechnical Engineering for Support of Planning Studies, 28 May 1999.

