HEAT AND MASS TRANSFER DOWNSTREAM OF ABRUPT NOZZLE EXPANSIONS IN TURBULENT FLOW

By D. J. TAGG, M. A. PATRICK (MEMBER) and A. A. WRAGG
Department of Chemical Engineering, University of Exeter

Axial local wall mass transfer distributions have been explored downstream of the nozzle plane for circular jets injected into axisymmetric circular ducts using the limiting electrolysis diffusion current technique. By using the Chilton–Colburn analogy the data for peak mass transfer rate were shown to be correlated along with independent heat transfer data by the equation $Sh = 0.27Re^{0.41} Sc^{0.32}$ thus demonstrating the usefulness of the technique in the modelling of recirculating flow heat transfer situations. Additionally, investigations of the relationship between mass transfer distribution and recirculation zone hydrodynamics have been made.

INTRODUCTION

Significant enhancement of heat and mass transfer rates generally occurs when fluids are transported through sudden enlargements in pipe or duct cross-sections. The increased transfer rates are associated with a region of flow separation and recirculation extending a few duct diameters downstream of the plane of enlargement.

The above situation arises frequently in engineering practice, notably in the process industries, and is of special importance in the design and operation of high throughput combustion systems. In this case the total heat-transfer is by a combination of radiation and convection and in high velocity separated flows the contribution of forced convection can become appreciable. An important mass transfer application is found in the design of entry sections for electrochemical reactors.

PREVIOUS WORK

There have been a number of investigations of heat and mass transfer in axisymmetric pipe expansions in recent years. Ede et al. made a comprehensive study of heat transfer to water under conditions of constant heat flux with the upstream or the downstream duct surfaces, or both, as active transfer area. Other workers have made similar measurements using air or water as the working fluid but with only the downstream section heated. Krall and Sparrow used orifice nozzles to provide flow separation and worked over a range of equivalent large to small duct diameters ($D/d$) of 1.5 to 4. As an alternative to these studies employing electrical heating, Louise used hot air together with water-cooled calorimeters along the duct wall, so approaching the constant temperature situation.

Mass transfer measurements for flow through axisymmetric sudden expansions have employed two main techniques. Read made measurements, for large diameter ratios, using the sublimation of naphthalene in air, while other workers have used the more accurate electrochemical “limiting diffusion current technique” (LDCT). Costello made use of the copper deposition reaction for two expansion ratios of 3:4 and 3:8. The various large cathodes used yielded only indirect information on the magnitude of local transfer rates. Runchalo employed the cathodic reduction of ferricyanide ions on nickel for an expansion ratio of 1:2 over a range of Schmidt number from 1400 to 2500. Although the measurements were made with small “local” electrodes, the active cathode area within which these were embedded only covered a part of the recirculation zone and thus there is some doubt as to the state of development of the mass transfer boundary layer in the region within which the results were obtained.

It should be noted that, in these previous studies, the abrupt changes of section have sometimes been formed using contoured nozzles, while in other cases simple pipe expansions have been employed. Detailed differences in results between these situations may be expected due to the different velocity profiles and turbulence structures of the flow in the plane of expansion.

The present study has endeavoured to produce well-characterised conditions for the flow into the expansion section by the use of standard nozzle shapes which give flat inlet velocity profiles and fairly low initial turbulence levels. The LDCT has been employed to provide true local measurements of mass transfer within and near the recirculation region of the axisymmetric sudden expansion over a wide range of expansion ratios. However, a detailed description of the influence of initial flow structure on the flow development for a series of different inlet geometries is outside the scope of this work and will be left for a further investigation.

EXPERIMENTAL

The use of the limiting diffusion current technique of mass transfer measurement in a variety of flow situations is well documented. The present investigation employed the cathodic reduction of ferricyanide ions at nickel electrodes. The electrolyte was made up to 0.5 M...
in NaOH with an equimolar K$_3$Fe(CN)$_6$/K$_4$Fe(CN)$_6$ mixture of concentration 0.005 M. Under limiting current conditions the mass-transfer coefficient was given by the simple expression

$$K = \frac{i_d}{zFAC_b}$$  \hspace{1cm} (1)

Figure 1 shows the main electrode assembly which was manufactured from a 50 cm long piece of 5.22 cm i.d. nickel tube split longitudinally to provide separate anode and cathode portions with an area ratio of approximately 2:1. The smaller segment forming the cathode was electrically insulated from the anode by means of a neoprene gasket and the whole arrangement carefully clamped together to form the downstream portion of an axisymmetric sudden expansion.

