U. S. DEPARTMENT OF COMMERCE
CHARLES SAWYER, *Secretary*

WEATHER BUREAU
F. W. REICHELDERFER, *Chief*

RESEARCH PAPER NO. 34

PREDICTING THE RUNOFF FROM STORM RAINFALL

by

M. A. KOHLER and R. K. LINSLEY

PROPERTY OF LIBRARY TENNESSEE VALLEY AUTHORITY

WASHINGTON
September 1951

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. - Price 5 cents

WEATHER BUREAU RESEARCH PAPERS

No. 1 **Progress Report on Development of an Electronic Anemometer.* W. R. Thickstun, July 1943.

No. 2 **A Project to Test the Potential Usefulness of Pressure Patterns for Forecasting.* H. W. Norton, G. W. Brier, and R. A. Allen, January 1944.

No. 3 **Preliminary Report on Duration of Stormy Periods at Selected Localities and Intervals Between Periods.* L. L. Weiss, January 1944.

No. 4 **Some Relationships Between Five-Day Mean Surface Maps and Their Component Daily Maps.* C. B. Johnson, January 1944.

No. 5 **Improvement of Trend Methods of Forecasting.* P. F. Clapp, July 1943.

No. 6 (Unassigned)

No. 7 **Formation of New Moving Centers South of Deep Lows.* R. C. Gentry, January 1944.

No. 8 **An Investigation of a Trajectory Method of Forecasting Flow Patterns at the 10,000-Foot Level.* H. G. Dorsey and G. W. Brier, January 1944.

No. 9 **Preliminary Report on Stagnant Highs Over Greenland, Iceland, and England, and Over the Bering Sea and Alaska in July and August.* R. C. Gentry and L. L. Weiss, January 1944.

No. 10 **Persistence in London Temperatures.* H. W. Norton and G. W. Brier, January 1944.

No. 11 **Skill in Selecting the "Best" Forecast.* G. W. Brier, January 1944.

No. 12 *Notes on Interdiurnal Pressure and Temperature Changes in the Upper Air.* R. C. Gentry, January 1944. (Unpublished.)

No. 13 *Investigation and Practical Use of a Method for Constructing Six-Hour Isallobars on Upper Level Charts.* E. M. Cason and P. F. Clapp, January 1944. (Unpublished.)

No. 14 *Weight Changes for the Atmosphere Divided Into Three Layers.* L. L. Weiss, February 1944. (Unpublished.)

No. 15 **Some Notes on Forecasting for Atlanta and Miami Districts (North and South Carolina, Georgia, and Florida).* Grady Norton, February 1944.

No. 16 **Verification of a Forecaster's Confidence and the Use of Probability Statements in Weather Forecasting.* G. W. Brier, February 1944.

No. 17 **Pressure Patterns Accompanying Cyclonic Activity in the Azores Area.* R. L. Pyle, March 1944.

No. 18 **Normal Mean Virtual Temperatures and Weights of the Air Column Between Sea Level and 10,000 Feet.* Staff, Extended Forecast Section, July 1944.

No. 19 **Temperature Changes During Formation and Dissipation of West Coast Stratus.* Morris Neiburger (University of California at Los Angeles), July 1944.

No. 20 *Empirical Studies of the Motion of Long Waves in the Westerlies.* P. F. Clapp, July 1944. (Unpublished.)

No. 21 **A Collection of Reports on the Preparation of Prognostic Weather Charts.* J. R. Fulks, H. B. Wobus, and S. Teweles, Edited by C. P. Mook, October 1944.

No. 22 **Relationship Between Surface Temperature and Mean Virtual Temperature in the Lower Troposphere.* W. M. Rowe, November 1944.

No. 23 **Forecasting the Time of Formation of Stratus Cloud Ceiling at Oakland, California, Airport.* Edward M. Vernon, April 1945.

No. 24 **Investigation of Polar Anticyclogenesis and Associated Variations of the Zonal Index.* Jerome Namias, September 1945.

No. 25 **Progress Report on Objective Rainfall Forecasting Research Program for the Los Angeles Area.* J. C. Thompson, July 1946.

No. 26 *A Study of Quantitative Precipitation Forecasting in the TVA Basin.* Glen W. Brier, November 1946. $0.25

No. 27 *Objective Methods of Forecasting Winter Minimum Temperatures at Washington, D. C.* C. P. Mook and Saul Price, August 1947. $0.35

No. 28 **Possibility of Long Range Precipitation Forecasting for the Hawaiian Islands.* Samuel B. Solot, January 1948.

No. 29 *An Objective Method of Forecasting Five-Day Precipitation for the Tennessee Valley.* William H. Klein, July 1948 $0.30

No. 30 *First Partial Report on the Artificial Production of Precipitation: Stratiform Clouds—Ohio, 1948.* Richard D. Coons, R. C. Gentry, and Ross Gunn, August 1948. $0.30

(*Continued on p. 10*)

*Out of print.

Note.—Nos. 2, 3, 4, 7, 8, 9, 10, and 11 are included in one publication under the title *A Collection of Reports on Extended Forecasting Research*, Weather Bureau, 1944.

