The equations that need to be used for the converion of nominal stress and strain to their true and logarithmic equivalents are provided in the ABAQUS/Standard User's Manuals, Volume III: Materials (Version 6.4~~5~~) DJ 5/19/05 Specifically,

$$\sigma_{true} = \sigma_{nom}(1+\epsilon_{nom})$$

$$\epsilon_{ln}^{pl} = \ln(1+\epsilon_{nom}) - \frac{\sigma_{true}}{E}$$

Or,

$$S_{y,true} = S_{y,nom}(1+S_{y,nom}/E)$$

$$\epsilon_{ln}^{pl} = \ln(1+S_{y,nom}/E) - \cancel{\sigma}\ \frac{S_{y,true}}{E}$$ DJ 5/19/05

and

$$S_{u,true} = S_{u,nom}(1+\epsilon_{nom})$$

$$\epsilon_{ln}^{pl} = \ln(1+\epsilon_{nom}) - \frac{S_{u,true}}{E}$$

Temperature (°C)	* Titanium Grade 7 Young's Modulus, E (GPa)	† Titanium Grade ~~24~~ 5 Young's Modulus, E (GPa)

DJ 5/19/05

Information potentially subject to copyright protection was redacted from this location.

The redacted material is from the information listed below.

* ASME B&PV code, Section II, Part D, Table TM-5

† Room temperature reference value obtained from Table 5.4.1.0(C1) and the temperature correction factor extracted from Figure 5.4.1.1.4 of the "Military Handbook: Metallic Materials and Elements for Aerospace Vehicle Structures."

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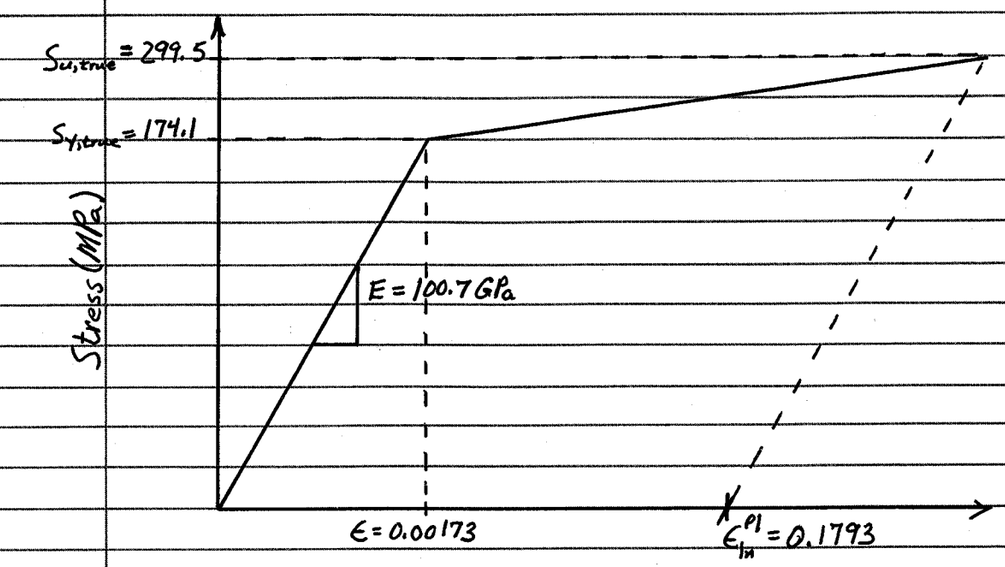
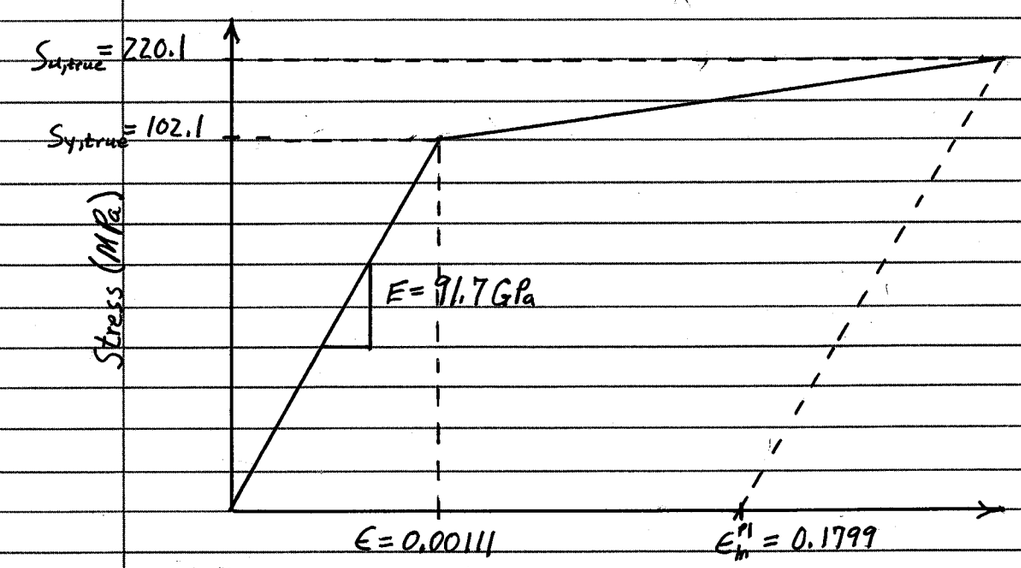
Based on the information provided, the bi-linear truestress and logarithmic strain curves as functions of temperature for the drip shield materials can be generated. These curves are illustrated in the following figures.

