# SCIENTIFIC NOTEBOOK # 918E
# Review of Rock Bulking Factor

by

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January 23, 2008

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## 1. Initial Entries

**CNWRA Scientific Notebook No. 918E**

**Title:** Bulking Factor Review

**Personnel Responsible:** R.S. Read (Contractor), G.I. Ofoegbu (CNWRA staff), F. Ferrante (CNWRA staff). Literature review and data analysis to be undertaken by R.S. Read and G.I. Ofoegbu. Statistical analysis of compiled data to be undertaken by F. Ferrante. Contractor work to be completed under SWRI subcontract 499883N (Task 09). Initials in subsequent entries are RSR (R.S. Read), GIO (G.I. Ofoegbu) and FF (F. Ferrante).

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**Required Qualifications:** The required qualifications to conduct the literature review and analysis of the compiled data includes experience in rock mechanics and geotechnical engineering, along with experience in statistical analysis of rock property data.

**Objectives and Approach:** To review bulking factor information from literature to determine the values of bulking factor applicable to the analysis of post-closure degradation of emplacement drifts at a potential Yucca Mountain nuclear waste repository. The review will examine the available bulking factor estimates, the approach used, and the geologic and engineering conditions of the estimate. The scientific notebook is intended to record any analysis of individual data sets and of the compiled data from the literature review.

**Software:** Software to be used for the analysis and reporting includes Microsoft EXCEL 2003 (Version11.8169.8172) SP3 and Microsoft WORD 2003 (Version 11.8169.8172) SP3. Other specialized software that may be required for statistical analysis, if needed, will be identified in subsequent entries in this scientific notebook.

**Hardware:** Computer hardware to be used for the analysis includes an IBM-compatible personal computer running Microsoft Windows Vista Business, Version 6.0.6000 Build 6000, or equivalent operating system.

**Issue Date:** This scientific notebook was issued January 22, 2008 once it was determined that there were calculations that required documentation.

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End of entries: *RSR 2008 Jan 23*

## 2. In-Process Entries

There are three in-process entries associated with this Scientific Notebook. The first is related to the calculation of caving height above emplacement drift excavations. The second is related to deriving curve-fit parameters to data published by Pappas and Mark (1993). The third is related to converting published data from Church (1981) and Hartman (1992) into bulking factor related quantities for tables. Calculations are contained in the attached EXCEL workbook entitled "Bulking Calculations.xls" and are described in the following sections.

Bulking Calculations.xls

### 2.1 Calculation of caving height based on two simplified models

Two scenarios were considered by OCRWM (2003) to estimate the extent of the caved zone above an emplacement drift. The first assumed a vertical chimney of collapsed rock directly above the circular drift. The second considered a caving mechanism originally proposed by Terzaghi where a volume of width $W$ and height $H$ above the drift collapses along slip lines angled at 45+$\phi$/2 degrees to vertical and tangent to the drift wall. The general configuration of the caved zone in relation to a cylindrical drift is shown in Figure 1.

Closed-form solutions for the height of the caved zone above the drift, and for the load on the drip shield and waste package, were provided by OCRWM (2003) based on a slightly different in-drift configuration of the waste package relative to that in Figure 1. For bulking factors ranging from 20 to 40%, cave height above the drift ranged from 2.6 to 1.2 times the drift diameter for the chimney (or piping) failure mechanism, and from 1.8 to 0.9 times the drift diameter for the Terzaghi type caving method. The corresponding loads on the drip shield ranged from 0.33 to 0.15 MPa for the chimney (or piping) failure mechanism, and from 0.23 to 0.12 MPa for the Terzaghi type caving mechanism. The implications of bulking factors less than 20% were not considered in this analysis. Comparative numerical analyses using continuum and discrete element codes indicated that the idealized closed-form solutions were conservative (i.e., predicted greater caved zone height and resulting load than the numerical models).

To extend the findings on caved zone height and load on the waste package for bulking factors less than 20%, closed-form solutions were developed based on the geometries shown in Figure 1. These relations differ slightly from those in OCRWM (2003) in that they account for the presence of ballast and the possibility of drift collapse prior to drip shield installation. The height $H$ of the caved zone above the drift is a function of the bulking factor $B$ and the size of the waste container in the drift. Assuming chimney collapse, the relation between normalized cave height and bulking factor is given by:

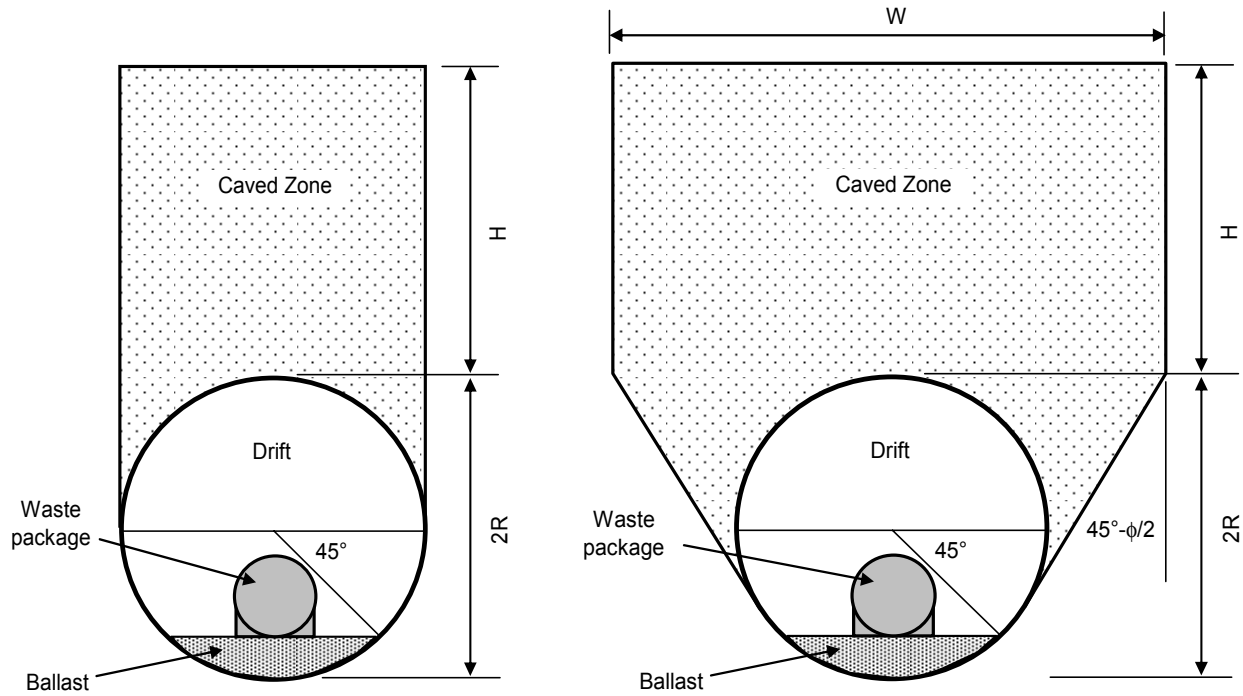



**Figure 1: Models of caving above an emplacement drift at Yucca Mountain. The chimney collapse model (left) is a special case of the more general Terzaghi collapse model (right) which considers friction angle (ϕ) in the calculation of caved zone height.**

$$\frac{H}{2R} = \left(\frac{\pi - 4}{8}\right) + \left[\frac{1}{16B}\left(3\pi + 2 - 2\pi a^2 - 8a^2\right)\right] \quad (1)$$

where $a$ is the ratio of waste container diameter to drift diameter $2R$. The ballast underlying the waste container and its cradle is assumed to fill the bottom of the drift to the point of intersection on the drift perimeter of a line through the drift centre angled down at 45°. The cradle supporting the waste container is assumed to be the same width as the container diameter.

For the more general case of the Terzaghi collapse model, the relation between normalized cave height and bulking factor is given by:

$$\frac{H}{2R} = \frac{-(W+b)h + \pi R^2 + 2y\sqrt{R^2 - y^2} - 2R^2 \operatorname{Arcsin}(y/R)}{4RW} + \left[\frac{R}{8BW}\left(3\pi + 2 - 2\pi a^2 - 8a^2\right)\right] \quad (2)$$

where

$$b = 2R\cos\theta$$
$$y = R\sin\theta$$
$$h = R(1+\sin\theta)$$
$$\theta = 45 + \phi/2$$
$$W = b + 2h\tan\theta$$

Analyses of potential rockfall impacts on waste packages (DOE 2001) considered a 21-PWR Absorber Plate waste package design, corresponding to an $a$ value of 0.3. Based on this value, the inverse relations between normalized cave height and bulking factor for a range of friction angles are shown in Figure 2. Note that the chimney collapse case is identical to a Terzaghi model case with a friction angle of 90°. It is evident from Figure 2 that large values of cave height are possible for rock with a bulking factor less than 20%.

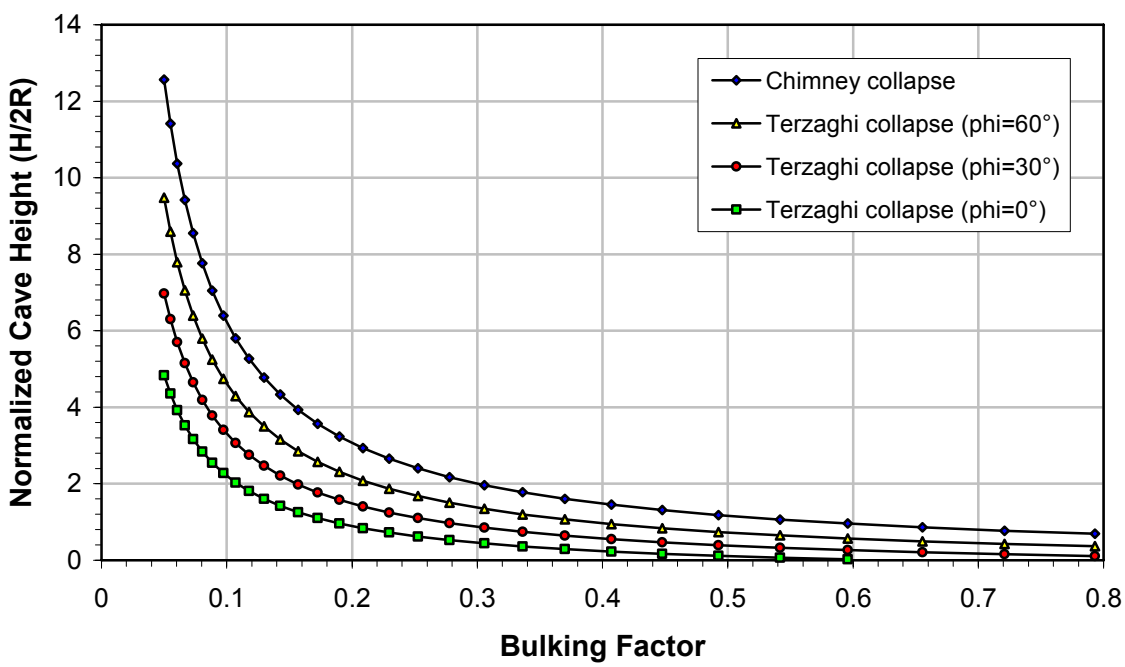



**Figure 2: Relations between normalized cave height and bulking factor for a range of possible friction angle (ϕ) values. Minimum bulking factor shown is 0.05 (i.e., 5%).**

The static vertical pressure applied to the waste package by gravity loading of the broken rock is calculated from the following equation:

$$p = (H + F)\frac{\rho g}{1 + B} \tag{3}$$

where $H$ is cave height, $F$ is the fall height from the drift crown to the top of the waste package, $\rho$ is mass density of the in situ rock, $g$ is acceleration due to gravity, and $B$ is bulking factor.