PREDICTING THE RUNOFF FROM STORM RAINFALL [1]

M. A. KOHLER AND R. K. LINSLEY [2]

Division of Climatological and Hydrologic Services, U. S. Weather Bureau, Washington, D. C.

ABSTRACT

The estimation of the volume of runoff to be expected from a given volume of rainfall is a fundamental problem in flood forecasting. Such estimates are necessary before the unit hydrograph [1] or other techniques can be used to predict the streamflow hydrograph. The authors describe the technique now used at the River Forecast Centers of the U. S. Weather Bureau for evaluating the effect of season, antecedent conditions, duration of rainfall and rainfall amount in determining the portion of the rainfall contributing to storm runoff [2]. Special problems encountered in flood forecasting are emphasized. The technique, developed and tested over several years, yields a high degree of accuracy in estimated runoff. Although prepared by empirical procedures, the close agreement between relations for basins of similar hydrologic characteristics suggests that rational parameters have been adopted. The similarity between relations also simplifies the work required for their preparation.

METHOD OF APPROACH

Many articles have appeared in the technical literature describing the application of infiltration theory to the problem of estimating storm runoff [3]. This is considered by many hydrologists to be the rational approach and, when considering heavy, intense rainfall over a small homogeneous area, it can be used to advantage for some specialized purposes. However, the hydrologic characteristics of a natural basin exceeding a few acres in area are so variable as to make such a rational approach exceedingly complex. When the usual variations in storm characteristics are superimposed, the solution becomes virtually impossible unless an unusually dense network of precipitation stations exists. Moreover, the direct application of the infiltration theory can be utilized to determine only the surface-runoff component of the flood hydrograph. River forecasting requires that the total flow, including interflow and groundwater flow, be estimated and these two latter components constitute a major portion of the flood hydrographs for some basins. An even more important consideration in forecasting, however, is speed. Time is not available for the detailed consideration of large basins by the rational infiltration approach.

The difficulties encountered in treating large natural basins in strict accordance with the infiltration theory have led to the use of infiltration indices such as the ϕ- and W-indices [3]. Since these indices must be correlated empirically to factors representing moisture deficiency of the basin, their use cannot be considered rational. There is no advantage in the use of such indices over a direct correlation of runoff with appropriate factors. The use of such arbitrary indices for computing runoff complicates the solution without enhancing the accuracy or rationalizing the approach. After extensive study the Weather Bureau has adopted a graphical correlation of runoff with selected parameters as the most satisfactory approach for forecasting purposes.

SELECTION OF PARAMETERS

The most important problem in developing a technique for forecasting runoff is the selection of the proper parameters to be used. Runoff is the factor which is required in the preparation of river forecasts. However, since runoff is the residual after the demands of interception, infiltration, and depression storage have been satisfied, there is some logic in using the difference between rainfall and runoff as the dependent variable. This difference is often called the "loss," but because of the ambiguity of this term the authors prefer the term "basin recharge." Knowing the basin recharge

[1] Paper presented at the 30th Annual Meeting of the American Geophysical Union, Washington, D. C., April 21, 1949.

[2] Now with Dept. of Civil Engineering, Stanford University, Palo Alto, Calif.

and the rainfall, runoff can be computed by direct subtraction.

For the purpose of forecasting, runoff is assumed to fall into two classes—(1) base or groundwater flow, and (2) direct runoff. Many methods have been suggested for the separation of these two components in the hydrograph. The selection of method is not as important as the consistent use of a single method throughout the study.

The method used by the Weather Bureau is shown in figure 1. The curve AB represents an extension of the recession existing prior to the storm, point B being directly under the peak. The straight line BC intersects the hydrograph at a point n-days after the crest or after the end of runoff-producing rainfall. The value of n is assumed constant for any basin, but is varied according to drainage area. While basin slope and other factors should be considered, the value of n is not particularly critical. If the derived relation is to be used in conjunction with a unit graph, then the same time base should, of course, be used in both analyses. The area bounded by the hydrograph and ABC converted to inches depth over the basin is considered to be the storm runoff. The basin recharge data are computed by direct subtraction of runoff from rainfall.

The amount of basin recharge resulting from a given storm depends upon (1) the moisture deficiency of the basin at the beginning of rainfall, and (2) the storm characteristics such as rainfall amount, intensity, etc. While storm characteristics can be determined from an adequate network of precipitation stations, the direct determination of moisture conditions throughout a basin is extremely difficult. Reliable point-observations of soil moisture are possible, but an integrated value (over area and throughout depth) is required in a medium recognized for its marked physical discontinuities, further emphasized by cultivation and vegetal cover. Moreover, conditions above the soil surface must be considered, i. e., storage capacity of depressions and vegetal cover (interception).

Numerous measureable factors have been used as indices to moisture conditions, notably (1) days since last rain, (2) discharge at beginning of the storm, and (3) antecedent precipitation. The first of these is obviously insensitive and should not be used if accurate results are required. The second, base flow, is a reasonably good index in humid and sub-humid regions, but it is affected by season and it does not necessarily reflect changes caused by rains during the previous week. Antecedent precipitation is universally applicable and yields good results provided the index is properly derived and is used in conjunction with season of the year or temperature.