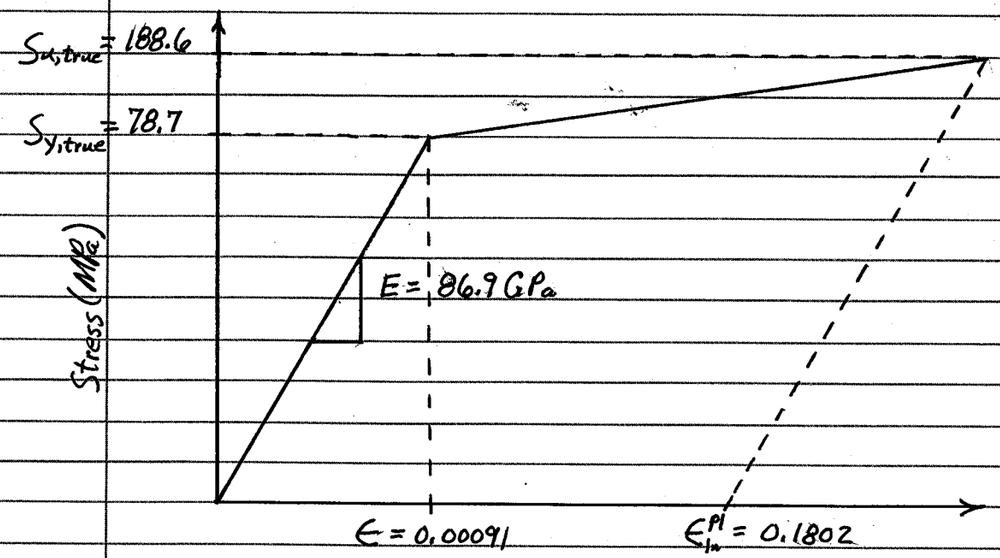
Ti-7 @ 150°C



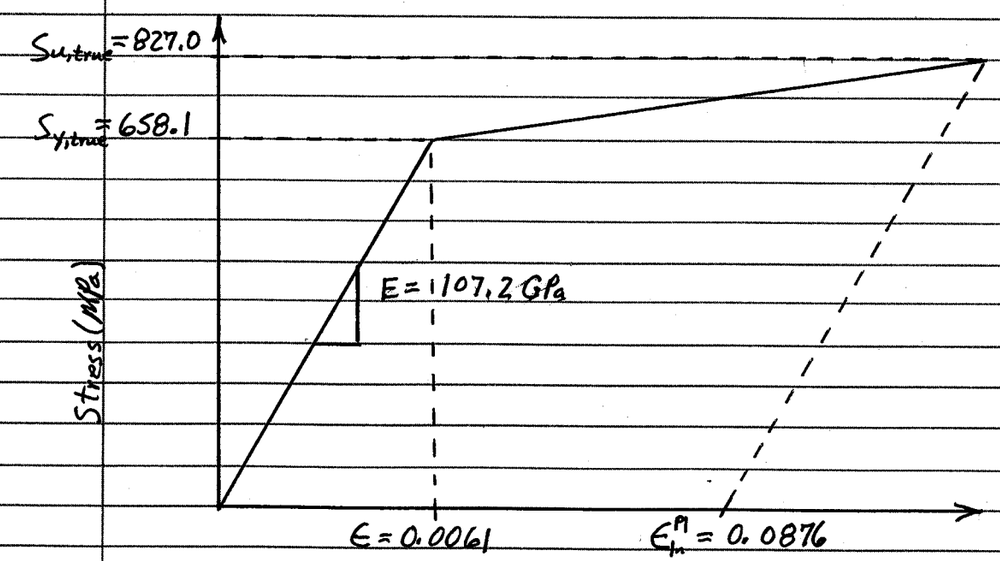
Ti-7 @ 260°C



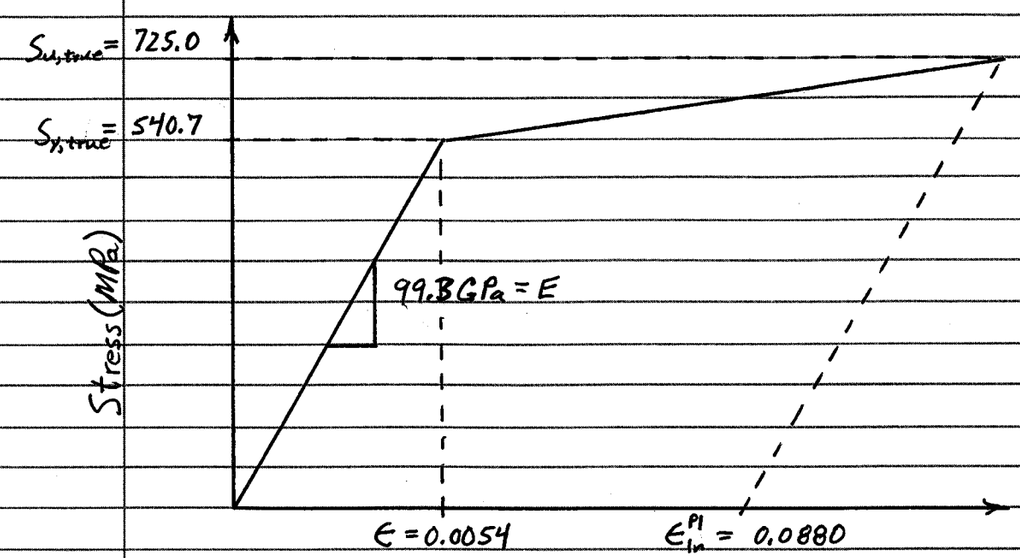
Ti-7 @ 316 °C



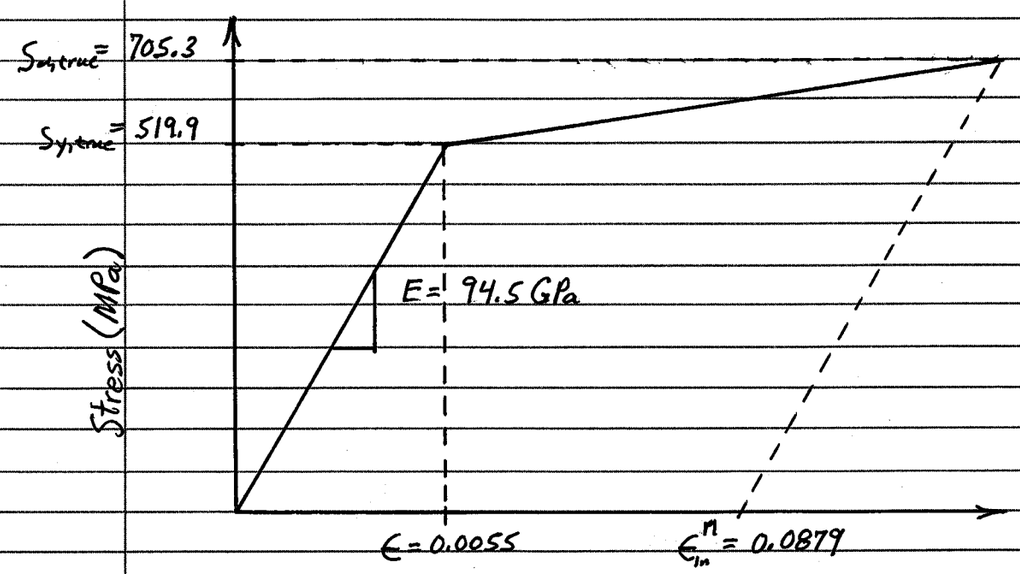
Ti-5 @ 150°C



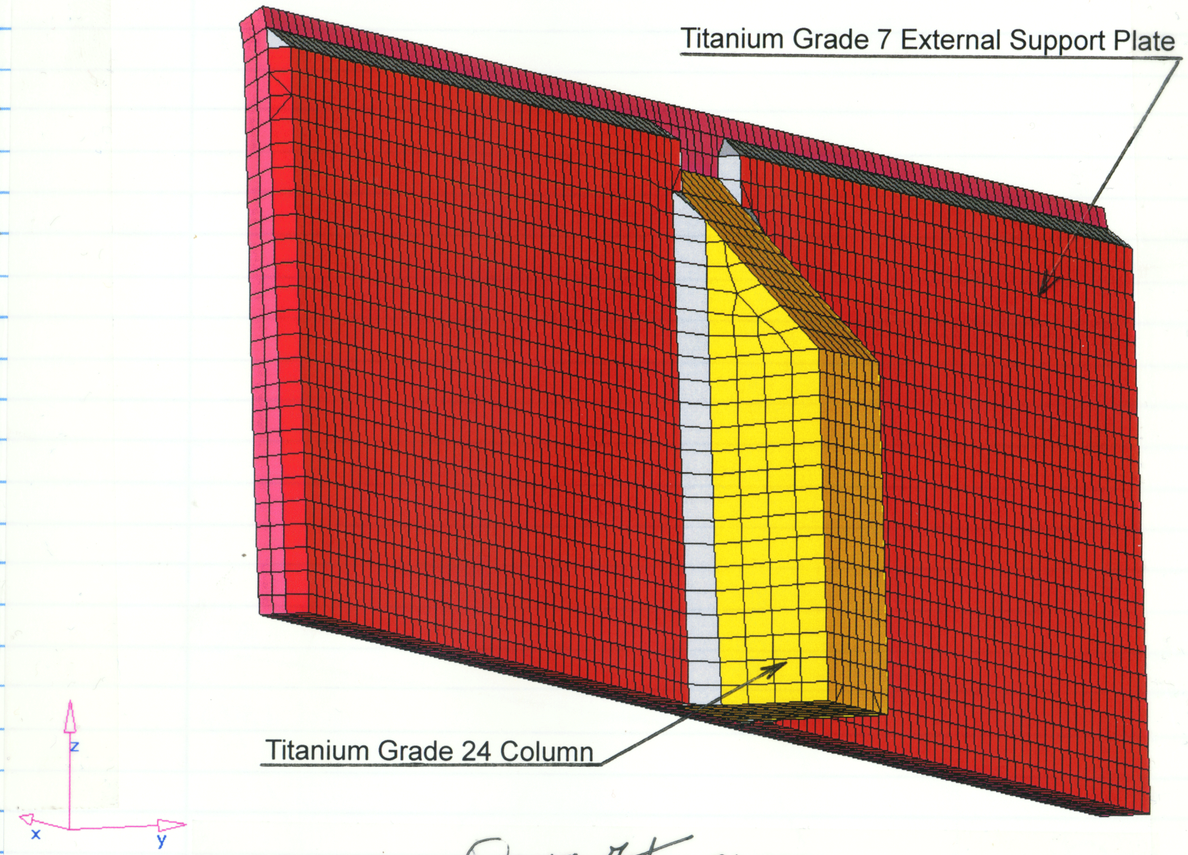
Ti-5 @ 260°C



Ti-5 @ 316°C



