

PROCEDURES FOR RANDOM VIBRATION THEORY BASED SEISMIC SITE RESPONSE ANALYSES

**A White Paper Report Prepared for the Nuclear
Regulatory Commission**

Ellen M. Rathje¹ and Albert R. Kottke²

¹ J. Neils Thompson Associate Professor, Department of Civil, Architectural, and Environmental Engineering, University of Texas at Austin, Austin, TX

² Graduate Research Assistant, Department of Civil, Architectural, and Environmental Engineering, University of Texas at Austin, Austin, TX

March 2008

**Geotechnical Engineering Report GR08-09
Geotechnical Engineering Center
Department of Civil, Architectural, and Environmental Engineering
The University of Texas at Austin**

Notice from the Authors

This publication was prepared by the University of Texas at Austin under contract to the United States Nuclear Regulatory Commission (US NRC). This publication is being provided as general information to the public by the United States Nuclear Regulatory Commission. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed in this report, or represent that its use would not infringe privately owned rights. Reference therein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. Any views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Legally binding regulatory requirements are stated only in laws; NRC licenses, including technical specifications; or orders, not in NUREG-series publications or their NRC sponsored contractor-prepared documents. The views expressed in contractor-prepared publications in this series are not necessarily those of the NRC.

This report should be cited as:

Rathje, E.M. and Kottke, A.R. 2008. "Procedures for Random Vibration Theory Based Seismic Site Response Analyses: A White Paper Prepared for the Nuclear Regulatory Commission," Geotechnical Engineering Report GR08-09, Geotechnical Engineering Center, Department of Civil, Architectural, and Environmental Engineering, University of Texas at Austin, Austin, TX.

CONTENTS

INTRODUCTION	1
BACKGROUND	2
Site Response Analysis	2
Random Vibration Theory	4
RVT and Ground Motion Simulation	7
RVT APPLIED TO SITE RESPONSE ANALYSIS.....	10
General Approach	10
Input Motion Characterization	10
<i>Seismological source theory</i>	10
<i>Inverse Random Vibration Theory</i>	12
<i>Input motion comparisons</i>	14
SENSITIVITY ANALYSES	18
Analyses Performed	18
Sensitivity of Results to Input Motion Characterization.....	20
Sensitivity of Results to Specification of Rock Motion Duration	21
Sensitivity of Results to Specification of Soil Motion Duration	24
Sensitivity of Results to Specification of Strain-Time History Duration	25
CONCLUSIONS.....	27
REFERENCES	29

INTRODUCTION

Seismic site response analysis is used to assess the effect of site-specific soil conditions on expected earthquake ground shaking. For this purpose, input rock motions are propagated through a site-specific soil column to obtain motions at the ground surface. The typical outcome of a site response analysis is a site-specific acceleration response spectrum at the ground surface or site amplification factors that represent the ratio of the soil surface spectral acceleration to rock spectral acceleration at each period. The input motions used in site response analysis are previously recorded, modified, or simulated rock motions. Due to the uncertainty in the expected input rock motions, a suite of motions must be used and the number of input rock motions should be relatively large to obtain a statistically stable estimate of the median surface response spectrum or amplification factors.

An alternative to conventional site response analysis is a Random Vibration Theory (RVT) approach, which has been proposed in the engineering seismology literature (e.g., Schneider et al. 1991). In this approach, time domain input motions are not required; rather, a single input motion is specified as a Fourier Amplitude Spectrum (FAS), the FAS is propagated through the soil column using the equivalent-linear approach, and RVT is used to predict peak time domain estimates of motion (i.e., peak ground acceleration, spectral acceleration) at the ground surface. Due to its stochastic nature, RVT analysis can provide median estimates of the site response with a single analysis and no time domain input motions. Therefore, RVT is a potentially powerful tool for site response analysis that can provide fast and accurate estimates of the surface ground motion or amplification factors at a site.

This white paper describes the fundamentals of RVT and its application to seismic site response analysis of soil deposits. The input requirements specific to RVT site response are outlined, and sensitivity analyses are performed to assess the sensitivity of RVT site response results to the RVT input parameters.

BACKGROUND

Site Response Analysis

In most cases, one-dimensional (1D) site response analysis is performed to assess the effect of soil conditions on ground shaking because vertically-propagating, horizontally-polarized shear waves dominate the earthquake ground motion wave field of engineering interest. The 1D propagation of shear waves can be computed using traditional equivalent-linear (EQL) analysis using time domain input motions, EQL analysis using random vibration theory (RVT) input, or fully nonlinear (NL) analysis. These three techniques are explicitly cited in NUREG/CR-6728 (McGuire et al. 2001) and Regulatory Guide 1.208 (U.S. Nuclear Regulatory Commission 2007) as appropriate techniques for site response analysis, although each has its own advantages and disadvantages, as discussed below (Figure 1).

EQL site response analysis is based on one-dimensional, linear elastic wave propagation through layered media, but incorporates soil nonlinearity through the use of strain-compatible soil properties (i.e., shear modulus, G , and damping ratio, D) for each soil layer. These soil properties are modified to be consistent with the shear strains generated in each layer by the earthquake shaking, and thus the strain-compatible properties model the shear modulus reduction and increased damping expected during strong shaking. The variations of shear modulus and damping ratio with shear strain are prescribed through modulus reduction (G/G_{\max}) and damping (D) curves, and the shear strain used to select G and D is taken as a fraction of the peak time-domain shear strain. This reduced strain level is called the effective shear strain (γ_{eff}). NL analysis models the fully nonlinear shear stress-shear strain response of the soil in the time domain, and does not incorporate strain-compatible properties.

The main advantages of traditional EQL site response analysis are the modest site characterization requirements (i.e., shear wave velocity profile, unit weight, G/G_{\max} and D curves) and the fast computation time (owing to the fact that the calculations are performed using frequency domain transfer functions). The disadvantages of the EQL approach include (1) the need for a large suite of input motions (this is an issue for any analytical approach using recorded earthquake motions as input), and (2) potentially biased estimates of site amplification, particularly at large input intensities. The large suite of input motions is required because the characteristics of individual input motions affect the predicted site response, such that a large number of motions allows one to develop statistically robust estimates of site amplification. The potential bias in amplification from EQL analysis is caused by the equivalent-linear approximation, which models soil nonlinearity by identifying strain-compatible properties for use in the linear elastic analysis. The fact that the final EQL iteration utilizes a single value of shear modulus and damping ratio for each soil layer leads to potential overamplification at the degraded natural period of the site. Random vibration theory (RVT) attempts to address the first shortcoming for EQL analysis, while NL analysis addresses the second (Figure 1). This document will focus on the RVT approach to EQL site response analysis.

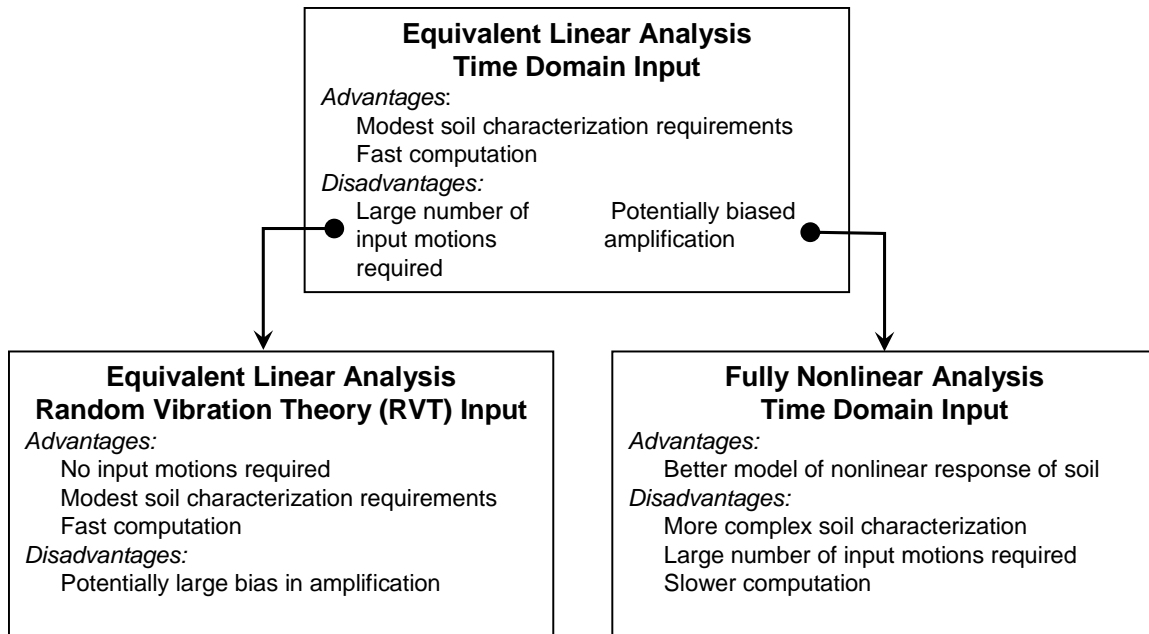


Figure 1. Advantages and disadvantages of different methods of site response analysis

RVT-based site response analysis is an extension of stochastic ground motion simulation procedures developed by seismologists to predict peak ground motion parameters as a function of earthquake magnitude and site-to-source distance (e.g. Hanks and McGuire 1981, Boore 1983). The RVT procedure consists of characterizing the Fourier Amplitude Spectrum (FAS) of a motion and using RVT to compute peak time domain values of ground motion from the FAS. When site response is included in the calculation, the FAS developed for rock is modified to account for the soil response before RVT is applied.

In geotechnical practice, the most commonly used analytical approach to site response involves one-dimensional, frequency domain transfer functions for layered soil deposits (e.g. SHAKE91, Idriss and Sun 1992). For this analysis, an outcropping rock acceleration-time history is specified to drive the analysis; therefore, it can be considered a time history site response analysis. A schematic of this procedure is shown in Figure 2(a), where the rock acceleration-time history is specified, propagated through the soil to the ground surface, and the time history surface motion is used directly to compute the acceleration response spectrum at the ground surface. The nonlinear response of the soil is approximated by the strain-compatible, EQL soil properties for each soil layer. Generally, a suite of scaled input motions is used that match, on average, a target input rock response spectrum. Alternatively, a suite of spectrally matched motions, each of which has been modified to match the input rock response spectrum at all periods, can be used. The target rock response spectrum for the input motions may be a deterministic response spectrum from a ground-motion prediction equation or, more likely, a uniform hazard spectrum from probabilistic seismic hazard analysis.

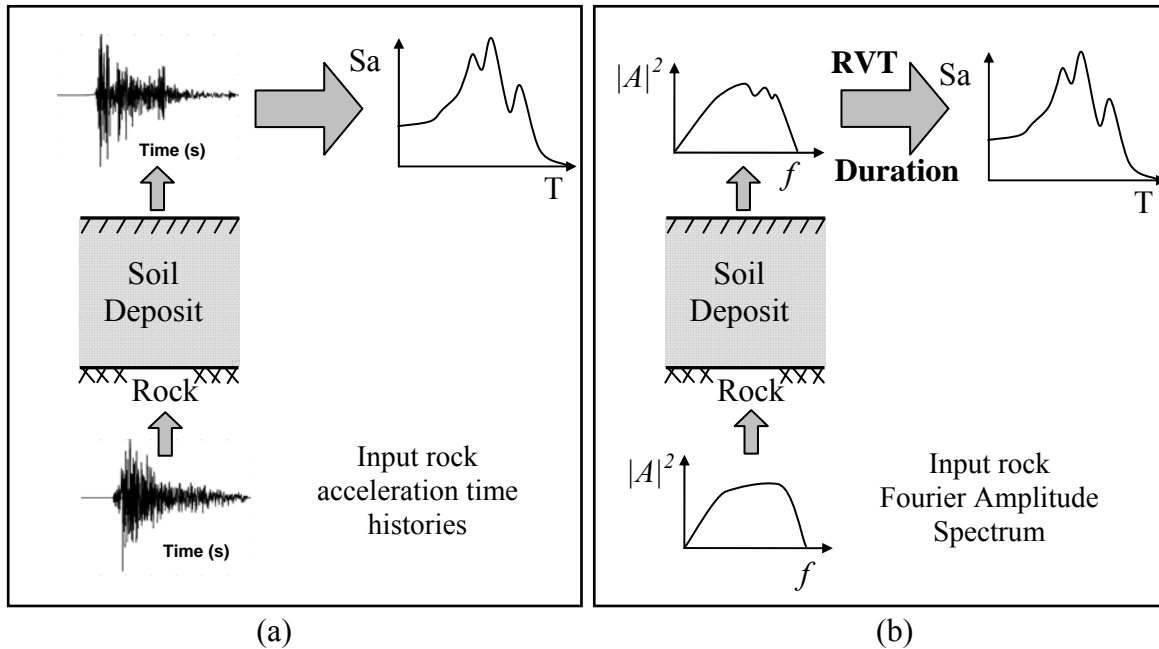


Figure 2. (a) Time History Seismic Site Response Analysis, and (b) Random Vibration Theory Based Seismic Site Response Analysis.