One theoretical disadvantage of this arrangement is that the system is not symmetrical about the centre line since mass transfer boundary layers of different characteristics will develop on the anode and cathode surfaces. However, due to the very thin layers obtained with the high Schmidt numbers involved, interaction with the main flow will be small and, in the absence of swirl, errors due to disturbances at the cathode/anode boundary will be negligible.

The cathode segment was fitted with "point" electrodes at intervals of 12.2 mm and in a staggered formation. These were formed by epoxy-cementing 1 mm diameter nickel wires into 1.4 mm diameter holes drilled at intervals along the length of the cathode and machining the protrusions flush with the inside wall. This method provided sufficient thickness of electrical insulation to give local mass transfer values within a large active area without significantly disrupting the developed concentration layer on the main electrode.

The abrupt expansion was formed by inserting, into the main tube, a perspex nozzle block with the nozzle contours shaped to a standard ASME long-radius profile. A series of interchangeable nozzle inserts gave expansion ratios of 2, 3, 4, 6 and 10. As shown in Figure 2, electrolyte was supplied to the test section from a reservoir using a centrifugal pump which circulated the fluid through a rotameter bank and control valve system to the test section and back to the reservoir.

Figure 3. Instrumentation.

The simple electrical circuit shown in Figure 3 was used to obtain the values of limiting current corresponding to the particular mass transfer rate for a given flow condition. Periodic checks were made to test the validity of the assumption of diffusion control but for all runs limiting currents were obtained with the applied potential between anode and cathode at 0.8 V. In obtaining local mass transfer rates, the main cathode surface was held at the same potential as the "point" electrodes, relative to the anode, using a stabilised power supply. The fluctuating current from a given small electrode was converted to a fluctuating voltage using a simple solid-state amplifier. This voltage was then time-averaged using a resistance capacitance network and the result read on a multi-range voltmeter.

Throughout the series of experiments the temperature was maintained at 20 ± 1 °C for which the corresponding physical properties of the electrolyte were

$$\rho = 1.02 \text{ g cm}^{-3}, \quad \mu = 1.05 \times 10^{-2} \text{ g cm}^{-1}s^{-1},$$

$$\nu = 7.09 \times 10^{-5} \text{ cm}^2s^{-1}$$

giving Sc ≈ 1450. The Reynolds number, based on the downstream duct diameter $D$, was varied over the range 1900 to 23 000. Prior to each set of runs, the electrode surfaces were polished with progressively finer grades of emery paper and finally degreased with carbon tetrachloride. It has been suggested that this provides an equivalent pre-treatment to that of adjusting the potential to give hydrogen evolution at the surface. In the present work, no noticeable influence on the results was found when using electrodes activated in either of these ways.

A further electrochemical experiment was carried out in order to investigate the length of the recirculation zone. This involved the use of a special "sandwich"
wall electrode (Figure 4) similar to that of Son & Hanratty\(^5\) which could be positioned at various distances downstream of the plane of enlargement by means of a sliding tube arrangement. The sandwich electrode was sensitive to the flow direction adjacent to the wall in that the "upstream" electrode of the pair always gave the higher reading, thus clearly indicating the mean local flow direction. The reattachment point was indicated when the differential output of the electrode pair dropped to zero. In addition, a 1 mm diameter nickel wire was set into the wall in the manner described previously at an identical downstream distance to the sandwich electrode. Operating this electrode as a cathode under diffusion control conditions, it was possible to investigate the axial distribution of mass transfer to a small active zone surrounded by an inert surface. In this case, the growth of the concentration boundary layer starts at the leading edge of the test electrode and due to its small size this does not result in the establishment of a fully developed profile. Runchal\(^9\) has reported that the mass transfer distribution is influenced strongly by the boundary conditions, i.e. whether the electrode is surrounded by an active or an inert surface.

