NUREG/CR-2489

Evaluation of the Buckling Stress Criteria for the Steel Containment of the Watts Bar Nuclear Reactor

Prepared by P. Seiede, V. Weingarten, S. Masri

International Structural Engineers

Prepared for
U.S. Nuclear Regulatory
Commission

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Evaluation of the Buckling Stress Criteria for the Steel Containment of the Watts Bar Nuclear Reactor

Manuscript Completed: June 1981
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Prepared for
Division of Engineering
Office of Nuclear Reactor Regulation
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
NRC FIN B6581

PREFACE

This report was prepared under Contract NRC-03-79-124, and completes our evaluation of the buckling stress criteria for the steel containment of the Watts Bar Nuclear Reactor.

Technical (and COAR) monitor for the Nuclear Regulatory Commission initially was A. Hafiz, who was subsequently changed to S. B. Kim. The Project Manager for ISE was B. Mossberg. The evaluation has been done by Drs. P. Seide, V. Weingarten and S. F. Masri

FINAL REPORT

6/30/81

Upon reviewing the Watts Bar Nuclear Plant Units 1 and 2 Containment Vessel Building analysis and the response to the questions sent to the applicant (see attached appendicies), we have reached the following conclusions:

1. The applicant's analysis and design methods appear to be in accordance with prevailing industry practices.

2. The present state of the art of static and dynamic structural analysis and design allows the applicant to perform a more accurate and refined analysis than was used by the applicant.

3. In the absence of these more accurate analyses, there still remain areas of uncertainty regarding the adequacy of the applicant's design. This does not necessarily imply that the present design is unsafe but only that a more accurate analysis is required to quantify the margin of safety of the design.

4. The more accurate analysis performed by the applicant should include attention to the questions still of concern to the contractor:

a) The behavior of the shell in the vicinity of the penetrations does not appear to have been modeled accurately in both the dynamic and buckling analysis. Thus, the effect of stress concentrations near the openings and the adequacy of the stiffening around the openings are uncertain. There is virtually nothing in the literature on the amount of stiffening required to nullify the opening from a buckling point of view. The present analysis therefore assumes that the stiffening has the desired result without any verification.

b) For the shell containing penetrations, it appears that no checks for convergence or accuracy were carried out for the stresses in the area of the penetration. Since the stresses around the penetration may trigger buckling, the solution accuracy should be investigated.

c) By doing a separate analysis for the supported equipment, the effect of the equipment on the containment shell is neglected. Thus, the effects of interaction between the motion of the shell and its attachments are not properly handled. Recent studies in the publishers literature (PO, ESSA, April 1979) have shown that significant interaction can develop under seismic excitation even if the mass ratio of the equipment is on the order of one (1) percent of the main structure.

5. The use of any general purpose computer code such as ANSYS, NASTRAN or MARC would provide the more accurate analysis procedures needed for the verification of containment design. The use of any of these programs would remove the need for many of the assumptions made in the applicant's analysis.

For International Structural Engineers

Dr. P. Seide
Dr. V. Weingarten
Dr. S. Masri

APPENDICES

A. TVA's 4/14/81 responses to ISE's second group of questions

B. ISE's 10/27/80 review of TVA's first responses and more questions

C. TVA's 8/15/80 response to ISE's first group of questions

D. ISE's 4/9/79 first group of questions to TVA

E. WBNP 2/23/79 buckling stress criteria supplied by TVA

SUPPLEMENT

Applicant's Response To The ISE Report

Appendix A

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

APR 3 1981

Mr. Bengt Mossberg
International Structural Engineers
P. O. Box 9595
Glendale, California 91206

Dear Mr. Mossberg:

Reference: NRC Contract #03-79-124

Enclosed please find response from the applicant (TVA) to the questions in your October 27, 1980 letter. Please call me if there is any questions.

Sincerely yours,

Sang Bo Kim

Enclosure: As stated

TVA

TENNESSEE VALLEY AUTHORITY

400 Chestnut Street Tower II

February 19, 1981

RECEIVED MAR 1 1 1981 U.S. NUCLEAR REGULATORY COMMISSION

Director of Nuclear Reactor Regulation
Attention: Mr. A. Schwencer, Chief
Licensing Branch No. 2
Division of Licensing
U.S. Nuclear Regulatory Commission
Washington, DC 20555

Dear Mr. Schwencer:

In the Matter of the Application of) Docket Nos. 50-390
Tennessee Valley Authority) 50-391

In a letter dated December 5, 1980, from R. L. Tedesco to me, TVA was requested to provide additional clarification with respect to the Watts Bar Nuclear Plant Containment Vessel Building Analysis. Enclosed is the requested information.

Very truly yours,

TENNESSEE VALLEY AUTHORITY

L. M. Mills

L. M. Mills, Manager
Nuclear Regulation and Safety

Sworn to and subscribed before me
this 19th day of Feb. 1981

Bryant M. Lowery
Notary Public
My Commission Expires 4/4/82

Enclosure

B001
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1/1
SEND Drawings to:
BC (After filming)
Aperture Dist.

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8103120544

ENCLOSURE

WATTS BAR NUCLEAR PLANT UNITS 1 AND 2
CONTAINMENT VESSEL BUILDING ANALYSIS

Introduction

Before proceeding with the responses to the revised Nuclear Regulatory Commission questions, additional design and analysis data on the containment vessel are presented to demonstrate the adequacy of the design. An overview of your comments to our responses indicates that your concern is primarily with the shell stability around major penetrations and the accuracy of decoupling the dynamic response of the locks and hatch from the primary structure under pressure transient and seismic excitation.

The Watts Bar containment is unstiffened longitudinally with the exception of the span between elevations 703'-9-3/8" and 716'-7-3/8" and around major penetrations. In the area of these penetrations, the containment shell is heavily reinforced with 1-3/8-inch by 22-inch stiffeners as shown on the attached Chicago Bridge and Iron (CBI) drawings 46 and 213 with supporting drawings 43, 55, 83, 84, 85, 89, 91, 92, 96, 99, 200, 202, and 209 to define stiffener sizes.

Table 1 (copy attached) gives a summary of maximum stress intensities in the shell around the major penetrations resulting from the most severe load combination. The column labeled "inertial stress" is the stress in the shell from the decoupled analysis of the locks and equipment hatch. The column labeled "initial stress" is the stress in the area of the penetrations from the combined affects of design basis accident (pressure transient), seismic, and dead loads. Note that even if the inertial stresses are multiplied by a factor of 2, the total stress intensity would be less than the ASME code allowables.

Apart from their role in carrying the general membrane stress around the penetrations, the local stiffening around the personnel locks and equipment hatch are very effective in suppressing the adverse effect of local buckling. This is especially true if the mesh of the reinforcement is smaller than the minimum local buckle region. Furthermore, the stiffening reduces the shell stresses in the panels between stiffeners whereas the allowable buckling stress for the panel is higher than for an unstiffened cylinder. The minimum allowable buckling stress for axial compression for the panels containing the equipment hatch and personnel locks is 14.9 kips per square inch (k/in^2) compared to an equivalent allowable buckling stress of 9.7 k/in^2 in areas of the shell without vertical stiffeners.

Table 1 is a list of the stress intensities in the containment shell in the area of the locks and hatch. These stress intensities are calculated from the maximum tension and compressive stresses and are less than the stress intensities in the general shell. The maximum compressive stresses in the area of the locks and hatch were not tabulated, but due to the additional stiffening, they will be less than the compressive stress in the general shell.

Figure 6 (copy attached) of the response to the original questions shows that the buckling ratios were less than 1 for all areas of the general shell. The additional membrane compressive stresses in the area of the locks and hatch due to the local response of the locks and hatch will be less than or equal to the maximum membrane (P_m) "inertial stress" of 1.6 k/in ^{2} shown in table 1. Even if this maximum compressive stress is multiplied by a factor of 2, it will still be much less than the incremental allowable buckling stress of 5.2 k/in^2 (14.9 - 9.7) between the local area around the locks and hatch and the general shell. Therefore, the buckling ratio of the shell in the area of the locks and hatch is less than 1 and this area of the containment meets the specification buckling criteria.

Question 2:

Provide a description of how the buckling curves contained in the report were applied to the buckling of the containment vessel. The description should include the application of these buckling curves to asymmetric dynamic loads in the areas where penetrations are present.

Review of answer 2:

We are still concerned for the reliability of the buckling analysis. The behavior of the shell in the vicinity of the penetrations does not appear to have been modeled accurately in both the dynamic and buckling analysis. Thus, the effect of stress concentrations near the openings and the adequacy of the stiffening around the opening are uncertain. There is virtually nothing in the literature on the amount of stiffening required to nullify the opening from a buckling point of view. The present analysis therefore assumes that the stiffening has the desired result without any verification.

Response:

The circumferential stiffeners on the Watts Bar containment are designed to have sufficient stiffnesses to enforce nodes at the circumferential stiffeners so as to preclude a general instability mode of buckling failure. Vertical and additional circumferential stiffening was designed (see CBI drawings 46 and 213) so as not to compromise the areas around the major penetrations from a stress intensity or buckling viewpoint. The area replacement of the opening, according to ASME code, section III, subsection NE, combined with the special stiffening essentially nullifies the effects of the opening. Refer to the introduction for further discussion of buckling relating to the locks and hatch.

Question 4:

Provide a description of the assumptions involved in modeling the containment vessel in order to use the programs identified in question 3. This description should include a discussion of any convergence and/or accuracy checks that were made.

Review of answer 4:

For the axisymmetric shell, the convergence check is acceptable. However, for the shell containing penetrations, it appears that no checks for convergence or accuracy were carried out for the stresses in the area of the penetration. Since the stresses around the penetration may trigger buckling, the solution accuracy should be investigated.

Response:

As described in the introduction, the shell stresses in the areas of the locks and hatch have been evaluated and meet the specifications and ASME code criteria.

Question 6:

Explain the procedure of obtaining the stress distribution in the shell using lumped mass beam model instead of a shell model for the dynamic seismic analysis.

Review of answer 6:

The use of the Timoshenko shear beam as an analog for a perfect (without penetrations) shell of revolution is acceptable; however, for the containment vessel under discussion there is no documentation or justification that this simplistic approach is applicable and that it will not suppress shell modes in the real structure that will be excited by seismic ground motion.

Response:

The large masses attached to the containment vessel are the two personnel locks and the equipment hatch. In the area of these penetrations, the containment shell is heavily reinforced with 1-3/8-inch by 22-inch stiffeners as shown on attached CBI drawings 46 and 213. These stiffeners restrict the response of the shell to dynamic movements of the locks and hatch. Therefore, these dynamic motions of the locks and hatch will be very local and not be associated with significant shell modes of the structure. Under these conditions, the shell modes of the real structure will not be significantly different from the shell modes of an axisymmetric model and will not be significantly excited by seismic loads.

Question 7:

Explain the justification for using an axisymmetric geometry computer program for the containment vessel.