Random Vibration Theory represents an alternative to selecting a suite of motions for EQL analysis. In the RVT approach, the input to the site response analysis is a single Fourier Amplitude Spectrum (FAS) that represents the input rock motion. This spectrum contains only the Fourier amplitudes, without the accompanying phase angles, and thus cannot be used to compute directly an acceleration-time history. However, RVT can be used to estimate peak time domain values (e.g., peak acceleration) from the Fourier amplitude information. A schematic of RVT-based site response analysis is shown in Figure 2(b). Transfer functions (the same frequency domain transfer functions used in EQL analysis with time history input motions) are used to propagate the rock FAS through the soil column to obtain the FAS of the motion at the ground surface, and RVT is utilized to calculate peak time domain parameters, such as peak ground acceleration and spectral acceleration, from the FAS. The RVT calculation requires an estimate of the ground motion duration, a parameter that is not required in traditional EQL analysis with time domain input motions. The product of an RVT-based site response analysis is an acceleration response spectrum calculated from the surface FAS, rather than an acceleration-time history.

Random Vibration Theory

The key to RVT analysis is the prediction of peak time domain motions from only an FAS representation of the motion and its duration. Parseval's theorem and extreme value statistics (EVS) are used to relate the frequency domain motion with the peak time domain motion. EVS was first used in seismology by Hanks and McGuire (1981) to predict peak ground acceleration (PGA) from the *rms* (root-mean-square) acceleration, a_{rms} . Parseval's theorem is used to compute a_{rms} from the FAS, and a peak factor is used to relate a_{rms} to the peak ground acceleration.

Before application to earthquake motions, let us consider any time varying signal $x(t)$ with its associated FAS, $X(f)$. The *rms* value of the signal (x_{rms}) is a measure of its average value

over a given time period, T_{rms} , and is computed from the integral of the time series over that time period using:

$$x_{rms} = \sqrt{\frac{1}{T_{rms}} \int_0^{T_{rms}} [x(t)]^2 dt} \quad (1)$$

Parseval's theorem relates the integral of a time series to the integral of its Fourier Transform, such that Equation (1) can be re-written in terms of the FAS of the signal:

$$x_{rms} = \sqrt{\frac{2}{T_{rms}} \int_0^{\infty} |X(f)|^2 df} = \sqrt{\frac{m_0}{T_{rms}}} \quad (2)$$

where m_0 is defined as the zero-th moment of the FAS. The n -th moment of the FAS is defined as:

$$m_n = 2 \int_0^{\infty} (2\pi f)^n |X(f)|^2 df \quad (3)$$

The peak factor (PF) represents the ratio of the maximum value of a signal (x_{max}) to its rms value (x_{rms}), such that if x_{rms} and PF are known, then x_{max} can be computed using:

$$x_{max} = PF \cdot x_{rms} \quad (4)$$

Cartwright and Longuet-Higgins (1956) studied the statistics of ocean wave amplitudes, and considered the probability distribution of the maxima of a signal to develop expressions for the PF in terms of the characteristics of the signal. Cartwright and Longuet-Higgins (1956) derived an integral expression for the expected value of the peak factor in terms of the number of extrema (N_e) and the bandwidth (ξ) of the time series (Boore 2003):

$$E[PF] = \sqrt{2} \int_0^{\infty} [1 - \{1 - \xi \exp(-z^2)\}^{N_e}] dz \quad (5)$$

$$\xi = \sqrt{\frac{m_2^2}{m_0 m_4}} = \frac{N_z}{N_e} \quad (6)$$

The form of the equation in (5) was derived by Boore (2003) based on the work of Cartwright and Longuet-Higgins (1956). The bandwidth, ξ , is based on various moments of the FAS and also represents the ratio of the number of zero crossings in the signal (N_z) to the number of extrema (N_e).

To understand the relationship between ξ , N_e , and N_z , consider harmonic signals of a single frequency and multiple frequencies. Figure 3(a) displays a time series for a 4 Hz signal with amplitude of 0.5, and the locations of the extrema and zero crossings are indicated on the time signal. In this case, $N_z = N_e = 4$ over the time interval shown, and thus, ξ is equal to 1.0. Figure 3(b) displays a time series when the original 4 Hz signal is combined with a 40 Hz signal of amplitude 0.1. Here, N_e is much larger than N_z ($N_z = 4$, $N_e = 40$) and the resulting value of $\xi = 0.1$. Thus, for signals with motion spread over a range of frequencies (such as earthquake motions), N_z is generally smaller than N_e and ξ is smaller than 1.0. Because ξ can be computed

directly from the moments of the FAS (equation 6), the only additional term required for equation (5) is N_e . N_e is derived from the frequency of extrema (f_e) and the ground motion duration (T_{gm}), and can be related to the moments of the FAS using (Boore 1983):

$$N_e = 2 \cdot f_e \cdot T_{gm} = \frac{1}{\pi} \sqrt{(m_4/m_2)} \cdot T_{gm} \quad (7)$$

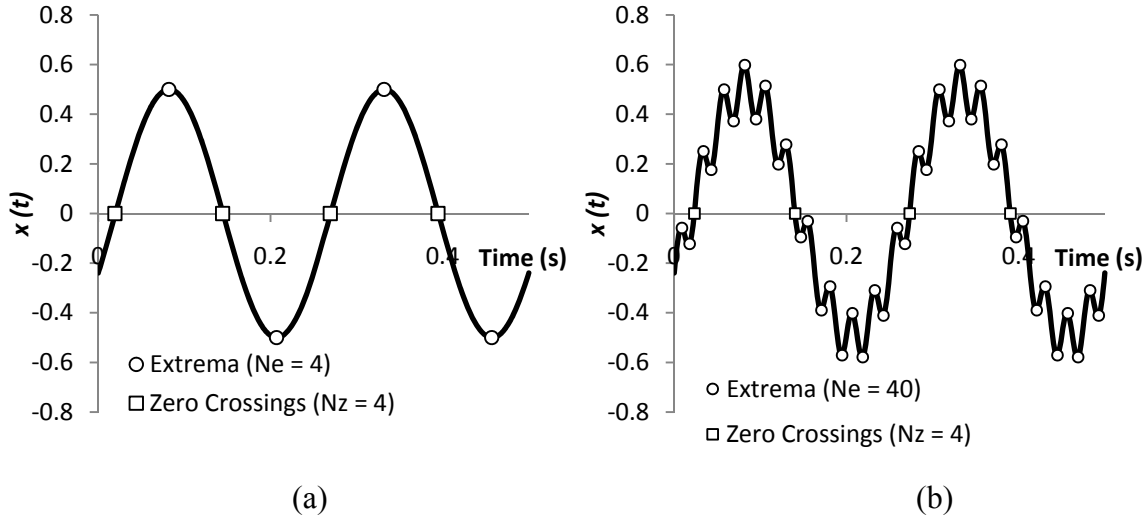


Figure 3. Examples of N_e and N_z for different signals: (a) time series at 4 Hz, (b) time series of combined 4 Hz and 40 Hz signals

For large values of N_e , equation (5) can be simplified into its asymptotic form (Boore 2003):

$$E[PF] = [2 \cdot \ln(\xi \cdot N_e)]^{1/2} + \frac{0.5772}{[2 \cdot \ln(\xi \cdot N_e)]^{1/2}} \quad (8)$$

Equation (8) is similar to the mean peak factor from a Type I Asymptotic equation (Gumbel 1958), which is valid for a narrow band signal ($\xi = 1.0$).

Figure 4 plots peak factors versus N_e computed from the full integral equation (equation 5) and from the asymptotic equation (equation 8) for two values of ξ . In Figure 4, the N_e values are plotted both in logarithmic and linear scales to illustrate differences at small and large values of N_e . A ξ value equal to 1.0 indicates that the initial signal is narrow band and that the peaks follow a Rayleigh distribution (Cartwright and Longuet-Higgins 1956). A ξ value of 0.45 represents a time series with motion spread over a range of frequencies and is a typical value for motions from moderate to large earthquakes. The values of PF generally range from 1.5 to 3.5 (Figure 4). The data in Figure 4 show that the PF increases more quickly at smaller values of N_e , but levels off for N_e greater than about 100. The curves indicate that the bandwidth has an important effect on the computed peak factor. Depending on the value of N_e , the pf for $\xi = 0.45$ is 5% to 50% smaller than for $\xi = 1.0$.

The integral equation is most accurate for calculating the expected value of PF , as it is valid for both large and small values of N_e . For $\xi = 1.0$, the asymptotic expression agrees well with the integral equation, with the difference between the integral and asymptotic equations less than

2% for all values of N_e greater than or equal to 2. For $\xi = 0.45$, the integral and asymptotic equations give similar results for N_e values larger than 20. For N_e values less than 20, the difference between the two equations becomes as large as 15%. Generally, it is preferred to use full integral expression (equation 5) to estimate the PF .

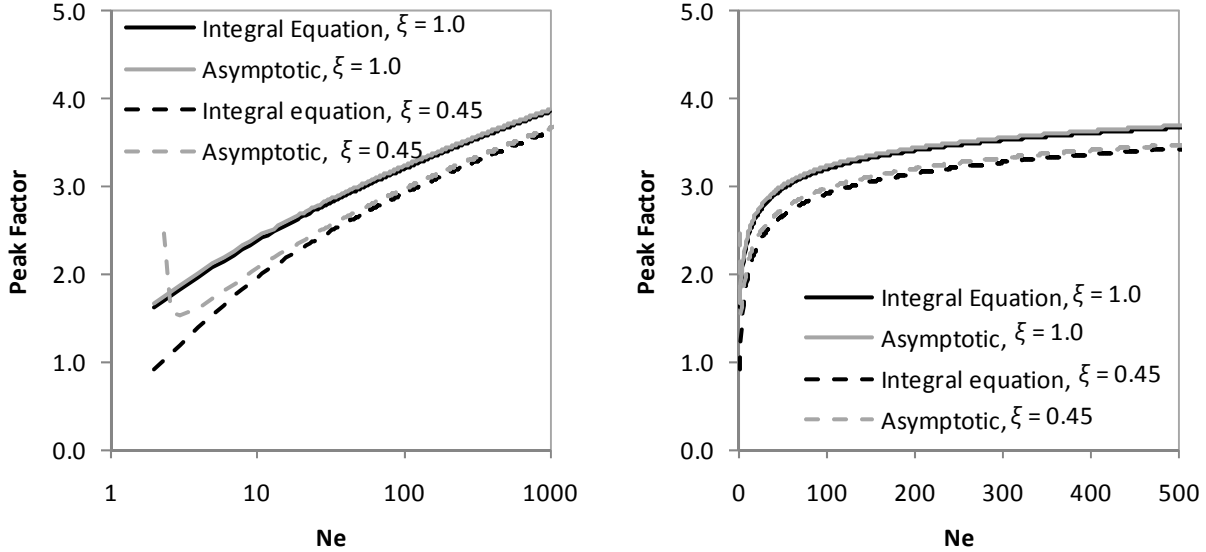


Figure 4. Comparison of peak factors derived from the integral expression (equation 5) and the asymptotic expression (equation 8) for different values of ξ .

Based on the description of Parseval's Theorem and EVS above, the following information is required to compute the peak time domain value of a signal (x_{max}):

- Fourier Amplitude Spectrum, $X(f)$
- Duration, T_{rms} , for rms calculation
- Ground motion duration, T_{gm} , for N_e calculation

These parameters and their selection for earthquake problems will be discussed further in the following sections.