The finite element discretization used to model the drip shield "side" and "crown" transition is shown in the figure below.



Doug Duke 5/19/05

The boundary conditions for the Moment-Rotation model are as follows:

- The nodes defined in the plane at the base of the drip shield column, external support plate, and plate are constrained to remain within this plane.
- A single node at the base of the column is fixed in all degrees of freedom to act as an "anchor", i.e., prevent unconstrained rigid body motion

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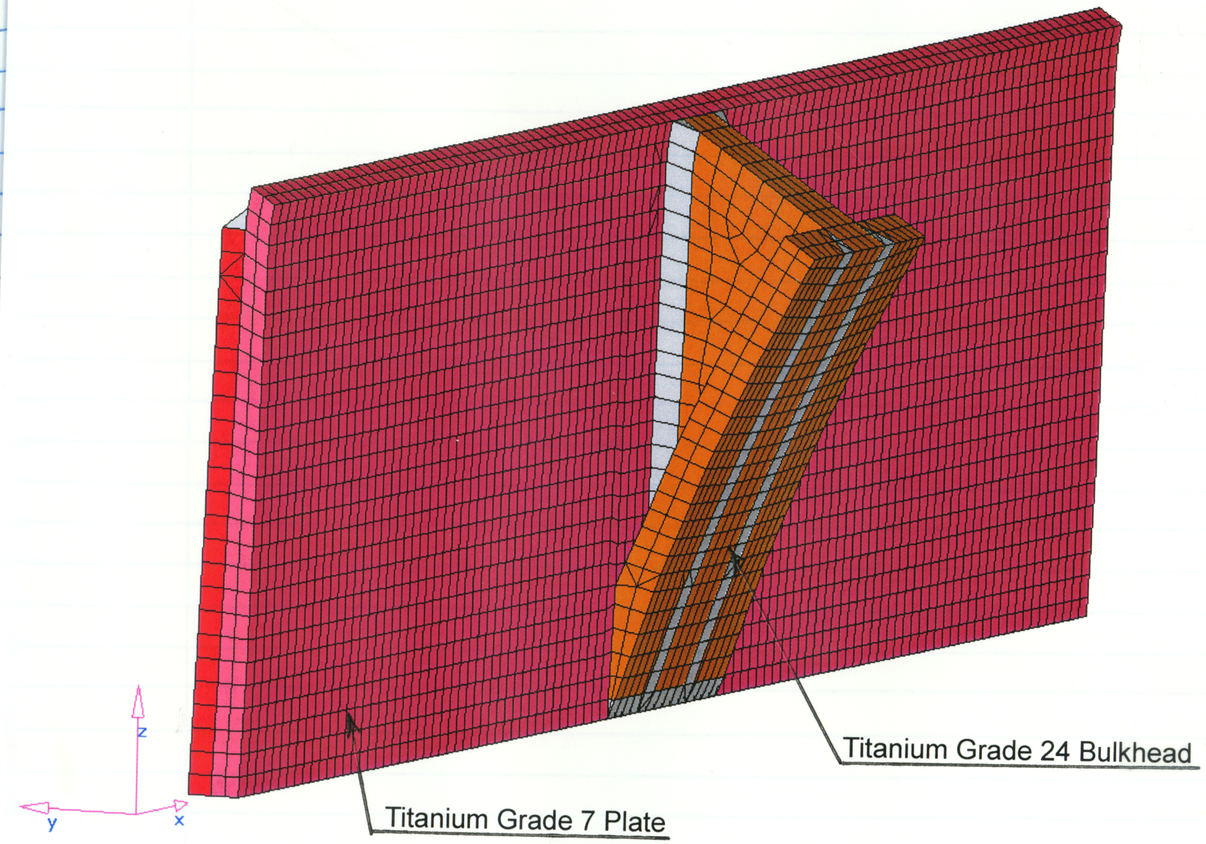
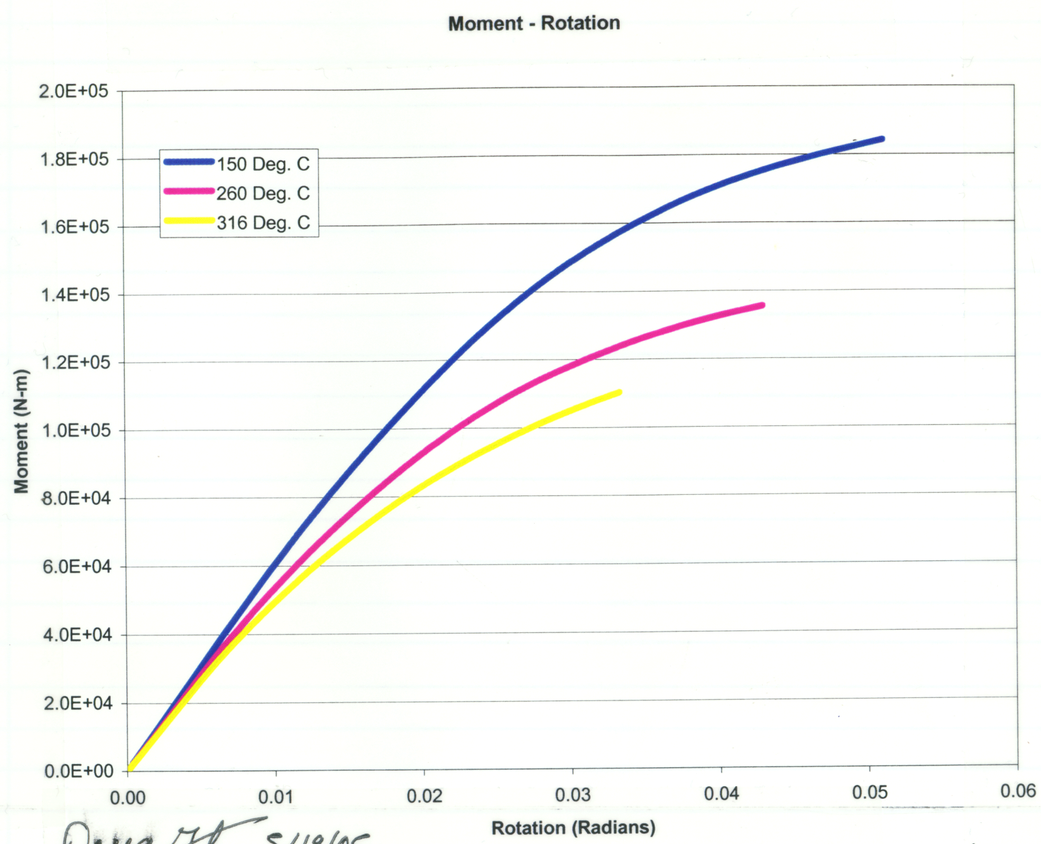