Review of answer 7:

Although the answer to question 7 states that the approach to analyzing the locks and hatches as a supported subsystem was used for the dynamic analysis of nuclear plants, it does not address the question of the accuracy of this approach for dynamic buckling analysis. Further justification of this approach is needed.

Response:

As shown on attached CBI drawings 46 and 213, the areas around the personnel locks and equipment hatch are heavily reinforced with 1-3/8-inch by 22-inch stiffeners. These stiffeners in conjunction with the 3-inch thick nozzle for the equipment hatch and the 2-inch thick nozzles for the personnel locks will preclude buckling of the containment shell in these areas.

Question 9:

Explain in detail the criteria and its justification for determining the interaction effects between the containment shell and the attached equipment.

Review of answer 9:

By doing a separate analysis for the supported equipment, the effect of the equipment on the containment shell is neglected. Thus, the effects of interaction between the motion of the shell and its attachments are not properly handled. Recent studies in the published literature (PO, BSSA, April 1979) have shown that significant interaction can develop under seismic excitation even if the mass ratio of the equipment is on the order of 1 percent of the main structure.

Response:

The study in the published literature reference (PO, BSSA, April 1979) shows a maximum error on the nonconservative side of 94 percent when an uncoupled analysis is performed in lieu of a coupled analysis. It was shown in the introduction that the stresses from the uncoupled analysis can be increased by 100 percent without exceeding specifications and ASME code allowables for stress intensity and buckling. This further substantiates the adequacy of the Watts Bar containment design.

TABLE 1
WATTS BAR NUCLEAR PLANT
MAXIMUM STRESS INTENSITY AT MAJOR PENETRATIONS

Description	Membrane (P_m) Surface ($P_m + P_b + Q$)	Location	Inertial Stress in Shell	Initial Stress in Shell	Total Stress	Allowable Stress
Upper personnel lock	P_m	At barrel - shell intersection (Point 7)	1.60 k/in^2	3.44 k/in^2	5.04 k/in^2	1.5 S_m 26.25 k/in^2
	$P_m + P_b + Q$		18.9 k/in^2	3.44 k/in^2	22.34 k/in^2	3.05 S_m 52.5 k/in^2
	P_m	At 0.5 Rt (Point 4)	1.03 k/in^2	13.07 k/in^2	14.10 k/in^2	1.1 S_m 19.25 k/in^2
	$P_m + P_b + Q$		2.7 k/in^2	13.07 k/in^2	15.77 k/in^2	3.0 S_m 52.5 k/in^2
Lower personnel lock	P_m	At barrel - shell intersection (Point 6)	0.66 k/in^2	16.44 k/in^2	17.10 k/in^2	1.5 S_m 26.25 k/in^2
	$P_m + P_b + Q$		16.6 k/in^2	16.44 k/in^2	33.04 k/in^2	3.0 S_m 52.5 k/in^2
	P_m	At 0.5 Rt (Point 2)	0.38 k/in^2	16.53 k/in^2	16.91 k/in^2	1.1 S_m 19.25 k/in^2
	$P_m + P_b + Q$		11.10 k/in^2	16.53 k/in^2	27.63 k/in^2	3.0 S_m 52.5 k/in^2
Equipment hatch	P_m	At insert barrel - shell intersection (Points 7 and 8)	0.66 k/in^2	15.04 k/in^2	15.70 k/in^2	1.5 S_m 26.25 k/in^2
	$P_m + P_b + Q$		3.30 k/in^2	15.36 k/in^2	18.66 k/in^2	3.0 S_m 52.5 k/in^2
	P_m	At 0.5 Rt (Point 4)	0.58 k/in^2	17.11 k/in^2	17.69 k/in^2	1.1 S_m 19.25 k/in^2
	$P_m + P_b + Q$		4.10 k/in^2	17.11 k/in^2	21.21 k/in^2	3.0 S_m 52.5 $k/in2$

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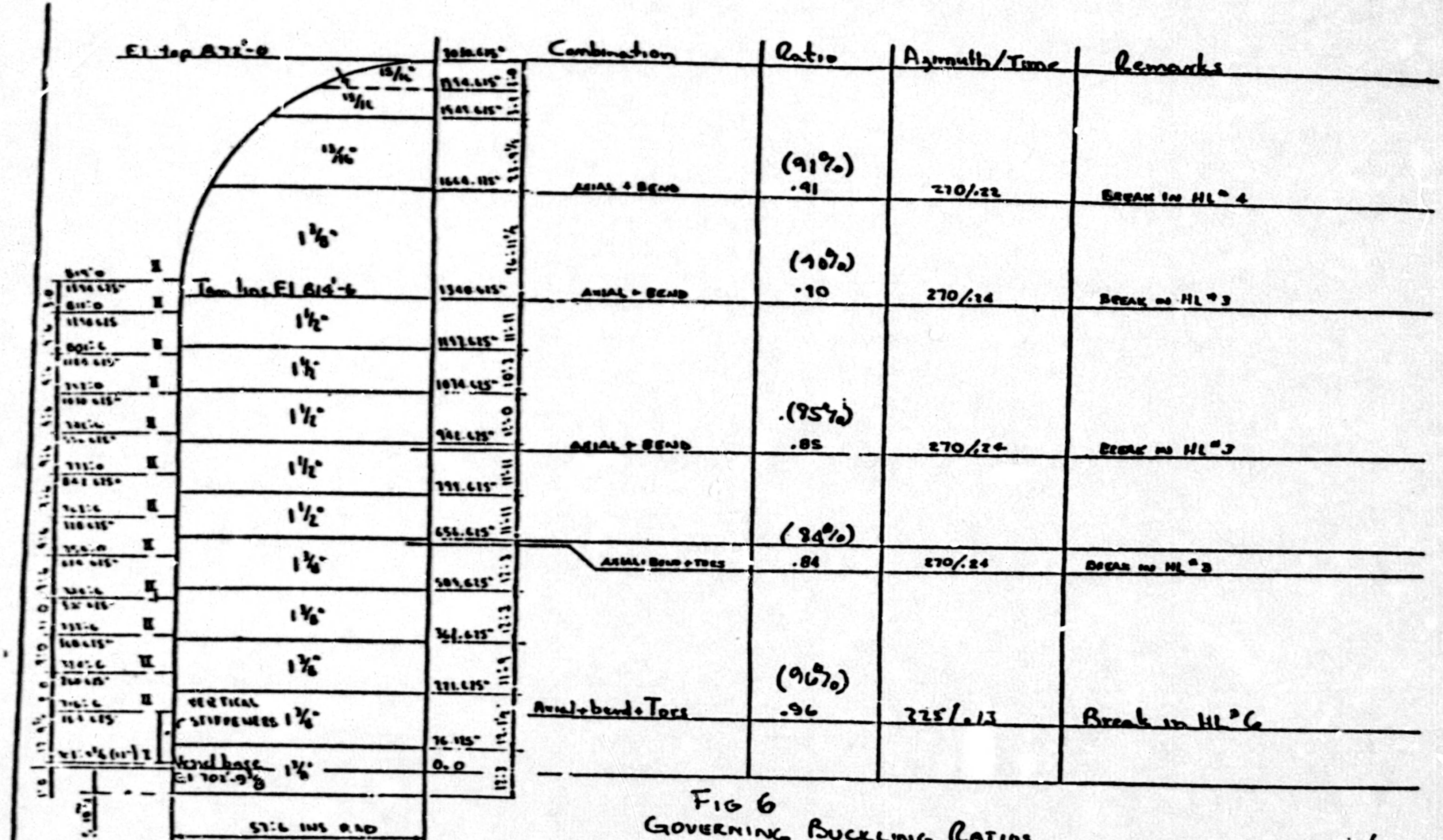


FIG 6
GOVERNING BUCKLING RATIOS

WML 1/6/1
6/8/79

Appendix B

International Structural Engineers

October 27, 1980

Mr. Sang Bo Kim
U.S. Nuclear Regulatory Commission
Office of Nuclear Regulation
Washington, D.C. 20555

Re: Watts Bar Containment Building
Contract No. NRC-03-79-124

Dear Mr. Kim:

We have concluded the review of TVA's responses to our questions. Enclosed is a copy of the results.

Task 2, as specified in the contract, calls for a second two-day meeting with the applicant in Tennessee or Maryland. The agenda would be the questions and our reservations as described in the attachment.

Task 3 specifies that a final report be submitted, encompassing the results of Task 2. We currently have enough information to proceed directly to Task 3. To do so, we would need a directive from you regarding if and when the second meeting in Task 2 will take place.

Please advise as soon as possible whether to prepare for a second two-day meeting or to start work on the final report. The final report will take a certain amount of time, and we wish to start as soon as possible to meet the December 31 deadline.

Sincerely,

Bengt Mossberg

nm
Enclosures

P. O. BOX 9595 • GLENDALE, CALIFORNIA 91206 • TELEPHONE (213) 837-8339

REVIEW OF TVA's
RESPONSES TO QUESTIONS
SUBMITTED APRIL 9, 1979

SUBJECT: Watts Bar Containment Building
Contract No. NRC-03-79-124

After a careful review of the TVA document entitled "Watts Bar Nuclear Plant - Containment Vessel Building Analysis" dated February 19, 1980, Drs. Seide, Weingarten and Masri find themselves in agreement with some, but not all, of TVA's explanations. In particular, the replies to questions 1, 3, 5, 8 and 10 are considered satisfactory. Drs. Seide, Weingarten and Masri find themselves in some disagreement with the replies to the remaining questions (2, 4, 6, 7 and 9). Their comments on the cited replies are listed below:

Question 2:
Provide a description of how the buckling curves contained in the report were applied to the buckling of the containment vessel. The description should include the application of these buckling curves to asymmetric dynamic loads in the areas where penetrations are present.

Review of Answer 2:
We are still concerned for the reliability of the buckling analysis. The behavior of the shell in the vicinity of the penetrations does not appear to have been modeled accurately in both the dynamic and buckling analysis. Thus the effect of stress concentrations near the openings and the adequacy of the stiffening around the opening are uncertain. There is virtually nothing in the literature on the amount of stiffening required to nullify the opening from a buckling point of view. The present analysis therefore assumes that the stiffening has the desired result without any verification.

Questions 4:
Provide a description of the assumptions involved in modeling the containment vessel in order to use the programs identified in Question 3. This description should include a discussion of any convergence and/or accuracy checks that were made.

Review of Answer 4:
For the axisymmetric shell, the convergence check is acceptable. However, for the shell containing penetrations, it appears that no checks for convergence or accuracy were carried out for the stresses in the area of the penetration. Since the stresses around the penetrating may trigger buckling, the solution accuracy should be investigated.

Question 6:
Explain the procedure of obtaining the stress distribution in the shell using lumped mass beam model instead of a shell model for the dynamic seismic analysis.