Analyses of potential rockfall impacts on waste packages (DOE 2001) considered a fall height of 3.1 m from a drift 5.5 m in diameter. Using these values, and assuming an average in situ bulk density of 2043 kg/m$^3$ for in situ tuff at Yucca Mountain based on bulk density analysis of core samples, the relation between static pressure on the waste package and bulking factor is shown in Figure 3. Note that the pressure values shown in Figure 3 do not account for possible dynamic impact loading due to rockfall. The key point from Figure 3 is that, for rock with low bulking factor, substantial static pressure may be imposed on the waste package.

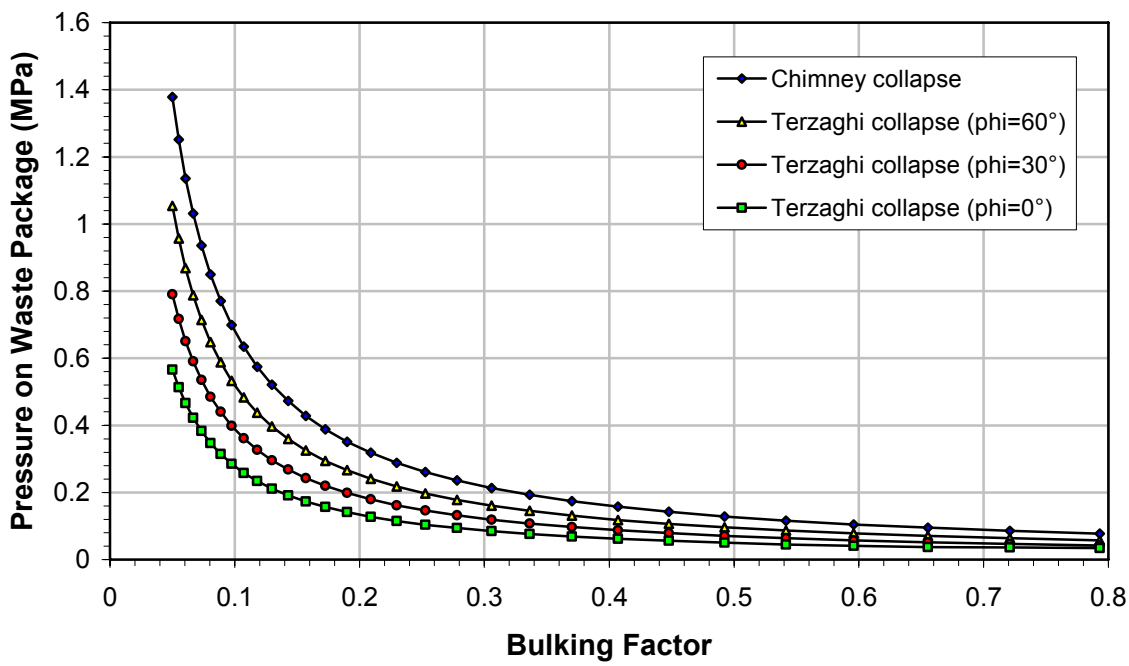



**Figure 3: Relations between vertical pressure on waste package and bulking factor for a range of possible friction angle (ϕ) values. Minimum bulking factor shown is 0.05 (i.e., 5%).**

## 2.2 Calculation of curve-fit parameters to Pappas and Mark (1993) data

Statistical analysis of the test results (Pappas and Mark, 1993) suggested that void ratio, secant modulus, tangent modulus, and most of the shape ratios were significantly affected by rock type, whereas changes in the maximum particle size did not affect any of the variables. Gradation curve shape was also found to affect the compressibility of the simulated gob material. Multiple regression analysis was used to develop relations between the various parameters. Bulking factor, tangent modulus and secant modulus were each found to be a function of rock strength and thickness-to-width shape ratio. The following relation describes the function associated with bulking factor:

$$B = ax_1 + bx_2 + c \tag{4}$$

where $x_1$ is rock strength (in MPa), $x_2$ is the thickness-to-width shape ratio (dimensionless), and $a$, $b$, and $c$ are derived factors that are dependent on stress level. Values for these factors are shown in Table 1. Note that the values for $a$ were adjusted from the original published values to accept rock strength expressed in MPa. Likewise, the values for $c$ were adjusted to express bulking factor as the change in volume associated with broken rock relative to the original in situ volume of rock rather than the ratio of broken rock volume to in situ rock volume.

**Table 1: Derived Factors Relating Bulking Factor to Rock Strength and Shape Ratio**

| Overburden Stress | | Multiple Regression Analysis Factors | | |
|---|---|---|---|---|
| (psi) | (MPa) | *a* | *b* | *C* |
| 400 | 2.758 | 2.669E-03 | 0.267 | 0.160 |
| 600 | 4.137 | 2.944E-03 | 0.274 | 0.060 |
| 800 | 5.516 | 2.712E-03 | 0.262 | 0.040 |
| 1000 | 6.895 | 2.683E-03 | 0.269 | -0.008 |
| 1500 | 10.342 | 2.321E-03 | 0.209 | 0.000 |
| 2000 | 13.790 | 2.176E-03 | 0.221 | -0.037 |
| 2500 | 17.237 | 1.973E-03 | 0.247 | -0.069 |

Note: Factors adjusted for rock strength to be expressed in MPa

To develop a general relation for bulking factor as a function of overburden stress, rock strength, and thickness-to-width ratio, curve-fitting using linear regression for parameters $a$ and $b$, and a Power Law for parameter $c$ was undertaken. The resulting expression for bulking factor is given as:

$$B = (a_1\sigma_v + b_1)x_1 + (a_2\sigma_v + b_2)x_2 + \left(a_3\sigma_v^{\,b_3} + c_3\right) \tag{5}$$

where $\sigma_v$ is the overburden stress (MPa), $x_1$ is rock strength (MPa), $x_2$ is the thickness-to-width ratio (dimensionless), and the curve-fit coefficients are as follows:

$$a_1 = -6.132 \cdot 10^{-5} \quad a_2 = -3.062 \cdot 10^{-3} \quad a_3 = 0.691$$
$$b_1 = 3.028 \cdot 10^{-3} \quad b_2 = 2.764 \cdot 10^{-1} \quad b_3 = -1.023$$
$$c_3 = -0.089$$

Applying Equation 5 over the range of shape ratios and rock strength values considered by Pappas and Mark (1993), a family of curves was generated (Figure 4). While these curves are based on laboratory tests for three specific rock types with known gradation, the results indicate a non-linear relation between bulking factor and overburden stress. It is evident that bulking factor is strongly affected by rock strength and to a lesser extent, rock fragment shape. For rocks with relatively low bulking factors under gravity loading in piles of loose material, significant reductions in bulking factor are possible if there is a load transfer from the overlying overburden to the accumulated rock debris in a drift.

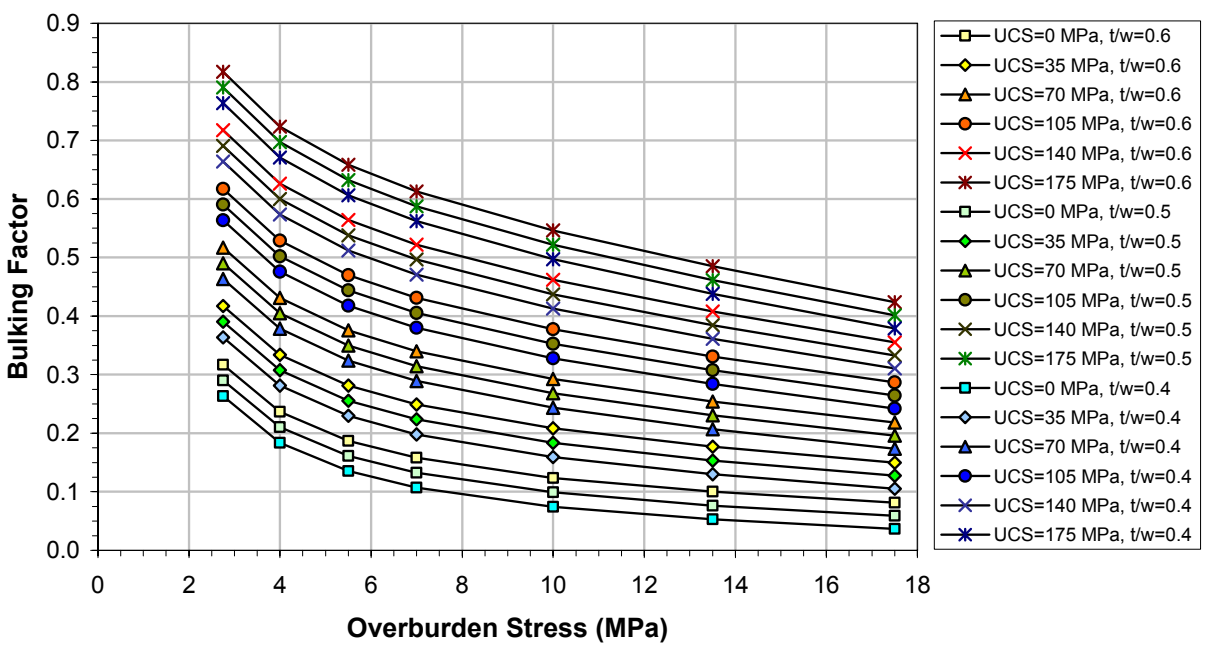



**Figure 4: Relation between overburden stress and bulking factor for a range of unconfined compressive strength and thickness-to-width ratios. Results are based on laboratory testing of simulated gob material from three longwall coal mines, including shale, weak sandstone, and strong sandstone (after Pappas and Mark, 1993)**

## 2.3 Calculation of bulking factors from published data

Bulk density data published by Church (1981) and Hartman (1992) were converted to SI units. Bulking factors related were derived from the converted data for the data review report. The original source data and the converted data are contained in the EXCEL workbook "Bulking Calculations.xls" attached to the Scientific Notebook.

## 2.4 Linear Averaging with Individual Uniform Distribution Assumption

The following MATLAB file 'weightaverageunidata.m' is developed in order to obtain a aggregate of the bulking factor ranges reported in literature, following a simple, widely used approach (Ayyub, 2001). The literature values were provided by G. Ofoegbu via Excel Spreadsheet. The approach is based on a weighted averaging technique, which combines the individual expert information based on probabilistic considerations.

The following assumptions are made:

- Given the ranges provided and the level of confidence in each data input, a probability distribution $f_i(B)$ with mean $\mu_i$ and standard deviation $\sigma_i$ is assumed for each set $i$ (which varies from 1 to $N$ = 50, the total number of expert inputs. In this program, uniform distributions with lower and upper limits $a_i$ and $b_i$ are chosen. See Figure 5.

- A weight $w_i$ is assigned based on the 'High', 'Medium', and 'Low' categories. 'Medium' is assumed to correspond to twice as much relevance as a 'Low' rating (i.e., $w_{medium} = 2\ w_{low}$). Similarly, a 'High' mark is assumed to represent three times the level of relevance of a 'Low' mark (i.e., $w_{high} = 3\ w_{low}$). See Figure 5.

- When only one value is provided (instead of a range), it is assumed that these entries represent the mean value of a probability distribution with unknown dispersion. The resulting aggregate can be tested by varying the ratio between the standard deviation and the mean (i.e., the coefficient of variance COV = $\sigma_i/\mu_i$), from which the range parameters $a_i$ and $b_i$ can be deduced.

