LABORATORY NOTEBOOK

GEOSCIENCES & ENGINEEING DIVISION

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NOTEBOOK NO. 824

ISSUED TO Debashis Basu

ON 9/22/06 20 06

DEPARTMENT Geo Sciences

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Purpose:

The purpose of this notebook is to document the technical activities and calculations related to the review of the two Electrical Power Research Institute (EPRI) reports. The calculations are preliminary in nature and mainly involve simple heat transfer and fluid mechanics related formulations.

Tasks:

1. Simple one-dimensional melting and solidification related analytical calculation.
2. Compressible fluid dynamics related analytical calculations involving normal shocks as well as nozzle flows, area-velocity relationship.
3. Sample analytical calculations related to bubble growth in magmas, depth of exsolution, and volume fraction.

Software:

Microsoft Excel for calculation of the volume fraction and depth of exsolution.

References used in the calculations:

Most of the calculations in the scientific notebook and the formulations that are used are based on the following book and technical articles or reports:

1. Andersom, John D., "Modern Compressible Flow: with historical perspective", McGraw-Hill Inc., 1990.

2. Woods, A. W., and Huppert, Hurbert, E., "On magma chamber evolution during slow eruptive eruptions", Journal of Geophysical Research, Vol. 108, No. B8, 2403, doi:10.1029/2002JB002019, 2003.

3. Sparks, R. S. J., "The dynamics of bubble formation and the growth in magmas-A review and analysis, Journal of Geophysical Research, Vol. 83, pp. 1-37, 1978.

4. EPRI (Electric Power Research Institute) 2004. Potential Igneous Processes Relevant to the Yucca Mountain Repository: Extrusive-Release Scenario. Analysis and Implications EPRI TR-1008169. Palo Alto, California: Electric Power Research Institute.

5. EPRI (Electric Power Research Institute) 2005. Program on Technology Innovation: Potential Igneous Processes Relevant to the Yucca Mountain

Repository: Intrusive Release Scenario. Analysis and Implications EPRI TR-1011165. Palo Alto, California: Electric Power Research Institute.

6. Carslaw, H. S., and Jaeger, J. C., "Conduction of Heat in Solids", Oxford Science Publications, 1989.

The heat transfer calculations are based on the formula provided in the book "Conduction of Heat in Solids", while the compressible nozzle flow and fluid dynamics calculations are based on the formulations provided in the book titled "Modern Compressible Flow: with historical perspective" by John D. Anderson. The formulas used for the calculation of volume fraction in magmas and depth of exsolution are based on the papers by Woods et al. [2] and Sparks [3].

7. Wilson, L., and Head, J. W., "Ascent and Eruption of Basaltic Magma on the Earth and Moon", Journal of Geophysical Research, Vol. 86, No. B4, pp. 2971-3001, 1981.

TITLE Some Calculations

Book No. 1

• This notebook provides calculations to verify some of the work of EPRI report.

Melting and Solidification in one dimension :→ Neumann's solution and its generalizations.

Surface of separation between the solid & liquid phases [X(t)] is given by, [Carslaw & Jaeger, 282-286]

$$X = 2\lambda . \sqrt{K_1 t}$$

$$\therefore \boxed{X \propto \sqrt{Kt}}$$

→ Thermal Diffusivity.

* [Conduction of Heat in Solids, by Carslaw and Jaeger. Page 282-286]

Assuming a thermal diffusivity value of magma in the range of $1-5 \times 10^{-7}\ m^2/s$ & the time (t) as 10 minutes we can have:-

$$X \propto \sqrt{1\times10^{-7} \times 600}$$

$$\Rightarrow \sqrt{6\times10^{-5}}$$

$$\Rightarrow 0.77\ cm.$$

For $K = 1\times10^{-7}\ m^2/s$.

For, $K = 5\times10^{-7}\ m^2/s$.

$$X \propto \sqrt{5\times10^{-7}\times600}$$

$$\propto \sqrt{3\times10^{-4}}$$

$$\propto 1.73\times10^{-2}\ m$$

$$\approx 1.73\ cm$$

∴ Range of chill thickness of the magma on the waste package ≈ 0.77 cm to 1.73 cm.