Review of Answer 6:
The use of the Timoshenkon shear beam as an analog for a perfect (without penetrations) shell of revolution is acceptable; however, for the containment vessel under discussion there is no documentation or justification that this simplistic approach is applicable, and that it will not suppress shell modes in the real structure that will be excited by seismic ground motion.

Question 7:
Explain the justification for using an axisymmetric geometry computer program for the containment vessel.

Review of Answer 7:
Although the answer to question 7 states that the approach to analyzing the locks and hatches as a supported subsystem was used for the dynamic analysis of nuclear plants, it does not address the question of the accuracy of this approach for dynamic buckling analysis. Further justification of this approach is needed.

Question 9:
Explain in detail the criteria and its justification for determining the interaction effects between the containment shell and the attached equipment.

Review of Answer 9:
By doing a separate analysis for the supported equipment, the effect of the equipment on the containment shell is neglected. Thus the effects of interaction between the motion of the shell and its attachments are not properly handled. Recent studies in the published literature (PO, BSSA, April 1979) have shown that significant interaction can develop under seismic excitation even if the mass ratio of the equipment is on the order of one percent of the main structure.

The questions 2, 4, 6, 7 and 9 are not yet completely answered and as our reviews indicate, they raise some serious reservations about the containment building design.

We feel that these areas should be addressed immediately at the next two day meeting (as specified in Task 2 of Contract NRC-03-79-124) between the applicant, ourselves and the COAR.

Appendix C

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

8/15/80

Bengt A. Mossberg
President
International Structural Engineering
P. O. Box 9595
Glendale, California 91206

Subject: Watts Bar Containment Building Contract No. NRC-03-79-124

Dear Mr. Mossberg:

Enclosed please find a copy of TVA's response to our questions regarding the subject matter. We are sending the response to you so that you may complete the contract work. The contract period is being extended from May 31, 1980 to December 31, 1980. Confirmation of the extension is expected to be issued by the Division of Contracts, NRC.

Sincerely,

S. B. Kim
Structural Engineering Branch
Division of Engineering
Office of Nuclear Reactor Regulation

WATTS BAR NUCLEAR PLANT

CONTAINMENT VESSEL
BUCKLING ANALYSIS

February 19,1980

Introduction

Before proceeding with the responses to the NRC questions, a brief description of the containment vessel follows. The containment is a low-leakage, free standing steel structure consisting of stiffened cylindrical walls with a hemispherical head. The stiffening is primarily circumferential; however, one of the vessel's lower bays is stiffened vertically. The circumferential stiffeners are spaced roughly four times Rt (10 foot intervals) and the vertical stiffeners are spaced at 5 degree intervals. Other locally stiffened areas are provided around the equipment hatch and two personnel locks. The base of the containment vessel is anchored in a concrete mat using pretensioned bolts on the inside and outside of the vessel extending down into the concrete mat. Equipment supported from the axisymmetric containment vessel include the personnel locks, equipment hatch and other minor attachments shown in Figure 1.

Figure 1 shows a schematic representation of the containment vessel geometry. Important information includes the shell thicknesses, vessel overall dimensions, ring locations, and the size of the stiffener rings. The location of the major supported equipment is also shown with dimensions roughly to scale.

Figure 2 shows a plot of the frequencies for the modes of natural vibration of the containment vessel. The natural frequency for the first and second axial modes for each of the Fourier harmonics of major interests are plotted.

NRC QUESTIONS

Question 1. Provide a description of the exact applied loads used for the buckling analysis. If any computer programs were used to obtain these loads, a complete description of the computer programs should be supplied. This description should include a discussion of the analytical and numerical methods used in the program, as well as a statement of its limitations and the methods used in its verification.

Response

The controlling load combinations for the buckling analysis are combinations No. 3 and No. 4 in section 3.8.2.3.2 of the Watts Bar FSAR. The hydrostatic and thermal loads in these combinations are not considered significant from a buckling standpoint and design internal pressure tends to increase the buckling strength. Therefore, for the buckling analysis dead weight and transient pressure loads were combined with Operating and Design Basis Earthquakes, since these combinations produce the greatest meridional and hoop compressive membrane stresses. For further discussion, see Appendix A.

A description of the computer programs used to determine loads is provided in our response to question 3. This description includes a discussion of program limitations and methods used. The Fourier series used to represent the specified asymmetric pressure transient was plotted for all elevations for representative transient times, and the plots were checked against the TVA pressure curves. Fourier coefficients also were calculated by Anamet Labs and a cross check on the coefficients was made.

<u>Question 2</u>. Provide a description of how the buckling curves contained in the report were applied to the buckling of the containment vessel. The description should include the application of these buckling curves to asymmetric dynamic loads in the areas where penetrations are present.

<u>Response</u>.

Membrane stresses in the containment due to deadweight, seismic and transient pressure loads were calculated. The deadweight stresses were determined using hand calculations and seismic stresses were determined using the beam model described in our response to Question 6. An asymmetric transient load analysis was performed using a shell of revolution model for the pressure transients from all of postulated breaks.

The buckling analysis was performed using CBI computer program E1391. The maximum deadweight and earthquake stresses were combined with the transient load stresses at each of 40 timesteps, at 123 elevations and at 24 points around the vessel, evenly spaced at 15 degree intervals.

Figures 3.8B-1 through -10 in the Watts Bar FSAR were used to determine the critical buckling stress based on the containment vessel geometry. In general, these curves are based on theoretical and experimental results for buckling of shell structures. The shell bays between the stiffeners are considered as simplysupported cylinders and the vertically stiffened bay assumes panel buckling in determining critical buckling stress at a given elevation.

The interaction equations given in Appendix 3.8.B-4 and -5 of the FSAR were evaluated taking the summation of the stress ratios of compressive membrane to critical buckling stress times the buckling load factors in Table 3.8.B-2. These summations were investigated at each timestep azimuth and elevation described above. In this evaluation, the longitudinal membrane stresses produced by the asymmetric pressure transient load (NASPL) and horizontal earthquakes were considered as caused by bending loads in the interaction equations. Deadweight and vertical earthquake loads cause axial compression in the equations.

These interaction equations provide the criteria for evaluating the interaction of multi-axial compressive stress and hoop compressive stress. The maximum stress ratio summation considering all 24 points around the circumference was tabulated for each elevation for each interaction equation specified. In the areas where there were major penetrations which interrupt the basic stiffening scheme on the vessel, stiffeners were designed to carry the stresses in the shell around the opening.

<u>Question 3</u>. Provide in-depth description of all computer programs used in the buckling analysis. The description should

state the origin of the program, its limitations, and the methods used to verify its validity.

Response

Appendix B presents abstracts of the computer programs employed in the buckling analysis of the Watts Bar containment vessels. These abstracts provide a description of the each program and its limitations. CBI programs 1374 and E0781A and Anamet's BALL program were verified by comparison with results of programs in the public domain. The other programs were verified by comparison with the results of hand calculations. The dynamic shell analysis was done using CBI program 1374, which was developed by CBI from the Kalmins shell of revolution statics program. The results of this analysis were verified by the results of Anamet's analysis using the BALL program.

Question 4. Provide a description of the assumptions involved in modeling the containment vessel in order to use the programs identified in Question 3. This description should include a discussion of any convergence and/or accuracy checks that were made.

Response

The vessel was modeled as an axisymmetric shell of revolution. The circumferential stiffeners were modeled discretely as ring stiffeners. The section of the vessel that has vertical stiffening was modeled as an orthotropic shell.

The representation of the specified loads required a total of 21 Fourier harmonics. This Fourier representation included ten sine and ten cosine terms plus the axisymmetric loading. The vessel was assumed to be fixed at the base, and the mass of any supported equipment was smeared over the circumference at the appropriate elevation.

In addition to the usual convergence and accuracy checks used with shell of revolution models, a completely independent analysis was done by Anamet Laboratories using a completely different computer program and model for one of the specified pressure transients. A comparison of the 1374 results with the Anamet results for a similar vessel geometry is shown in Figures 3a and 3b.

Question 5. Provide a complete step-by-step description of which and how the buckling stress criteria were applied.

Response

The buckling criteria in Appendix 3.8B of the FSAR was applied to the Watts Bar Containment design. The step-by-step approach used for the application of these criteria has been described in our response to Question 2 and in Appendix A.

Question 6. Explain the procedure of obtaining the stress distribution in the shell using lumped mass beam model instead of a shell model for the dynamic seismic analysis.

Response

The Timoshenko shear beam is a realistic analog for the response of a shell of revolution to seismic ground motion. A complete discussion of the use of the beam model for the seismic analysis of containment vessel is provided in Appendix C. "The Design of a Thin Shell Nuclear Containment Vessel for Seismic Loading" by Jon Hagstrom.

Question 7. Explain the justification for using an axisymmetric geometry computer program for the containment vessel.

Response

The main aspects of the containment geometry that are not handled precisely by the shell of revolutions model are the vertical stiffening and the attached equipment, primarily the locks and hatch. The spacing of the vertical stiffening used for the Watts Bar containment vessel is such that the orthotropic representation is a very reasonable one. This can be verified by results in the literature or static analysis.

For the attached equipment, some additional consideration is necessary. In the analysis of the Watts Bar containment vessel this aspect was handled by doing a separate dynamic analysis for the response of the locks and hatch. The procedure for doing this additional analysis is spelled out in the FSAR, Par. 3.8.2.4.7. In effect, the locks and hatch are treated as a supported subsystem and they are evaluated using a separate dynamic model. This general approach is commonly used in the dynamic analysis of nuclear plants.

Question 8. Provide the criteria used in the computer program to calculate the buckling loads, description of the mass matrix formulation and how the maxima at each time point were chosen in the CBI containment shell analysis using the finite element model.

Response

The criteria used in the bucking program is described in our response to Question 2 and in Appendix A. As discussed, for each timestep 24 locations around the circumference were considered at each elevation investigated.

The mass matrix formulation for the 1374 program is described in the program abstract for 1374 in Appendix B.

Question 9. Explain in detail the criteria and its justification for determining the interaction effects between the containment shell and the attached equipment.

Response

As described above, interaction effect between the containment shell and the attached equipment was determined by doing a separate analysis for the supported system. A time history response of the motion of the axisymmetric shell at the point of attachment of a supported system was determined from the shell of revolution results. A response spectra for this calculated motion was generated; the frequencies of the supported system was calculated; and a spectral response was calculated.

An experimental and theoretical investigation of mass loading effects has been reported by North American Rockwell Corporation in Report SD 68-29. Excerpts from this report are included in Appendix "D" of this response.

Question 10. Was a thermal analysis conducted? If the answer is yes, describe step by step the procedure that was followed.

Response

Two thermal analyses were performed on the containment vessel. One analysis was an axisymmetric thermal of the embedment region to investigate the effect of the base restraint. This analysis utilized axisymmetric elements to model the lower containment region with the temperature distribution shown in Figure 3.8A-2 of the FSAR. Figure 4 shows the summation of stresses due to coincident loads.