- The moment load is applied to the reference node that controls the analytical rigid surface which defines the plane at the top of the bulkhead, external support plate, and plate.
- All of the nodes that lie on the plane at the top of the bulkhead, external support plate, and plate must remain on the analytical rigid surface plane as it rotates.

The Moment-Rotation curves obtained from the model are plotted in the following figure:



Doug Dale 5/19/05

For all three cases, i.e., at temperatures of 150, 260, and 316 °C; the failure of the drip shield "side" and "crown" transition is characterized by significant yielding of the Ti-7 plate and external support plate (see the following figure). Note that the Ti-24 components and welds have been removed from the figure to clearly convey the extent of yielding that the Ti-7 components have experienced.

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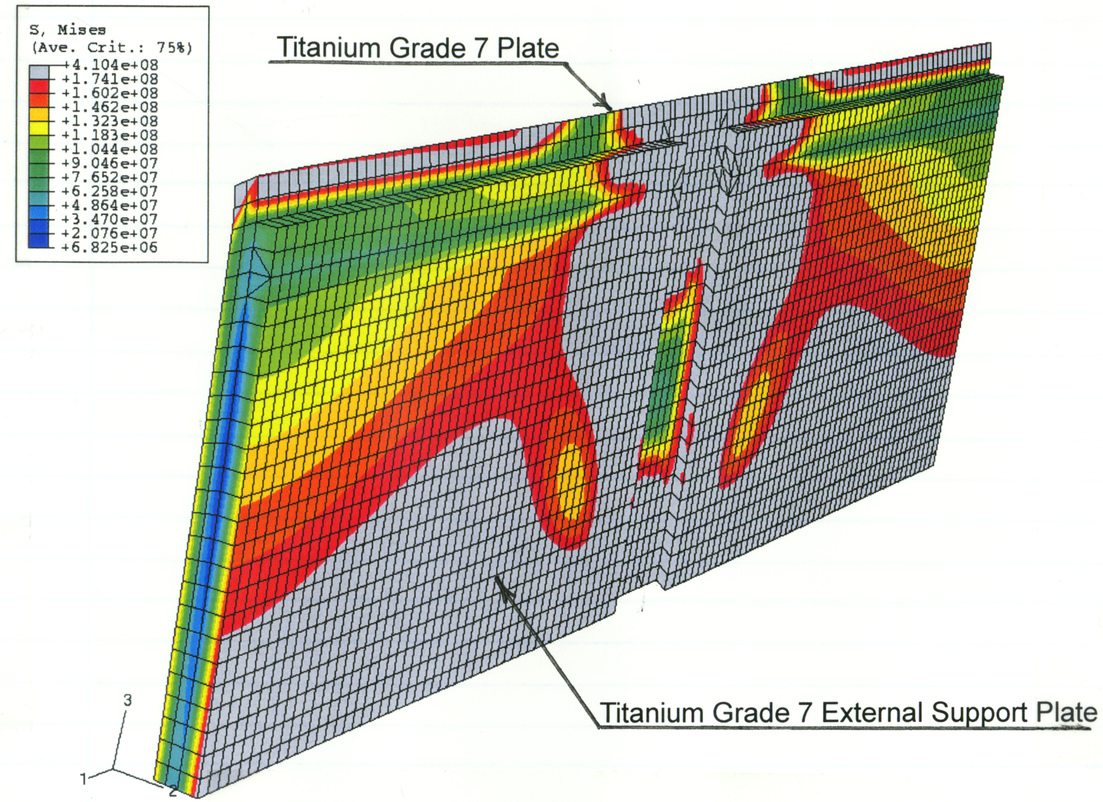
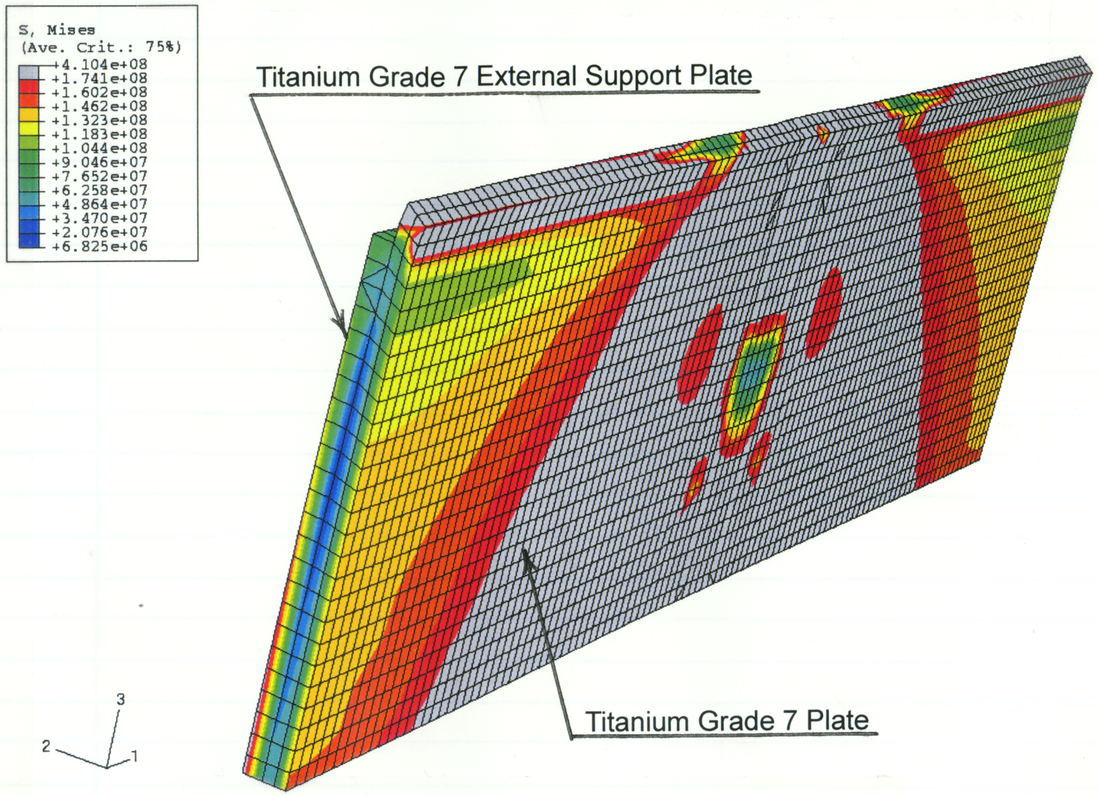
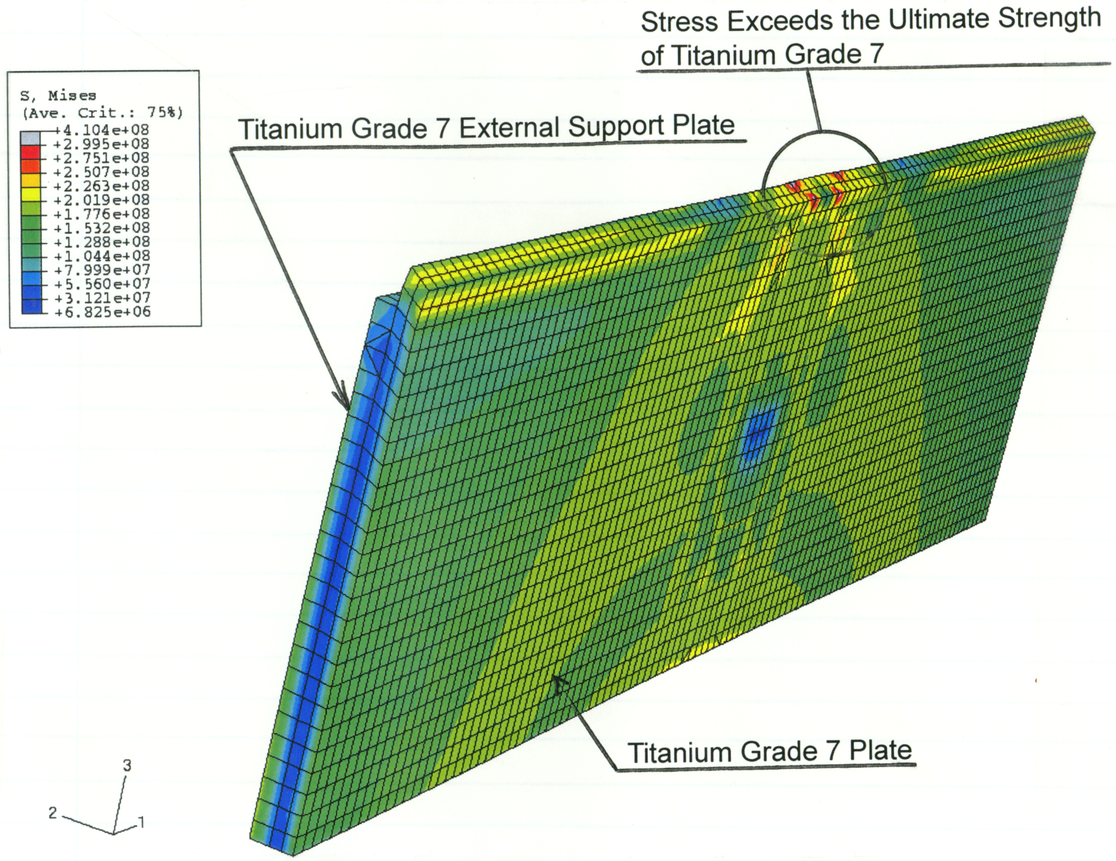