The antecedent precipitation index is generally defined by an equation of the type

$$I=b_1P_1+b_2P_2+b_3P_3+\ .\ .\ .\ +b_iP_i \qquad (1)$$

Where P_i is the amount of precipitation which occurred i days prior to the storm under consideration, b_i is a constant which is assumed to be some function of time such as $b_i=1/i$, and the number of terms is arbitrarily selected. If a day-to-day value of the index I is required, as is the case in river forecasting, there is considerable advantage in assuming that b_i decreases with time (prior to the storm of interest) according to a logarithmic recession rather than as a reciprocal. In other words, during periods of no precipitation,

$$I_t=I_0k^t \qquad (2)$$

where t is the number of days between I_t and the initial index I_0. Letting t equal unity,

$$I_1=kI_0 \qquad (3)$$

Thus, the index for any day is equal to that of the previous day multiplied by the factor k. If rain occurs on any day, the amount of rain observed is added to the index as is shown in figure 2. Since storm runoff does not, of itself, add to the residual moisture of the basin, it is evident that an antecedent index of "precipitation minus runoff," or basin recharge, should be more satisfactory than precipitation only. This refinement requires con-

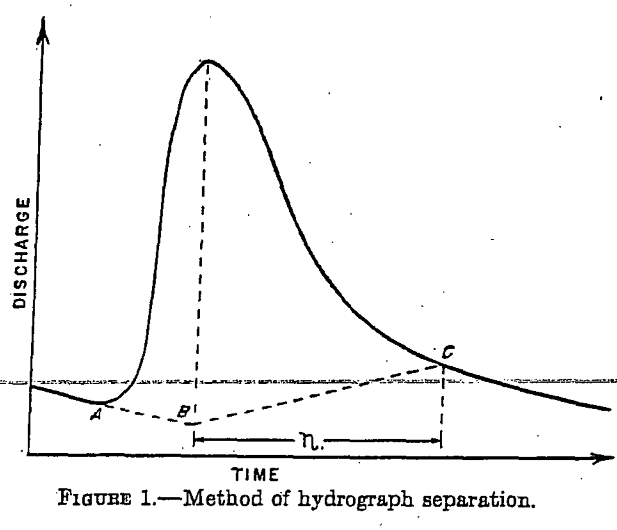

FIGURE 1.—Method of hydrograph separation.

siderably more computations, however, and its use is probably not justified.

The effect of a given amount and distribution of antecedent precipitation upon storm runoff obviously depends upon the extent to which it has been dissipated through evaporation, transpiration, etc. While k could be assumed to vary as a function of pan evaporation, air temperature, dewpoint or vapor pressure deficiency, much of the variation in evapo-transpiration is of a seasonal nature and the introduction of season (or week of year) into the correlation has been found highly satisfactory. There is an added advantage in using season as a parameter in that it reflects variations in surface conditions as related to farming practices, vegetation, etc.

Theoretically, the value of the recession factor k should also be a function of the physiographic characteristics of the basin, but experience has shown that the factor is not critical—values range from 0.85 to 0.90 over most of the eastern and central portions of the United States.

The antecedent precipitation index can be computed either (1) from average daily values over the basin, or (2) from daily precipitation at the various stations, and then averaged.

To utilize the advantages of the logarithmic recession, the computation of the index must be carried forward throughout the period of record being analyzed. The index value for any day theoretically depends upon antecedent precipitation over an infinite period. However, if some reasonable initial value is assumed, the computed index will closely approach the true value within several weeks. It has been the practice either (1) to begin the computations at the end of a dry spell (prior to the first storm analyzed) with an assumed low value of the index, or (2) to begin the computations two or three weeks in advance of the first storm with an assumed value equal to the normal 10-day precipitation for the season (which approximates the average index value for the area).

In computing the data for a particular storm, the index at the beginning of the first day of rain is used. For example, an index value of 1.8 would be used for the storm of the 9th and 10th in figure 2. The computation can be rapidly performed with the aid of a chart (fig. 3), or comparable table. By entering the chart with an initial index, the value t days later (assuming no rainfall) can be read directly.

In any discussion of antecedent precipitation, a question immediately arises regarding snowfall. If the water equivalent of snowfall is added to the index at the time of its occurrence, its effect on a subsequent rain storm will be over-emphasized if removed from the basin through evaporation and underestimated if melted at a later date. In the usual sequence of events, evaporation from the snow surface is not far different from surface evaporation following a rain and, consequently, snowfall can probably best be considered to have been applied to the basin on the day it melted rather than when it fell.

PREPARATION OF DATA

In general, extended complex storms should be broken into as many short, unit storms as can successfully be accomplished through hydrograph

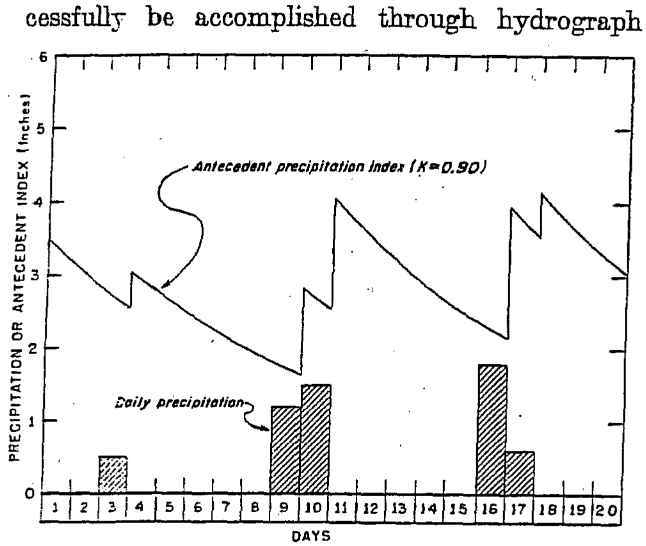

FIGURE 2.—Variation of antecedent index with daily precipitation.