**RESULTS AND DISCUSSION**

**Local Mass Transfer Profiles**

Figures 5 and 6 show examples of the detailed variation of mass transfer coefficient along the duct wall downstream of the plane of enlargement for the 1 : 3 and 1 : 10 expansion ratios. Included on each of the graphs, for comparison purposes, are asymptotic values for fully developed mass transfer in turbulent pipe-flow as predicted by the Dittus-Boelter equation:

\[
Sh = 0.023Re^{0.8}D^{0.33}
\]

(2)

For all the expansions tested, the mass transfer rate increased to a maximum value within a few duct diameters of the plane of enlargement and then decayed slowly towards the predicted fully developed value. These mass transfer profiles are very similar to those obtained by other workers\(^5,6,7\) and confirm that the region prior to the peak in the transfer rate is also subject to mass transfer in excess of that expected for fully developed flow.
Peak Transfer Rates

In Figure 7, the ratio of the measured peak Stanton number to the fully developed value is plotted as a function of Reynolds number for each nozzle arrangement. This illustrates the increase in mass transfer rate at constant Reynolds number $Re_D$ for increasing expansion ratio corresponding to the higher jet velocities involved. It can also be seen that the relative mass transfer enhancement increases with decrease in Reynolds number.

The good agreement between these two sets of data for such widely differing fluid property numbers illustrates the applicability of the LDCT approach to heat-transfer modelling. However, it is interesting to note that if the present data are plotted alone they are better fitted by a 0.6 power on $Re_N$, this value agreeing with that suggested by Spalding for separated flows.

Relation of Peak Transfer Position to Recirculation Zone Length

The positions at which the peak transfer rates occur in Figures 5, 6, and the rest of the data are in good agreement with those determined by Runchal and by Krall and Sparrow, though somewhat shorter than those obtained by Ede, where the nozzle to peak distance was approximately equal to the product of the expansion ratio and downstream diameter. In the present work the location of the peak can be seen to remain sensibly constant for the higher expansion ratios at a given Reynolds number and to drift upstream as the Reynolds number is increased.

Figure 8 shows the present results and those of Krall and Sparrow for heat transfer plotted as peak Sherwood/Nusselt number against the nozzle Reynolds number. A one-third power on the Schmidt/Prandtl number is seen to reduce both sets of data to a single line, in accord with the Chilton-Colburn analogy, the equation being

$$Sh = 0.27Re_N^{0.67} \cdot Sc^{0.33} \quad (3)$$

A typical output from the sandwich electrode for the 1:3 expansion is given as Figure 9. Results of the determination of the length of the recirculation zone from such readings are shown in Figure 10 for the range of expansion ratios and a Reynolds-number dependency can be seen to exist. The recirculation zone length is markedly greater than the distance from nozzle to peak transfer position which is also indicated on Figure 10. Examination of Figure 9 reveals a further interesting feature, namely the existence of a second stagnation point closer to the plane of enlargement. This provides firm evidence for the existence of a secondary, smaller recirculation zone immediately downstream of the enlargement as has been suggested by Abbot and Klein as a result of flow visualisation studies on a two-dimensional step. A single mass transfer profile obtained with the local electrode surrounded by inert surface has been included for comparison in Figure 11.
much greater. Asymptotic conditions are reached very much sooner after the peak with the inert surroundings, but the shapes of the profiles prior to this are similar enough to refute the suggestion of Runchal that the boundary conditions strongly affect the location of the peak (at least for high values of $Sc$).

The results of the determination of recirculation zone length and the position of the mass transfer peak for both sets of boundary conditions are fully presented in the Table. The slightly greater distance between the nozzle and the peak transfer position in the developed mass transfer case may be a result of unavoidable slight differences in the cell geometry and inlet conditions for the two experiments. From this Table, however, it is very clear that for all expansion ratios and Reynolds numbers the peak transfer position does not coincide with the end of the recirculation zone, as was widely suggested in earlier work, but lies well within it.