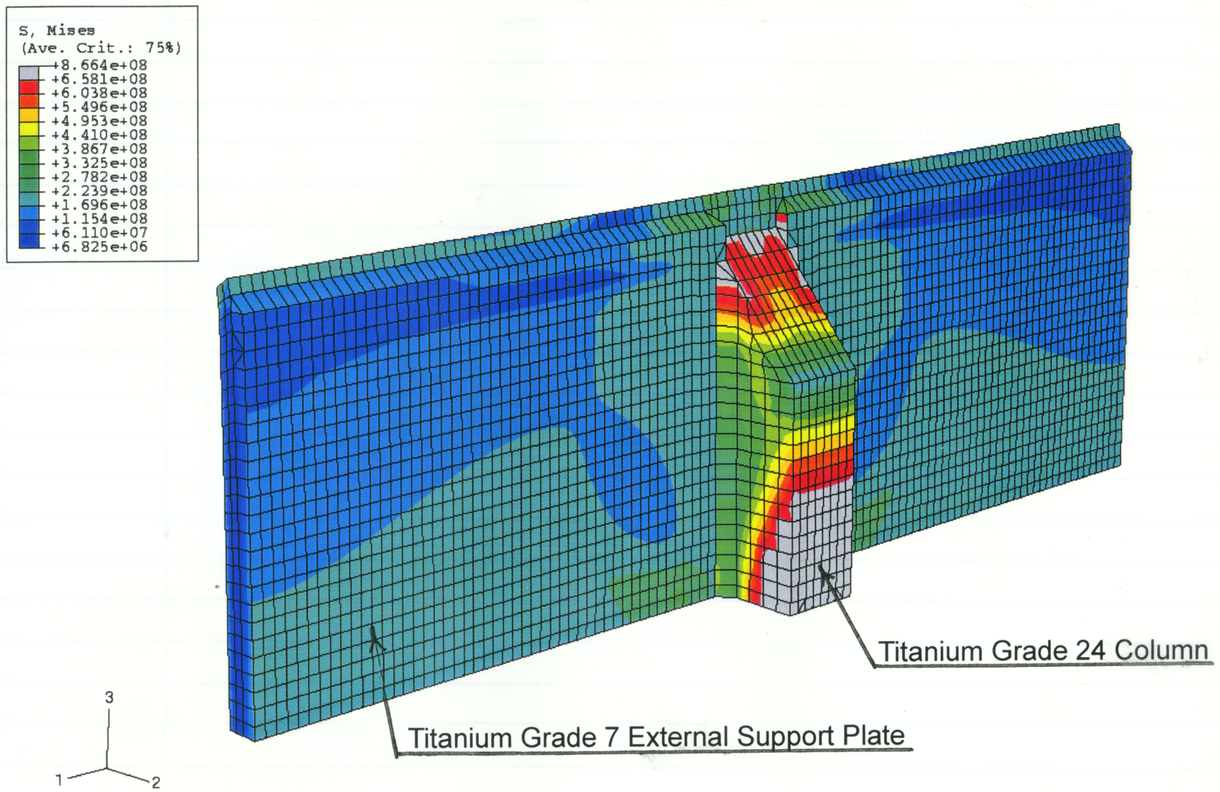
Doug Sate 5/19/05



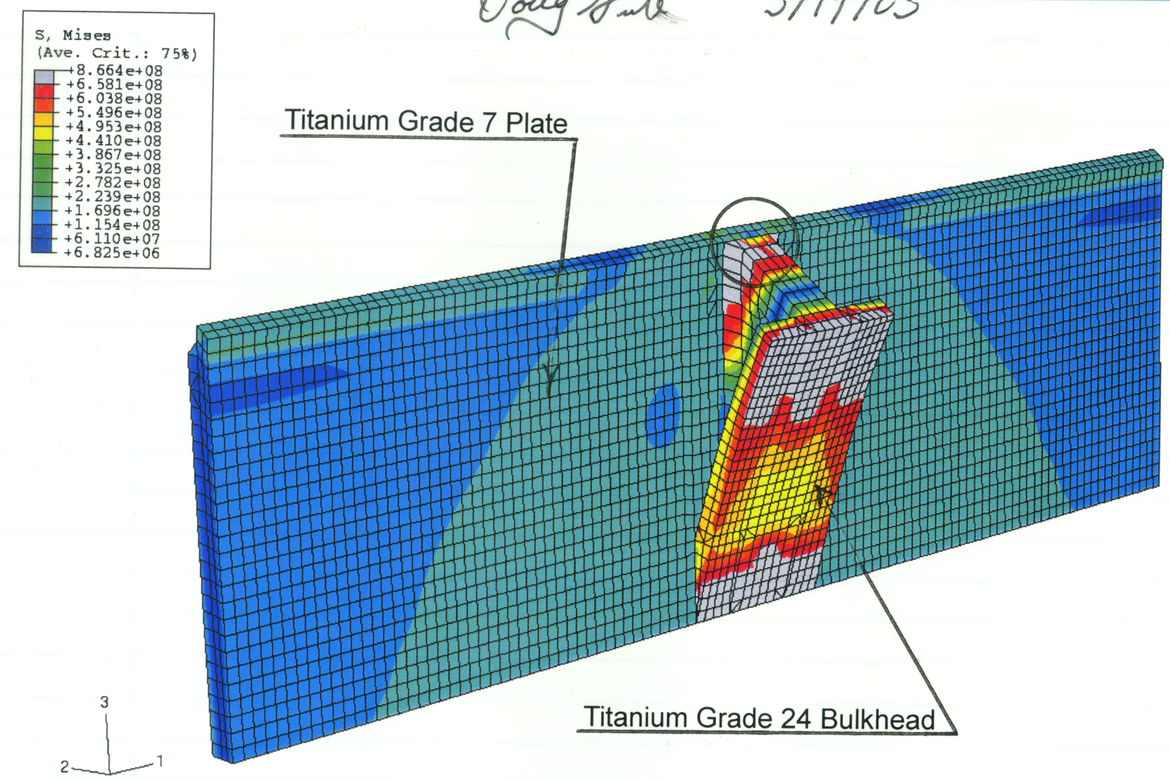
Doug Sate 5/19/05



Doug Ante 5/19/05



Doug Ante 5/19/05



DA 5/19/05

Note that the region of the ~~support~~ bulkhead circled in the figure above indicates where a small, localized volume of the Ti-24 has exceeded the ultimate strength of this material.

To facilitate the assessment of potential creeping of the Ti-7 plate in the drip shield "side" and "crown" transition region, a table (see below) that conveys the moment required to cause the plate to experience a stress equal to the yield strength was compiled. Specifically,

Temp. (°C)	Ti-7 Yield Stress, Sy (MPa)	Moment Required to Cause the Ti-7 Plate to ~~Exceed~~ DJS 5/19/05 Experience Stress > Sy (kN-m)
150	174.1	39
260	102.1	26
316	78.7	19

The moment required to cause structural failure of the drip shield "side" and "crown" transition region corresponds to the moment required to cause the Ti-7 plate to experience a stress equal to its ultimate strength.

Temp. (°C)	Ti-7 Ultimate Strength, Su (MPa)	Moment Required to Cause the Ti-7 Plate to Experience Stress > Su (kN-m)
150	299.5	184.2
260	220.1	135.4
316	188.6	110.2

It needs to be emphasized that for the analyses performed for 260 and 316 °C that the volume of Ti-24 material that has yielded is extremely small and highly localized.

Doug Sente 5/19/05

The next task to be completed is the development of a methodology for approximating the effective spring constants that will represent the effect of the rock rubble accumulated along the sides of the drip shield. Two approaches were taken. The first is a closed-form solution derived using a standard theory of elasticity approach. The second approach was using the finite element method.

Rock Rubble Spring Constants by way of Theory of Elasticity

The following information is based on material presented in

Timoshenko and Goodier. Theory of Elasticity. McGraw-Hill Book Co. NY, NY. 3'rd Edition. 1970.

Constitutive Relationships:

Information potentially subject to copyright protection was redacted from this location

The redacted material is from the information listed above.