It should be noted that the specification of durations for RVT analysis of strong ground motion has evolved since the pioneering work of Hanks and McGuire (1981) and Boore (1983). The original work used the ground motion duration for both the rms and N_e calculations, and this duration was taken as the source duration. Boore and Joyner (1984) illustrated the need to modify the duration used in the rms calculation when considering oscillator responses, and they introduced the concept of an rms duration. Current practice described in Boore (2003) recommends that the ground motion duration (T_{gm}) be derived from the sum of the source and path durations, and that a different duration (T_{rms}) be used for the rms calculation. This issue is addressed further in the next sections.

RVT and Ground Motion Simulation

Random vibration theory was first used in seismology by Hanks and McGuire (1981) to predict peak ground acceleration (PGA) from the rms acceleration, a_{rms} . This work was followed

by the landmark papers of Boore (1983) and McGuire et al. (1984) that demonstrated the power of RVT to predict various ground motions parameters (PGA, peak ground velocity, spectral acceleration). These works generally use the Brune (1970, 1971) omega-squared (ω^2) point source spectrum to describe the FAS of earthquake motions and single-degree-of-freedom oscillator transfer functions are applied to this FAS to describe spectral accelerations and spectral velocities. Boore (1983) compared peak estimates of ground motion from RVT with values computed from time domain simulations and showed that spectral values developed by RVT compared well with median values from the time domain simulations. McGuire et al. (1984) compared RVT-predicted spectral velocities with those from recorded motions from the 1971 San Fernando ($M_w = 6.6$) earthquake (Figure 5). The RVT-simulated spectral velocities agreed well with the recorded data. The work by McGuire et al. (1984) is considered landmark because it provided the most convincing evidence that RVT could be coupled with a seismological source spectrum to develop estimates of earthquake ground motions consistent with observations (Silva and Lee 1987), despite the various RVT assumptions violated by earthquake ground motions (e.g., non-stationary, non-Gaussian, etc.).

The application of RVT to ground motion simulation has evolved since the early 1980's, as summarized in Silva et al. (1997) and Boore (2003). These studies have considered the appropriate descriptions of the FAS of motion (e.g., point source vs. finite source, single corner frequency vs. multiple corner frequency) and the appropriate values of ground motion and *rms* durations, T_{gm} and T_{rms} respectively. Boore (2003) recommends the following description of T_{gm} for WNA:

$$T_{gm} = \frac{1}{f_c} + 0.05 \cdot R \tag{9}$$

where f_c is the corner frequency from the FAS and R is distance in km. The first term in equation (9) represents the source duration and the second term represents the path duration.

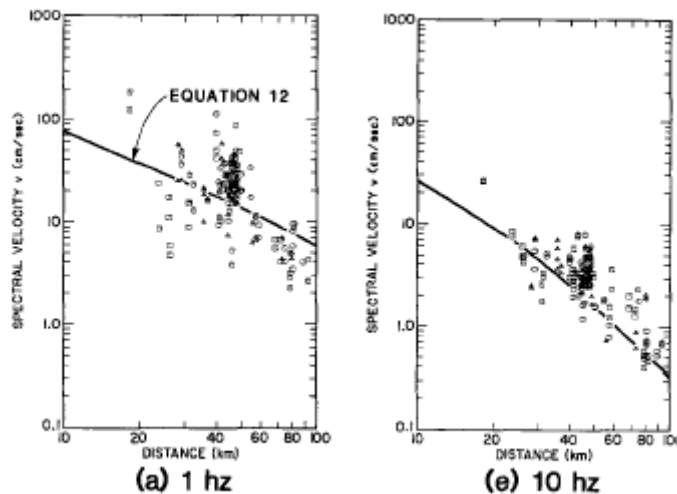


Figure 5. Comparison of observed (points) and RVT-predicted spectral velocities at frequencies of 1 and 10 Hz for the 1971 San Fernando earthquake (from McGuire et al. 1984).

The *rms* duration requires modification for spectral acceleration to account for enhanced duration due to the oscillator response. Generally, adding the oscillator duration to the ground motion duration will suffice, except in cases where the ground motion duration is short (Boore and Joyner 1984). Boore and Joyner (1984) recommend the following expressions to define T_{rms} :

$$T_{rms} = T_{gm} + T_o \left(\frac{\gamma^n}{\gamma^n + \alpha} \right) \quad (10)$$

$$\gamma = \frac{T_{gm}}{T_n} \quad (11)$$

$$T_o = \frac{T_n}{2\pi\beta} \quad (12)$$

where T_o is the oscillator duration, T_n is the oscillator natural period, and β is the damping ratio of the oscillator. Based on numerical simulations, Boore and Joyner (1984) proposed $n = 3$ and $\alpha = 1/3$ for the coefficients in equation (10).

RVT APPLIED TO SITE RESPONSE ANALYSIS

General Approach

RVT can be applied to EQL site response analysis to predict the response spectrum at the top of a soil deposit. The input rock motion FAS is specified and this spectrum is propagated to the ground surface using frequency domain transfer functions (i.e. $|F_{ij}(f)|^2$, where i and j are soil layers) that represent 1D wave propagation through the soil deposit. The transfer functions are similar to those used in traditional EQL site response programs such as SHAKE91 (Idriss and Sun 1992), except that only the amplitudes of the complex-valued transfer functions are used. In any EQL analysis, the shear strains in each layer must be computed to select the EQL soil properties that account for soil nonlinearity. In RVT-based analysis, these strains are computed using transfer functions that compute the shear strain FAS in each layer from the input rock FAS. Here, the FAS for shear strain is defined for each layer and RVT is used to calculate the peak time domain shear strain from the shear strain FAS. Similar to traditional EQL procedures, the peak shear strain is reduced to an effective shear strain to choose strain-compatible soil properties. Iterations are performed until the EQL soil properties are compatible with the shear strains generated in the soil. Comparisons between EQL site response analysis using RVT input and traditional, time domain input motions are provided by Ozbey (2006) and Rathje and Ozbey (2006).

Input motion characterization

There are different methods available to describe the input rock FAS for RVT site response analysis. One approach involves the use of seismology theory to compute the radiated FAS from a point source in terms of various source, path, and site parameters (EPRI 1993). Other techniques involve more complex seismological simulations (e.g., Silva et al. 1997, Beresnev and Atkinson 1998) or deriving the FAS from an acceleration response spectrum (e.g., Gasparini and Vanmarcke 1976, Rathje et al. 2005). It is critical that whatever procedure is used to develop the input rock FAS, that the corresponding acceleration response spectrum agrees with a developed target response spectrum.

Seismological source theory

This section represents a brief introduction to the terminology in seismological source theory and earthquake motion characterization, based on the description given by Boore (2003). The FAS of acceleration, $Y(f)$, at a rock site can be described analytically as a function of the source, propagation path, and site characteristics (the site characteristics in this case represent the effect of the near-surface rock layers and not the effect of the overlying soil layers). The Brune (1970, 1971) omega-squared (ω^2) point source spectrum is the most common and simplest representation of the radiated FAS from an earthquake. This source spectrum, $E(M_o, f)$, is coupled with the effects of the propagation path, $P(R, f)$, high frequency diminution, $D(f)$, and crustal amplification, $A(f)$, resulting in what is often called a Brune spectrum:

$$Y(f) = E(M_o, f) \cdot P(R, f) \cdot D(f) \cdot A(f) \quad (13a)$$

$$Y(f) = \left[0.78 \cdot \frac{\pi}{\rho_o \beta_o^3} \cdot M_o \frac{f^2}{1 + \left(\frac{f}{f_c}\right)^2} \right] \cdot \left[Z(R) \cdot \exp\left(\frac{-\pi f R}{Q(f) \beta_o}\right) \right] \cdot [\exp(-\pi \kappa_o f)] \cdot A(f) \quad (13b)$$

where f is frequency (Hz), ρ_o is the mass density of the crust (g/cm^3), β_o is the shear wave velocity of the crust (km/s), R is the distance from the source (km), $Z(R)$ is the geometric attenuation, $Q(f)$ is the anelastic attenuation, κ_o is the diminution parameter (seconds) and M_o is the seismic moment (dyne-cm). Seismic moment is related to moment magnitude (M_w) by:

$$M_o = 10^{1.5M_w + 16.05} \quad (14)$$

Finally, f_c in equation (13b) is the corner frequency (Hz), which represents the frequency below which the FAS of acceleration decays. The corner frequency is defined as:

$$f_c = 4.9 \cdot 10^6 \cdot \beta_o \left(\frac{\Delta\sigma}{M_o} \right)^{1/3} \quad (15)$$

where $\Delta\sigma$ is the stress drop (bars). The expressions above assume a point source for the earthquake and include only a single corner frequency.

Typical values of the source parameters required to describe $Y(f)$ are given in Table 1 for Western North America (WNA) and Eastern North America (ENA) based on Campbell (2003). The amplification function, $A(f)$, in equation 13b accounts for the propagation of waves from the deeper crust, where the shear wave velocity of the rock is equal to β_o ($\sim 3,500$ m/s), to the near surface, where the shear wave velocity of competent rock is generally 620 to 750 m/s in WNA and 2,800 m/s in ENA (Campbell 2003). Suggested values of $A(f)$ can be found in Boore and Joyner (1997) for generic rock sites in WNA and in Campbell (2003) for generic rock sites in ENA. These amplification values generally range between 1.0 and 4.0 in WNA and between 1.0 and 1.15 in ENA.

Table 1. Baseline seismological source and path parameters

Parameter	WNA	ENA
Density, ρ (g/cc)	2.8	2.8
Shear wave velocity, β_o (km/s)	3.5	3.6
Stress drop, $\Delta\sigma$ (bar)	100	150
Diminution parameter, κ_o (s)	0.04	0.006
Geometric attenuation, $Z(R)$	R^{-1} for $R < 40$ km $R^{-0.5}$ for $R \geq 40$ km	R^{-1} for $R < 70$ km R^0 for $R = 70$ to 130 km $R^{-0.5}$ for $R \geq 130$ km
Anelastic attenuation, $Q(f)$	$180 * f^{0.45}$	$680 * f^{0.36}$

Beyond earthquake magnitude and site-to-source distance, the most important parameters affecting the shape of the Brune spectrum are the stress drop ($\Delta\sigma$) and the diminution parameter (κ_o). The stress drop affects the corner frequency (equation 15), which in turn affects the moderate to high frequency portions of the spectrum (i.e., above about 0.3 Hz). The diminution

parameter affects higher frequencies (i.e., above about 10 Hz) through its frequency dependent exponential form (equation 13b).

Figure 6 displays the FAS for WNA and ENA ($M_w = 7$, $R = 20$ km) based on equation 13b and Table 1. The effects of crustal amplification (i.e., the term $A(f)$ in equation 13b) are not included. The most striking differences in the FAS in Figure 6 are the amplitudes at high frequencies for ENA due to the smaller value of κ_0 and the larger value of $Q(f)$. These differences illustrate that ENA ground motion have more high frequency content than WNA motions.

More advanced models that simulate the finite dimensions of the fault can be used to describe the rock FAS of motion (Silva et al. 1997). These simulations model the larger fault rupture with a number of smaller point sources with $M \sim 5$ (for crustal events) to 6.5 (for subduction events). Each point source is modeled using the Brune (1970, 1971) spectrum and the resulting spectra combined to generate the FAS at the site. More sophisticated models could also be used to generate the FAS of motion, but these simulations are not often performed on engineering projects.

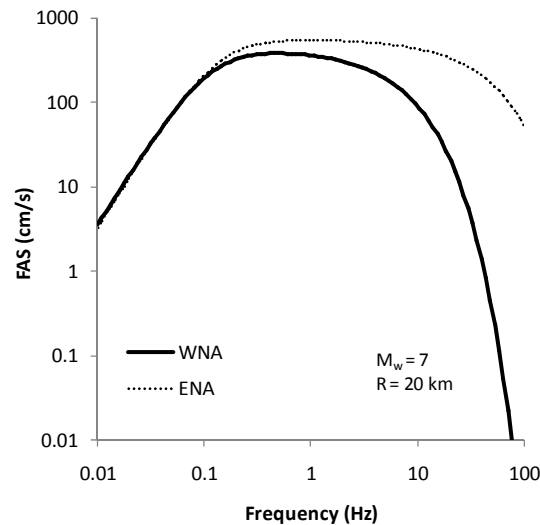


Figure 6. FAS for rock site conditions in WNA and ENA ($M_w = 7$, $R = 20$ km) using the point source spectrum and excluding crustal amplification effects ($A(f)$).