The other thermal analysis was an asymmetric analysis of the containment vessel using an axisymmetric shell of revolution model. The temperature distribution varies significantly in the circumferential and vertical directions 1000 seconds after the pressure transient (LOCA) creating internal stresses in containment from self-constraint. Fourier representations were used to simulate the circumferential temperature distribution. A step-by-step discussion of this analysis is given in Appendix E.

The stresses observed from these analyses are significantly less than the ASME allowable stress intensities. Therefore it can be determined by observatión that buckling is not a problem.

Question 11. Provide a list of all the loading combinations and the stress allowables (stress intensity and buckling stress) which had been used in the design of the steel containment.

Response

The loading combinations used in the design of the containment vessels are given in Section 3.8.2.3.2 of the FSAR. TAble 3.8.2-1 in the FSAR gives the allowable stress criteria for these loading combinations. Appendix 3.8B of the FSAR give the allowable buckling criteria for all loading combinations.

Question 12. Indicate the critical loading combinations which control the design of the steel containment shell with regard to stress intensity and buckling. Identify also the regions and/or regions of the steel containment which was controlled by these critical loadings.

Response

Only those loading combinations with pressure transients (NASPL) govern the shell with regard to stress intensity and/or buckling. See attached Figures 5 and 6 for a summary.

Question 13. Indicate, approximately, the contribution (as a percentage of the allowable stress intensity and allowable buckling loads) of each of the loadings identified in Question 12.

Answer

Refer to figures 5 and 6. Percentages are in parenthesis.

CBI

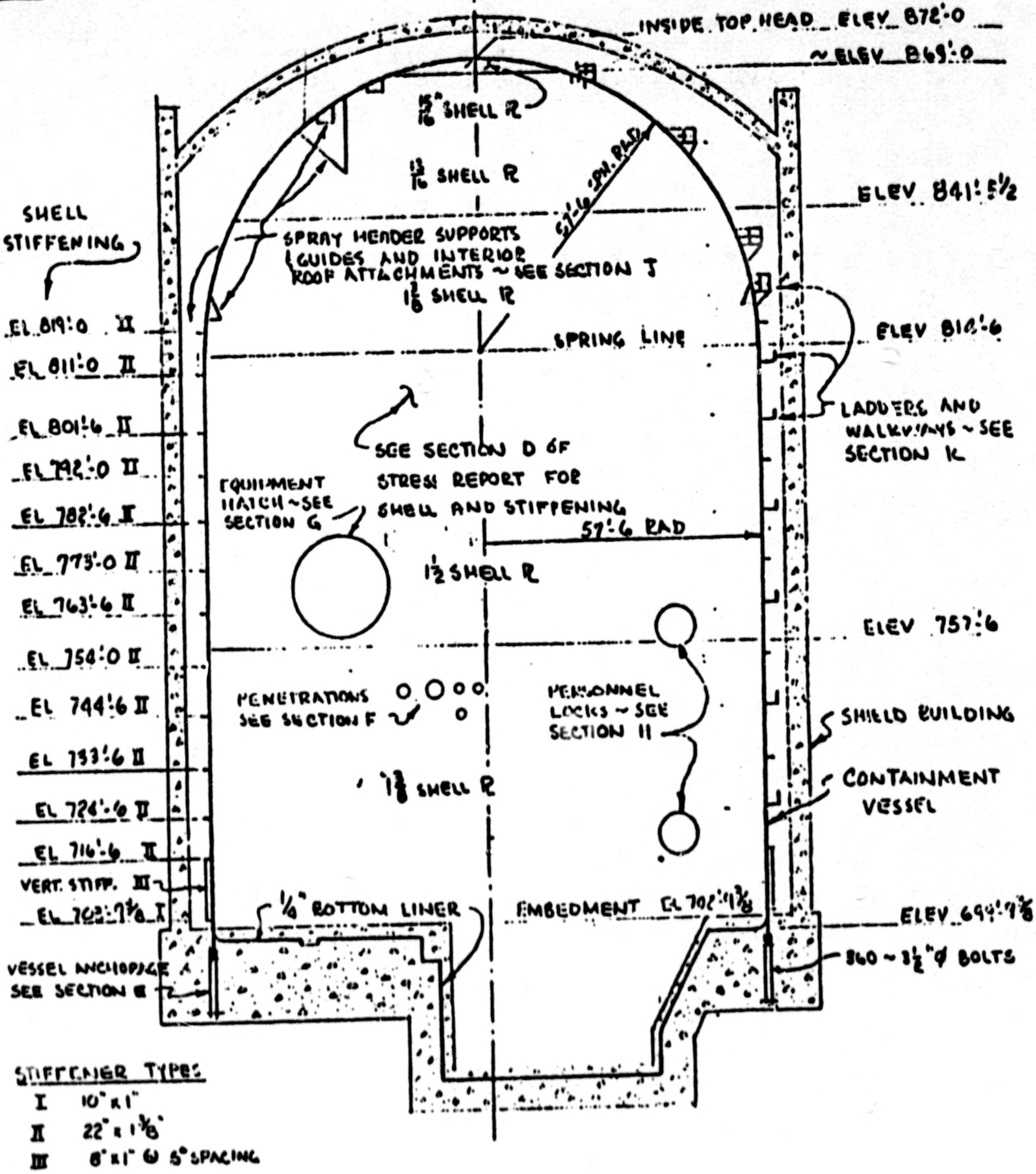
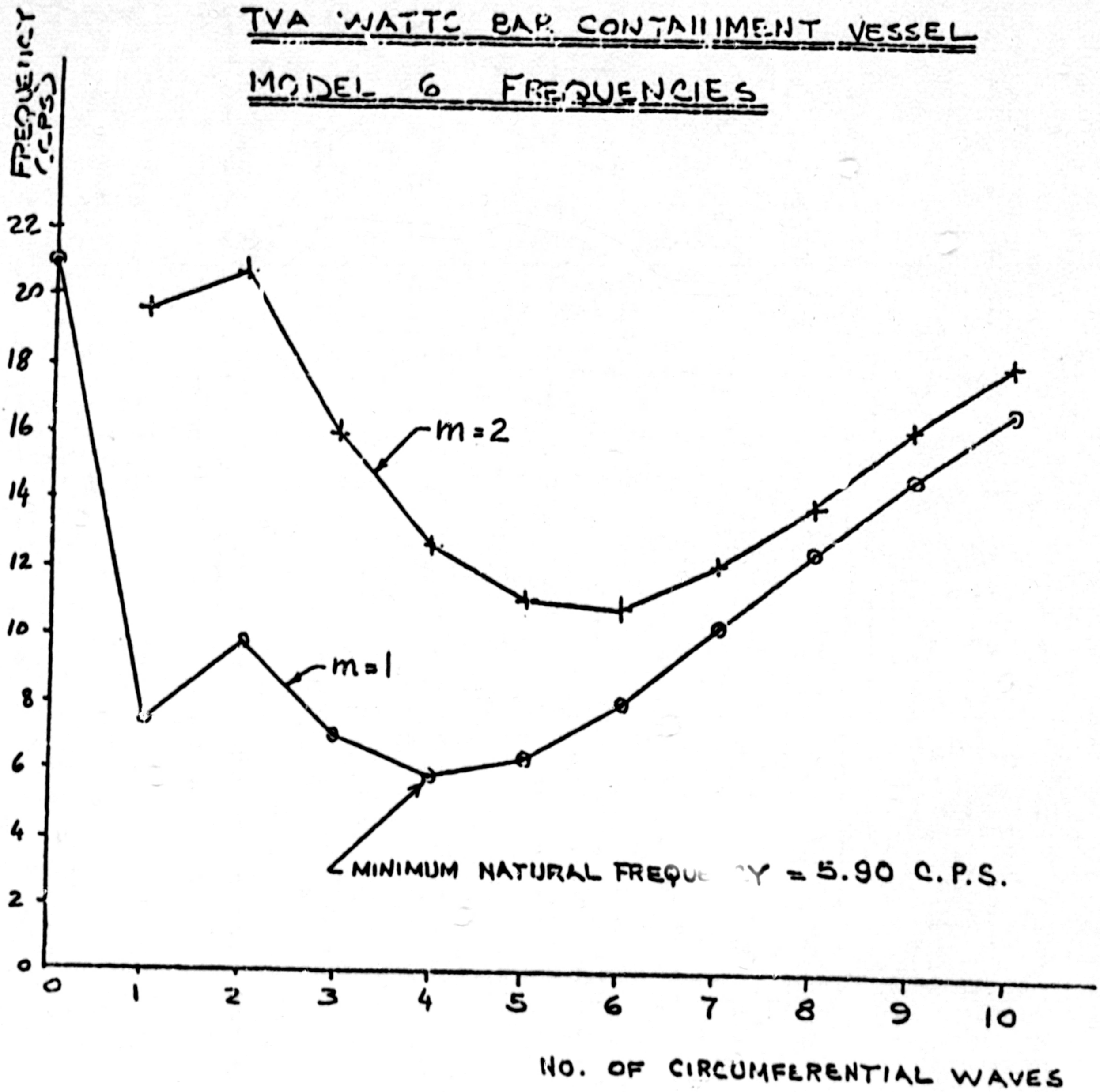


FIGURE 1
SECTION MIRU VESSEL

SUBJECT	MADE BY	CHKD BY	REV	By		CHARGE NO.
INTRODUCTION	REA	WUL		Chkd		72-4333
	DATE	DATE		Date		SHT A-3 OF

Location________________



(m REFERS TO LONGITUDINAL MODE SHAPES)

FIG. 2

SUBJECT	MADE BY	CHKD BY	REV			CHARGE NO.
TRANSIENT ANALYSIS	FES	CJP		By		724333
WATTS BAR CONTAINMENT	DATE 2/7[illegible]	DATE [illegible]/74		Chkd		[illegible]
				Date		