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Book No. 1 TITLE

For $K = 2 \times 10^{-7}\ m^2/s$

$$X \sim \sqrt{2 \times 10^{-7} \times 600}$$

$$\sim \sqrt{12 \times 10^{-5}}$$

$$\sim \sqrt{1.2 \times 10^{-4}}$$

$$\sim 1.095 \times 10^{-2}\ m$$

$$\sim 1.095\ cm$$ ✓

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This is related

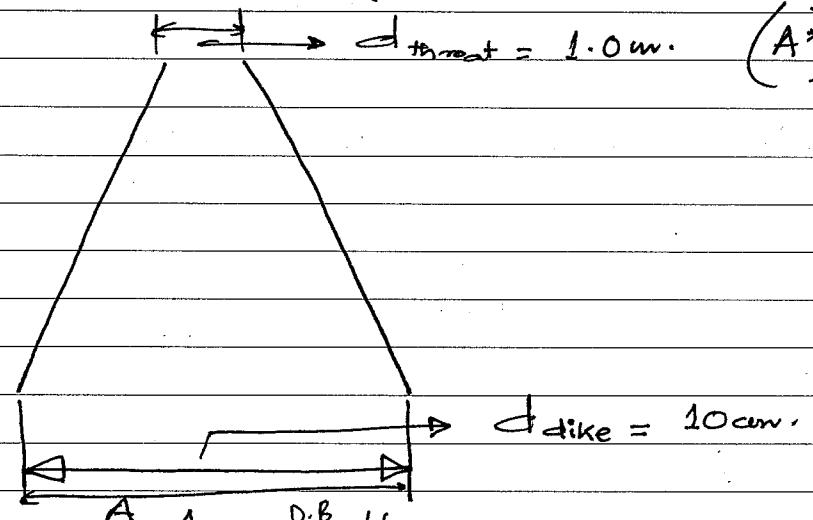
Normal Shock Relation

$$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right]\left[\frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right].$$

$T_1 \rightarrow$ Before shock.

$T_2 \rightarrow$ After shock.

Throat Calculation (EPRI)



$$\frac{A}{A^*} = \frac{\cancel{A}\ A_{dike}}{A_{throat}} = \left(\frac{10}{1}\right)^2 \approx 100.$$

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From Table A-1 (Anderson, Compressible flow).

$A/A^* = 104.1 \longrightarrow M = 7,\ P_0/P = 0.444 \times 10^4,\ \rho_0/\rho = 0.38 \times 10^3,$

$T_0/T = 0.1080 \times 10^2.$

$A/A^* = 97.70,\quad M = 6.9,\quad P_0/P = 0.3779 \times 10^4,\ \rho_0/\rho = 0.3591 \times 10^3,$

$T_0/T = 0.1052 \times 10^2$

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At $\frac{A}{A^*} = 100$,

$$\frac{100 - 97.7}{104.1 - 97.7} = \frac{M - 6.9}{7 - 6.9} = \frac{P_0/P - 0.3779\times10^4}{0.4140\times10^4 - 0.3779\times10^4} = \frac{T_0/T - 10.52}{10.80 - 10.52}$$

$M = 6.936$

$P_0/P = 0.3908 \times 10^4$

$T_0/T = 10.62$

Convergent portion

(Supersonic)

Subsonic Calculation in convergent Part

$\frac{A}{A^*} = 100$

M & u will be very small

(from the table) ✓

A.1 ✓

~~$\frac{100 - 96.66}{144.8 - 96.66}$~~

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Because the convergent portion is subsonic ✓

$$\frac{100 - 28.94}{14.48 - 28.94} = \frac{M - 0.02}{0.04 - 0.02}$$

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$100 - 14.48 - 28.94$

$$\frac{71.06}{-14.46} = \frac{M - 0.02}{0.02}$$

$-4.914 * 0.02 = M - 0.02$

?? $\boxed{\approx 0}$ ✓

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$$\frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1}$$

$$\frac{T_0}{T} = \left(1 + \frac{\gamma-1}{2} M^2\right)$$

← Relationships.

$$\frac{P_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$$

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$$\frac{10}{1} = \left[1 + 2 M^2\right]^{3.5}$$

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$$\boxed{M = 2.157}$$ ✓

∴ Area - Velocity Relation :-

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{dU}{U}$$

$0 \leq M < 1, \rightarrow \frac{dA}{A} = -f\left(\frac{dU}{U}\right)$ (subsonic flow)

u decreasing → u decreasing

$M > 1 \rightarrow \frac{dA}{A} = +f\left(\frac{dU}{U}\right)$

u increase → u decrease.