Inverse Random Vibration Theory

Inverse RVT (IRVT) develops a frequency domain FAS that is compatible with a specified acceleration response spectrum (Figure 7) and specified duration. While it is relatively straight forward to produce a response spectrum from a FAS, an iterative procedure is required to perform the inverse. There are two complications for the inversion. First, the spectral acceleration at a given frequency is influenced by a range of frequencies in the FAS, such that the spectral acceleration at a given period cannot be related solely to the Fourier amplitude at the same period. To solve this problem, the IRVT procedure takes advantage of the narrow-band properties of lightly damped, single-degree-of-freedom (SDOF) oscillator transfer functions (TF). The second complication is that the peak factor cannot be determined apriori because it is based on the moments of the FAS (equations 6 and 7), and the FAS initially is unknown. However, a peak factor can be assumed to develop an initial estimate of the FAS and then this

FAS can be used in a second iteration to compute peak factors for use in the inversion. The IRVT methodology described below is based on the procedure proposed by Gasparini and Vanmarcke (1976) and is described by Rathje et al. (2005).

Using the characteristics of SDOF oscillator transfer functions and an assumed, or known peak factor, the square of the Fourier amplitude at the SDOF oscillator natural frequency f_n ($|Y(f_n)|^2$) can be written in terms of the spectral acceleration at f_n (S_{a,f_n}), the peak factor (PF), the *rms* duration of motion (T_{rms}), the Fourier amplitudes ($|Y(f)|^2$) at $f < f_n$, and the integral of the SDOF oscillator transfer function ($|H_{f_n}(f)|^2$):

$$|Y(f_n)|^2 \approx \frac{1}{\int_0^\infty |H_{f_n}(f)|^2 df - f_n} \left(\frac{T_{rms}}{2} \cdot \frac{S_{a,f_n}^2}{PF^2} - \int_0^{f_n} |Y(f)|^2 df \right) \quad (16)$$

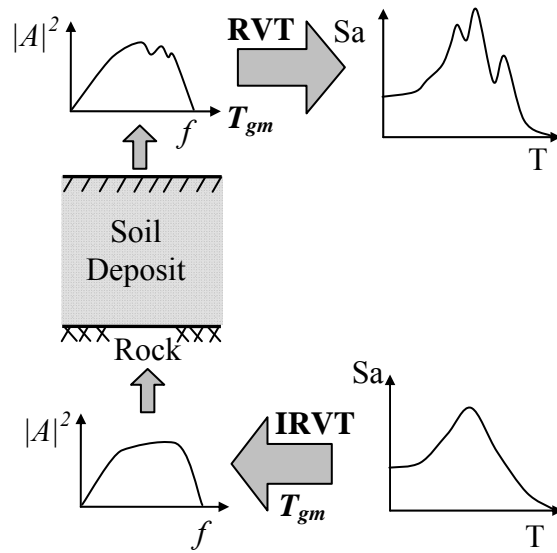


Figure 7. The application of IRVT to derive the input FAS for an RVT site response analysis.

Within equation (16), the integral of the transfer function is a constant for a given natural frequency and damping ratio, allowing the equation to be simplified to (Gasparini and Vanmarcke 1976):

$$|Y(f_n)|^2 \approx \frac{1}{f_n \left(\frac{\pi}{4\beta} - 1 \right)} \left(\frac{T_{rms}}{2} \cdot \frac{S_{a,f_n}^2}{PF^2} - \int_0^{f_n} |Y(f)|^2 df \right) \quad (17)$$

Equation (17) is applied first to predict $|Y(f)|$ at very low frequencies ($f_n \sim 0.01$ Hz), where the FAS integral term in (17) can be assumed equal to zero. The equation then is applied at successively higher frequencies using the previously computed values of $|Y(f)|$ to assess the integral. In previously performed work (Rathje et al. 2005), the frequency values consisted of 500 points equally spaced in log space. The minimum and maximum frequencies correspond to the minimum and maximum periods in the target response spectrum.

Equation (17) is used first to develop an initial estimate of the FAS using the target response spectrum and an assumed peak factor of 2.5 for all frequencies. The resulting FAS is then used

in a second iteration to develop more accurate values of peak factors that vary with frequency (equations 5 through 7 provide the PF values). Generally, after this second iteration the computed FAS from IRVT produces a response spectrum that deviates 5 to 10% from the target response spectrum. To improve the match with the target response spectrum, a correction is applied to the FAS based on the error between the target response spectrum and the response spectrum derived from the FAS. Although the spectral acceleration at a given frequency does not directly correlate to the FAS at that frequency, the ratio of spectral accelerations is used to modify the FAS and appears to work well. Using this correction, when the spectral value from the FAS is smaller than the target value, the FAS is increased, and when the spectral value from the FAS is greater than the target value, the FAS is reduced. The spectral ratio is defined as:

$$r = \left(\frac{Sa_{target}}{Sa_{FAS}} \right) \quad (18)$$

where Sa_{target} is the target spectral acceleration at a given period and Sa_{FAS} is the calculated spectral acceleration at the same period computed from the FAS using RVT. The FAS from iteration i is corrected using:

$$|Y(f)_{i+1}| = r \cdot |Y(f)_i| \quad (19)$$

This process is repeated until the desired accuracy is achieved. Additionally, some constraints are applied to the FAS at low and high frequencies to ensure that the tails of the FAS do not curve up in these frequency ranges. The correction is applied until an average error of 2% is reached or 25 iterations are performed.

Input motion comparisons

To compare the various RVT input motion characterizations, input rock FAS were developed for WNA for two earthquake scenarios: $M_w = 6.5$ and $R = 5$ km and $M_w = 7.5$, $R = 50$ km, where R represents the closest distance to the fault rupture plane. To develop IRVT input motions, target acceleration response spectra were developed using the Abrahamson and Silva (1997) ground motion prediction equations for rock. For the seismological point source characterization, the parameters in Table 1 were used in equation (13b) along with the crustal amplification factors recommended by Campbell (2003). A fictitious depth (h) of 10 km was combined with the closest distance measure (i.e., $R_{adj} = \sqrt{R^2 + h^2}$, Rathje and Ozbey 2006) and this value of R_{adj} was used in the point source FAS to be consistent with ground motion prediction equations. Ground motion durations (T_{gm}) were derived from seismological considerations and taken as the sum of the source duration and the path duration (e.g., equation 9 for WNA). For $M_w = 6.5$ and $R = 5$ km, T_{gm} was equal to 5.6 s, and for $M_w = 7.5$, $R = 50$ km T_{gm} was 18.4 s.

The derived rock FAS from the point source seismological spectrum and from IRVT, along with their corresponding acceleration response spectra, are shown in Figure 8. The target Abrahamson and Silva (1997) response spectra are also shown. The IRVT response spectra for both earthquake events are in excellent agreement with the target response spectrum from Abrahamson and Silva (1997), with the spectra being indistinguishable. For the point source spectrum, the resulting response spectra deviate from the target response spectra. For the $M_w = 6.5$, $R = 5$ km event, the point source FAS provides short period spectral accelerations that are as much as 40% smaller than the target spectrum, although the match is adequate at periods greater

than about 2.0 s. For the $M_w = 7.5$, $R = 50$ km event, the point source FAS provides an adequate match to the target spectrum over most periods. However, the response spectrum from the point source FAS is about 15% larger than the target between $T = 0.1$ and 0.2 s and is about 30% to 50% larger at periods greater than 2.5 s. A better match to the target spectrum potentially could be achieved by varying the parameters in Table 1 or the fictitious depth term within reasonable ranges.

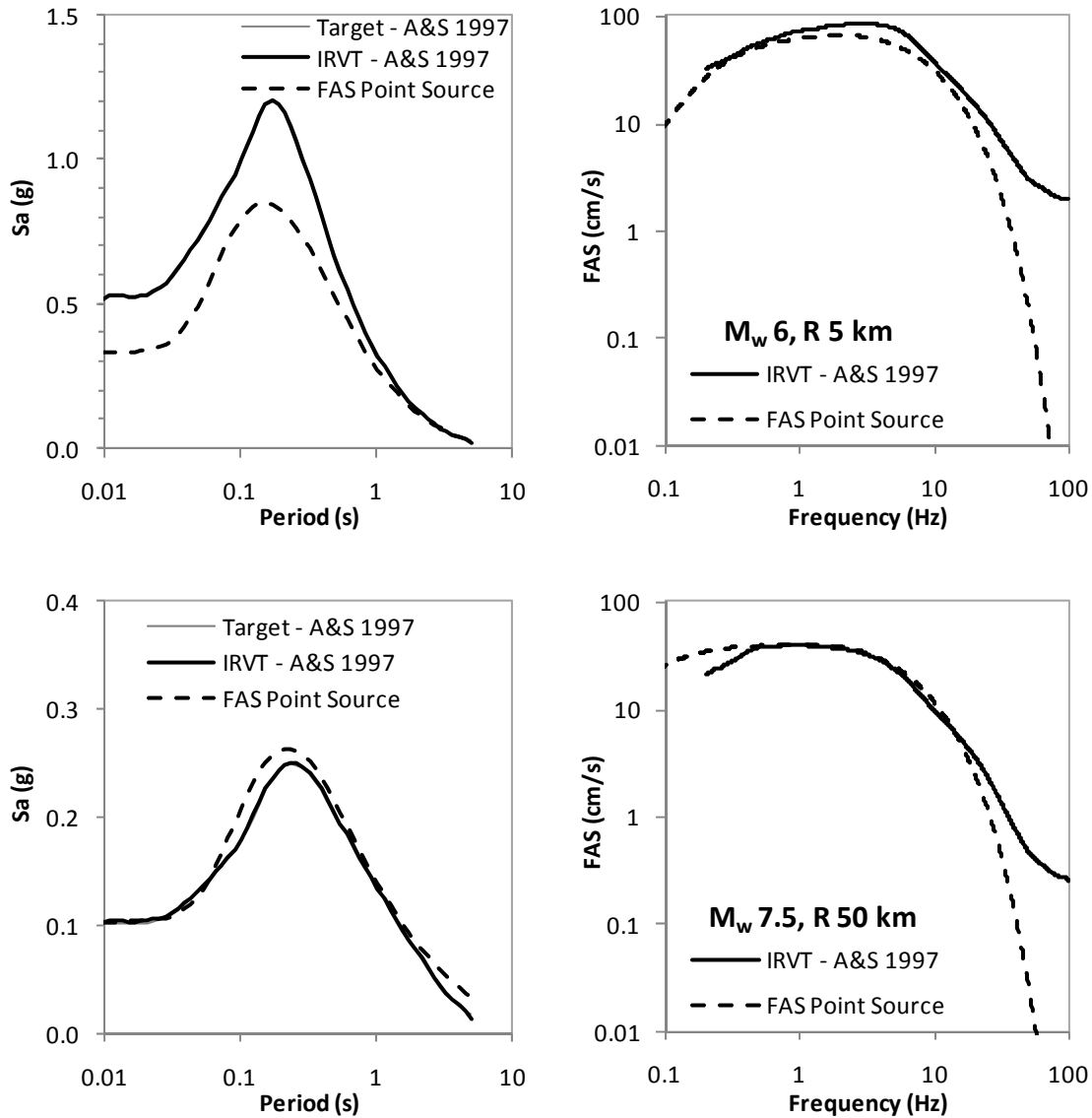


Figure 8. Comparison of Fourier amplitude spectra and acceleration response spectra from the seismological point source model and IRVT.

In comparing the FAS from IRVT and the point source model for the $M_w 6.5$, $R = 5$ km event (Figure 8), the FAS match at lower frequencies but start to diverge at frequencies greater than 2.0 Hz. These differences lead to the significant underprediction in spectral acceleration from the point source FAS for this event. As noted previously, the parameters from Table 1 or the fictitious depth term could be modified to provide a better match, but the resulting response

spectrum still would not necessarily match the target spectrum at all periods (Rathje et al. 2005). For the larger magnitude event (M_w 7.5, $R = 50$ km), the point source FAS matches the IRVT FAS over frequencies from 0.4 to 15 Hz, but it is significantly larger than the IRVT FAS at lower frequencies. This difference results in significantly larger spectral accelerations at long periods. The overprediction of the FAS at low frequencies for larger magnitude earthquakes occurs because of the breakdown of the point source assumption (Boore 1983). To address this issue, a two corner frequency model could be used (Atkinson and Silva 1997) or finite fault simulations could be performed (e.g., Silva et al. 1997, Beresnev and Atkinson 1998).