Location

BRK IN ELEMENT 3

MAXIMUN COMPRESSIVE VALUE OF LONGITUDINAL FORCE VS ELEVATION

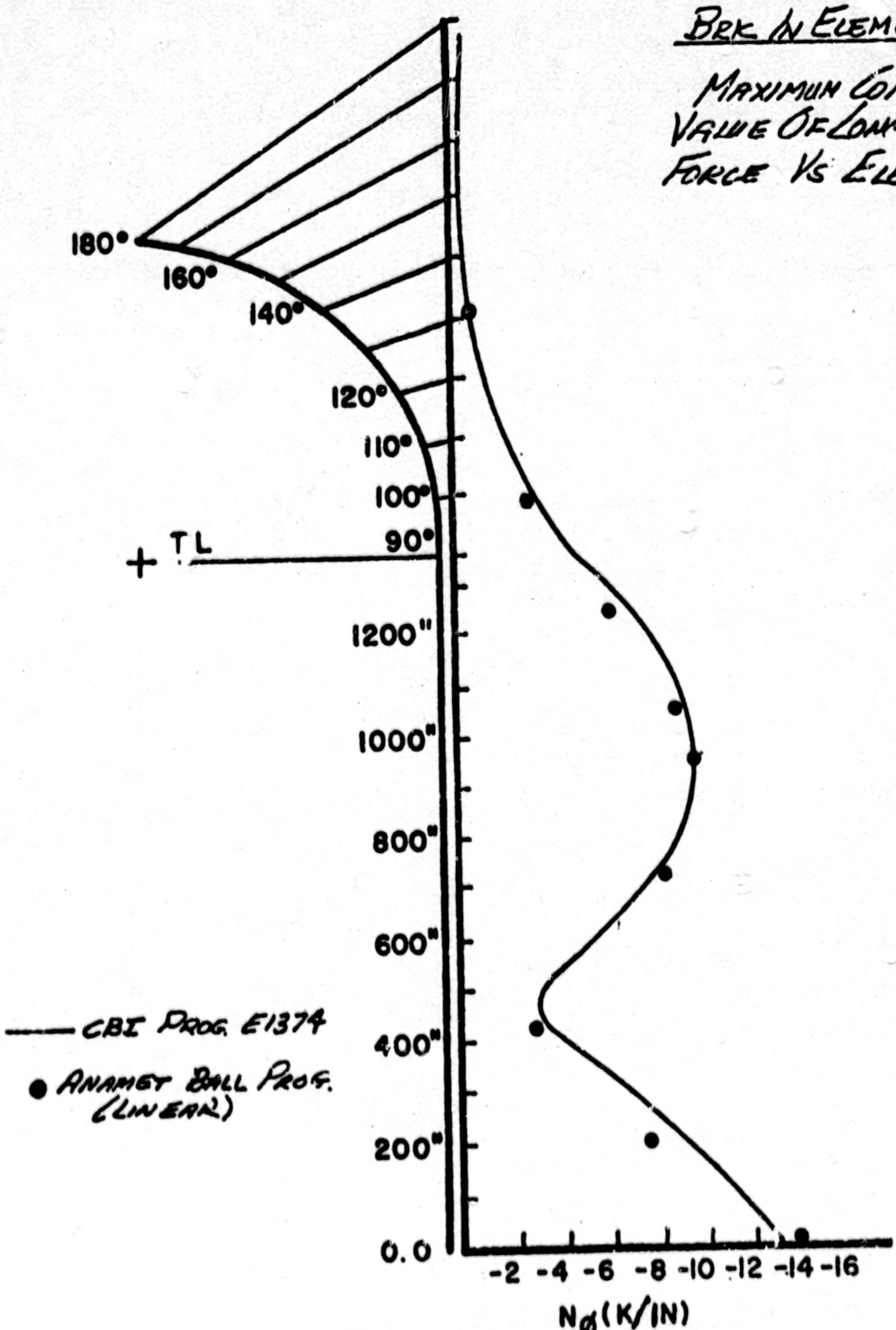
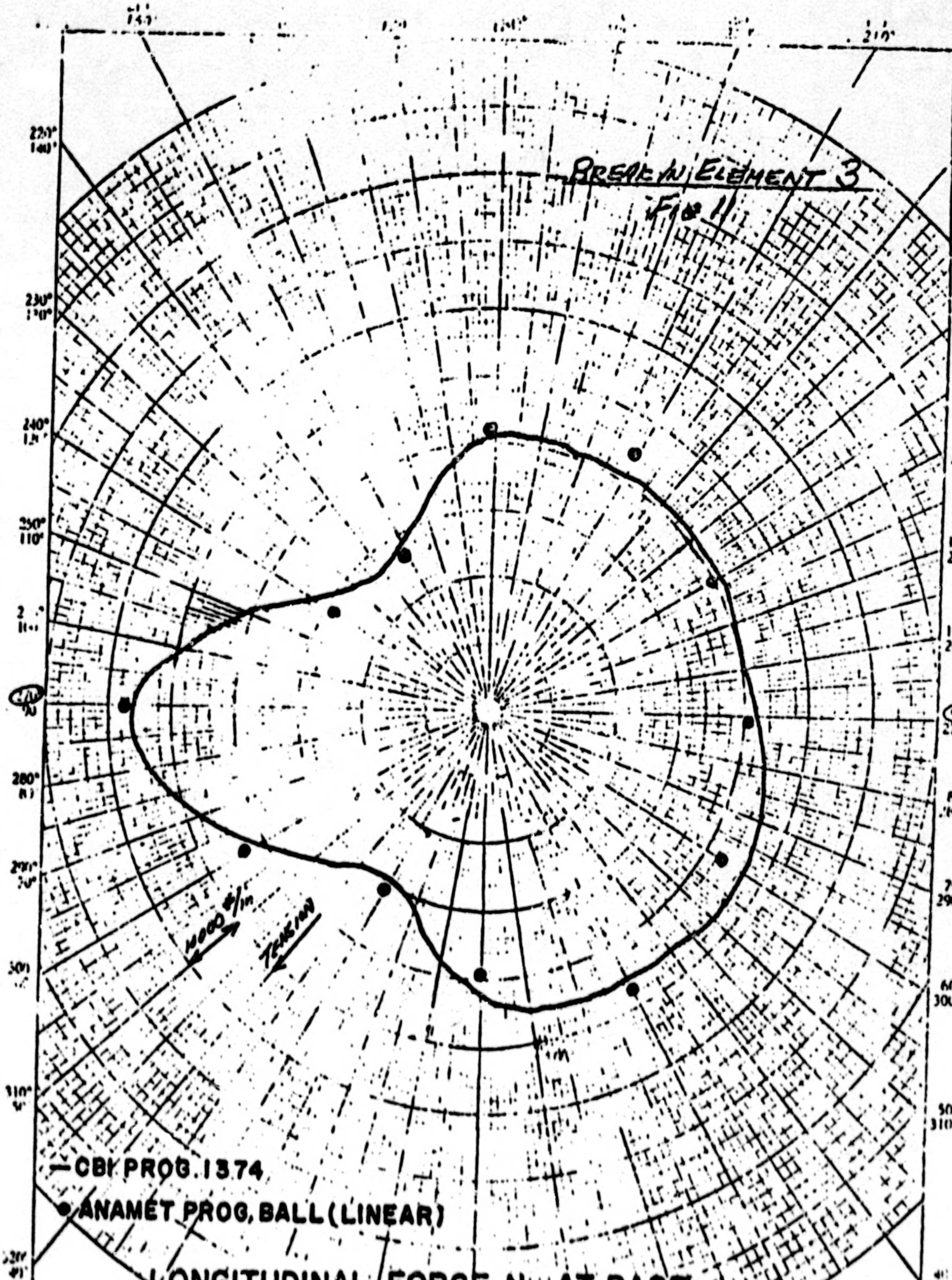


FIGURE 3a

SUBJECT	MADE BY	CHKD BY	REV	By		CHARGE NO.
FIG 7		FES		Chkd		69-5545
SEQUOYAH CONTAINMENT	DATE 10/73	DATE 10/73		Date		BN25 SHT ___ OF ___



LONGITUDINAL FORCE N_ϕ AT BASE VERSUS AZIMUTH AT .25 SEC

FIGURE 3b

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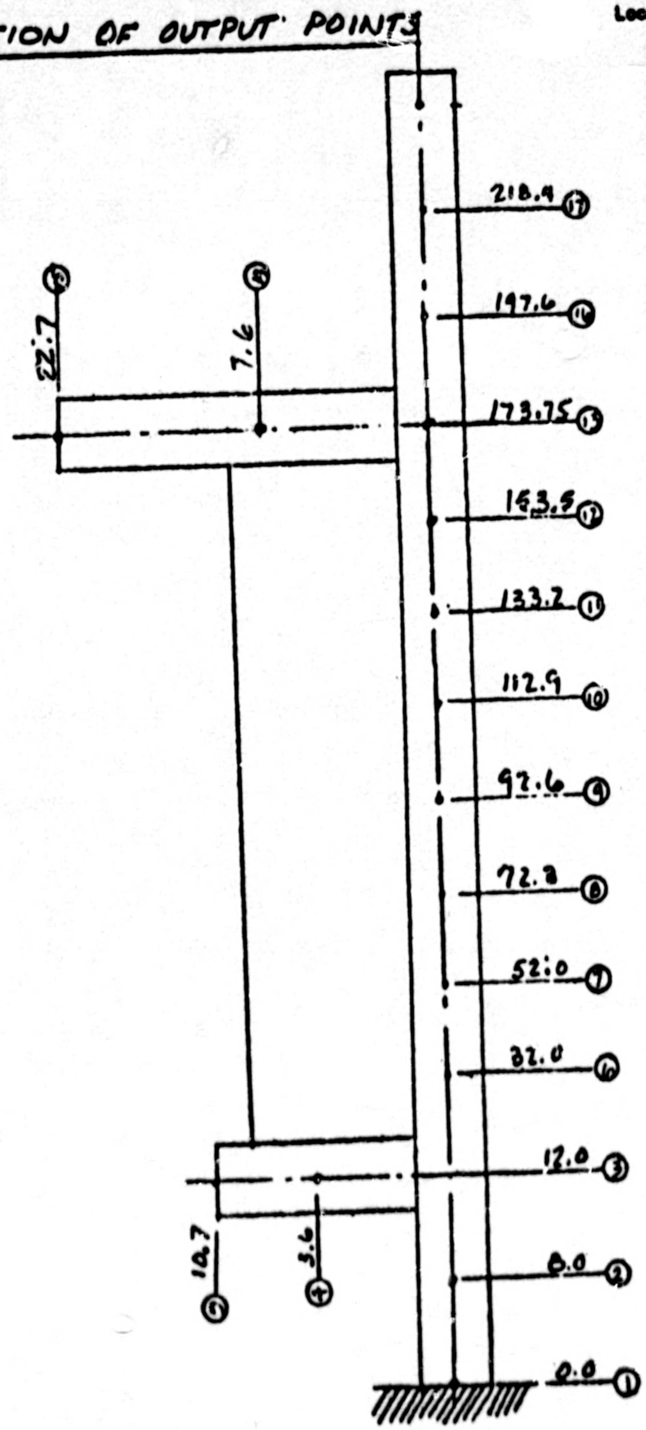
CBI

LOCATION OF OUTPUT POINTS



Location BIRMINGHAM DESIGN

σ_ϕ	σ_θ	w
3382	6581	9954
3382	6427	9916
3382	6947	1.005
3083	8024	1.035
↓	8532	1.052
	7668	1.035
	4637	.9629
	-1093	.8204
	-5561	.6086
↓	13240	.3549
3382	2568	.0891
↓	704	.0430
↓	1034	.0