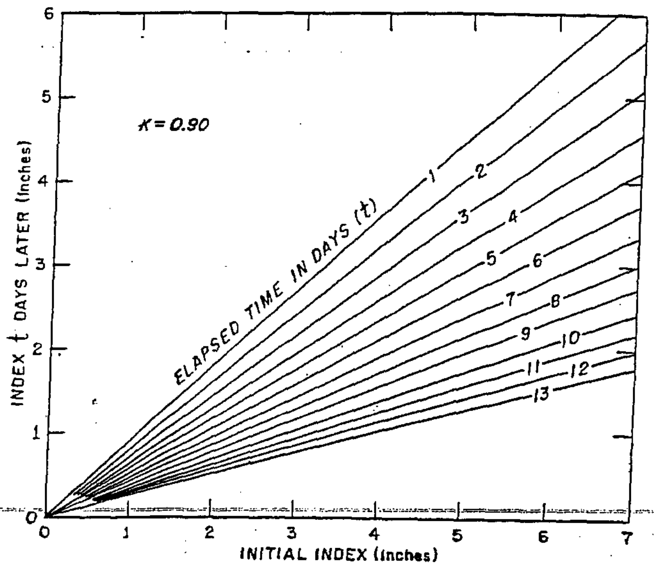

FIGURE 3.—Chart for computing antecedent precipitation index.

analysis. Having decided upon the storm period, the amount and duration of rainfall are computed and tabulated for each storm. While data are generally insufficient to accurately determine the average duration of rainfall over a basin, this factor is not critical and can be adequately derived by examination of available six-hourly rainfall data. In the development of the relations to be described, the duration was defined as the sum of those six-hourly periods with more than 0.2 inch of rain plus one-half the intervening periods with less than 0.2 inch. While experimental infiltration data indicate rates commonly in excess of 0.10 inch per hour after saturation, relations developed to date consistently show that the portion of basin recharge which seems to be correlated with duration takes place at rates in the order of 0.01 inch per hour. The difference between these rates is largely accountable to interflow, intercorrelations, and the method of hydrograph separation.

COAXIAL GRAPHICAL CORRELATION ANALYSIS

In the previous discussion reasons were advanced for the selection of five variables to be included in the correlation—basin recharge, antecedent precipitation index, season or week of year, storm duration, and storm rainfall. While analytical correlation could be used, the existence of joint functions complicates the problem to such an extent that the selection of an appropriate equation is extremely difficult. Ezekiel [4] describes a method of graphical correlation which yields excellent results for some problems, but the coaxial method is more flexible and yields correspondingly better results for runoff correlations because of the joint relations involved.

The coaxial method [2] of graphical correlation is based on the premise that if any important factor is omitted from a relation then the scatter of points in a plotting of observed values of the dependent variable *vs.* those computed by the relation will be at least partially explained by the omitted factor. In other words, if the points of such a plotting are labeled with corresponding values of the omitted factor, a family of curves fitting the data can be used to modify or correct the values computed from the original relation.

In applying the coaxial method to the selected parameters, a three-variable relation is first developed (fig. 4, Chart A) by (1) plotting antecedent precipitation *vs.* basin recharge, (2) labeling the points with week number, and (3) fitting a smooth family of curves representing the various weeks. Chart B, for plotting computed *vs.* observed basin recharge, is placed with horizontal scale (computed) matching that of Chart A to facilitate plotting. Points labeled with duration are then plotted in Chart B at the observed recharge on the vertical scale and at a computed value on the horizontal scale corresponding to that determined by entering Chart A with antecedent index and week number. A smooth family of curves is then drawn which represent the effect of duration upon basin recharge. The combination of Charts A and B constitutes a graphical relation for estimating recharge from antecedent index, week, and storm duration. Storm precipitation is then introduced (Chart C) by (1) plotting computed recharge (from Charts A and B) *vs.* observed recharge (on horizontal scale), (2) labeling the points with rainfall amount, and (3) fitting a family of curves. Charts A, B, and C constitute the first approximation of the relation involving the selected parameters. Chart D, a plotting of observed recharge *vs.* that computed from Charts A, B, and C, is shown to indicate the over-all correlation of the relation.