The discrepancies between the input rock response spectra and the FAS from the seismological point source model are certainly affected by the shortcomings of the point source model. A better fit to the target spectrum could be obtained from more sophisticated source models or finite fault simulations. Nonetheless, the differences in Figure 8 reveal that it is critical that the input rock FAS used in RVT site response calculations be converted to a rock response spectrum such that its match to a target spectrum can be assessed.

Duration Estimates

Based on seismological theory, the ground motion duration (T_{gm}) can be specified based on a source duration (T_{source}), which is equal to the inverse of the corner frequency, and a path duration (T_{path}), which is related to the distance from the source (e.g., equation 9 for WNA, see Campbell (2003) for ENA). Generally, this duration corresponds well with the duration between the occurrences between 5% and 75% of the Arias Intensity (D_{5-75}), and thus empirical predictions for D_{5-75} can also be used to specify T_{gm} for acceleration (Ozbey 2006). The T_{gm} value is used for the calculation of number of extrema (N_e) in RVT (equation 7), and is used to develop the *rms* duration (T_{rms}) used in the *rms* calculation for acceleration and spectral acceleration (equations 2 and 10). When considering site response analysis, two additional considerations are required: the influence of site response on the ground motion duration and the appropriate duration for the RVT calculation of peak shear strain.

Empirical observations of ground shaking have demonstrated that the duration of shaking at soil sites is longer than the duration at rock sites. The Abrahamson and Silva (1996) empirical model for duration predicts a 2 second difference between the values of D_{5-75} for soil and rock sites for shallow crustal events, while Alarcon (2007) predicts D_{5-75} soil motion durations about 25% to 50% larger than rock motion durations. These differences will influence RVT results, predominantly through the *rms* calculation (equation 2).

The other consideration is the appropriate duration to use in the RVT calculation for shear strain. The peak shear strain computed in each soil layer via RVT is converted to an effective shear strain and used to select strain-compatible soil properties for the EQL approximation. A shear strain-time history is related to a velocity-time history, and it is clear that a velocity time series evolves differently over time than an acceleration time series. For example, Figure 9 displays the acceleration and velocity time histories of the LA Temple and Hope recording (PGA = 0.13 g, PGV = 14 cm/s) from the 1994 Northridge earthquake. Also shown are plots of the build-up of normalized Arias Intensity. Here the Arias Intensity is simply defined as the integral over time of the squared values of the time series. Figure 9 reveals that the development of the normalized Arias Intensity over time for the acceleration and velocity-time histories is quite different. As a result, the D_{5-75} of the acceleration time history is 7.3 s, while the D_{5-75} of the velocity time history is 12.6 s. These data indicate that different durations potentially should be

used in the RVT calculations for shear strain and acceleration. To date, the same durations have been used, and the influence of using different durations has not been investigated.

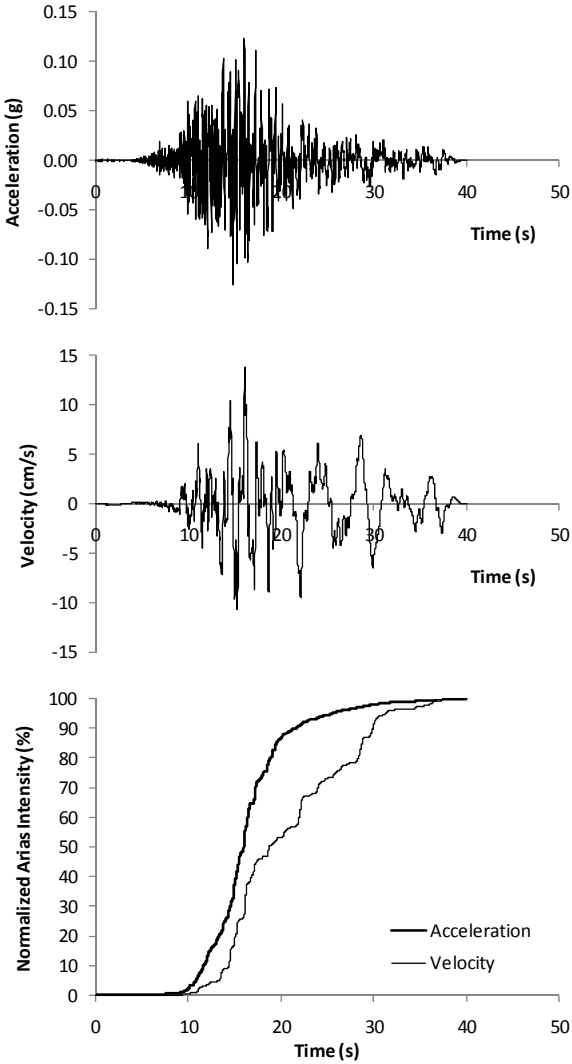


Figure 9. Acceleration and velocity time histories, and associated values of normalized Arias Intensity, for the LA Temple and Hope record from the Northridge earthquake.

SENSITIVITY ANALYSES

Analyses Performed

RVT site response analysis is fundamentally different than traditional EQL site response analysis in two main respects: (1) the input motion is characterized via a FAS rather than an acceleration-time history, and (2) Parseval's theorem and extreme value statistics (EVS) are utilized to compute PGA, spectral acceleration, and peak shear strains from FAS. The RVT input motion characterization can be derived via seismological simulation or from a rock acceleration response spectrum computed from an empirical ground motion prediction equation. Parseval's theorem and EVS both require an estimate of ground motion duration, a parameter that is not required in traditional EQL site response analysis. To assess the sensitivity of RVT site response results to the various parameters required for the analysis, a suite of site response analyses were performed. In these analyses, the input motion characterization and various duration parameters used in RVT were varied in an attempt to assess how the selection of these parameters affects RVT site response results.

The deep soil site at the Sylmar County Hospital (SCH) parking lot site in Southern California was used for all analyses. The site profile and shear wave velocity profile proposed by Chang (1996) were used in the analyses and are shown in Figure 10. The site consists of 91 m of sandy alluvium overlying bedrock, and the initial site period is about 0.7 s. The average shear wave velocity over the top 30 m is 273 m/s, which classifies the site as Site Class D (stiff soil) in the IBC (2006) site classification system. The equivalent linear response of the soil was modeled using the modulus reduction and damping curves proposed by Darendeli and Stokoe (2001). The Darendeli and Stokoe (2001) curves for mean effective confining pressures of 0.5 atm (middle of the surface layer), 2.5 atm (middle of the second layer), 5.5 atm (middle of the third layer), and 7.5 atm (middle of the bottom layer) are shown in Figure 11.

The FAS input motions for the RVT site response analyses were specified using two approaches: (1) derived directly from a point source seismological model and (2) FAS derived using IRVT and a rock acceleration response spectrum computed from a ground motion prediction equation for WNA. Two earthquake scenarios were considered: $M_w = 6.5$, $R = 5$ km ($\text{PGA}_{\text{rock}} \sim 0.5$ g) and $M_w = 7.5$, $R = 50$ km ($\text{PGA}_{\text{rock}} \sim 0.1$ g). The baseline values of rock motion duration (T_{gm}) for these scenarios are 5.6 s and 18.4 s, respectively. The input rock FAS and response spectra for these two scenarios were shown in Figure 8.

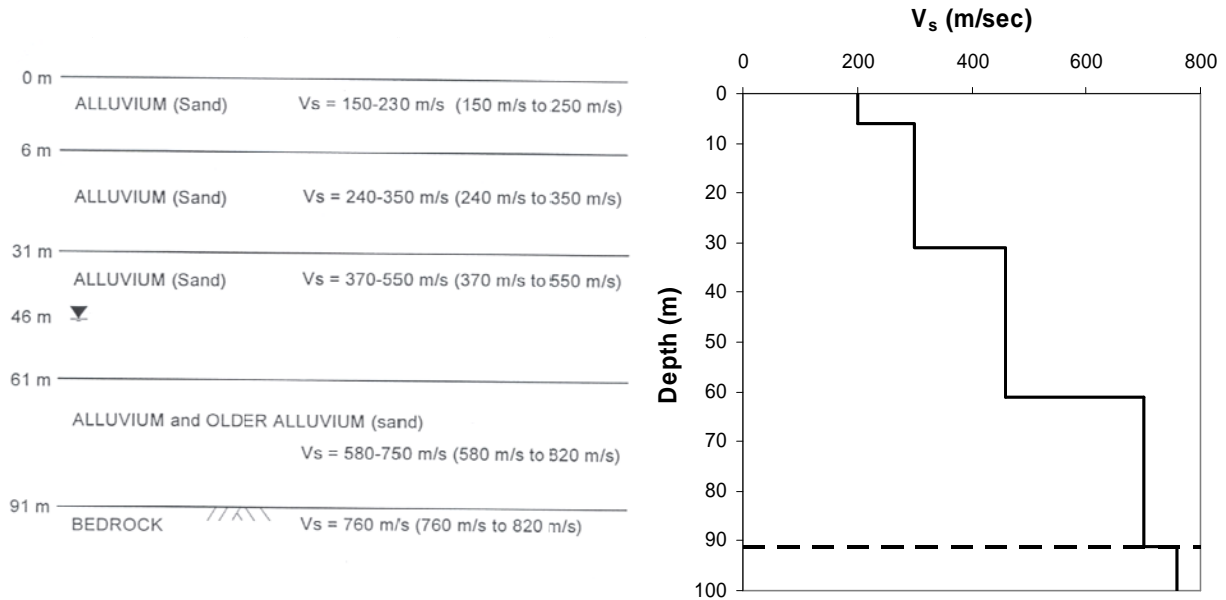


Figure 10. Site profile and shear wave velocity profile for the Sylmar County Hospital site (from Chang 1996).

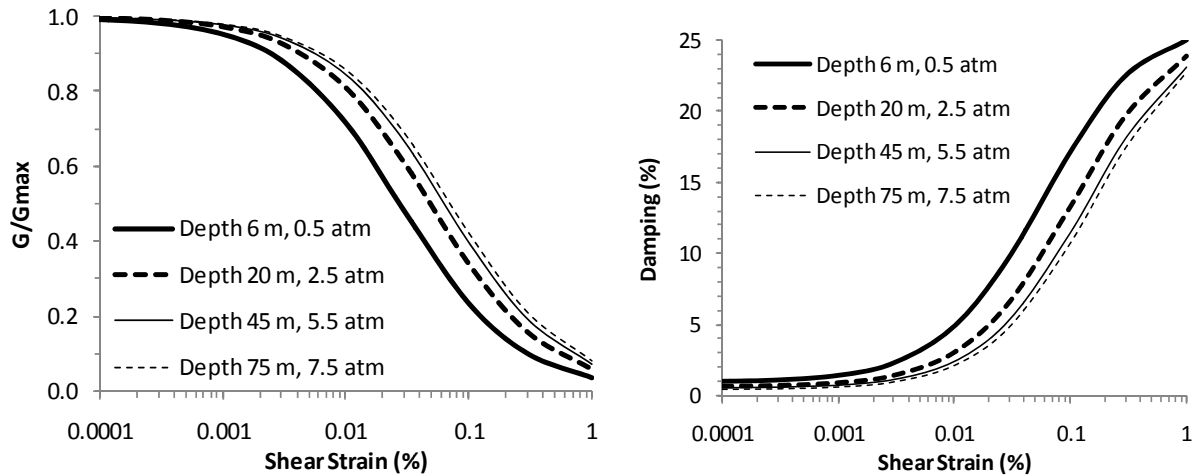


Figure 11. Modulus reduction and damping curves used in site response analyses.