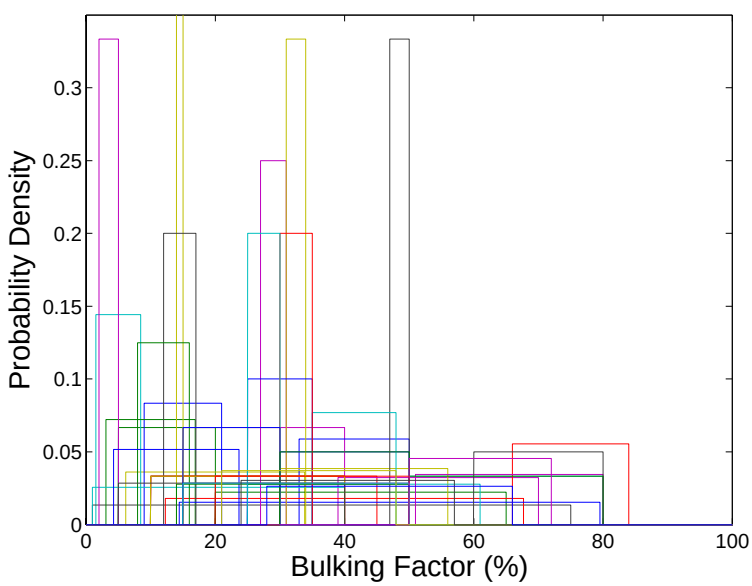
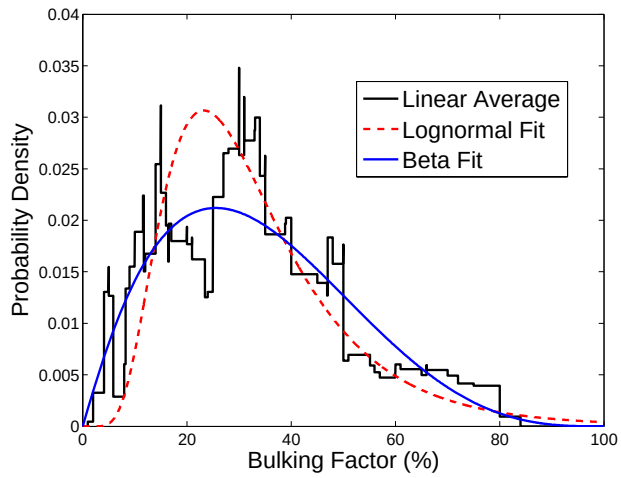
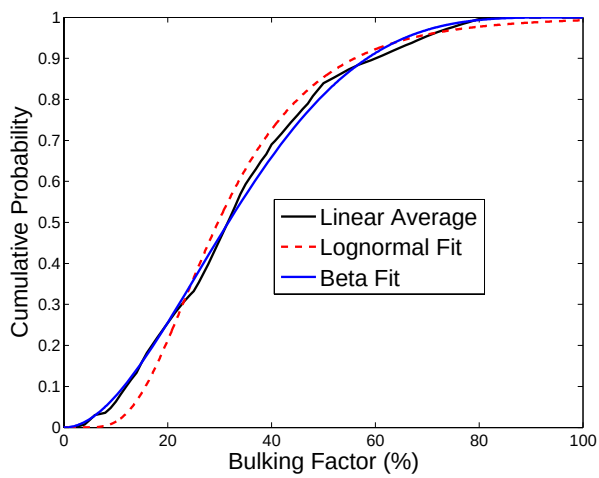



**Figure 5: Range of bulking factor probability density functions assuming uniform distributions**

Performing the linear weighted averaging with the assumptions above regarding probability distributions and weight values results in the aggregate distribution $f(B)$ shown in Figure 6(a), along with the corresponding cumulative distribution $F(B)$ in Figure 6(b). In both 7(a) and (b), a COV of 10% is assumed for all the single point values, following the previously described approach. A lognormal and beta fitted distributions are also shown, derived from the calculated mean $\mu$ = 34% and standard deviation $\sigma$ = 18% of $f(B)$. The corresponding input parameters for the beta distribution shown in Figure 6 are α = 2 and β = 4.




**(a)**




**(b)**

**Figure 6: Aggregate (a) probability density *f*(*B*) and (b) cumulative distribution *F*(*B*)**

## *Main MATLAB file 'weightaverageunidata.m'*

```
clear all

gofoegbu;

I = find(data(:,1) == data(:,2));
COV = 0.1;
data(I,1) = data(I,1) - sqrt(3)*(data(I,1)*COV);
data(I,2) = data(I,2) + sqrt(3)*(data(I,2)*COV);

A = data(:,1)';
B = data(:,2)';
wc = data(:,3)';

a = 0;b = 100;
x = a:0.001:b;
n = length(A);

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';

figure;
plot(x,P);hold on;
axis([0 100 0 0.35])
xlabel('Bulking Factor (%)')
ylabel('Probability Density')

save run A B w n

figure;
ps = myfunsuni(x);
plot(x,ps,'k');hold on
axis([0 100 0 0.045])

my = quad(@myfunsmeanuni,a,b)
mean((A+B)/2)
sy = sqrt(quad(@myfunsstduni,a,b) - quad(@myfunsmeanuni,a,b)^2)
sqrt(sum(((B-A)/sqrt(12)).^2))
beta = sqrt(log((sy/my)^2 + 1));
medy = my*exp(-0.5*beta^2);
logy = lognpdf(x,log(medy),beta);
plot(x,logy,'r--','Linewidth',2);

[X,F,f,x] = betagen(my,sy,0,100,10);
plot(x,f,'b','Linewidth',2);

legend('Linear Average','Lognormal Fit','Beta Fit')
xlabel('Bulking Factor (%)')
ylabel('Probability Density')

figure;
for i = 1:100;
    pcdf(i) = quad(@myfunsuni,a,i);
end
plot(1:100,pcdf,'k');hold on
logcy = logncdf(x,log(medy),beta);
plot(x,logcy,'r--','Linewidth',2);
plot(x,F,'b','Linewidth',2);
xlabel('Bulking Factor (%)')
```

```
ylabel('Cumulative Probability')
legend('Linear Average','Lognormal Fit','Beta Fit')
```

## *Input MATLAB file with data 'gofoegbu.m'*

```
data = [30  50  2
5   20  3
10  40  3
50  80  1
25  40  2
20  20  3%
10  40  3
33  50  2
20  65  3
15  45  3
5   5   2
39  70  2
30  56  2
1   75  2
9   21  3
10  10  3%
40  40  2%
1   40  1
2   5   1
14  15  1
12  17  1
47  47  2%
8   16  3
10  40  3
25  61  2
27  31  2
31  34  2
47  50  2
14  14  2%
14  50  1
66  84  2
35  48  3
51  80  2
10  40  3
60  80  3
25  35  3
50  80  3
30  35  3
25  30  3
15  50  3
15  30  3
5   40  3
15  30  3
30  50  3
20  50  3
15  50  3
50  72  2
21  48  2
24  57  2
28  66  2];
```

## *MATLAB function that calculates linear average 'myfunsuni.m'*

```
function y = myfunsuni(x)

load run

n = length(w);

W = repmat(w',1,length(x));

P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';

y = sum((P.*W));
```

## *MATLAB function that calculates first moment of linear average 'myfunsmeanuni.m'*

```
function y = myfunsmeanuni(x)

load run

n = length(w);

W = repmat(w',1,length(x));

P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';

y = (x.^1).*sum((P.*W));
```

## *MATLAB function that calculates second moment of linear average 'myfunsstduni.m'*

```
function y = myfunsstduni(x)

load run

n = length(w);

W = repmat(w',1,length(x));

P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';

y = (x.^2).*sum((P.*W));
```

## *MATLAB function used to calculate beta distribution parameters 'betagen.m'*

```
function [X,F,f,x] = betagen(mu,sigma,a,c,N);

x = a:((c-a)/1000):c;

A = (mu-a)*(-c*mu + a*c - mu*a + sigma^2 + mu^2)/((a-c)*sigma^2);
B = (a*mu^2 - 2*a*c*mu + a*c^2 - mu^3 - mu*sigma^2 + c*sigma^2 - mu*c^2 + 2*c*mu^2)/((a-
c)*sigma^2);

f = betapdf((x-a)/(c-a),A,B)/(c-a);
F = betacdf((x-a)/(c-a),A,B);

X = betarnd(A,B,1,N)*(c-a) + a;
```

[FF, January 24th, 2008]

## 2. 5 Linear Averaging with Individual Normal Distribution Assumption

A similar program to the one previously described is developed in MATLAB ('weightaverageunidata.m') for the normally distributed individual data assumption. Aside from the distribution change, the following assumptions are adjusted from the previous case:

- The ranges provided are defined to represent the 5$^{th}$- and 95$^{th}$-percentile values (from which mean and standard deviation can be derived). See Figure 7.

- When only one value is provided (instead of a range), it is assumed that the mean is now that of a normal distribution with unknown COV. See Figure 7.

Figure 7 and 9 show the results (with COV = 20% for single data values) of this analysis. The following probability density areas for different bulking factors are shown next for this aggregate distribution:

P(B < 10%) = 8%
P(B < 15%) = 16%
P(B < 60%) = 91%
P(10% < B < 90%) = 92%
P(B < 90%) ≈ 100%

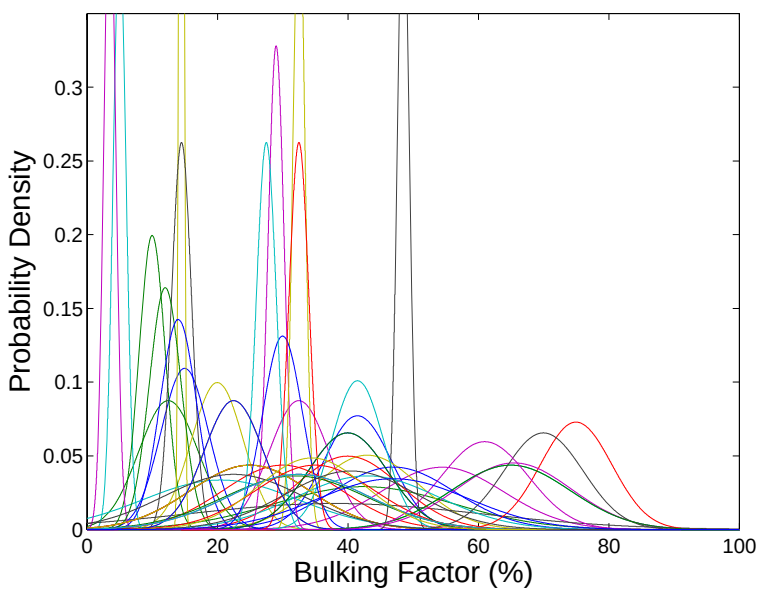
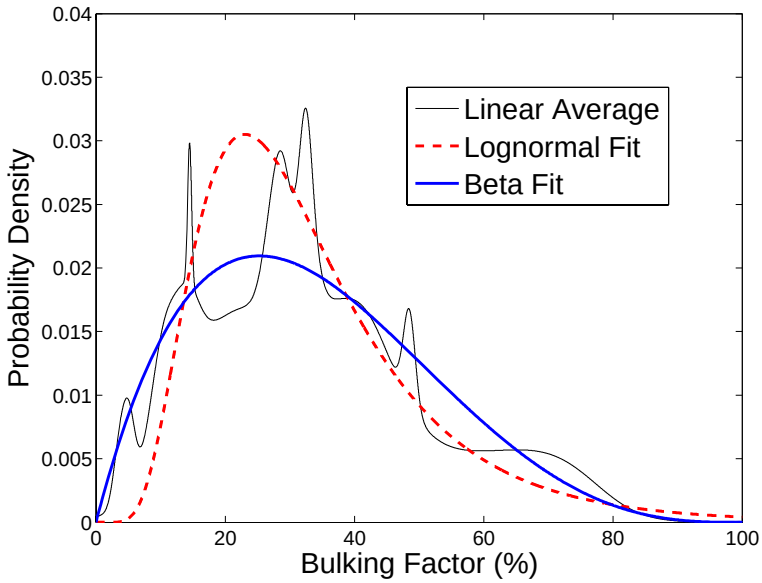
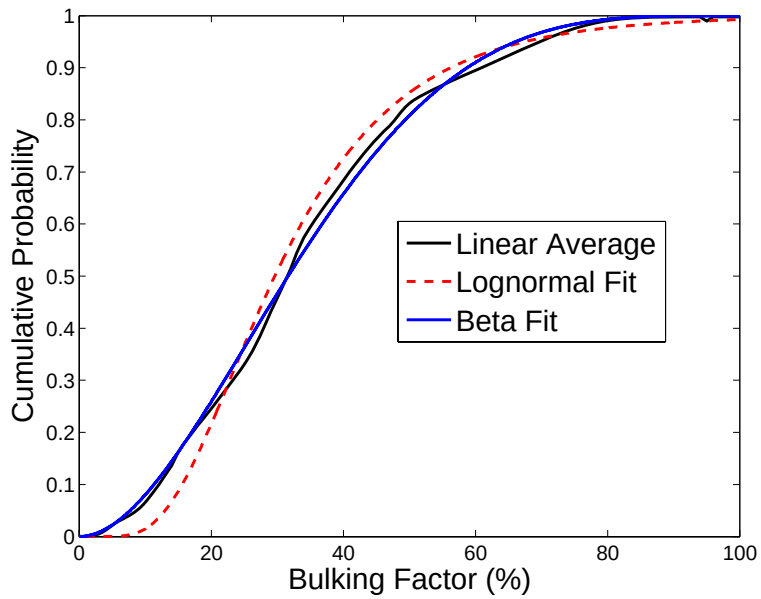