Strain-Displacement Relationships:

Information potentially subject to copyright protection was redacted from this location

The redacted material is from the information listed above.

Governing Differential Equation:

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The redacted material is from the information listed on page 127.

Employing the requisite strain-displacement compatibility relationships allows the Governing Differential Equations (for 2-D formulations) to be recast as

$$\frac{\partial^4 \phi}{\partial x^4} + \frac{2\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

where

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} - X \quad ; \quad X = 0$$

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$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} - \cancel{Y}\,{}_0\sigma_y \quad ; \quad Y = \rho g \cancel{(y-h)},$$ stress created by accumulated rockfall rubble.

$$\tau_{xy} = \frac{-\partial^2 \phi}{\partial x \partial y}$$

$h \equiv$ height of the rockfall rubble pile.

$${}_0\sigma_y @ y = \rho g (y-h)$$

Assuming Plane Strain Conditions:

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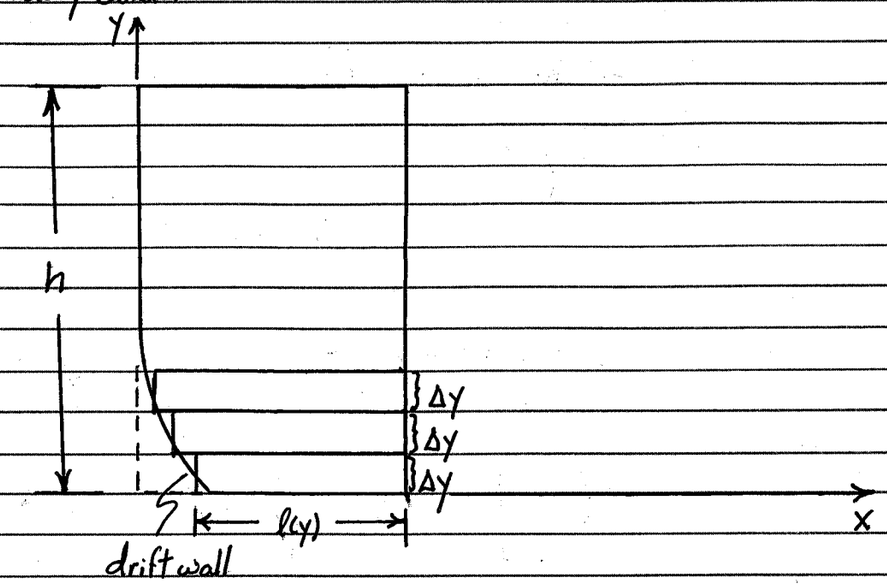
The redacted material is from the information listed on page 127

Boundary Conditions:



$h \equiv$ total rock rubble height defining the vertical pressure acting on the pile at a given vertical location, i.e.

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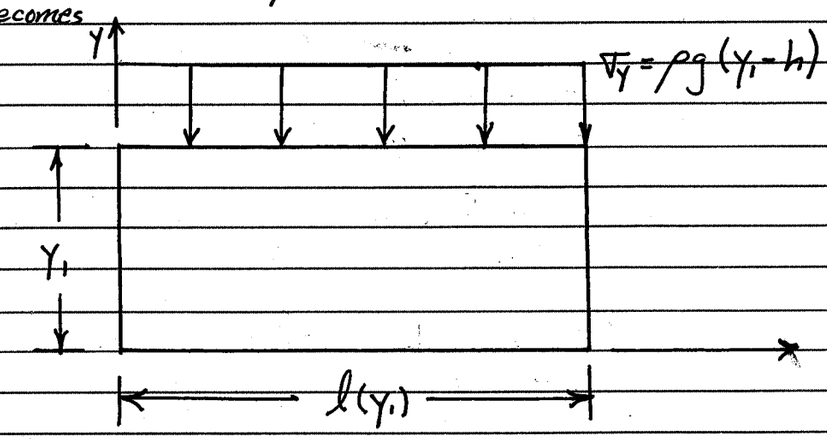
$${}_0\sigma_y = \rho g (y-h)$$ where $\rho \equiv$ rock rubble density $\approx 2{,}500\ kg/m^3$

$g \equiv$ acceleration due to gravity $\approx 9.81\ m/s^2$

Although the length of the rockfall rubble, ℓ, is clearly a function of y because of the driftwall curvature, it is assumed that it is a constant once a specific y has been defined. Thus, the problem becomes



The specific boundary conditions for this problem are

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$$\sigma_y = \cancel{y\sigma_0}\ \sigma_y = \rho g(y_1 - h) \quad @\ y = y_1$$
$$u = 0 \quad @\ x = 0$$
$$v = 0 \quad @\ y = 0$$
$$u = u^* \quad @\ x = \ell$$; represents drip shield side wall displacement

Now, let

$$\phi = \frac{a}{2}x^2 + \frac{b}{2}y^2$$

Then

$$\frac{\partial\phi}{\partial x} = ax \qquad \frac{\partial\phi}{\partial y} = by \qquad \frac{\partial^3\phi}{\partial x^2\partial y} = 0$$
$$\frac{\partial^2\phi}{\partial x^2} = a \qquad \frac{\partial^2\phi}{\partial y^2} = b \qquad \frac{\partial^4\phi}{\partial x^2\partial y^2} = 0$$
$$\frac{\partial^3\phi}{\partial x^3} = 0 \qquad \frac{\partial^3\phi}{\partial y^3} = 0$$
$$\frac{\partial^4\phi}{\partial x^4} = 0 \qquad \frac{\partial^4\phi}{\partial y^4} = 0$$

Thus, the governing diff. eq'n is satisfied and

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = b$$
$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = a$$; note that it has been assumed that σ_y does not vary within the body, i.e., it is constant.
$$\tau_{xy} = \frac{-\partial^2\phi}{\partial x\partial y} = 0$$

Substituting these values for stress into the constitutive relations for plane strain conditions give

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E}\left[b(1-\nu^2) - a\nu(1+\nu)\right]$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{1}{E}\left[a(1-\nu^2) - b\nu(1+\nu)\right]$$

Solving for u and v respectively gives

DJ 5/23/05

$$u = \frac{1}{E}\left[b(1-\nu^2) - \cancel{b}\,a\nu(1+\nu)\right]x + f(y)$$
$$v = \frac{1}{E}\left[a(1-\nu^2) - b\nu(1+\nu)\right]y + g(x)$$

Applying the boundary conditions gives,

$$\sigma_y = a = \rho g(y_1 - h)$$
$$u = f(y) = 0 \quad @\ x = 0$$
$$v = g(x) = 0 \quad @\ y = 0$$
$$u = u^* = \frac{1}{E}\left[b(1-\nu^2) - a\nu(1+\nu)\right]\ell$$
$$\frac{Eu^*}{\ell} = b(1-\nu^2) - \rho g(y_1 - h)\nu(1+\nu)$$
$$\therefore\ b = \frac{1}{(1-\nu^2)}\left[\frac{Eu^*}{\ell} + \rho g(y_1 - h)\nu(1+\nu)\right]$$