$M = 1 \rightarrow \frac{dA}{A} = 0$

↳ $\boxed{A \text{ is minimum}}$ ✓

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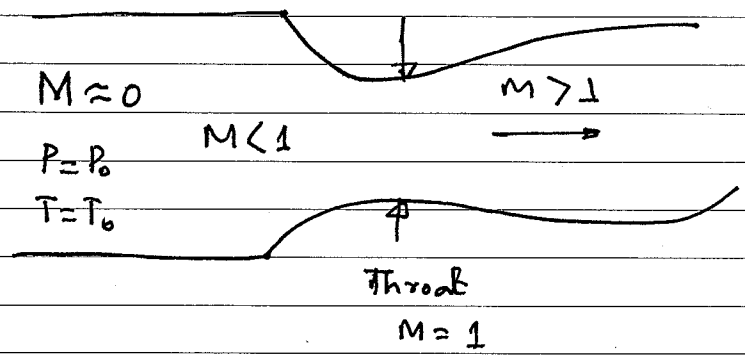


$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$\frac{A}{A^*} = \frac{\text{Local Area}}{\text{Throat Area.}}$$

If $\frac{A}{A^*} \rightarrow \infty$, $M \sim 0$.

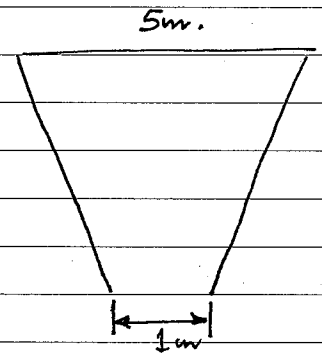
(Stagnation condition) ✓

Here, $A/A^* = 100$ $\therefore (A/A^*)^2 = 10000$

(for EPRI configuration) $\therefore M \sim 0$.

$\therefore$ EPRI considering magma to be stagnant.

Diverging portion



$$\frac{A}{A^*} = 25.$$

(Supersonic)

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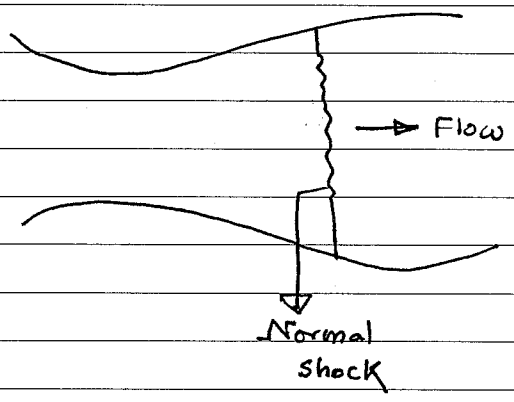
From Table A.1 (Anderson),

for $\frac{A}{A^*} = 25$ $M = 5$, $P_0/p = 529.1$. $P_0/p = 88.18$

$$\frac{T_0}{T} = 6.$$

* In conventional nozzle flow, as exit pressure is reduced, the normal shock moves further downstream.



$$\therefore \frac{A_e}{A^*} = 25.$$

This area ratio corresponds to a Mach number of ~5 in the diverging portion [Anderson, Compressible flow] ✓

solubility of H_2O in basalt (Woods and Huppert, 2003) $n = 3 \times 10^{-6} p^{1/2}$

p = pressure (Pascals)
n = mass fraction
[wt% = 100·n]