Fig 4

SUBJECT	MADE BY DMB	CHKD BY	REV	By		CHANGE NO. 72-1333
	DATE	DATE		Chkd		

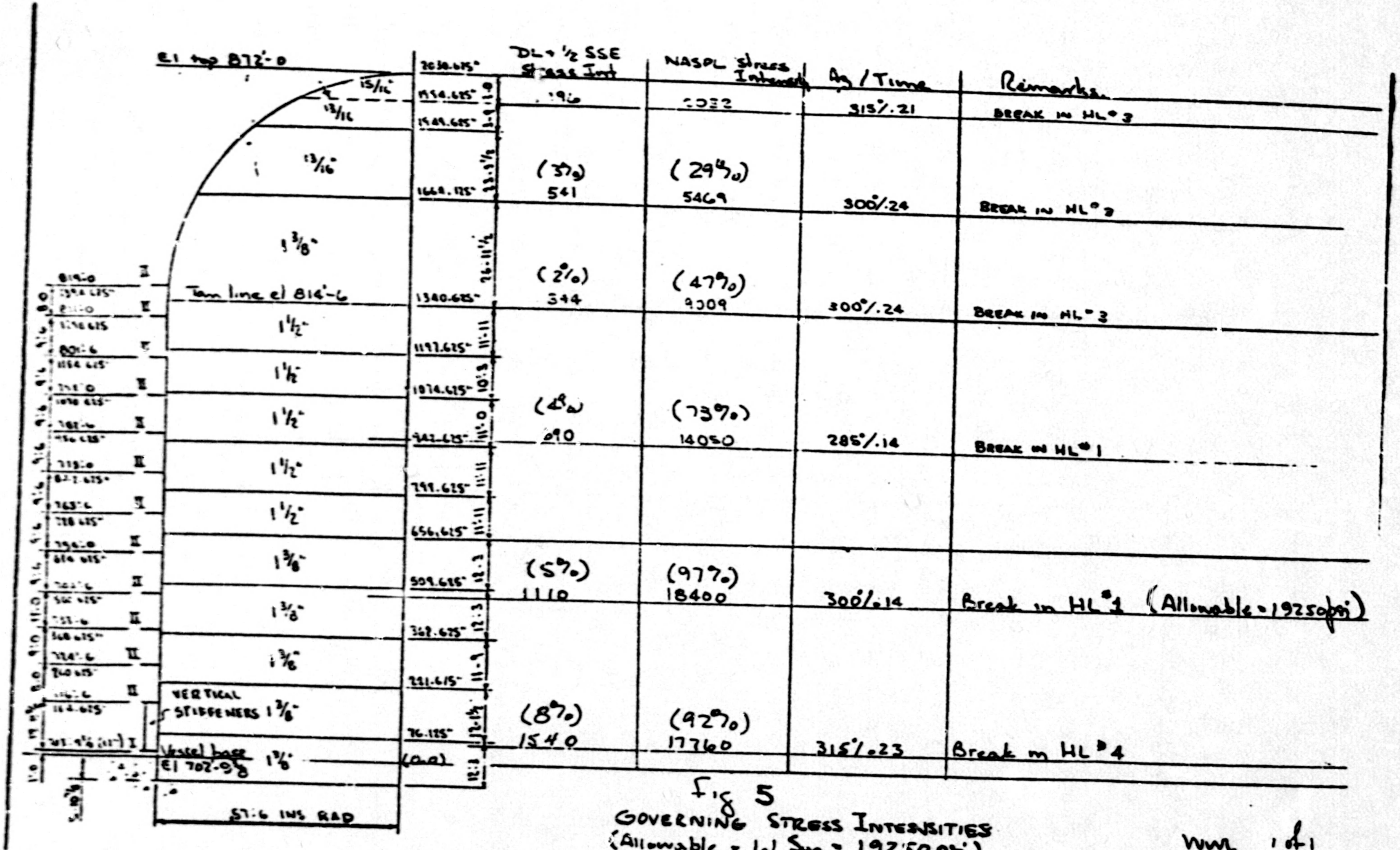
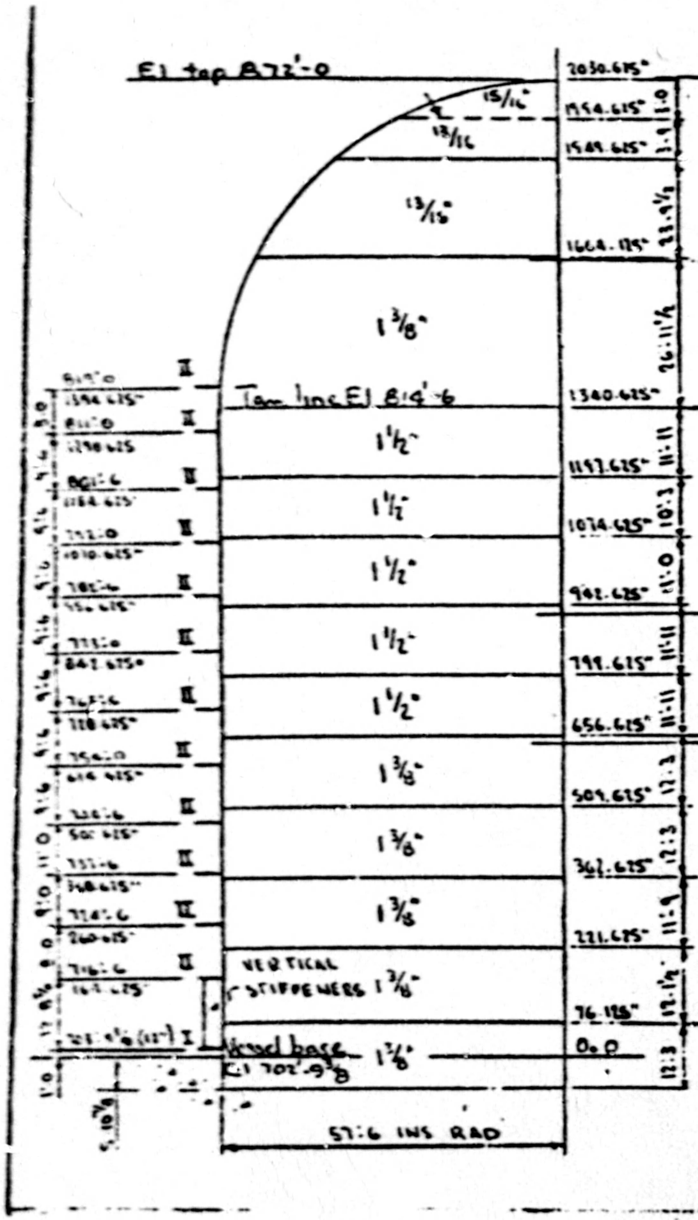


Fig 5
GOVERNING STRESS INTENSITIES
(Allowable = 1.1 Sm = 19250 psi)

WWR 1 of 1
6/8/79



Combination	Ratio	Azimuth/Time	Remarks
AXIAL + BEND	(91%) .91	270/.22	BREAK IN HL#4
AXIAL + BEND	(90%) .90	270/.24	BREAK IN HL#3
AXIAL + BEND	(85%) .85	270/.24	BREAK IN HL#3
AXIAL+BEND+TORS	(84%) .84	270/.24	BREAK IN HL#3
Axial+bend+Tors	(96%) .96	225/.13	Break in HL#6

FIG 6
GOVERNING BUCKLING RATIOS

WWR
6/8/79

LIST OF APPENDICES

APPENDIX A'

Discussion of Buckling Analysis

DISCUSSION OF BUCKLING ANALYSIS

This section presents the buckling analysis of the Watts Bar Containment Vessel for the pressure transient loading condition. This buckling analysis is performed in accordance with TVA Specification 1440.

Shell loads for the buckling analysis are developed in the section presenting the pressure transient shell analysis. In addition to these pressure transient loads, this analysis also considers the effects of the shell weight, miscellaneous loads, and loads resulting from vertical and horizontal seismic action.

The buckling analysis is performed using CBI computer program E1391. The pressure transient loads developed by the pressure transient shell analysis are stored by the computer and then read in directly by program E1391. The other loads considered in the buckling analysis are derived in various portions of the analysis of the containment vessel. These loads are assembled and input separately into the program. The geometry of the vessel describing the shell and stiffening and the material properties are also read into the program.

The program performs the buckling analysis for the vessel at each point considered in the pressure transient shell analysis. For the Watts Bar Containment Vessel the following points are checked:

6 Hot Leg Breaks (½ SSE)

6 Cold Leg Breaks (½ SSE)

1 Hot Leg Break (SSE)

123 Elevations for each break

24 Azimuths for each elevation

40 Timesteps for each azimuth

The analysis is performed using all of the breaks for the one-half safe shut down earthquake (½ SSE) and for one break for the safe shut down earthquake (SSE). It can be noted that by comparing the results of the buckling analysis for

Location Birmingham Design

the SSE break with the corresponding break using ½ SSE the buckling ratios are larger for the ½ SSE condition. The load factor of ~~safety~~ for the ½ SSE condition is 1.25 while for the SSE condition it is 1.1. Since the seismic loads are small compared to the pressure transient loads it can be seen that the ½ SSE condition is the critical condition. The one SSE condition is reported to confirm this fact and it is not necessary to investigate any other breaks for the SSE condition.

The pressure transient loads are input at each point under consideration while the other loads are input at fewer locations. The program performs a linear interpolation to obtain the values of these other loads at each point where the buckling stress is calculated.

In order to determine the critical buckling stress at any point, it is necessary to determine the vessel geometry in the region of that point. The program considers several cases depending on the shell and stiffening arrangement. Once the geometrical properties are determined the program calculates the critical buckling stress in accordance with Appendix H, Revision 1 of TVA Specification 1440.

By multiplying the actual stress by the load factor ~~of safety~~ and dividing the product by the critical buckling stress the buckling ratio is determined. The buckling ratio is calculated for meridional stresses due to axial and bending loads, circumferential stress, and shear stress. These individual buckling ratios are then combined to form the final buckling ratios in accordance with Appendix H.

For each elevation under consideration the program calculates the ratios for all azimuths and times and finds the maximum buckling ratio. This ratio, with its corresponding azimuth and time is then recorded.

A further discussion of the methods used may be found on the following pages.

ANALYSIS | DATE | DATE | Date

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Description of Program E1391Ø

Shell Buckling Analysis for TVA Containment Vessel

Introduction

Program #1391Ø performs the buckling check of the TVA ice condenser containment vessel for Non-axisymmetric pressure loading combining with the dead loads and seismic loads occuring in the vessel. The non-axisymmetric loads are developed by CBI program E1374. The resulting stresses are compared to critical buckling stress and by using a specified load factor ~~of safety~~ the buckling ratio is found.

Calculation of Dead Loads and Seismic Loads

As a general rule, dead loads and seismic loads are calculated at a few points along the vessel and the vessel is checked for buckling at many more points. As a result the program performs a linear interpolation to derive the dead loads and seismic loads at any given point along the vessel.