In Summary,

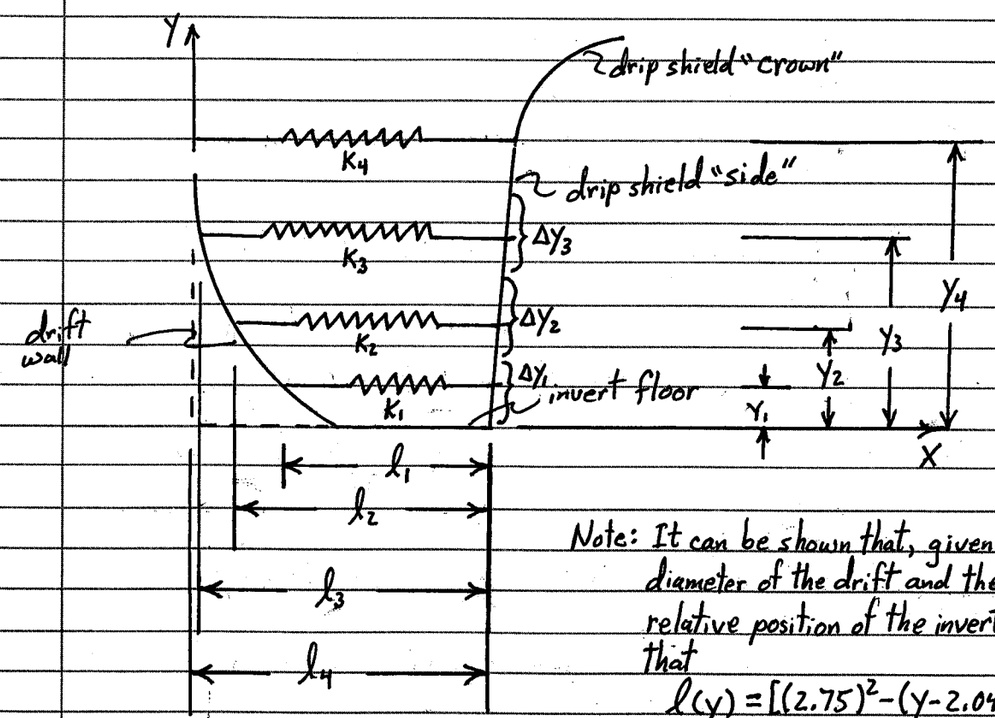
$$\sigma_x = \frac{1}{(1-\nu^2)}\left[\frac{Eu^*}{\ell} + \rho g(y_1 - h)\nu(1+\nu)\right]$$
$$\sigma_y = \rho g(y_1 - h)$$
$$\tau_{xy} = 0$$

$$u = \frac{1}{E}\left[\frac{Eu^*}{\ell} + \rho g(y_1 - h)\nu(1+\nu) - \rho g(y_1 - h)\nu(1+\nu)\right]x$$

$$\therefore u = \frac{u^* x}{\ell}$$

$$v = \frac{1}{E}\left\{\rho g(y_1 - h)(1-\nu^2) - \frac{\nu(1+\nu)}{(1-\nu^2)}\left[\frac{Eu^*}{\ell} + \rho g(y_1 - h)\nu(1+\nu)\right]\right\}y$$

This solution is used to calculate the effective spring constants for the rock rubble in the following manner. First, the vertical location of the springs on the drip shield "side" are defined (see the figure below).



Note: It can be shown that, given the diameter of the drift and the relative position of the invert, that

$$\ell(y) = [(2.75)^2 - (y - 2.045)^2]^{1/2} = 1.28$$

Next, the effective area that each spring is transmitting its force to is calculated. For example, for spring K_1, the effective area is

$$A_1 = W\left[\left(\frac{y_1 + y_2}{2}\right) - 0\right]$$

where

$W \equiv$ width of the rock rubble ≈ 1.04 m (approx. spacing between drip shield columns + bulkheads)

Similarly, for K_2,

$$A_2 = W\left[\left(\frac{y_2 + y_3}{2}\right) - \left(\frac{y_1 + y_2}{2}\right)\right]$$

and so on.

Third, the net force acting on the drip shield over the effective surface area is

$$F_{x_1} = \int_{\partial A} \sigma_x \, dA$$

$$= \int_0^{\left(\frac{y_1+y_2}{2}\right)} \frac{W}{(1-\nu^2)}\left\{\frac{Eu^*}{\ell} + \rho g\underbrace{\left[\frac{1}{2}\left(\frac{y_1+y_2}{2} + 0\right)\right.}_{y_{ave}} - h\Big]\nu(1+\nu)\right\}dy$$

$$= \frac{W}{(1-\nu^2)}\left[\frac{Eu^*}{\ell} + \rho g(y_{ave} - h)\nu(1+\nu)\right]\left(y\Big|_0^{\left(\frac{y_1+y_2}{2}\right)}\right)$$

$$= \frac{W}{(1-\nu^2)}\left[\frac{Eu^*}{\ell} + \rho g(y_{ave} - h)\nu(1+\nu)\right]\left(\frac{y_1+y_2}{2}\right)$$

If only the confining pressure is considered, i.e., $u^* = 0$, then

$$F_{x_1} = \frac{W}{(1-\nu^2)}\rho g(y_{ave} - h)\nu(1+\nu)\left(\frac{y_1+y_2}{2}\right)$$

Given that the spring constant can be written mathematically as

$$K_1 = \frac{\Delta F_{x_1}}{\Delta u}$$

where

$$\Delta u = u^* - 0$$

and

$$\Delta F_{x_1} = \frac{W}{(1-\nu^2)}\left[\frac{E u^*}{l} + \rho g (y_{ave} - h)\nu(1+\nu)\right]\left(\frac{y_1+y_2}{2}\right)$$
$$- \frac{W}{(1-\nu^2)}\rho g (y_{ave} - h)\nu(1+\nu)\left(\frac{y_1+y_2}{2}\right)$$

$$= \frac{W}{(1-\nu^2)}\left(\frac{y_1+y_2}{2}\right)\frac{E u^*}{l}$$

Thus

$$\therefore\ K_1 = \frac{W}{(1-\nu^2)}\left(\frac{y_1+y_2}{2}\right)\frac{E}{l}$$

Note that the final spring constant is not dependent on the confining pressure nor the prescribed displacement.

In general terms, the spring constant can be written as

$$\boxed{\therefore\ K_i = \frac{E}{(1-\nu^2)}\frac{W}{l_i}\Delta y_i}$$ closed form solution

To assess the potential non-linear geometry effects on this solution, a finite element model was created. The finite element discretization for this model is shown on the next page.