Since the parameters are intercorrelated and since the first charts were developed independent of factors subsequently introduced, tests should be made to determine if revisions of the charts could improve the relation, i. e., the process is necessarily one of successive approximations. To check the curves of Chart A, the assumption is made that the other charts are correct. Therefore, the horizontal coordinate for an adjusted point (in Chart A) can be determined by entering Charts B and C in reverse order with observed recharge, rainfall amount and duration. The ordinate for the adjusted point corresponds to the observed antecedent precipitation index. In other words, the week-curves must be revised to fit the point adjusted in this manner if the relation is to yield a computed value equal to the observed. The second-approximation curves for duration and storm precipitation and all subsequent approximations are made in a similar manner. In each case the points are plotted by entering the chart sequence from both ends with observed values to determine the adjusted coordinates.

The method of performing the correlation presented in previous paragraphs is of general application and can be used as described. In developing the relation for basin recharge, however, certain modifications simplify the procedure and re-

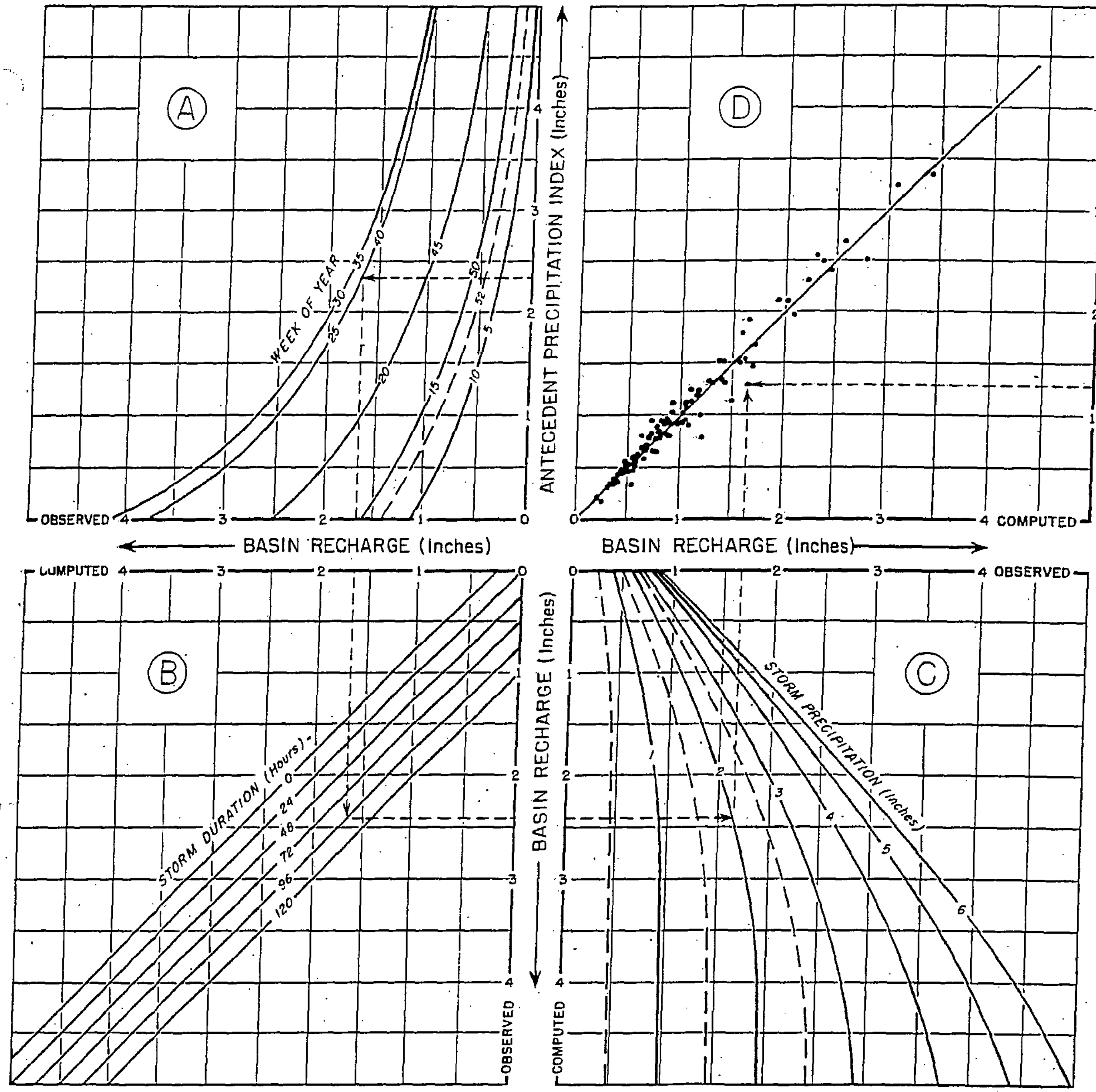

FIGURE 4.—Basin recharge relation for the Monocacy River at Jug Bridge, Md.

sult in the derivation of the final relation with fewer approximations. Since storm precipitation is extremely important, the first plotting of Chart A will show so little correlation that the construction of the curve family is extremely difficult. Introducing storm rainfall in the first plotting would improve the correlation, but there is also an important advantage in having this parameter in the last chart of the sequence—namely, the possibility of computing runoff in excess of rainfall and of computing negative values of runoff is eliminated. Moreover, the arrangement shown in figure 4 results in the determination of a unified index of initial moisture conditions in the first chart—a decided advantage in forecast application.