The RVT sensitivity studies for duration focused on varying the duration parameters that are used for the rock motion ($T_{gm,rock}$), the soil surface motion ($T_{gm,soil}$), and the shear strain-time history ($T_{gm,strain}$). Three conditions were considered and are listed in Table 2. Generally, Condition 1 assumes all durations are the same and investigates the influence of the rock motion duration, Condition 2 investigates the influence of the soil surface motion duration, and Condition 3 investigates the influence of the strain duration. The baseline durations were varied by factors of 0.5 and 2.0 in an attempt to encompass the possible range of each value. For each condition, site response results were compared in terms of the input rock motion, soil surface motion, and spectral amplification ratio ($S_{a,soil} / S_{a,rock}$).

Table 2. Duration conditions modeled in sensitivity study

Condition 1 Rock Motion Duration	Condition 2 Soil Motion Duration	Condition 3 Strain Duration
$T_{gm,rock} = T_{source} + T_{path}$ $T_{gm,soil} = T_{gm,rock}$ $T_{gm,strain} = T_{gm,rock}$ $T_{gm,rock}$ varied by factors of 0.5 and 2.0.	$T_{gm,rock} = T_{source} + T_{path}$ $T_{gm,soil} = 0.5 T_{gm,rock}$ $= 1.0 T_{gm,rock}$ $= 2.0 T_{gm,rock}$ $T_{gm,strain} = T_{gm,rock}$	$T_{gm,rock} = T_{source} + T_{path}$ $T_{gm,soil} = T_{gm,rock}$ $T_{gm,strain} = 0.5 T_{gm,rock}$ $= 1.0 T_{gm,rock}$ $= 2.0 T_{gm,rock}$

Sensitivity of Results to Input Motion Characterization

To investigate the sensitivity of RVT site response results to the input motion characterization, analyses were performed using the point source FAS and IRVT FAS for the M_w 6.5, R 5 km scenario. This scenario was selected because it provided the largest discrepancies between the IRVT rock response spectrum and the rock response spectrum derived from the point source FAS (Figure 8).

The rock response spectra, the soil surface response spectra, and the spectral amplification from the site response analyses are shown in Figure 12. As noted previously, the input motions differ significantly in intensity (Figure 12a), and as a result the soil surface spectra and spectral amplification also differ (Figures 12b, c). The point source results show a smaller PGA and smaller spectral accelerations at moderate periods than the IRVT results, as well as the maximum spectral acceleration at a slightly shorter period. At some periods, the differences are as large as 30%. The differences in surface spectra are due to differences in spectral amplification (Figure 12c) induced by the different input intensities (Figure 12a), coupled with the different input intensities themselves. The smaller input intensity from the point source model induces smaller strains, which leads to less damping, more low period amplification (Figure 12c), and a slightly shorter site period as evidenced by the locations of the peaks in spectral amplification in Figure 12c. The results in Figure 12 are not surprising, and similar differences would be observed using time history input motions fit to the different input rock response spectra. Nonetheless, these results emphasize the importance of knowing the response spectral representation of the FAS input used in RVT site response analyses.

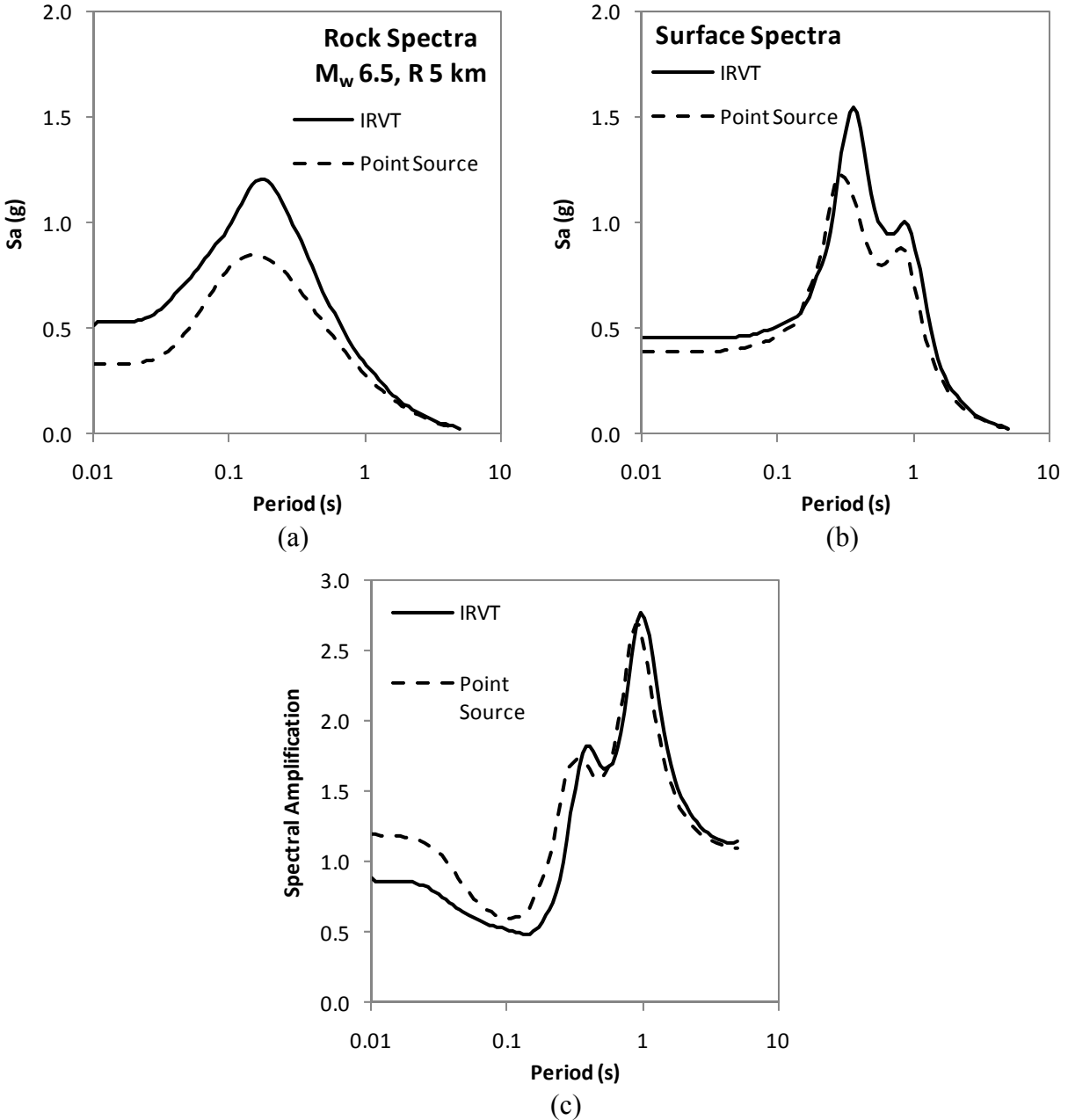


Figure 12. (a) Input rock response spectra, (b) soil surface response spectra, and (c) spectral amplification for input motions specified via a point source seismological spectrum and IRVT.

Sensitivity of Results to Specification of Rock Motion Duration

Site response analyses were performed for seismological point source and IRVT input motions using variable rock motion durations ($T_{gm,rock}$), as specified in Table 2 (Condition 1). The value of $T_{gm,rock}$ influences the *rms* and peak factor calculations, both of which influence computed RVT peak values (i.e., peak acceleration, spectral acceleration, peak shear strain). The *rms* and peak factor calculations are affected differently by changes in T_{gm} . An increase in T_{gm} increases the *rms* duration (T_{rms}) which reduces the *rms* acceleration due to the term $\sqrt{1/T_{rms}}$ in

the *rms* equation (equation 2). On the other hand, an increase in T_{gm} increases the peak factor due to the increase in the number of extrema (equation 7, Figure 4). However, the change in peak factor is not directly related to the change in T_{gm} (see equation 5, Figure 4), and thus the increase in the peak factor generally is less than the associated decrease in the *rms* value. As a result, an increase in T_{gm} generally reduces peak acceleration and higher frequency spectral accelerations. This trend is somewhat reversed at longer periods, where the number of extrema (N_e) is small. Here, an increase in N_e leads to larger increases in the peak factor (Figure 4) that compensate for the decrease in *rms* value. Thus, at longer periods an increase in T_{gm} may result in only a minor change, or even an increase, in spectral acceleration.

Rock motion durations affect differently the input representation of the seismological point source model and IRVT. When using the seismological point source model, the input FAS is not affected by $T_{gm,rock}$, but the corresponding acceleration response spectrum is affected due to the influence of $T_{gm,rock}$ on the *rms* and peak factor calculations. When using IRVT input, the acceleration response spectrum is not affected by $T_{gm,rock}$, but the corresponding FAS is affected; again due to the influence of $T_{gm,rock}$ on the *rms* and peak factor calculations. Figure 13 displays the FAS and acceleration response spectra derived for the M_w 6.5, R 5 km event from the point source model and IRVT using different values of $T_{gm,rock}$. The baseline value of $T_{gm,rock}$ for this event is 5.6 s. The changes in $T_{gm,rock}$ result in changes of +/- 30% in the FAS for IRVT input, and similar changes are observed in the acceleration response spectra for the point source FAS. However, the observed changes spectra are in the opposite direction: for IRVT an increase in $T_{gm,rock}$ results in an increase in FAS, while for the point source an increase in $T_{gm,rock}$ results in a decrease in spectral acceleration. These trends are expected based on how Parseval's theorem and EVS are applied in RVT versus IRVT procedures. Similar results were found for the M_w 7.5, R 50 km event and its associated rock motion duration ($T_{gm,rock} = 18.4$ s).

The differences in input characterizations due to different values of $T_{gm,rock}$ will affect the site response results. Figure 14 displays the surface response spectra and spectral amplifications derived from RVT site response analyses using a point source FAS input and IRVT input for M_w 6.5, R 5 km. Curves are shown for the different values of $T_{gm,rock}$. For the point source input characterization, changes in $T_{gm,rock}$ result in changes of +/- 20% in the surface response spectra and +/- 40% for the spectral amplification. For the IRVT input characterization, the differences are more moderate, in the range of +/- 15%. These differences are most significant at shorter periods. Again, changes in $T_{gm,rock}$ affect the point source FAS and IRVT site response results in opposite directions (i.e., an increase in $T_{gm,rock}$ results in a smaller response for the point source input FAS and a larger response for the IRVT input). Changes in $T_{gm,rock}$ resulted in larger differences in surface spectra and smaller differences in spectral amplification for the less intense M_w 7.5, R 50 km event.

These results reveal that input rock motion duration can affect the results from RVT site response analyses. The differences appear to be less pronounced when using IRVT input characterization, but differences as large as 30% are still possible.