**Figure 7 Range of bulking factor probability density functions assuming normal distributions**




**(a)**




**(b)**

**Figure 8: Aggregate (a) probability density *f*(*B*) and (b) cumulative distribution *F*(*B*)**

## *Main MATLAB file 'weightaverageunidata.m'*

```
clear all

gofoegbu;

I = find(data(:,1) == data(:,2));
COV = 0.2;
data(I,1) = norminv(0.05,data(I,1),data(I,1)*COV);
data(I,2) = norminv(0.95,data(I,2),data(I,2)*COV);

p05a1 = data(:,1)';
p95a1 = data(:,2)';
wc = data(:,3)';

mu = (p95a1+p05a1)/2;
s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));

a = 0;b = 100;
x = a:0.001:b;
n = length(p05a1);

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';

figure;
plot(x,P);hold on;
axis([0 100 0 0.35])
xlabel('Bulking Factor (%)')
ylabel('Probability Density')

save run p05a1 p95a1 w

figure;
ps = myfuns(x);
plot(x,ps,'k');hold on
axis([0 100 0 0.045])

my = quad(@myfunsmean,a,b)
mean(mu)
sy = sqrt(quad(@myfunsstd,a,b) - quad(@myfunsmean,a,b)^2)
sqrt(sum(s.^2))

beta = sqrt(log((sy/my)^2 + 1));
medy = my*exp(-0.5*beta^2);
logy = lognpdf(x,log(medy),beta);
plot(x,logy,'r--','Linewidth',2);

[X,F,f,x] = betagen(my,sy,0,100,10);
plot(x,f,'b','Linewidth',2);

legend('Linear Average','Lognormal Fit','Beta Fit')
xlabel('Bulking Factor (%)')
ylabel('Probability Density')

figure;
for i = 1:100;
    pcdf(i) = quad(@myfuns,a,i);
end
plot(1:100,pcdf,'k');hold on
```

```
logcy = logncdf(x,log(medy),beta);
plot(x,logcy,'r--','Linewidth',2);
plot(x,F,'b','Linewidth',2);
plot(x,F,'b','Linewidth',2);
xlabel('Bulking Factor (%)')
ylabel('Cumulative Probability')
legend('Linear Average','Lognormal Fit','Beta Fit')
```

## *MATLAB function that calculates linear average 'myfuns.m'*

```
function y = myfuns(x)

load run

mu = (p95a1+p05a1)/2;
s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));

n = length(w);

W = repmat(w',1,length(x));

P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';

y = sum((P.*W));
```

## *MATLAB function that calculates first moment of linear average 'myfunsmean.m'*

```
function y = myfunsmean(x)

load run

mu = (p95a1+p05a1)/2;
s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));

n = length(w);

W = repmat(w',1,length(x));

P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';

y = (x.^1).*sum((P.*W));
```

## *MATLAB function that calculates second moment of linear average 'myfunsstd.m'*

```
function y = myfunsstd(x)

load run

mu = (p95a1+p05a1)/2;
s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));

n = length(w);

W = repmat(w',1,length(x));

P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';

y = (x.^2).*sum((P.*W));
```

[FF, January 25th, 2008]

## 2.6 Comparison between Linear Averaging Fit and TPA distribution

Figure 9 is developed using the previously described 'betagen.m' function to compare the current input beta distribution assumed in the TPA code for the bulking factor parameter $B_1$ and the beta fit for aggregate distribution derived for the normally distributed individual data inputs. In the TPA code, this distribution is constrained between 1.01 and 2, which corresponds to 1% to 100% in the plots shown here (i.e., $B = (B_1 - 1) \times 100\%$).

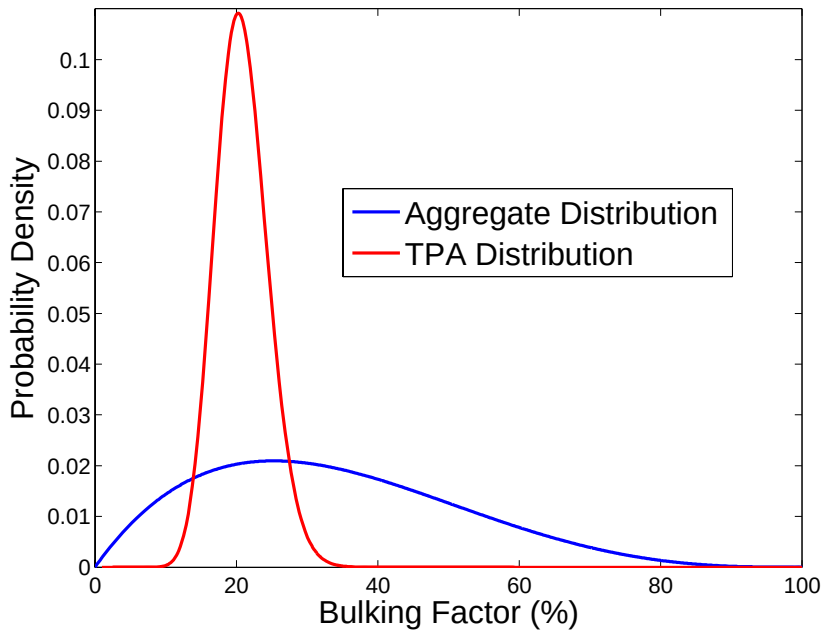



**Figure 9: Comparison between aggregate and TPA bulking factor probability densities**

[FF, January 25th, 2008]

## 2.7 New Data Set

A second data set was provided by Kevin Smart to Goodluck Ofoegbu based on an independent analysis of the collected literature. In MATLAB, this file is titled 'gofoegbu1.m' and is shown below. A comparison MATLAB script 'weightaveragecompare.m' is developed to compare TPA, the 1st data set, and the 2nd data set. The resulting figures are developed based on the use of this program.

*MATLAB file 'gofoegbu1.m'*

```
data = [30  50  1
5   20  1
10  40  2
50  80  3
25  40  3
20  20  1
10  40  3
33  50  3
20  65  3
15  45  3
5   5   1
39  70  3
30  56  1
1   75  1
9   21  2
10  10  2
40  40  1
1   40  1
47  47  2
8   16  2
10  40  2
25  61  2
27  31  1
31  34  2
47  50  3
14  14  1
66  84  2
35  48  3
51  80  2
10  40  1
60  80  2
25  35  1
50  80  3
30  35  1
25  30  1
5   40  1
15  30  2
30  50  3
20  50  2
15  50  1
50  72  3
21  28  3
31  45  2
48  66  1];
```

*MATLAB file 'weightaveragecompare.m'*

```
clear all

optiondata = input('Set (1) or (2) = ');
if optiondata == 1
    gofoegbu;
else
    gofoegbu1;
end
optiondist = input('Set Uniform (1) or Normal (2) = ');
COV = input('Set Coefficient of Variation = ');

I = find(data(:,1) == data(:,2));
if optiondist == 1
    data(I,1) = data(I,1) - sqrt(3)*(data(I,1)*COV);
    data(I,2) = data(I,2) + sqrt(3)*(data(I,2)*COV);
    A = data(:,1)';
    B = data(:,2)';
else
    data(I,1) = norminv(0.05,data(I,1),data(I,1)*COV);
    data(I,2) = norminv(0.95,data(I,2),data(I,2)*COV);
    p05a1 = data(:,1)';
    p95a1 = data(:,2)';
end
wc = data(:,3)';

if optiondist == 1
    n = length(A);
else
    mu = (p95a1+p05a1)/2;
    s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));
    n = length(p05a1);
end
a = 0;b = 100;
x = a:0.001:b;

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

if optiondist == 1
    P =
(unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';
    save run A B w n
else
    P =
(normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';
    save run p05a1 p95a1 w
```

```
end

%figure;
if optiondist == 1
    ps = myfunsuni(x);
else
    ps = myfuns(x);
end

if optiondist == 1
    my = quad(@myfunsmeanuni,a,b)
    mean((A+B)/2)
    sy = sqrt(quad(@myfunsstduni,a,b) - quad(@myfunsmeanuni,a,b)^2)
    sqrt(sum(((B-A)/sqrt(12)).^2))
else
    my = quad(@myfunsmean,a,b)
    mean(mu)
    sy = sqrt(quad(@myfunsstd,a,b) - quad(@myfunsmean,a,b)^2)
    sqrt(sum(s.^2)-mean(mu)^2)
end

[X,F,f,x] = betagen(my,sy,0,100,10);

A = 23.17;B = 92.70;
a = 1;c = 100;
ft = betapdf((x-a)/(c-a),A,B)/(c-a);

%plot(x,ft,'k','Linewidth',2);hold on
plot(x,f,'b','Linewidth',2);
set(gca,'FontSize',14)
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)
axis([0 100 0 0.115])
%axis([0 100 0 0.025])

%legend('TPA Distribution','1^{st} set Aggregate Distribution','2^{nd} set
Aggregate Distribution')
```