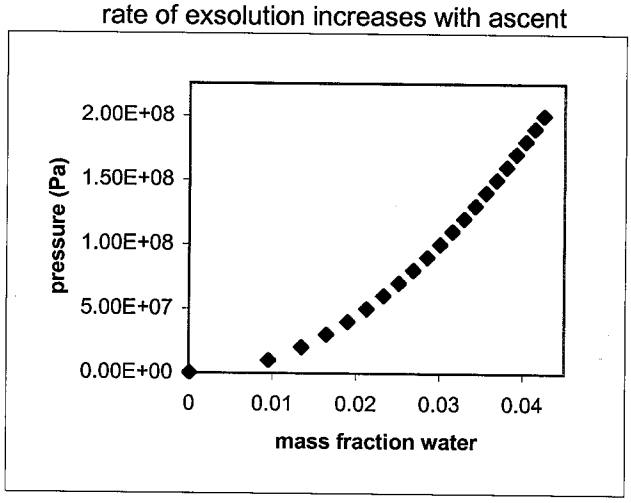
based on Henry's law $x = k p^{1/2}$

x = mass of volatiles dissolved in a liquid as a function of pressure
k = solubility constant

k in Henry's law

s_i	p (Pa)	n
3.00E-06	2.00E+08	0.042426
	1.90E+08	0.041352
	1.80E+08	0.040249
	1.70E+08	0.039115
	1.60E+08	0.037947
	1.50E+08	0.036742
	1.40E+08	0.035496
	1.30E+08	0.034205
	1.20E+08	0.032863
	1.10E+08	0.031464
	1.00E+08	0.03
	9.00E+07	0.02846
	8.00E+07	0.026833
	7.00E+07	0.0251
	6.00E+07	0.023238
	5.00E+07	0.021213
	4.00E+07	0.018974
	3.00E+07	0.016432 ← 0.015
2.5E+07 →	2.00E+07	0.013416
	1.00E+07	0.009487
	0.00E+00	0
	5.00E+06	0.006708

rate of exsolution increases with ascent



gas density (ρ_g) = $\frac{P}{RT}$

P = pressure (6 - 10 MPa)
R = 461 J/kg·K
T = temperature = 1100 K

ρ_g range: ~~6~~ 10 - 16 kg/m³

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Stokes : $v_s = \frac{2}{9} r^2 g \frac{(\rho_p - \rho_f)}{\eta}$

units: $\frac{m^2 \cdot \frac{m}{sec^2} \cdot \frac{kg}{m^3}}{Pa \cdot sec} = \frac{\frac{kg}{sec^2}}{\frac{kg}{m \cdot sec^2} \cdot sec} \Rightarrow \frac{m}{sec}$

$Pa = \frac{kg}{m \cdot sec^2}$

ρ_p = density of particle or gas bubble ($\sim 12\ kg/m^3$)
ρ_f = density of fluid or basaltic magma ($2400 - 2650\ kg/m^3$)
r = radius ($0.5 - 1.0\ cm$)
η = viscosity ($10^5\ Pa \cdot s$)
g = gravitational acceleration ($9.8\ m/sec^2$)

if $\rho_f = 2400\ kg/m^3$
$r = 0.5\ cm$
$v_s = -1.3 \times 10^{-6}\ m/sec$

if $\rho_f = 2650\ kg/m^3$
$r = 1.0\ cm$
$v_s = \cancel{-1.4 \times 10}$ NKA 11/1/06 $-5.7 \times 10^{-6}\ m/sec$

if $\rho_f = 2650\ kg/m^3$
$r = 0.5\ cm$
$v_s = -1.4 \times 10^{-6}\ m/sec$

if $\rho_f = 2400\ kg/m^3$
$r = 1.0\ cm$
$v_s = -5.2 \times 10^{-6}\ m/sec$

DEPTH OF EXSOLUTION

n_i = initial mass fraction dissolved volatiles
p = pressure = lithostatic = $\rho_c g d$
ρ_c = density of country rock = $2200\ kg/m^3$