If the first plotting in Chart A is limited to those storms having an amount of rainfall within a specified class interval (2 to 4 inches, for example), the construction of the curves is simplified provided there are sufficient data. Actually, only limited data are required since the general type of curvature and convergence can be determined from theoretical reasoning. Moreover, the relations are quite similar throughout any general area, and once such a relation is developed, all curve-families but one can be used as the first-approximation curves for any other basin in the area. In fact, a single relation has been found applicable to as many as six or eight tributary drainages within a river basin.

As stated previously, correlations made to date indicate that storm duration, as determined in an arbitrary manner, is not particularly effective in determining basin recharge. An assumed spacing of one to two hundredths inch per hour generally proves satisfactory, but the assumed curves should be checked by plotting after the curve families of Charts A and C have been finally determined.

Examination of figure 4 will show that the errors of the points with little runoff (recharge approaching precipitation) are considerably magnified when routed back through the chart sequence as described for the development of the second-approximation curves. Therefore, if this approach is used, it will be found that the curves can be more readily determined if low-runoff points are omitted in the plotting. As an alternate approach, the required revisions of the curves can be determined qualitatively by labeling the points of Chart D with week number or duration to determine if there is any residual correlation. A third approach, also qualitative, is illustrated in figure 5, where the errors of the relation are plotted against antecedent precipitation with week number as a parameter. Either of these supplementary plottings indicate in which direction the curves should be shifted. For example, figure 5 indicates that weeks numbered about 5 through 8 should be shifted to the right for high antecedent index and to the left for low. The degree of shift indicated by the plottings can be reflected back through the chart sequence to determine approximately how much the curve should be shifted.

APPLICATIONS OF DERIVED RELATIONS

In preparing river forecasts, runoff is the controlling factor rather than basin recharge. Since rainfall and recharge determine runoff, however, the curves of Chart C in figure 4 can be converted to read runoff directly as shown in figure 6. Moreover, the charts can be superimposed (fig. 7) to conserve space without reducing the scale.

The proper application of the unit hydrograph requires that runoff increments be estimated for successive time periods throughout an extended storm. This can be accomplished by computing

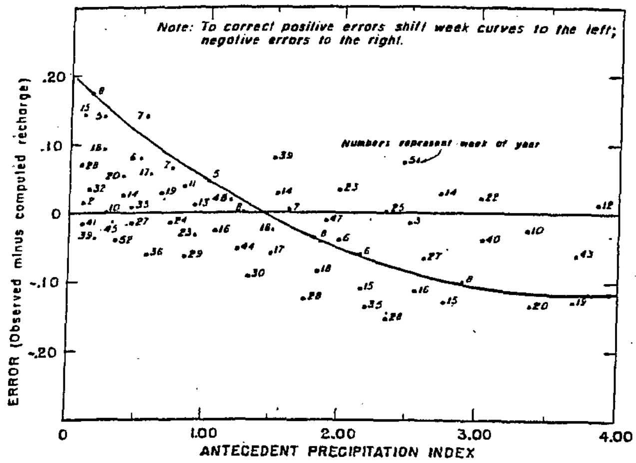

FIGURE 5.—Illustration of method for revision of week curves.

runoff depths from accumulated precipitation up to the termini of the designated periods, and subtracting successive values of runoff. As an alternative, all precipitation prior to the period of interest can be considered to be antecedent precipitation, and the storm rainfall for the period used to compute the corresponding increment of runoff. For forecast purposes, where time is of the essence, the first method may be preferable. The second method, on the other hand, gives more significance to time variations of rainfall intensity and may, therefore, provide for more accurate computations. However, the relative accuracies of the two techniques are also dependent upon the adequacy of the assumed weights for antecedent precipitation, since the first method is in accord with the analysis used in developing the basin relation.

Since it is impossible to segregate the water passing the gaging station according to the portion of the basin in which it fell, statistically derived runoff relations must necessarily be determined from basin averages of the parameters. Unfortunately, because of the higher order and joint functions involved, a relation which is based on storms of uniform areal distribution will yield runoff values which are too low when applied to storms with extremely uneven distribution. This can be demonstrated by computing the runoff for four, six, and eight inches of storm precipitation, assuming all other factors to remain constant. While six is the average of four and eight, the runoff depths computed from these three values of precipitation do not bear a corresponding relation. An uneven distribution of antecedent precipitation produces similar results.