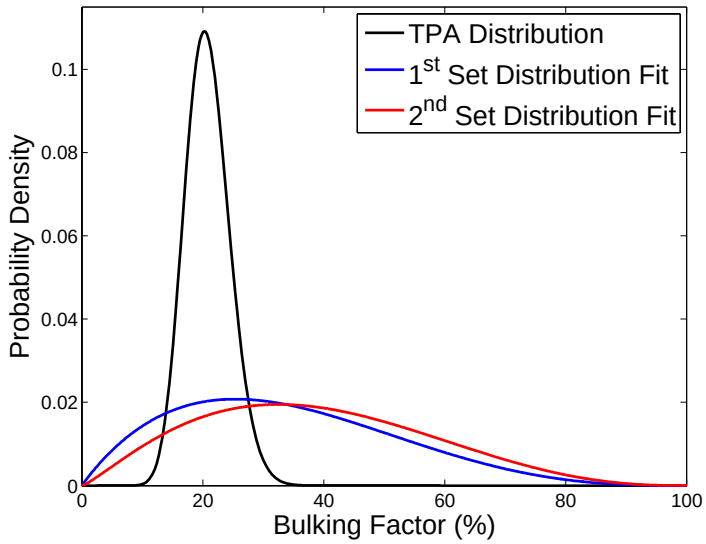



**Figure 10: Comparison between the bulking factor probability for TPA code and the aggregate distribution fits**

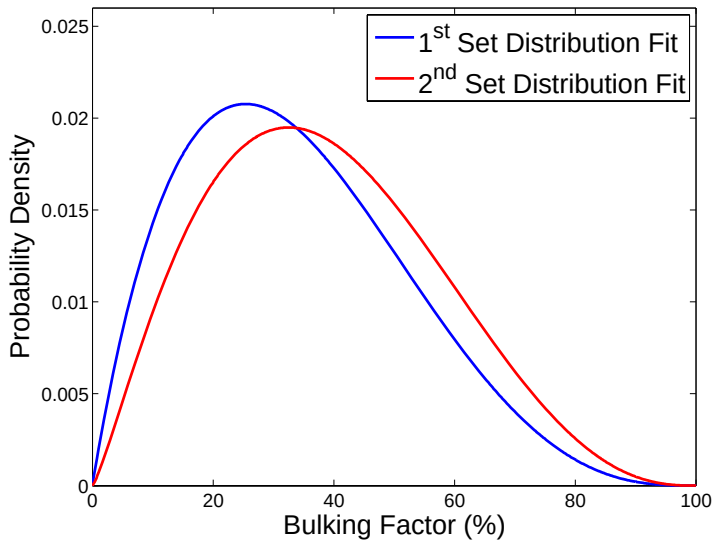



**Figure 11: Comparison between aggregate distribution fits of the 1st and 2nd data sets**

[FF, January 31st, 2008]

## 2.8 Updated Results

The results above are updated to reflect a change in one input data range (from '28 to 66' to '48 to 66'). No significant changes occur in either the numerical or the resulting figures. The previous programs were consolidated into the following main file:

*Main MATLAB file 'weightaveragedata.m'*

```
clear all

optiondata = input('Set (1) or (2) = ');
if optiondata == 1
    gofoegbu;
else
    gofoegbu1;
end
optiondist = input('Set Uniform (1) or Normal (2) = ');
COV = input('Set Coefficient of Variation = ');

I = find(data(:,1) == data(:,2));
if optiondist == 1
    data(I,1) = data(I,1) - sqrt(3)*(data(I,1)*COV);
    data(I,2) = data(I,2) + sqrt(3)*(data(I,2)*COV);
    A = data(:,1)';
    B = data(:,2)';
else
    data(I,1) = norminv(0.05,data(I,1),data(I,1)*COV);
    data(I,2) = norminv(0.95,data(I,2),data(I,2)*COV);
    p05a1 = data(:,1)';
    p95a1 = data(:,2)';
end
wc = data(:,3)';

if optiondist == 1
    n = length(A);
else
    mu = (p95a1+p05a1)/2;
    s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));
    n = length(p05a1);
end
a = 0;b = 100;
x = a:0.001:b;

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

if optiondist == 1
    P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';
    save run A B w n
else
    P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';
    save run p05a1 p95a1 w
end

figure
plot(x,P);hold on;
axis([0 100 0 0.35])
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)
set(gca,'FontSize',14)
```

```
figure;
if optiondist == 1
    ps = myfunsuni(x);
else
    ps = myfuns(x);
end
plot(x,ps,'k');hold on
axis([0 100 0 0.045])

if optiondist == 1
    my = quad(@myfunsmeanuni,a,b)
    mean((A+B)/2)
    sy = sqrt(quad(@myfunsstduni,a,b) - quad(@myfunsmeanuni,a,b)^2)
    sqrt(sum(((B-A)/sqrt(12)).^2))
else
    my = quad(@myfunsmean,a,b)
    mean(mu)
    sy = sqrt(quad(@myfunsstd,a,b) - quad(@myfunsmean,a,b)^2)
    sqrt(sum(s.^2)-mean(mu)^2)
end

beta = sqrt(log((sy/my)^2 + 1));
medy = my*exp(-0.5*beta^2);
logy = lognpdf(x,log(medy),beta);
plot(x,logy,'r--','Linewidth',2);

[X,F,f,x] = betagen(my,sy,0,100,10);
plot(x,f,'b','Linewidth',2);

set(gca,'FontSize',14)
legend('Linear Average','Lognormal Fit','Beta Fit')
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)

figure;
nint = 200;
if optiondist == 1
    for i = 1:nint;
        i
        pcdf(i) = quad(@myfunsuni,a,i);
    end
else
    for i = 1:nint;
        i
        pcdf(i) = quad(@myfuns,a,i);
    end
end
plot(1:(length(pcdf)),pcdf,'k');hold on
logcy = logncdf(x,log(medy),beta);
plot(x,logcy,'r--','Linewidth',2);
plot(x,F,'b','Linewidth',2);
plot(x,F,'b','Linewidth',2);
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Cumulative Probability','Fontsize',20)
set(gca,'FontSize',14)
legend('Linear Average','Lognormal Fit','Beta Fit','Location','NorthWest')
axis([0 100 0 1])
```

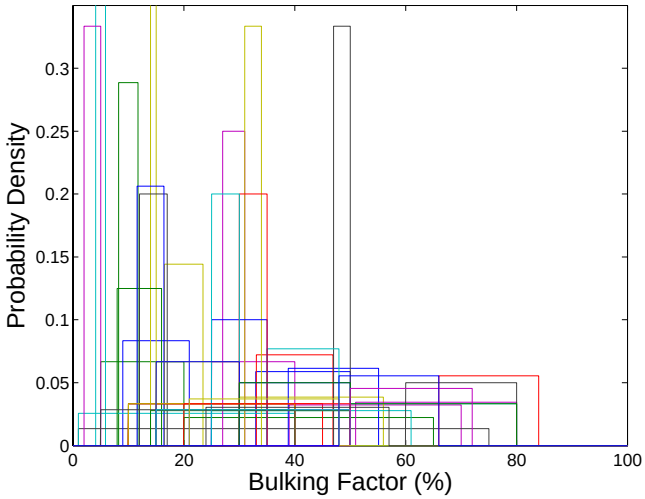



**Figure 12: Update of Figure 5**

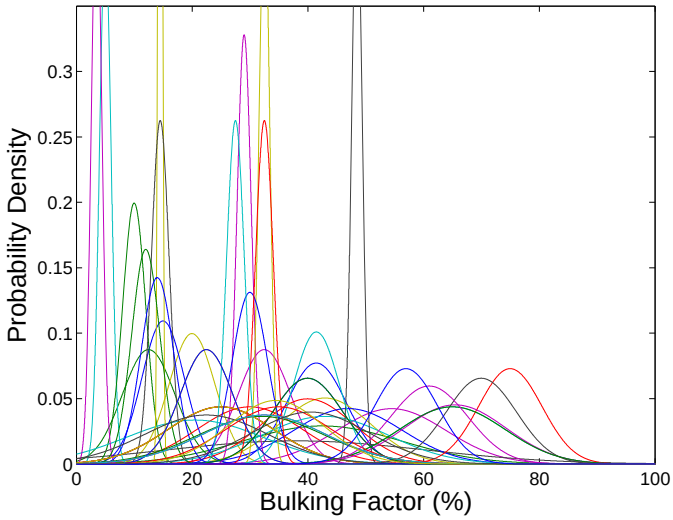
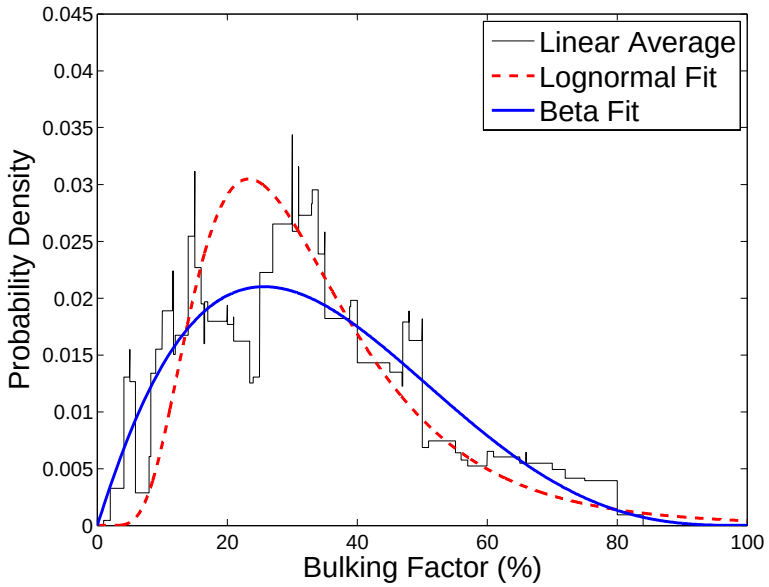
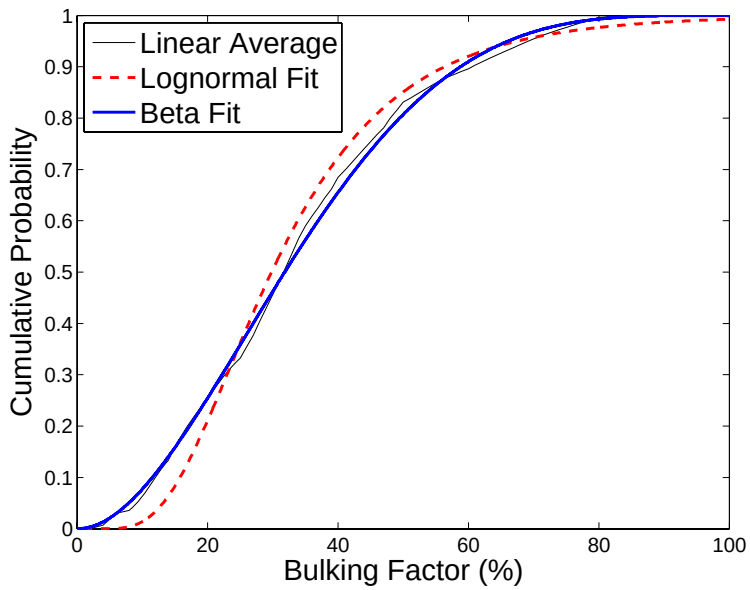