Calculation of Critical Buckling Stresses

In order to determine the critical buckling stress at a given point it is necessary to determine the geometry in the region of that point. The program considers four cases:

a) cylinders stiffened meridionally and circumferentially
b) cylinders stiffened circumferentially
c) spheres stiffened circumferentially and meridionally
d) unstiffened spheres

For each of these conditions certain constants must be determined based on known geometry. This information is in the form of graphs and is input into the computer as a series of straight lines. Once these constants are known, the critical buckling stresses can be determined using the applicable formulae. These calculations are based on Appendix H of TVA specification 1440.

SUBJECT	MADE BY	CHKD BY	REV	By		CHARGE NO.
Description of Program E1391Ø	REA			Chkd		72-4333 DCA-3
	DATE 10/24/73	DATE		Date		SHT ___ OF ___

Calculation of Stress and Buckling Ratios

The program calculates stresses and buckling ratios at any number of elevations and azimuths and for any number of time periods and then finds the maximum buckling ratio at each elevation. The program calculates four stresses at each point:

a) meridional stress due to axial loads
b) meridional stress due to bending loads
c) circumferential stress
d) shear stress

The program considers only compressive stresses. If a stress is tensile it is set equal to zero. For each of the stresses a buckling ratio is calculated using the stress multiplied by a factor of safety and divided by the critical buckling stress. These buckling ratios are then combined and five ratios result:

a) Axial + Circumferential
b) Axial + Bending
c) Axial + Shear
d) Axial + Shear + Bending
e) Axial + Shear + Circumferential

After these combined ratios are calculated, the maximums regardless of time or azimuth are recorded for each elevation.

In areas where vertical stiffening is present the program calculates the buckling ratio of the stiffener acting with the shell as a column simply supported at each end. The portion of the shell used in the column is determined using the rules of the Shell Analysis Manual referenced in Appendix H of Specification 1440. The buckling ratio is calculated in the manner discussed previously and the maximum is recorded.

Program Output

The program prints out the following data at each elevation considered:

a) Basic shell data and geometry of stiffeners
b) Load such as static pressure, dead loads, and seismic loads
c) Buckling stress coefficient
d) The five combined buckling ratios.
e) Vertical stiffener buckling ratios

APPENDIX B'

Description of Computer Programs

SECTION C

DESCRIPTION OF COMPUTER PROGRAMS

COMPUTER PROGRAMS USED BY CBI

1374	SHELL DYNAMIC ANALYSIS PROGRAM	Modified Kalnins
E0781A	SHELLS OF REVOLUTION PROGRAM	
1017	MODAL ANALYSIS PROGRAM	
E1622	LOAD PREPROCESSOR FOR 1374	
E1623	POST PROCESSOR FOR E1374	
E1624	SPECTRAL CURVE GENERATOR	
E 1391ϕ	SHELL BUCKLING ANALYSIS PROGRAM	
E 1668	SPECTRAL ANALYSIS FOR ACCELERATION RECORDS	

COMPUTER PROGRAMS USED BY ANAMET

PROGRAM BALL ANALYSIS OF NONLINEAR DYNAMIC SHELLS OF REVOLUTION

PROGRAM E1374 10/31/73

1. Introduction

Program E1374 is CBI's shell dynamic analysis program. Presently, it is capable of extracting eigenvalues and performing undamped transient analyses. Non-axisymmetric loads can be handled through the use of appropriate Fourier series.

The equation of motion for a particular Fourier harmonic n of an undamped system is

$$[M_n]\{\ddot{U}_n\} + [K_n]\{U_n\} = \{P_n\}$$

where
$[M_n]$ = Mass matrix
$[K_n]$ = Stiffness matrix
$\{P_n\}$ = Applied load
$\{U_n\}$ = Displacement
$\{\ddot{U}_n\}$ = Acceleration

Note that all of the above are functions of n.

In order to calculate free vibration frequencies and mode shapes the applied load is set equal to zero, $\{U_n\}$ is assumed to be a harmonic function of time, and the eigenvalues and eigenvectors of the resulting equation obtained using the method shown in Section 4.

If the transient response due to a time-varying load is required, the numerical integration technique outlined in Section 5 is used.

2. Stiffness matrices*

In general the procedure for forming the stiffness matrices is to first find influence values for each segment (or element) using Runge-Kutta numerical integration and then manipulate these values mathematically as follows:

Starting with the influence values

$$\{U_2\} = [Y_1]\{U_1\} + [Y_2]\{F_1\} + \{Z_1\}$$
$$\{F_2\} = [Y_3]\{U_1\} + [Y_4]\{F_1\} + \{Z_2\}$$

which are obtained by setting each element of $\{U_1\}$ and $\{F_1\}$ to one in succession, while all the other elements of these vectors are zero, and integrating the thin shell differential equations to the other end of the segment. The vectors $\{Z\}$ are obtained by setting $\{U_1\} = \{F_1\} = \{0\}$ while applying the distributed loads and integrating to the end of the segment.

To change this to stiffness matrix form, one needs only switch $\{F_1\}$ and $\{U_2\}$, thus

$$\{F_1\} = [K_1]\{U_1\} + [K_2]\{U_2\} + \{C_1\}$$

$$\{F_2\} = [K_3]\{U_1\} + [K_4]\{U_2\} + \{C_2\}$$

* See References (1) and (2)

where $\{F_1\}$ = forces at start of segment

= $\{Q, N\phi, M\phi, N\}_{@1}$

$\{F_2\}$ = forces at end

$\{U_1\}$ = displacements at start of segment

= $\{W, U\phi, \beta\phi, U_\theta\}_{@1}$

$\{U_2\}$ = displacements at end

$[K_1] = [-Y_2^{-1}Y_1]$

$[K_2] = [Y_2^{-1}]$

$[K_3] = [Y_3 - Y_4Y_2^{-1}Y_1]$

$[K_4] = [Y_4Y_2^{-1}]$

$\{C_1\} = \{Y_2^{-1}Z_1\}$

$\{C_2\} = \{Z_2 - Y_4Y_2^{-1}Z_1\}$.

Since Program 1374 is not set up to handle longitudinal stiffeners, the integration for this portion of the shell is performed using Program 781. The influence values are then converted to stiffness matrix form and stored on disc. After Program 1374 has set up the stiffness matrices for the unstiffened shell, the matrices for segments with stiffening are replaced with the Program 781 matrices from disc. The solution in Program 1374 then continues in the standard manner. This consists of assembling the overall stiffness matrix $[Kn]$ and load vector $\{Cn\}$, reducing to upper triangular form, and back-substituting.

3. Ring Matrices

In order to develop a stiffness matrix for the ring stiffeners the following assumptions were made:

1. Thin beam theory is applicable, i.e. a normal to the neutral axis remains straight and normal after deformation and the thickness in negligible compared to the radius.

2. The stiffness of the ring out of its plane is negligible.

3. The ring is attached to a cylinder.

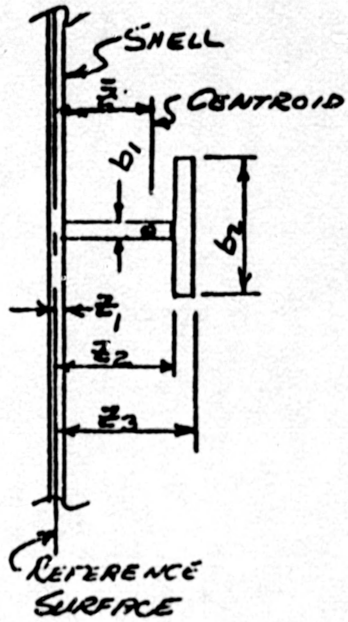

FIG. 1

4. The ring is made of one isotropic material with Poisson's ratio zero.

5. The ring can be divided into a series of cylinders of constant thickness. (See Fig. 1)

The ring is then treated as a series of m layers which start a distance Z_i from the reference surface, which is the mid surface of the shell, and ends at Z_{i+i}. The width of the layer is b_i. Then using the equations from Kalnin's paper*:

* See Reference (3).

$$A = \sum_{i=1}^{m} b_i (z_{i+i} - z_i)$$

$$A\bar{Z} = \frac{1}{2} \sum_{i=1}^{m} b_i (z_{i+i}{}^2 - z_i{}^2)$$

$$I = \frac{1}{3} \sum_{i=1}^{m} b_i (z_{i+i}{}^3 - z_i{}^3)$$

where A = area of section

$\bar{Z}$ = centroid of section, measured from reference surface

I = moment of inertia of section about reference surface

and

$$M_{\theta n} = EIk_{\theta n} + EA\bar{Z} e_{\theta n} \quad (1)$$

$$N_{\theta n} = EA e_{\theta n} + EA\bar{Z} k_{\theta n} \quad (2)$$

$$e_{\theta n} = \frac{1}{r} (nu_{\theta n} + w_n) \quad (3)$$

$$k_{\theta n} = \frac{n}{r^2} (n w_n + u_{\theta n}) \quad (4)$$

where

$( )_n$ = amplitude of a variable with n circumferential waves, e.g. $M_\theta = M_{\theta n} \cos n\theta$.

$M_{\theta n}$ = moment about the reference surface

$N_{\theta n}$ = thrust in ring

$e_{\theta n}$ = strain at reference surface

$k_{\theta n}$ = curvature

w_n = radial displacement of reference surface

$u_{\theta n}$ = circumferential displacement of reference surface

r = radius of reference surface

E = modulus of elasticity

Note that the equations above assume that all variables except $u_{\theta n}$ vary as cos $n\theta$. The equations also apply to sine series but the sign of n must be changed.

The potential energy of the ring is:

$$V = \int_0^{2\pi} \left[\frac{1}{2}\left(M_\theta k_\theta + N_\theta e_\theta\right) + Qw + Nu_\theta \right] r d\theta$$

where Q = radial force on ring at reference surface
N = circumferential force on ring at reference surface

Performing the integration, after having expanded the displacements and forces into Fourier series, yields

$$V = \pi r \sum_{n=0}^{\infty} \left[\frac{1}{2}\left(M_{\theta n} k_{\theta n} + N_{\theta n} e_{\theta n}\right) + Q_n w_n + N_n u_{\theta n} \right] \qquad (5)$$

$$\text{since} \int_0^{2\pi} \sin n\theta \cos m\theta \, d\theta = \int_0^{2\pi} \sin n\theta \sin m\theta \, d\theta =$$

$$= \int_0^{2\pi} \cos n\theta \cos m\theta \, d\theta = 0 \text{ if } n \neq m$$

Substituting eqs. (1) to (4) into eq. (5) yields:

$$V = \pi r \sum_{n=0}^{\infty} \left\{ \frac{EA}{r^2} \left[\frac{1}{2} \frac{I}{Ar^2} (n^2 w_n + n u_{\theta n})^2 + \right. \right.$$

$$+ \frac{\bar{Z}}{r} n(n w_n + u_{\theta n})(w_n + n u_{\theta n}) + \frac{1}{2}(n u_{\theta n} + w_n)^2 \Big] +$$

$$\left. + Q_n w_n + N_n u_{\theta n} \right\}$$

$$\text{Since } \frac{\partial V}{\partial w_n} = \frac{\partial V}{\partial u_{\theta n}} = 0, \text{ then}$$

$$Q_n = \frac{-EA}{r^2} \left\{ \left[1 + \frac{\bar{Z}}{r} 2n^2 + \frac{I}{Ar^2} n^4 \right] w_n + n \left[1 + \frac{\bar{Z}}{r} (n^2+1) + \frac{I}{Ar^2} n^2 \right] u_{\theta n} \right\} \quad (6)$$

$$N_n = \frac{-EA}{r^2} \left\{ n \left[1 + \frac{\bar{Z}}{r} (n^2+1) + \frac{I}{Ar^2} n^2 \right] w_n + n^2 \left[1 + 2\frac{\bar{Z}}{r} + \frac{I}{Ar^2} \right] u_{\theta n} \right\} \quad (7)$$

For each ring, A, I, and $\bar{Z}$ (See Fig. 1) are input directly into the program. The stiffness terms are then calculated using equations (6) and (7).