### Table. Tabulation of peak transfer positions and recirculation zone lengths

<table>
<thead>
<tr>
<th>$D/d$</th>
<th>$Re_D$</th>
<th>Recirculation zone $x/D$</th>
<th>Peak point mass transfer $x/D$</th>
<th>Peak developed mass transfer $x/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1984</td>
<td>2.50</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>3858</td>
<td>2.875</td>
<td>1.375</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>6270</td>
<td>3.0—3.125</td>
<td>1.375</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>10126</td>
<td>3.0</td>
<td>1.50</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>15912</td>
<td>2.875—3.00</td>
<td>1.50</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>22180</td>
<td>2.875</td>
<td>1.50</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>1984</td>
<td>3.75</td>
<td>2.0</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>3858</td>
<td>3.75</td>
<td>2.0</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>6270</td>
<td>3.625</td>
<td>2.0</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>10126</td>
<td>3.3</td>
<td>2.0</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>15430</td>
<td>3.25—3.375</td>
<td>1.875</td>
<td>2.125</td>
</tr>
<tr>
<td></td>
<td>21340</td>
<td>3.25—3.375</td>
<td>1.875</td>
<td>2.125</td>
</tr>
<tr>
<td>4</td>
<td>1984</td>
<td>3.875—4.10</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>3858</td>
<td>3.875</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>6269</td>
<td>3.75—3.875</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>10125</td>
<td>3.75</td>
<td>2.125</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>14950</td>
<td>3.625</td>
<td>2.0</td>
<td>2.375</td>
</tr>
<tr>
<td></td>
<td>20010</td>
<td>3.5</td>
<td>1.875</td>
<td>2.375</td>
</tr>
<tr>
<td>6</td>
<td>1984</td>
<td>4.0—4.125</td>
<td>2.375</td>
<td>2.625</td>
</tr>
<tr>
<td></td>
<td>3858</td>
<td>4.0</td>
<td>2.375</td>
<td>2.625</td>
</tr>
<tr>
<td></td>
<td>6269</td>
<td>3.75—3.875</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>10125</td>
<td>3.75</td>
<td>2.125</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>15430</td>
<td>3.3</td>
<td>2.0</td>
<td>2.25</td>
</tr>
<tr>
<td>10</td>
<td>3375</td>
<td>3.75—3.875</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>4940</td>
<td>3.75</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>6270</td>
<td>3.75</td>
<td>2.125</td>
<td>—</td>
</tr>
</tbody>
</table>

Furthermore, recent numerical modelling work$^{20,21}$ shortly to be published in fuller form$^{22,23}$ confirms this and demonstrates the relationship of the peak transfer position to the flow structure, notably the turbulence intensity near the wall.

### CONCLUSIONS

A comprehensive set of mass transfer distribution data has been obtained for a range of sudden expansions.
Comparison with independent heat transfer data has confirmed the usefulness of the Chilton-Colburn analogy for this flow situation. The location of the peak wall mass transfer rate corresponds neither to the “eye” of the recirculation zone (i.e. the centre of the recirculation vortex) nor to the point of flow reattachment.

SYMBOLS USED

- \( A \) electrode area (cm\(^2\))
- \( c_b \) bulk ferricyanide ion concentration (mol dm\(^{-3}\))
- \( d \) nozzle diameter, upstream (cm)
- \( D \) duct diameter, downstream (cm)
- \( \Phi \) diffusion coefficient (cm\(^2\) s\(^{-1}\))
- \( F \) Faraday number (C mol\(^{-1}\))
- \( i_l \) limiting electrolysis current (A)
- \( K \) mass transfer coefficient (cm s\(^{-1}\))
- \( V \) mean fluid velocity (cm s\(^{-1}\))
- \( x \) distance downstream from enlargement (cm)
- \( z \) number of electrons exchanged (-)
- \( \mu \) fluid viscosity (g s\(^{-1}\) cm\(^{-1}\))
- \( \rho \) fluid density (g cm\(^{-3}\))

Dimensionless groups

\[
Re_N = \frac{dV\rho}{\mu} \quad Re_D = \frac{DV\rho}{\mu} \\
St = \frac{K}{V} \quad Sh = \frac{KD}{\Phi} \quad Sc = \frac{\mu}{\rho\Phi}
\]

REFERENCES


ADDRESS

Correspondence in connection with this paper should be addressed to Dr M. A. Patrick, Department of Chemical Engineering, University of Exeter, North Park Road, Exeter EX4 4QF.

The manuscript of this paper was received 3 July 1978. The revised manuscript was received 15 January 1979.