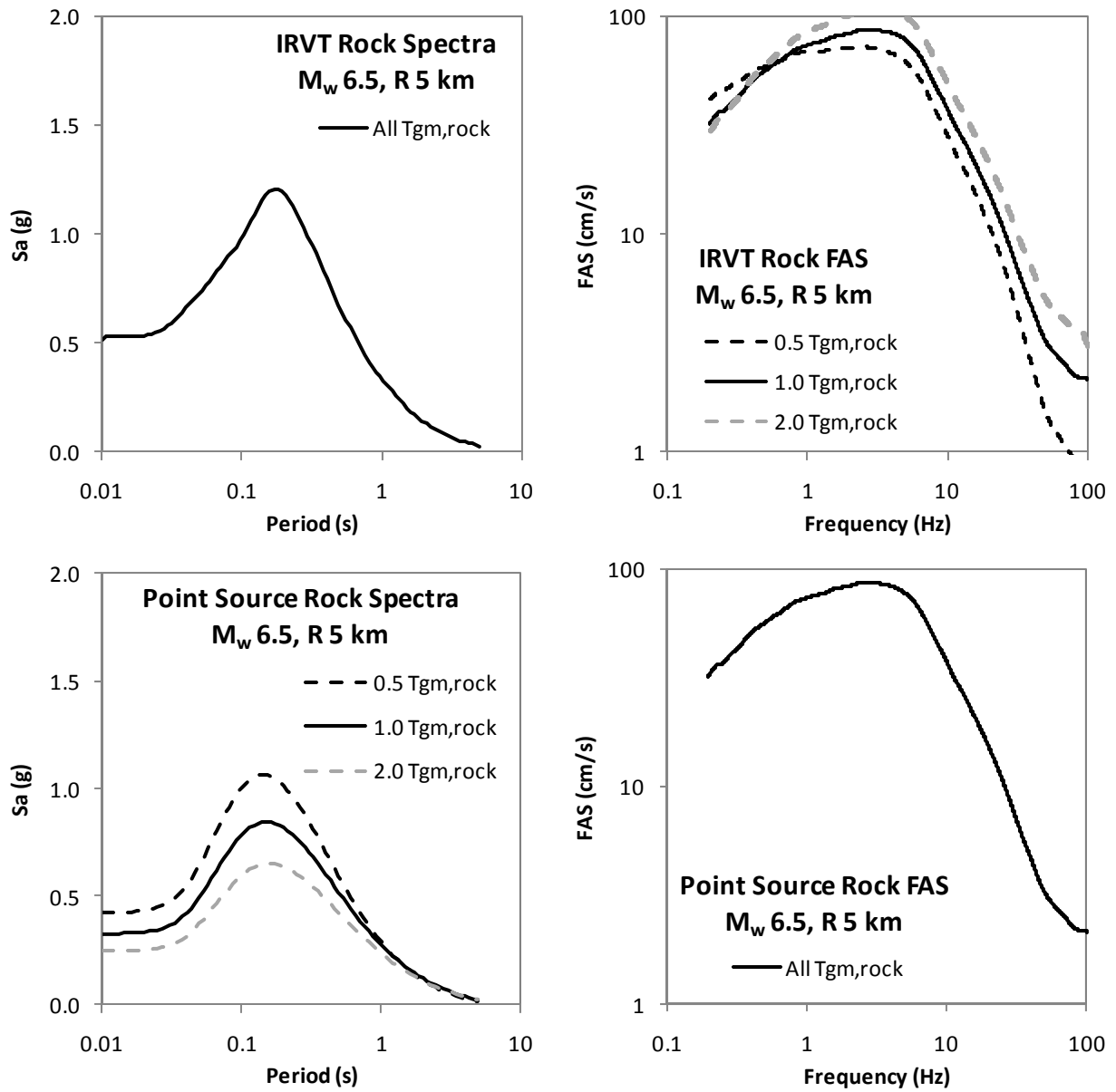


Figure 13. Influence of rock motion duration on input rock response spectra and input rock FAS derived from point source seismological spectrum and IRVT.

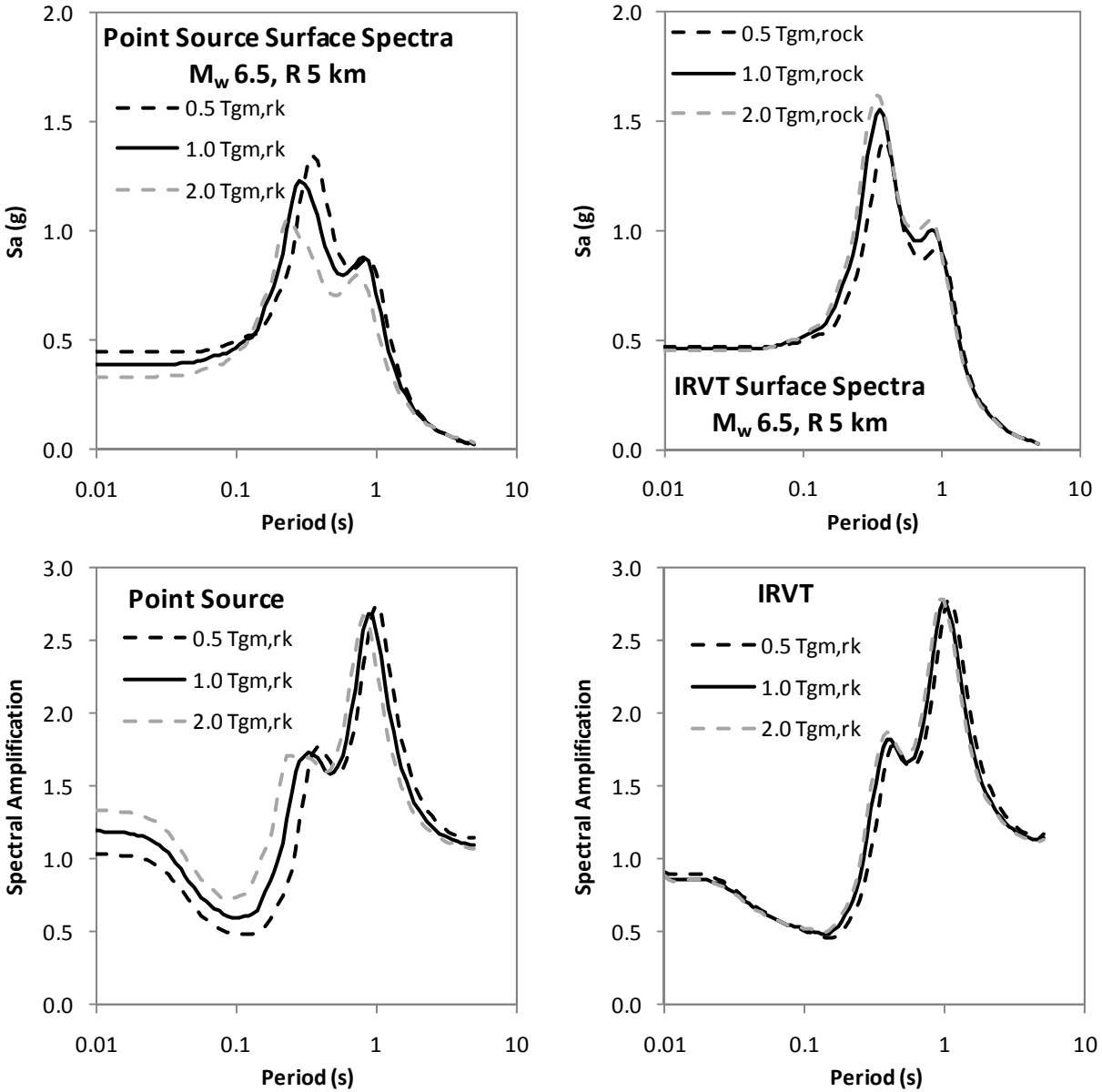


Figure 14. Influence of rock motion duration on soil surface spectrum and spectral amplification when using input motion based on point source spectrum and IRVT.

Sensitivity of Results to Specification of Soil Motion Duration

Site response analyses were performed using variable durations for the soil surface motion ($T_{gm,soil}$) associated with a constant $T_{gm,rock}$, as specified in Table 2 (Condition 2). In these analyses, T_{gm} was modified only for the RVT calculation that converts the soil surface FAS to soil surface acceleration response spectrum. The RVT calculations for rock response spectra and shear strain used the baseline value of $T_{gm,rock}$. The point source FAS was used as the input characterization for all analyses in this section, although results were similar for IRVT input.

Figure 15 displays soil surface spectra for different values of $T_{gm,soil}$ and the two earthquake events (M_w 6.5, R 5 km and M_w 7.5, R 50 km). At periods less than about 1.0 s, the differences in soil surface spectra are +/- 30%, with larger values of $T_{gm,soil}$ resulting in smaller spectral

accelerations. At longer periods, the differences are less significant, generally less than 15%. The observed differences for spectral amplification were exactly the same as for soil surface spectra because changes in $T_{gm,soil}$ do not affect the rock response spectra, and thus these results are not shown.

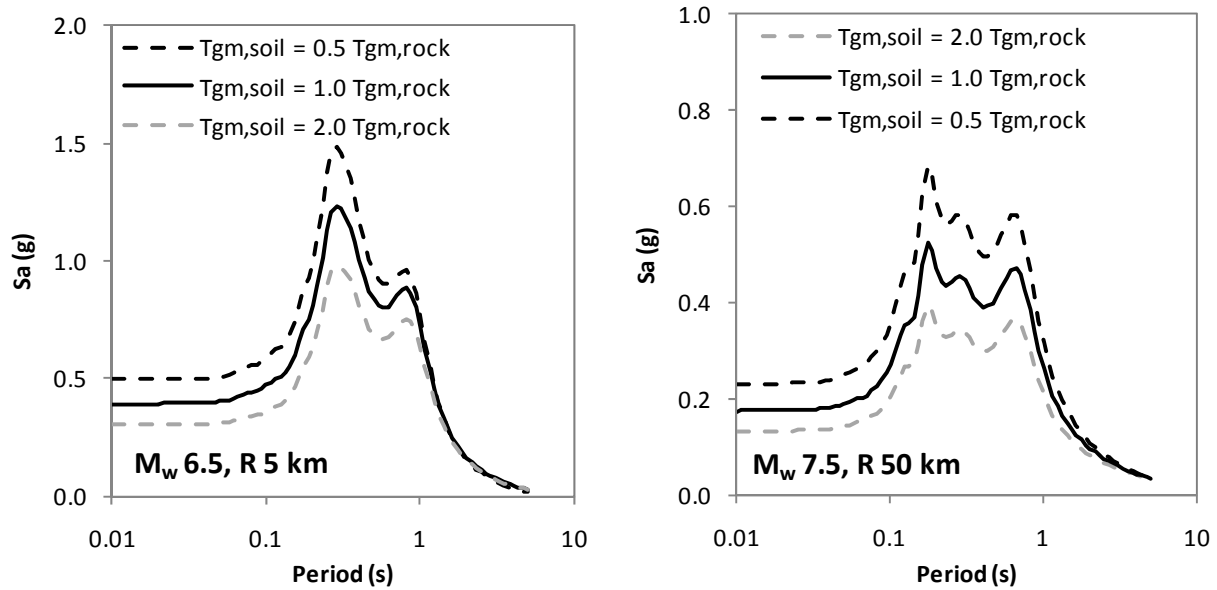


Figure 15. Influence of soil motion duration on soil surface response spectra.

Sensitivity of Results to Specification of Strain-Time History Duration

Site response analyses were performed using variable durations for the shear strain-time history motion ($T_{gm,strain}$), as specified in Table 2 (Condition 3). In these analyses, T_{gm} was modified only for the RVT calculation that converts the shear strain FAS in each soil layer to peak shear strain for use in the selection of EQL soil properties. The RVT calculations for rock response spectra and soil surface response spectra used the baseline value of $T_{gm,rock}$. The point source FAS was used as the input characterization for all analyses in this section, although results were similar for IRVT input.

Changing $T_{gm,strain}$ results in changes in the peak shear strain, and thus the level of nonlinearity induced by a given input motion. Generally, increasing $T_{gm,strain}$ results in smaller strains, less modulus reduction, and less damping within the soil deposit. As a result, increasing $T_{gm,strain}$ induces less period lengthening (i.e., smaller site period) and increases spectral accelerations at shorter periods (due to less damping). The opposite is true when decreasing $T_{gm,strain}$.