**Figure 13: Update of Figure 7**

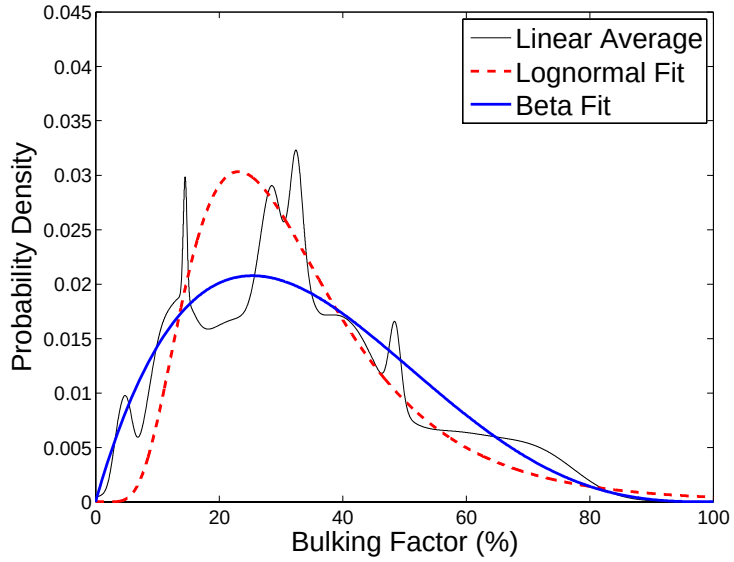



**(a)**

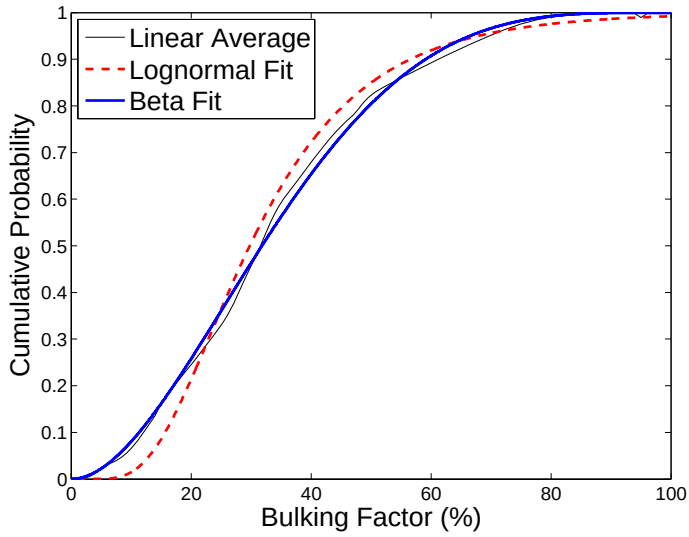



**(b)**

**Figure 14: Update of Figure 6**




**(a)**




**(b)**

**Figure 15: Update of Figure 8**

[FF, January 31st, 2008]

**New Updated Results**

The previous results were updated to reflect (i) a revised relevance estimation set of input data and (ii) an alternate method of considering dispersion uncertainty. The updated relevance estimation resulted in 15 data values considered relevant to emplacement drifts in welded tuff (see updated 'gofoegbu1.m' file). The "1,2,3" ranking approach, though giving the same results as the "0,1" approach, were abandoned because some of the geologists involved in the reported opined that several data values in Table 3-1 are inappropriate and should be given zero weight [Goodluck Ofoegbu, February, 19, 2008]. The alternate dispersion method for the individual uniform distribution assumption, resulted in a range being developed around the mean with a constant value (i.e., $a_i = \mu_i - \xi_i$ and $b_i = \mu_i + \xi_i$). For the individual normal distribution, the 5$^{th}$- and 95$^{th}$-percentile values are assumed to correspond to $\mu_i \pm \xi_i$ for all data with one bulking factor value. In both cases, $\xi_i$ = 5 percent is assumed for all $i$. Using this set of assumptions; the resulting updated plots are shown in Figures 16 to 19. The updated matlab files are 'gofoegbu1.m', 'weightaveragedatanew.m', and 'weightaveragecomparenew.m'.

Also, for the entire data set, a new curve is developed for comparison with the aggregates from the subset of 15 and the TPA code distribution. This set is equivalent to the original set prescribed but with equal weights assigned. This is shown in Figure 19. The following relevant statistical data is derived from this analysis:

Mean of Subset of 15 Aggregate = 36.2224
Standard Deviation of Subset of 15 Aggregate = 18.2486

Mean of Entire Set Aggregate = 34.1696
Standard Deviation of Subset of 15 Aggregate = 18.6136

For the subset of 15: P(B < 10%) = 6%
P(10% < B < 60%) = 82.6%
P(10% < B < 90%) = 94.0 %
P(B < 90%) = 99.9

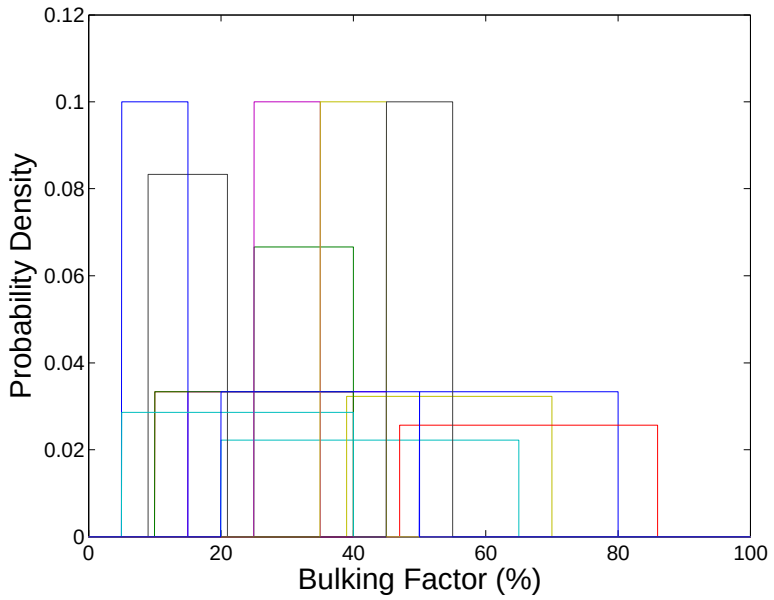



**Figure 16: Update of Figure 12**

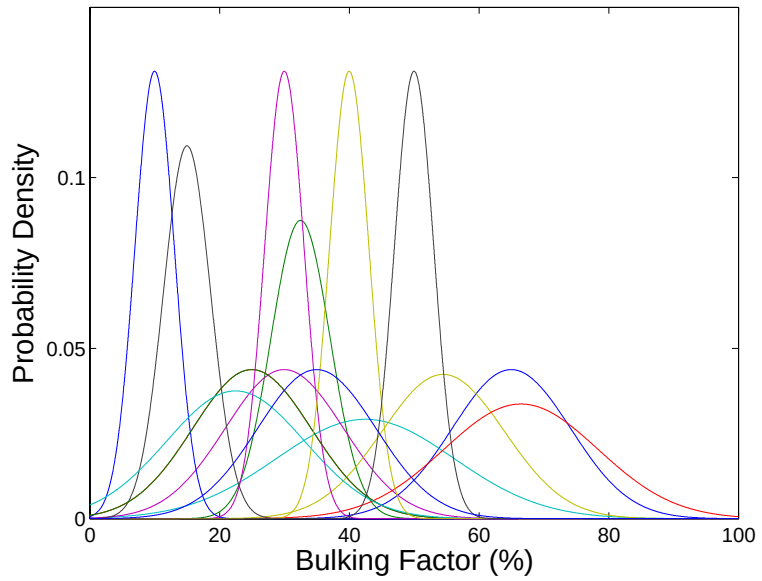
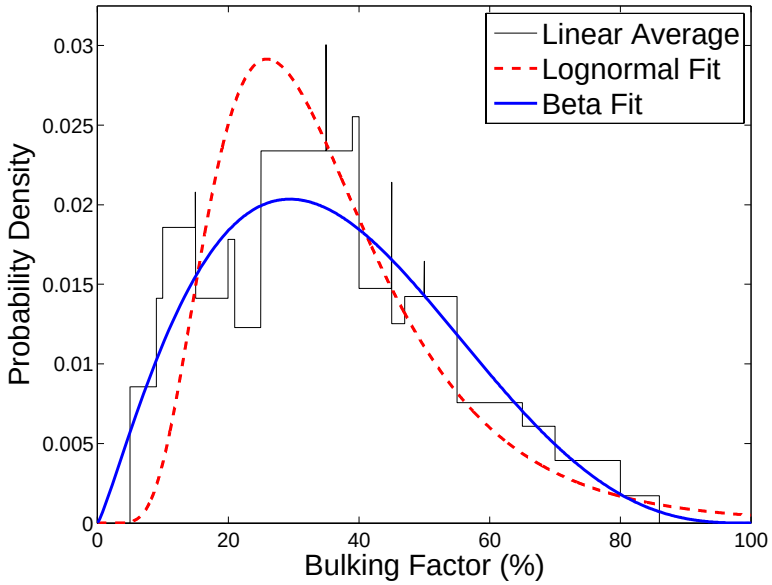
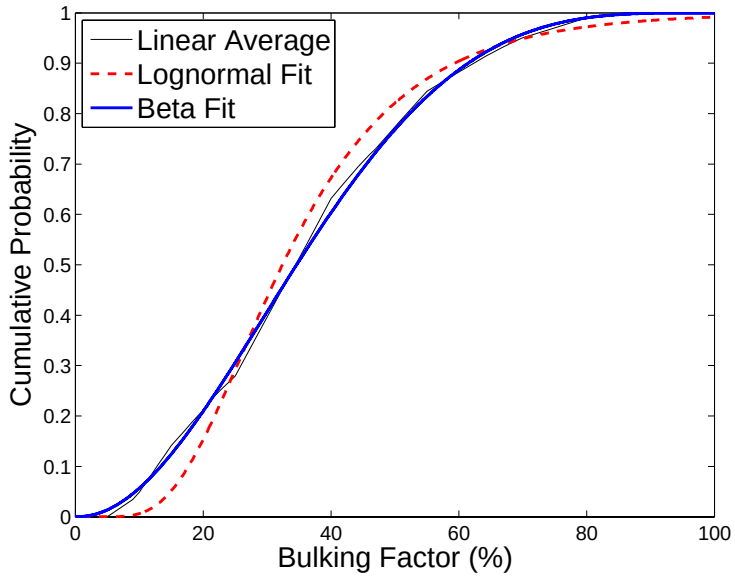



**Figure 17: Update of Figure 13**

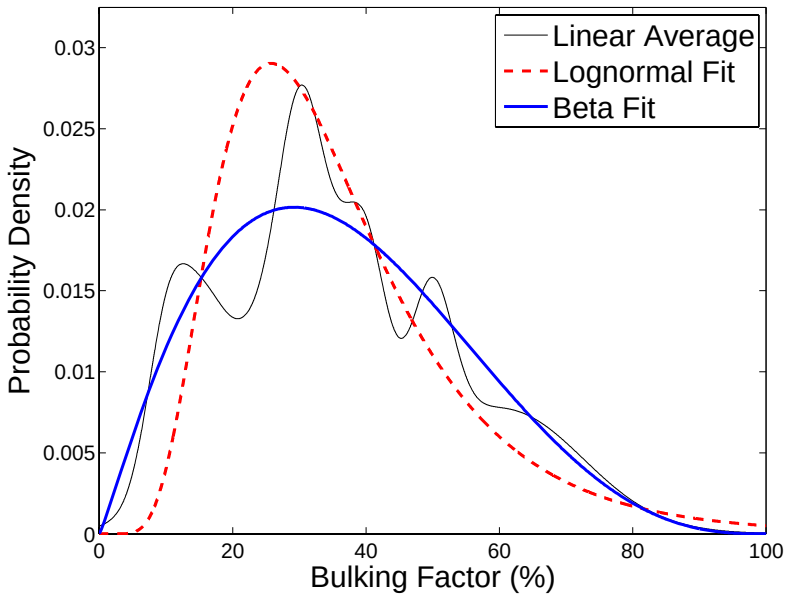



**(a)**

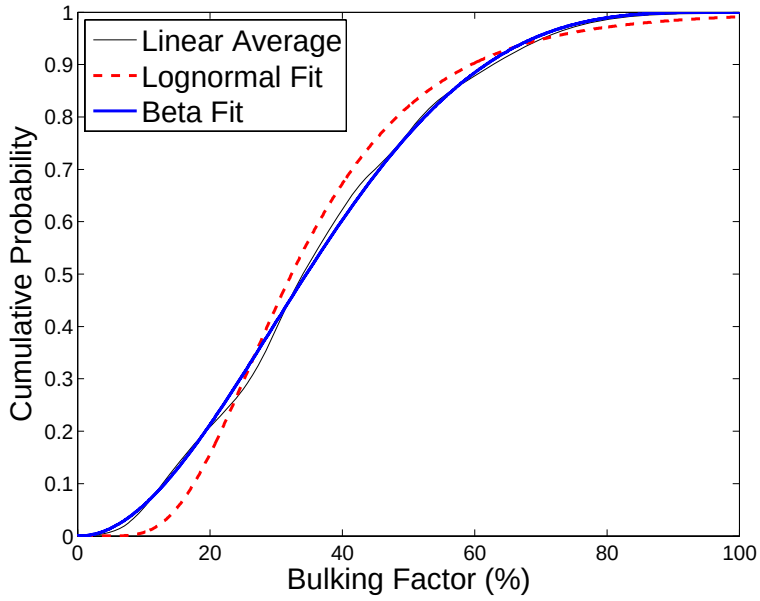



**(b)**

**Figure 18: Update of Figure 14**




**(a)**




**(b)**

**Figure 19: Update of Figure 15**

*MATLAB file* ~~*'gofoegbu1.m'*~~ [FF, February, 15, 2008] *'gofoegbu2.m'*

```
data = [50  80  1
25  40  1
10  40  1
20  65  1
15  45  1
39  70  1
9   21  1
10  10  1
10  40  1
47  86  1
5   40  1
30  30  1
40  40  1
50  50  1
20  50  1];
```