In order to develop a mass matrix it was assumed that the ring behaved as though it were a lump mass located at the reference surface but including rotational inertia proportional to its eccentricity. This yields a simple diagonal mass matrix.

$$\rho A \begin{bmatrix} 1. & & & \\ & 1. & & \\ & & \bar{z} & \\ & & & 1. \end{bmatrix} \begin{bmatrix} w_n \\ u_{\phi n} \\ \beta_{\phi n} \\ u_{\theta n} \end{bmatrix}$$

where ρ = mass density of the ring material.

These mass and stiffness matrices are then added to the gross matrices as they are assembled.

4. Natural Modes and Frequencies

The eigenvalues are extracted using an iterative procedure*. In order to avoid time consuming integration at each iteration, the influence of 8 independent loading systems is determined before the iterative procedure starts. The results of each loading system are converted to $\{C_1\}$ and $\{C_2\}$ as per above, and these vectors constitute a column of the influence value matrix $[B]$. The loading systems used, in order, are:

Col. 1- $P_n = 1 - \xi^2(3-2\xi)$

2- $P_\ell = 1 - \xi$

3- $P_n = -\xi\,(1-2\xi + \xi^2)/L$

4- $P_c = 1 - \xi$

5- $P_n = \xi^2(3-2\xi)$

6- $P_\ell = \xi$

7- $P_n = \xi^2(1-\xi)/L$

8- $P_c = \xi$

where
- L = length of segment
- ξ = S/L
- S = coordinate along meridian
- P_n = normal pressure
- P_ℓ = longitudinal load
- P_c = circumferential load

(* See Ref. (4)

Thus $\{C_1\}$ and $\{C_2\}$ can be generated by multiplying the $|B|$ matrix by the pressure intensity at the two ends. Assuming that the distribution along the segment is not radically different from functions used to generate the $|B|$ matrix, these load vectors will be reasonably accurate. In the eigenvalue problem, the pressures are functions of the displacements and mass. Since the displacements must be continuous and smooth, then the pressures will be well behaved and the above approximation is very good.

$$\{C_1\} = |B_1| \{P_1\} + |B_2| \{P_2\}$$

$$\{C_2\} = |B_3| \{P_1\} + |B_4| \{P_2\}$$

where $\{P_1\} = \rho\omega^2 h \begin{bmatrix} W_1 \\ U_{\phi 1} \\ \beta_{\phi 1} \\ U_{\theta 1} \end{bmatrix}$, etc.

where ρ = mass density

h = thickness

ω = circular frequency

The iterative procedure follows the cycle.

1. Take the last calculated deflections as assumed deflections for next try (to start with, assume $W = 1$, $U_\phi = \beta_\phi = U_{\theta_2} = 0$, everywhere).

2. Calculate load vector for each segment, $\{C\} = \rho h [B] \{U_i\}$

3. Solve for new displacements, $\{U_{i+1}\}$

4. Scale new displacements so that largest value is one.

5. Compare new displacements with assumed displacements; stop if

$$\sum_{j=1}^{M} \left| U_{j,i+1} - U_{j,i} \right| < \epsilon M, \text{ where } \epsilon \text{ is}$$

error criterion supplied by user and M is total number of displacements; otherwise continue iterating with step 1.

The above procedure converges to the lowest longitudinal mode. In order to find higher modes the lower modes must be swept out during each iteration. The sweeping is done between steps 3 and 4. The lower modes are eliminated by utilizing the well known fact that

$$\{\phi_\ell^T\} [M_n] \{\phi_m\} = 0 \qquad \ell \neq m$$

where ϕ_m = displacement vector of mth mode, etc.

Since $[B_n]$, the assembled influence matrix, is approximately proportional to $[M_n]$, it is used in place of $[M_n]$. If the total number of modes already found is K, then after $\{U_{i+1}\}$ has been obtained in step 3, it is assumed that

$$\{U_{i+1}\} = \{\bar{U}_{i+1}\} + \sum_{m=1}^{K} \alpha_m \{\phi_m\}$$

where $\bar{U}_{i+1}$ = vector which is orthogonal to $\{\phi_m^T B_n\}$, m = 1,K

Premultiplying by $\left\{\phi_m^T\right\}\left[B_n\right]$ yields

$$\alpha_m = \frac{\phi_m^T B\ U_{i+1}}{\phi_m^T B\ \phi_m} \qquad m = 1,K$$

and $\left\{\overline{U}_{i+1}\right\}$ can be determined.

Then $\left\{\overline{U}_{i+1}\right\}$ is scaled in step 4, and the process continues.

5. Direct Time Integration Solutions (Prog. 1374)

The direct time integration subroutine uses the same influence matrix $|B|$ that is used in the eigenvalue subroutine. To determine the accelerations at a given time, Houbolts'* scheme is used

$$\{\ddot{U}_i\} = 2\{U_i\} - 5\{U_{i-1}\} + 4\{U_{i-2}\} - \{U_{i-3}\} / \Delta t^2$$

where Δt = time increment.

Thus for each segment,

$$\{\bar{F}_{1,i}\} = \frac{\rho h}{\Delta t^2}\{2U_{1,i} - 5\,U_{1,i-1} + 4\,U_{1,i-2} - U_{1,i-3}\} + \{P_{1,i}\}$$

where $\{P_{1,i}\}$ = applied pressure intensity at end 1 at time t_i

ρh = mass/sq. inch of surface

Then

$$|C_1| = |B_1||\bar{F}_{1,i}| + |B_2||\bar{F}_{2,i}|$$

$$|C_2| = |B_3||\bar{F}_{1,i}| + |B_4||\bar{F}_{2,i}|$$

Having obtained the new load vector $|C|$, the displacements and forces at time t_i are solved for in the usual manner.

* See Reference (5)

CHICAGO BRIDGE & IRON COMPANY

Location Oak Brook Engineering

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(1) A. Kalnins, "Analysis of Shells of Revolution Subjected to Symmetrical and Non-Symmetrical Loads", Trans. of the ASME, Journal of Applied Mechanics, Sept. 1964, pp 467-476.

(2) H. Kraus, Thin Elastic Shells, John Wiley & Sons, Inc., New York, 1967. pp 419-430

(3) A. Kalnins, "On Free and Forced Vibration of Rotationally Symmetric Layered Shells", Trans. of the ASME, Journal of Applied Mechanics, Dec. 1965, pp. 941-943.

(4) W. C. Hurty & M. F. Rubenstein, Dynamics of Structures Prentice-Hall Book Company, Inc., Englewood Cliffs, New Jersey, 1964.

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PROGRAM E0781A

The Shells of Revolution Program is the Chicago Bridge & Iron Company Program E0781A. The program calculates the stresses and displacements in thin-walled elastic shells of revolution when subjected to static edge, surface and/or temperature loads with arbitrary distribution over the surface of the shell. The geometry of the shell must be symmetric, but the shape of the median is arbitrary. It is possible to include up to three branch shells with the main shell in a single model. In addition, the shell wall may consist of four layers of different orthotropic materials, and the thickness of each layer and the elastic properties of each layer may vary along the median.

Program E0781A numerically integrates the eight ordinary first order differential equations of thin shell theory derived by H. Reissner. The equation are derived so that the eight variables which appear on the boundaries of the axially symmetric shell are chosen so that the entire problem can be expressed in these fundamental variables.

Chicago Bridge & Iron Company has extensively revised the Kalnins Program. The program has been altered so that a 4 x 4 force-displacement relation can be used as a boundary condition as an alternative to the usual procedures of specifying forces or displacements. This force-displacement relation can be used to describe the forces at the boundary in terms of displacements at the boundary, or the displacements at the boundary in terms of forces or some compatible combination of the two. In this manner, it is possible to study (CONTINUED) the behavior of a large complex structure. It is also possible to introduce a "spring matrix" at the end of any part of the stress model. This matrix must be expressed in the form, force = spring matrix x displacement. In this manner it is possible to model the restraint of the sand cushion in the transition zone at the point of the embedment. In addition to the above changes, the Kalnins Program has been modified to increase the size of the problem that can be considered and to improve the accuracy of the solution.

PROGRAM 781

METHOD OF MODELING VERTICAL STIFFENERS

N = No. of vertical stiffeners around

E = Modulus of elasticity

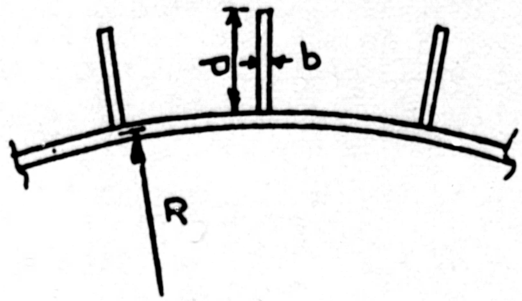

FIG. 1

The shell shown in Fig. 1 is modeled using 2 layers. The inside layer represents the shell and, therefore, has the normal isotropic material properties. The outer layer, on the other hand, is described as an orthotropic material having the following properties.

$$t_2 = d$$

$$E\phi_2 = \frac{bN}{2\pi R} E$$

$$E\theta_2 = 0$$

$$G\phi\theta_2 = 0$$

where

t_2 = Thickness of outer layer

$E\phi_2$ = Modulus of elasticity of outer layer in longitudinal direction

$E\theta_2$ = Modulus of elasticity of outer layer in circumferential direction.

$G\phi\theta_2$ = Shear modulus of outer layer.

SUBJECT: Prog. E781 PROGRAM DESCRIPTIONS	MADE BY: GJP	DATE: 10/73	CHKD BY: [illegible]	DATE: 10/11/74	REV	BY	CHKD	DATE	CHANGE NO. 72 4333	SHT CB-17 OF