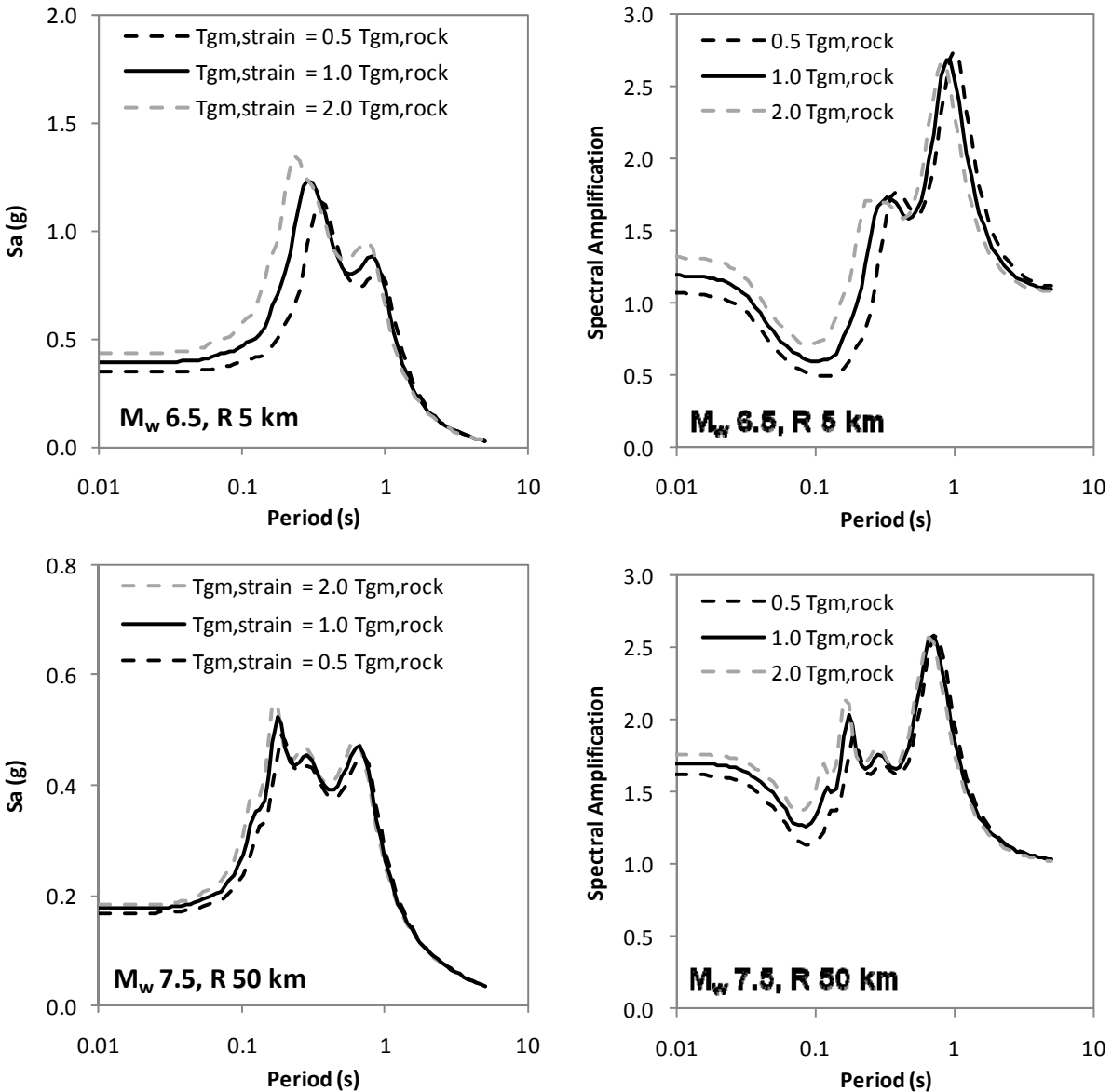


Figure 16. Influence of strain duration on soil surface response spectra and spectral amplification.

The surface soil response spectra and spectral amplification factors for variable $T_{gm, strain}$ are plotted in Figure 16. Figure 16 reveals that changing $T_{gm, strain}$ induces changes of +/- 35% in spectral accelerations at periods less than about 0.3 s for the M_w 6.5, R 5 km event, and induces changes of +/- 15% for the M_w 7.5, R 50 km event. At longer periods, the differences are more moderate (less than 10%). The effect of $T_{gm, strain}$ on period lengthening and high frequency damping can be observed more easily in the spectral amplification plots of Figure 16. For the more intense M_w 6.5, R 5 km event, the differences are more pronounced because larger strains, and thus more nonlinearity, are induced.

CONCLUSIONS

Random vibration theory (RVT) based seismic site response analysis is a potentially powerful simulation tool that can provide median estimates of soil surface response spectra and spectral amplification factors for soil deposits subjected to earthquake motions. The main advantage of RVT site response analysis is the fact that a suite of input rock motions is not required to perform the analysis. However, RVT site response analysis can only be performed using the equivalent-linear approximation, and thus cannot truly model the fully nonlinear stress-strain characteristics of the soil.

RVT site response analysis uses the same soil characterization used by traditional EQL analysis (i.e., shear wave velocity profile, unit weight, modulus reduction and damping curves). The main difference is in how the input rock motion is specified. The input rock motion is specified only as a Fourier Amplitude Spectrum (FAS), and this FAS is propagated to the ground surface using linear-elastic, frequency-domain transfer functions. The FAS is converted to an acceleration response spectrum using Parseval's theorem and extreme value statistics (EVS). This process requires that a ground motion duration be specified; duration is the one RVT parameter that is distinct from those used in traditional EQL site response analysis. The input rock FAS for RVT site response analysis can be specified using seismological simulations or derived from a rock acceleration response spectrum.

When performing RVT site response analysis, it is important to evaluate the equivalent input rock response spectrum used in analyses, particularly if one is using seismological simulations to generate the FAS of the input rock motion. The input motion characterization, including the rock motion duration ($T_{gm,rock}$), affects the level of nonlinearity induced, the spectral amplification, and the absolute values of spectral acceleration computed from RVT. Thus, it is critical that the input rock response spectrum be consistent with a previously determined target rock response spectrum.

Sensitivity analyses demonstrated that ground motion duration can significantly influence the results from RVT site response analysis. Currently, most RVT site response analyses use the same duration for the rock motion, soil surface motion, and shear strain-time histories in each soil layer. Under these conditions, modifying $T_{gm,rock}$ by a factor of two can change the surface spectra and spectral amplification factors by as much as 40%. This effect is most pronounced at periods less than 1 to 2 s. Observations of ground shaking reveal that the duration of soil motions differ from those of rock motions, indicating that the duration of the soil surface motion should be modified from $T_{gm,rock}$. Under these conditions, modifying $T_{gm,soil}$ by a factor of two, while keeping $T_{gm,rock}$ constant, can change the surface spectra and amplification factors by as much as 30%. Again, this effect is most pronounced at periods less than 1 to 2 s. Finally, shear strain-time histories display different characteristics than acceleration-time histories, and thus the RVT shear strain calculation potentially should use a duration different than $T_{gm,rock}$. Under these conditions, modifying $T_{gm,strain}$ by a factor of two, while keeping $T_{gm,rock}$ and $T_{gm,soil}$ constant, can change the surface spectra and amplification factors by as much as 30%. These changes are most significant at periods less than 0.3 s and for more intense input intensities that induce larger strains and more nonlinearity.

Specification of the appropriate duration parameters for RVT site response analysis is critical to obtaining results consistent with time domain simulations. This study revealed that modification to the duration parameter used in RVT site response analyses can change the results by as much as 40%. A comprehensive comparison between RVT and time domain EQL site

response analyses is required to evaluate the appropriate durations to be used in future simulations.

REFERENCES

- Abrahamson, N. A., and Silva W. J. (1997). "Empirical response spectral attenuation relationships for shallow crustal earthquakes." *Seism. Res. Lett.*, Vol. 68, No. 1, pp. 94-127.
- Abrahamson, N.A. and Silva, W.J. 1996. "Empirical ground motion models," Report prepare for *Brookhaven National Laboratory*, New York, New York, 144 pp.
- Alarcón, J.E. (2007). Estimation of Duration, Number of Cycles, Peak Ground Velocity, Peak Ground Acceleration and Spectral Ordinates for Engineering Design. A thesis submitted to the University of London for the Degree of Doctor of Philosophy, Department of Civil & Environmental Engineering, Imperial College London, April, pp. 340
- Beresnev, I.A., and Atkinson, G.M. (1998) "FINSIM: A FORTRAN program for simulating stochastic acceleration time histories from finite faults," *Seismological Research Letters*, 69, pp. 27-32.
- Boore, D. (1983). "Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra." *Bull. Seism. Soc. Am.*, Vol. 73, No. 6, pp. 1865-1894.
- Boore, D. M. (2003). "Simulation of ground motion using the stochastic method." *Pure Appl. Geophys.*, Vol. 160, No. 3-4, pp. 635-676.
- Boore, D. M., and Joyner, W. B. 1984. "A note on the use of random vibration theory to predict peak amplitudes of transient signals." *Bull. Seism. Soc. Am.*, Vol. 74, No. 5, pp. 2035-2039.
- Boore, D. M., and Joyner, W. B. (1997). "Site amplification for generic rock sites." *Bull. Seism. Soc. Am.*, Vol. 87, No. 2, pp. 327-341.
- Brune, J. (1970). "Tectonic stress and the spectra of seismic shear waves from earthquakes." *Journal of Geophysics Research*, Vol. 75, No. 26, pp. 4997-5009.
- Brune, J. (1971). "Correction." *Journal of Geophysics Research*, Vol. 76, No. 20, pp. 5002.
- Campbell, K. W. (2003). "Prediction of strong ground motion using the hybrid empirical method and its use in the development of ground-motion (attenuation) relationships in Eastern North America." *Bull. Seism. Soc. Am.*, Vol. 93, No. 3, pp.1012-1033.
- Cartwright, D. E., and Longuet-Higgins, M. S. (1956). "The statistical distribution of the maxima of a random function." *Proc. Roy. Soc. London*, Ser. Vol. A237, pp. 212-232.
- Chang, S. Wa-Yne C. (1996) "Seismic Response of Deep Stiff Soil Deposits.", *PhD. Dissertation*, Univ. of California, Berkeley.
- Darendeli, M. B. and Stokoe, K. H., II (2001). "Development of a new family of normalized modulus reduction and material damping curves," *Geotechnical Engineering Report GD01-1*, University of Texas at Austin, Austin.
- Electric Power and Research Institute (1993) "Guidelines for Determining Design Basis Ground Motions. Volume 1: Method and Guidelines for Estimate Earthquake Ground Motion in Eastern North America," EPRI TR-102293, Electric Power and Research Institute, Palo Alto, CA.
- Gasparini, D. A., and Vanmarcke, E. H., (1976) *SIMQKE: Simulated earthquake motions compatible with prescribed response spectra*, Mass. Institute of Tech., Cambridge, Massachusetts.
- Gumbel, E. J. (1958). *Statistics of Extremes*, Columbia University Press, New York.
- Hanks, T. and McGuire, R. (1981). "The character of high-frequency strong ground motion." *Bull. Seism. Soc. Am.*, Vol. 71, No. 6, pp. 2071-2095.
- IBC (2006), "International Building Code." by *International Code Council*, Delmar Publishers.

- Idriss, I. M., and Sun, J. I. (1992). *SHAKE91: A Computer Program for Conducting Equivalent Linear Seismic Response Analyses of Horizontally Layered Soil Deposits*, Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California, Davis.
- McGuire, R. K., Becker, A. M., Donovan, N. C. (1984). "Spectral estimates of seismic shear waves." *Bulletin of Seismological Society of America*, Vol. 74, No. 4, pp. 1427-1440.
- McGuire, R., Silva, W.J., and Costantino, C.J. 2001. "Technical basis for revision of regulatory guidance on design ground motions: hazard- and risk consistent ground motion spectra guidelines, U.S. Nuclear Regulatory Commission Report NUREG/CR-6728.
- Ozbey, M.C. 2006. *Site Specific Comparisons of Random Vibration Theory-based and Traditional Seismic Site Response Analysis*, Ph.D. Dissertation, University of Texas, Austin, TX.
- Rathje, E.M., Kottke, A.R., and Ozbey, M.C. 2005. "Using Inverse Random Vibration Theory to Develop Input Fourier Amplitude Spectra for Use in Site Response," *16th International Conference on Soil Mechanics and Geotechnical Engineering: TC4 Earthquake Geotechnical Engineering Satellite Conference*, Osaka, Japan, September, pp. 160-166.
- Rathje, E.M., and Ozbey, M.C. 2006. "Site Specific Validation of Random Vibration Theory-Based Site Response Analysis," *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE (accepted for publication).
- Schneider, J. F., Silva, W. J., Chiou, S. J., Stepp, J. C. (1991). "Estimation of ground motion at close distances using the band-limited-white-noise model." Proc., Fourth Int. Conf. on Seismic Zonation, EERI, Stanford, CA, Vol. 4, pp. 187-194.
- Silva, W. J., Abrahamson, N., Toro, G., and Costantino, C. (1997). "Description and validation of the stochastic ground motion model, Final Report." *Brookhaven National Laboratory*, Contract No. 770573, Associated Universities, Inc. Upton, New York
- Toro, G.R., Abrahamson, N.A., and Schneider, J.F. (1997). "Model of Strong Ground Motions from Earthquakes in Central and Eastern North America: Best Estimates and Uncertainties." *Seism. Res. Lett.*, Vol. 68, No. 1, pp. 41-57.
- U.S. Nuclear Regulatory Commission 2007. *A Performance-Based Approach to Define the Site-Specific Earthquake Ground Motion*, Regulatory Guide 1.208.