*MATLAB file 'weightaveragedatanew.m'*

```
clear all

optiondata = input('Set (1) or (2) = ');
if optiondata == 1
    gofoegbu;
else
    'gofoegbu1;' [FF, February, 15, 2008] 'gofoegbu2;'
end
optiondist = input('Set Uniform (1) or Normal (2) = ');
delta = input('Plus/Minus Delta = ');

I = find(data(:,1) == data(:,2));
if optiondist == 1
    data(I,1) = data(I,1) - delta;
    data(I,2) = data(I,2) + delta;
    A = data(:,1)';
    B = data(:,2)';
else
    data(I,1) = data(I,1) - delta;
    data(I,2) = data(I,2) + delta;
    p05a1 = data(:,1)';
    p95a1 = data(:,2)';
end
wc = ones(size(data(:,3)'));
```

```
if optiondist == 1
    n = length(A);
else
    mu = (p95a1+p05a1)/2;
    s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));
    n = length(p05a1);
end
a = 0;b = 100;
x = a:0.001:b;

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

if optiondist == 1
    P =
(unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';
    save run A B w n
else
    P =
(normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';
    save run p05a1 p95a1 w
end

figure
plot(x,P);hold on;
axis([0 100 0 0.35])
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)
set(gca,'FontSize',14)

figure;
if optiondist == 1
    ps = myfunsuni(x);
else
    ps = myfuns(x);
end
plot(x,ps,'k');hold on
axis([0 100 0 0.045])

if optiondist == 1
    my = quad(@myfunsmeanuni,a,b)
    mean((A+B)/2)
    sy = sqrt(quad(@myfunsstduni,a,b) - quad(@myfunsmeanuni,a,b)^2)
    sqrt(sum(((B-A)/sqrt(12)).^2))
else
    my = quad(@myfunsmean,a,b)
    mean(mu)
    sy = sqrt(quad(@myfunsstd,a,b) - quad(@myfunsmean,a,b)^2)
    sqrt(sum(s.^2))
```

```
end

beta = sqrt(log((sy/my)^2 + 1));
medy = my*exp(-0.5*beta^2);
logy = lognpdf(x,log(medy),beta);
plot(x,logy,'r--','Linewidth',2);

[X,F,f,x] = betagen(my,sy,0,100,10);
plot(x,f,'b','Linewidth',2);

set(gca,'FontSize',14)
legend('Linear Average','Lognormal Fit','Beta Fit')
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)

figure;
nint = 200;
if optiondist == 1
    for i = 1:nint;
        i
        pcdf(i) = quad(@myfunsuni,a,i);
    end
else
    for i = 1:nint;
        i
        pcdf(i) = quad(@myfuns,a,i);
    end
end
plot(1:(length(pcdf)),pcdf,'k');hold on
logcy = logncdf(x,log(medy),beta);
plot(x,logcy,'r--','Linewidth',2);
plot(x,F,'b','Linewidth',2);
plot(x,F,'b','Linewidth',2);
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Cumulative Probability','Fontsize',20)
set(gca,'FontSize',14)
legend('Linear Average','Lognormal Fit','Beta Fit','Location','NorthWest')
axis([0 100 0 1])
```

*MATLAB file 'weightaveragecomparenew.m'*

```
clear all

optiondata = input('Set (1) or (2) = ');
if optiondata == 1
    gofoegbu;
else
    ~~'gofoegbu1;'~~ [FF, February, 15, 2008] ~~'gofoegbu2;'~~
end
```

```
optiondist = input('Set Uniform (1) or Normal (2) = ');
delta = input('Plus/Minus Delta = ');

I = find(data(:,1) == data(:,2));
if optiondist == 1
    data(I,1) = data(I,1) - delta;
    data(I,2) = data(I,2) + delta;
    A = data(:,1)';
    B = data(:,2)';
else
    data(I,1) = data(I,1) - delta;
    data(I,2) = data(I,2) + delta;
    p05a1 = data(:,1)';
    p95a1 = data(:,2)';
end
wc = ones(size(data(:,3)'));

if optiondist == 1
    n = length(A);
else
    mu = (p95a1+p05a1)/2;
    s = (p05a1 - mu)/(sqrt(2)*erfinv(2*0.05 - 1));
    n = length(p05a1);
end
a = 0;b = 100;
x = a:0.001:b;

w = (1/sum(wc))*wc;
W = repmat(w',1,length(x));

if optiondist == 1
    P = (unifpdf(repmat(x,n,1)',repmat(A,length(x),1),repmat(B,length(x),1)))';
    save run A B w n
else
    P = (normpdf(repmat(x,n,1)',repmat(mu,length(x),1),repmat(s,length(x),1)))';
    save run p05a1 p95a1 w
end

%figure;
if optiondist == 1
    ps = myfunsuni(x);
else
    ps = myfuns(x);
end

if optiondist == 1
    my = quad(@myfunsmeanuni,a,b)
    mean((A+B)/2)
    sy = sqrt(quad(@myfunsstduni,a,b) - quad(@myfunsmeanuni,a,b)^2)
    sqrt(sum(((B-A)/sqrt(12)).^2))
else
    my = quad(@myfunsmean,a,b)
    mean(mu)
```

```
    sy = sqrt(quad(@myfunsstd,a,b) - quad(@myfunsmean,a,b)^2)
    sqrt(sum(s.^2))
end

[X,F,f,x] = betagen(my,sy,0,100,10);

A = 23.17;B = 92.70;
a = 1;c = 100;
ft = betapdf((x-a)/(c-a),A,B)/(c-a);

%plot(x,ft,'k','Linewidth',2);hold on
plot(x,f,'b','Linewidth',2);hold on
set(gca,'FontSize',14)
xlabel('Bulking Factor (%)','Fontsize',20)
ylabel('Probability Density','Fontsize',20)
axis([0 100 0 0.115])
%axis([0 100 0 0.025])

%legend('TPA Distribution','1^{st} set Aggregate Distribution','2^{nd} set
Aggregate Distribution')
```

[FF, February 4, 2008]

**Results including laboratory result**

During technical review of the 'Bulking Factor of Rock for Underground Openings' report, Dr. Wes Patrick indicated that the experimental values reported in Appendix B should be included in the statistical analysis. Figures previously presented are updated to include the range 58.3 to 83.2 percent. The following are the results for the individual normal assumption:

Mean of Subset of 16 Aggregate = 38.4
Standard Deviation of Subset of 16 Aggregate = 19.5

Mean of Entire Set Aggregate = 34.9
Standard Deviation of Subset of 15 Aggregate = 19.1

Entire Set: P(B < 10%) = 8.6%
P(10% < B < 60%) = 80.0%
P(10% < B < 90%) = 91.3 %
P(B < 90%) = 99.9%

In order to quantitatively evaluate the closeness of fit between the aggregate and lognormal/beta distribution fits, the maximum absolute difference $max(|F(B) - F_{beta}(B)|)$ and $max(|F(B) - F_{lognormal}(B)|)$ are calculated. For the total set with individual normal assumption, the results are $max(|F(B) - F_{beta}(B)|) = 0.03217768$ and $max(|F(B) - F_{log}(B)|) = 0.08923967$. For subset of 16 with the individual uniform assumption, these values correspond to $max(|F(B) - F_{beta}(B)|) = 0.03557524$ and $max(|F(B) - F_{log}(B)|) = 0.0899962$. For subset of 16 with the individual normal assumption, these values correspond to $max(|F(B) - F_{beta}(B)|) = 0.02807595$ and $max(|F(B) - F_{log}(B)|) = 0.08344505$.

Figures previously presented are updated to include the range 58.3 to 83.2 percent.