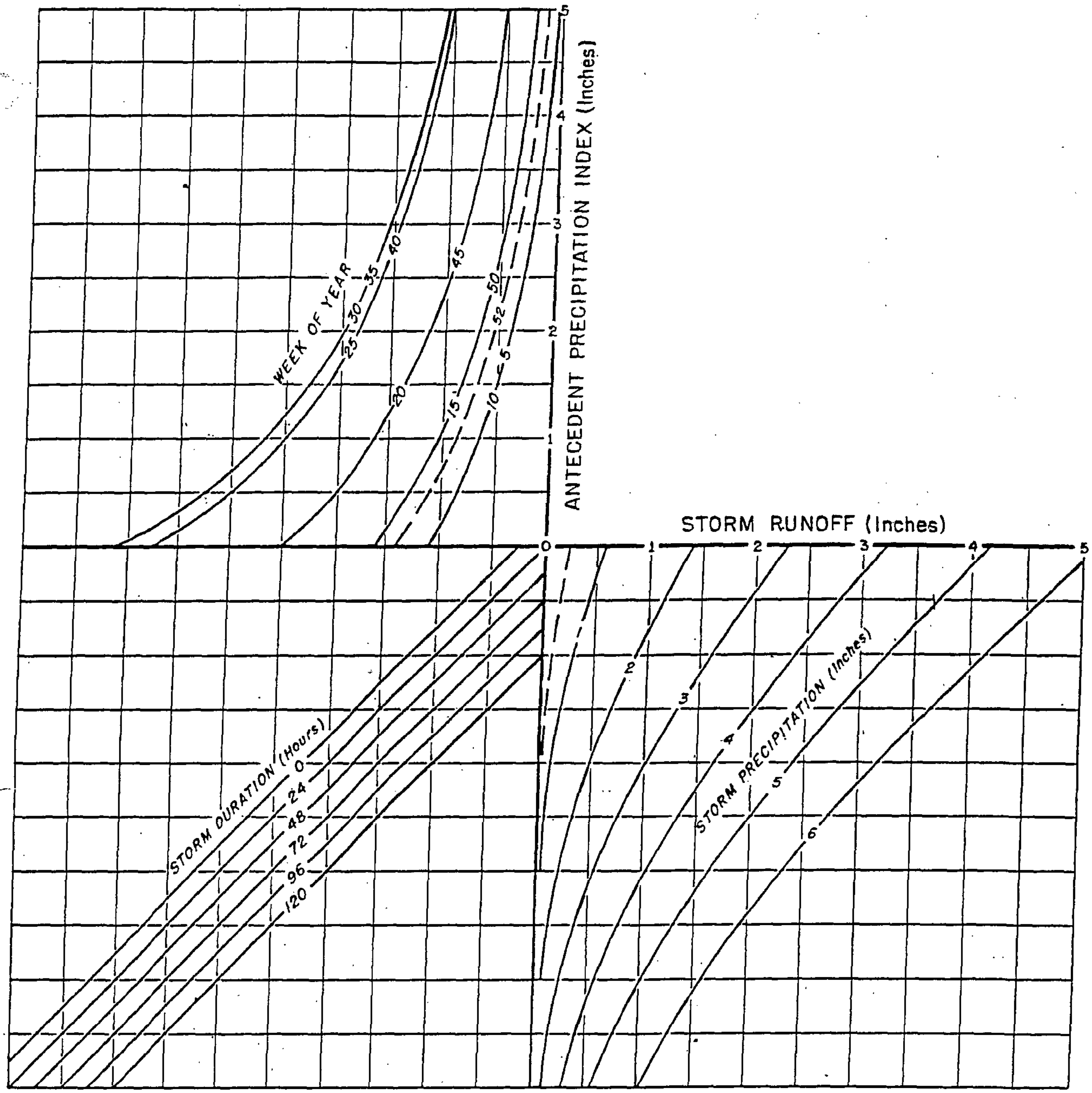

Figure 6.—Runoff relation for Monocacy River at Jug Bridge, Md.

If, however, the runoff relations are based on data representing reasonably uniform conditions, they can properly be used to compute the runoff in the vicinity of each of the rainfall stations. The average of such computed values will, in general, more nearly approach the observed runoff. In other words, if either storm or antecedent precipitation is highly variable from one portion of the basin to another, then computed runoff depths, rather than precipitation, should be averaged.

DEFICIENCIES OF DERIVED RELATIONS

Relations of the type described yield high correlation for most basins and provide a simple method of computing runoff, but they, nevertheless, have certain deficiencies which should not be overlooked. First, rainfall intensity is omitted; second, frozen soil obviates their direct use; and third, snowfall has not been considered. Since both rainfall amount and duration are considered, average intensity for the entire storm