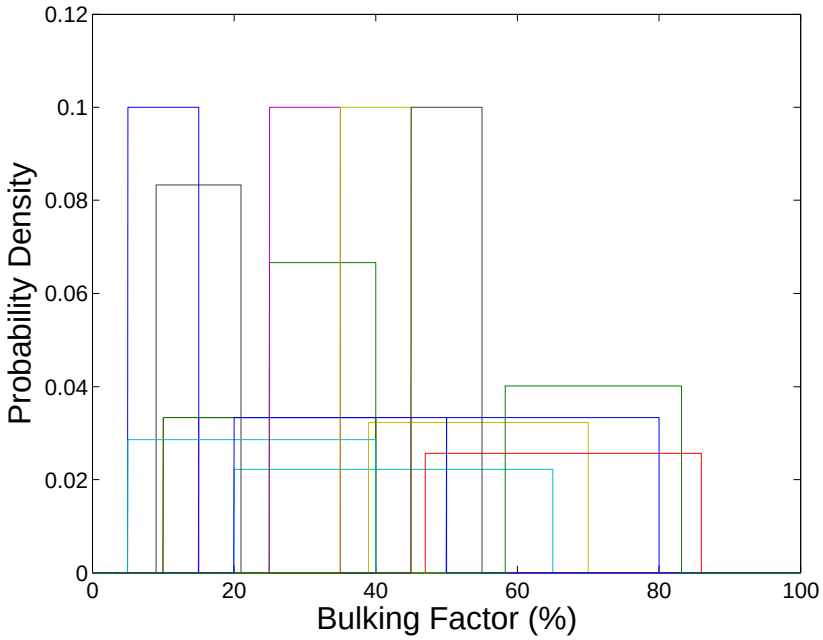



**Figure 20: Update of Figure 16**

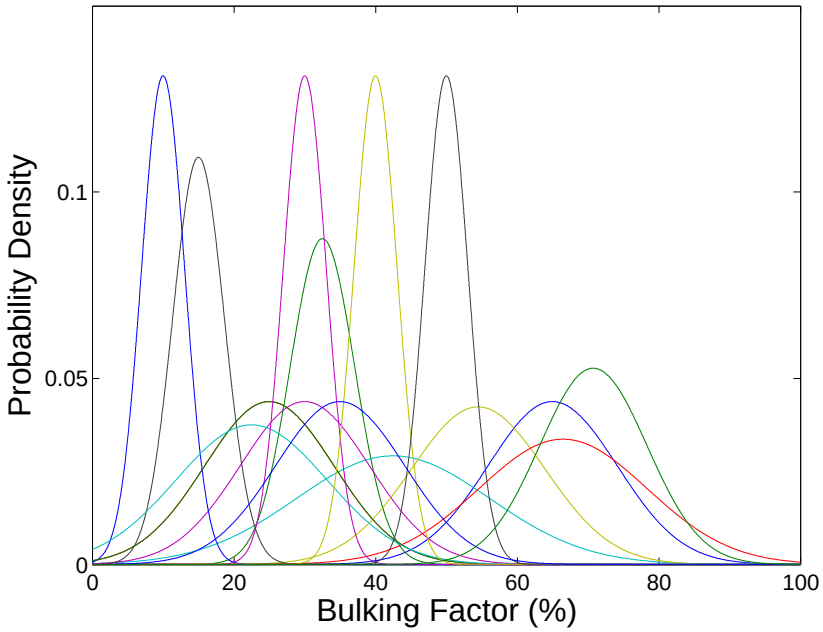
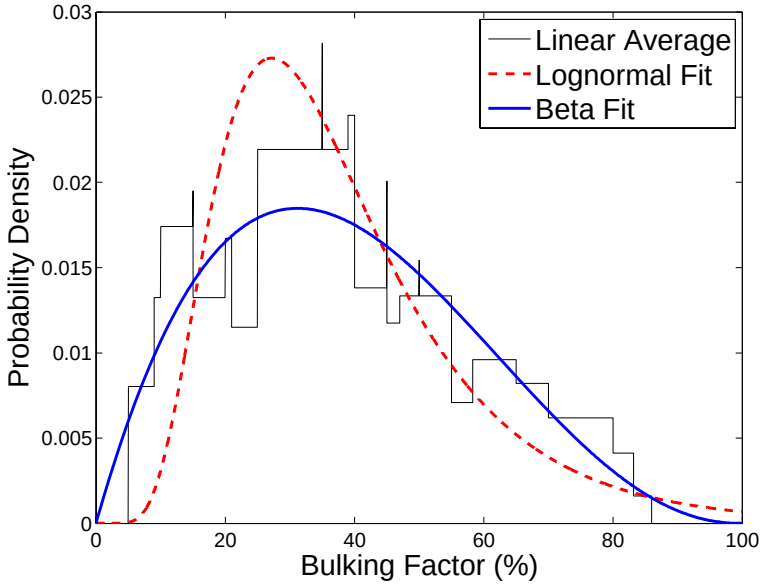
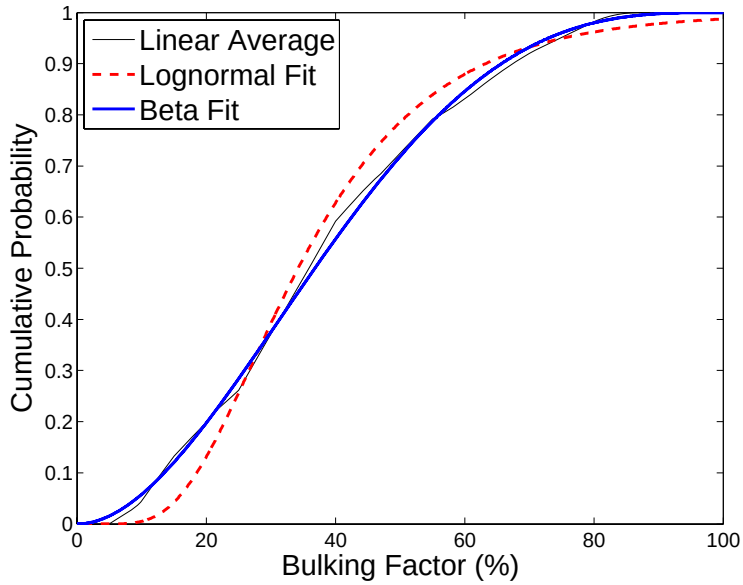



**Figure 21: Update of Figure 17**

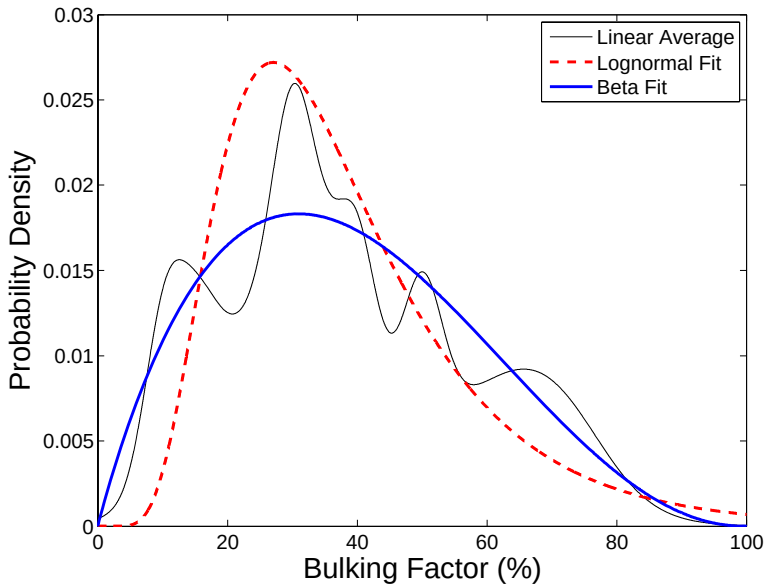



**(a)**

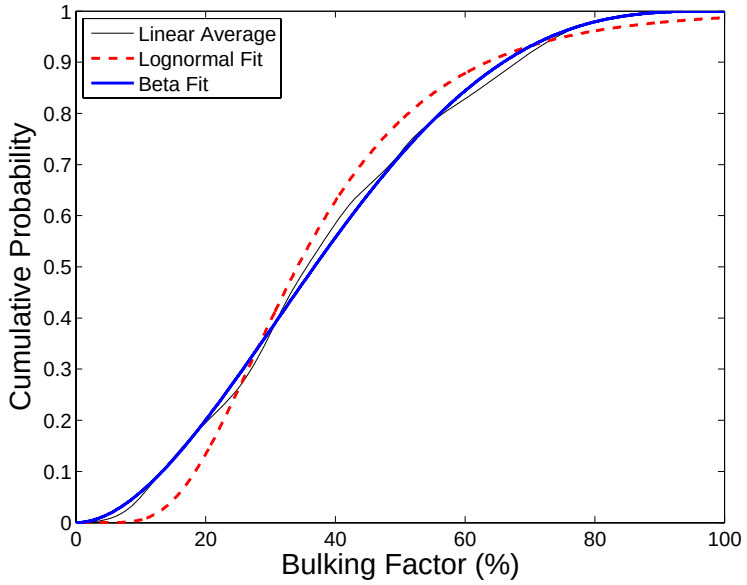



**(b)**

**Figure 22: Update of Figure 18**




**(a)**




**(b)**

**Figure 23: Update of Figure 19**

[FF, February 21, 2008]

### Update on the Total Set Data

The total set data was reviewed to include published data from Church (1981), based on discussions between Wes Patrick, Kevin Smart, Danielle Wyrick, and Goodluck Ofoegbu. An additional 10 data points was added to the previous set of 51 values considered. The following MATLAB input file represents the updated information used in the development of the aggregate distribution through an equally-weighted linear average.

*MATLAB file 'gofoegbuf.m'*

```
data = [30  50  1
5   20  1
10  40  1
50  80  1
25  40  1
20  20  1
10  40  1
33  50  1
20  65  1
15  45  1
5   5   1
39  70  1
30  56  1
1   75  1
9   21  1
10  10  1
40  40  1
1   40  1
2   5   1
14  15  1
12  17  1
47  47  1
8   16  1
10  40  1
25  61  1
27  31  1
31  34  1
47  50  1
14  14  1
14  50  1
66  84  1
35  48  1
51  80  1
10  40  1
60  80  1
25  35  1
50  80  1
30  35  1
25  30  1
15  50  1
15  30  1
5   40  1
15  30  1
30  50  1
20  50  1
15  50  1
50  72  1
21  48  1
24  57  1
28  66  1
58.3 83.2  1
67  67  1
64  64  1
72  72  1
61  61  1
77  77  1
75  75  1
50  86  1
75  75  1
65  65  1
39  50  1];
```

The analysis results in the updated Figure 24, and the following distribution parameters:

Mean of Entire Set Aggregate = 39.9597
Standard Deviation of Entire Set Aggregate = 21.4660

Beta Distribution Fit values: α = 1.6810, β = 2.5257

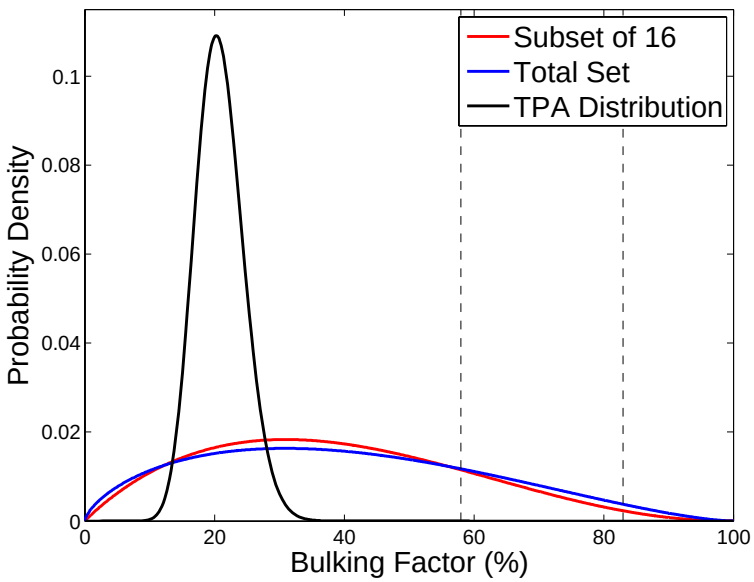



**Figure 24: Update of Figure 10**

[FF, February 26, 2008]

### Closure of Scientific Notebook SN918E

This scientific notebook was provided to the cognizant manager Asad Chowdhury for closure on June 10, 2008; as required by QAP-01. MATLAB and EXCEL files are provided on a compact disk labeled “Attachment to SN918E”.

[FF, June 10, 2008]

## 3. References:

Ayyub, B.M. *Risk Analysis in Engineering and Economics.* 1st Edition. College Park, Maryland: Chapman & Hall/CRC. 2003.

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Church, H.K., 1981, Excavation Handbook: McGraw-Hill, 982 p.

Hartman, H.L (Ed.). 1992. SME Mining Engineering Handbook. Second edition, Volume 2. Society for Mining, Metallurgy and Exploration, Inc., Lttleton, CO.

# ADDITIONAL INFORMATION FOR SCIENTIFIC NOTEBOOK NO. 918E

| | |
|---|---|
| **Document Date:** | 01/23/2008 |
| **Availability:** | Southwest Research Institute®<br>Center for Nuclear Waste Regulatory Analyses<br>6220 Culebra Road<br>San Antonio, Texas 78228 |
| **Contact:** | Southwest Research Institute®<br>Center for Nuclear Waste Regulatory Analyses<br>6220 Culebra Road<br>San Antonio, TX 78228-5166<br>Attn.: Director of Administration<br>210.522.5054 |
| **Data Sensitivity:** | ■"Non-Sensitive" ☐ Sensitive<br>☐"Non-Sensitive - Copyright" ☐ Sensitive - Copyright |
| **Date Generated:** | 06/10/2008 |
| **Operating System:** (including version number) | Microsoft Windows V.5.1 |
| **Application Used:** (including version number) | MATLAB V.7.4.0.287; Microsoft Office Excel 2003 |
| **Media Type:** (CDs, 3 ½, 5 1/4 disks, etc.) | 1 CD |
| **File Types:** (.exe, .bat, .zip, etc.) | m, xls |
| **Remarks:** (computer runs, etc.) | |