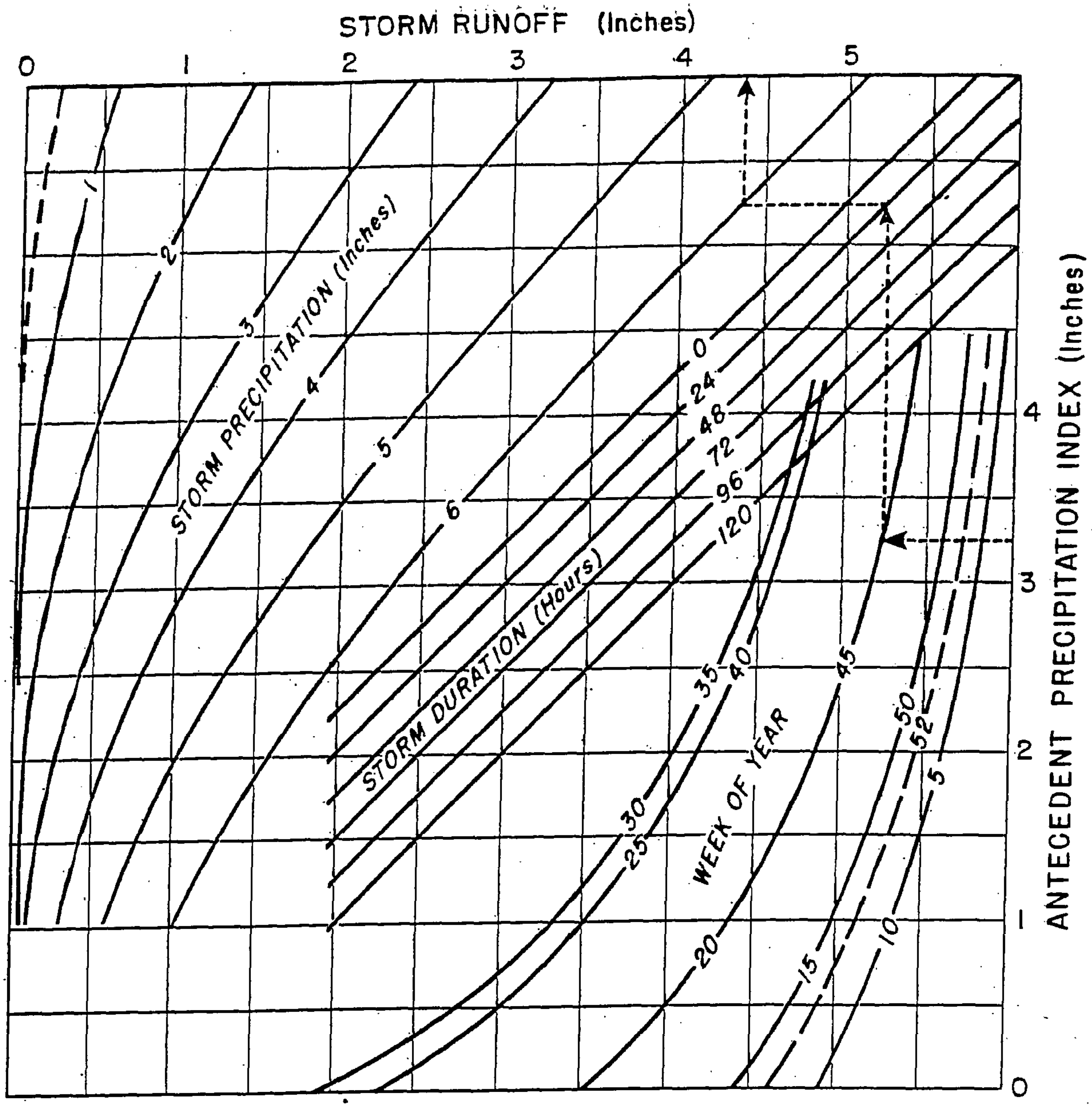

FIGURE 7.—Runoff relation for Monocacy River at Jug Bridge, Md., with curve-families superimposed.

period is an integral part of the relations. However, the computed runoff for a 5-inch, 24-hour storm is independent of intensity variations within the period. As mentioned previously, the storm can be treated as several short periods of rainfall, considering all rainfall occurring prior to any specific period as antecedent precipitation. While intensity variations can be given consideration in this manner, neglecting intensity apparently causes serious error in total storm runoff only when intensities are so great throughout the entire storm that rainfall runs off too rapidly to alleviate the moisture deficiency of the basin. Experience has shown that the relations yield fair results during frozen conditions, provided that the weekly curve representing maximum runoff conditions is used, regardless of the date of the storm. Storms which are predominantly snow present an entirely different problem and are not considered here. If only a slight snow cover remains at the end of the storm, the estimated water equivalent can be subtracted from the observed storm precipitation. Snow on the ground at the beginning of the storm should be included in the storm precipitation (rather than antecedent precipitation) if it is dissipated during the storm.

REFERENCES

1. L. K. Sherman, "Streamflow from Rainfall by the Unit-graph Method," *Engineering News-Record*, vol. 108, No. 14, April 7, 1932, pp. 501–505.
2. R. K. Linsley, M. A. Kohler, and J. L. H. Paulhus, *Applied Hydrology*, McGraw-Hill, New York, 1949.
3. H. L. Cook, "The Infiltration Approach to the Calculation of Surface Runoff," *Transactions of the American Geophysical Union*, vol. 27, No. 5, Oct. 1946, pp. 726–747.
4. M. Ezekiel, *Methods of Correlation Analysis*, 2d ed., Wiley, New York, 1941.

WEATHER BUREAU RESEARCH PAPERS

(Continued from inside self cover)

No. 31 *Second Partial Report on the Artificial Production of Precipitation: Cumuliform Clouds—Ohio, 1948.* Richard D. Coons, Earl L. Jones, and Ross Gunn, January 1949. $0.30

No. 32 *Further Studies in Hawaiian Precipitation.* Samuel B. Solot, January 1950. $0.30

No. 33 *Artificial Production of Precipitation. Third Partial Report: Orographic Stratiform Clouds—California, 1949. Fourth Partial Report: Cumuliform Clouds—Gulf States, 1949.* Richard D. Coons, Earl L. Jones, and Ross Gunn, September 1949.

No. 34 *Predicting the Runoff From Storm Rainfall.* M. A. Kohler and R. K. Linsley, September 1951. $0.40

Weather Bureau Research Papers for sale by Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

U. S. GOVERNMENT PRINTING OFFICE: 1951