$n_i = n$ if $d \geq d_{exs}$ $\quad d_{exs}$ = depth of exsolution
↑ solubility

$n_i = 3 \times 10^{-6} (\rho_c g d_{exs})^{1/2}$

$d_{exs} = (n_i / 3 \times 10^{-6})^2 / \rho_c g$

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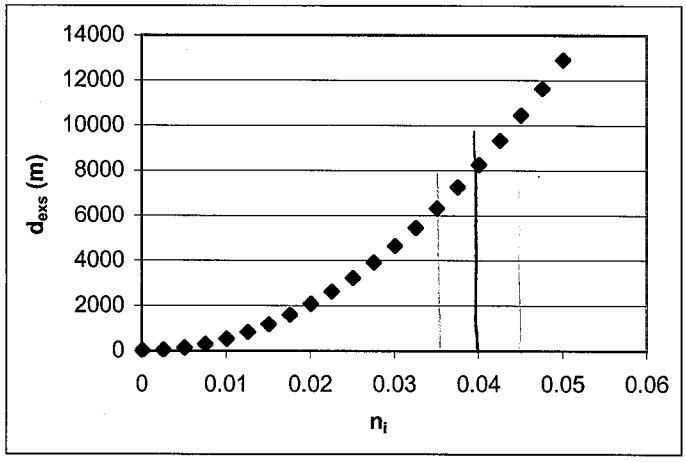
ρ_c (kg/m^3)	n_i	d_{exs} (m)
2200	0.05	12883.94
	0.0475	11627.76
g (m/sec^2)	0.045	10435.99
9.8	0.0425	9308.648
	0.04	8245.723
	0.0375	7247.217
	0.035	6313.131
	0.0325	5443.465
	0.03	4638.219
	0.0275	3897.392
	0.025	3220.985
	0.0225	2608.998
	0.02	2061.431
	0.0175	1578.283
	0.015	1159.555
	0.0125	805.2463
	0.01	515.3577
	0.0075	289.8887
	0.005	128.8394
	0.0025	32.20985
	0	0



DEPTH VS. VOLUME FRACTION

(Wilson & Head, 1981)

gas volume fraction $\phi = \dfrac{\rho_{mix} - \rho_m}{\rho_g - \rho_m}$

ρ_g = gas density $\dfrac{\rho_c g h}{RT}$

density of mixture (ρ_{mix}): $\dfrac{1}{\rho_{mix}} = \dfrac{n_{exs}}{\rho_g} + \dfrac{(1-n_{exs})}{\rho_m}$

$\rho_c = 2200\ kg/m^3$

$T = 1283K$

ρ_m = melt density $2650\ kg/m^3$

n_{exs} = mass fraction exsolved volatiles

$= n_i - n$

$n_i = 0.04$

$n_{exs} = 0.04 - n$

$n = 3\times10^{-6}\rho^{1/2}$

$= 3\times10^{-6}(\rho_c g h)^{1/2}$

$= 3\times10^{-6}(21560h)^{1/2}$

$n_{exs} = 0.04 - [3\times10^{-6}(21560h)^{1/2}]$

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$$\phi = \frac{\left(\dfrac{1}{\dfrac{n_{exs}}{\rho_g} + \dfrac{(1-n_{exs})}{\rho_m}}\right) - \rho_m}{\rho_g - \rho_m}$$

$$\rho_g = \frac{2200\,\frac{kg}{m^3} \cdot 9.8\,\frac{m}{sec^2} \cdot h}{461\,J/kg\cdot K \cdot 1283K}$$

$$\rho_g = 0.03645h$$

$$\phi_{0.04} = \frac{\dfrac{1}{\left(\dfrac{0.04 - [3\times10^{-6}(2156\,0h)^{1/2}]}{0.03645h}\right) + \left(\dfrac{1 - \{0.04 - [3\times10^{-6}(2156\,0h)^{1/2}]\}}{2650}\right)} - 2650}{0.03645h - 2650}$$

let n_i range between 0.01 - 0.05

critical volume fraction 0.70 - 0.75

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d (m)	$\Phi_{\text{initial water content (ni)}}$ 0.05	0.04	0.03	0.02	0.01
15000	-0.01944	-0.071447	-0.127861	-0.189268	-0.25636
14500	-0.015447	-0.069011	-0.127313	-0.191012	-0.260892
14000	-0.011111	-0.066313	-0.126617	-0.192765	-0.265651
13500	-0.006396	-0.063321	-0.12575	-0.194523	-0.270656
13000	-0.001258	-0.059999	-0.124688	-0.196276	-0.275929
12500	0.00435	-0.056304	-0.123398	-0.198014	-0.281493
12000	0.010484	-0.052188	-0.121846	-0.199725	-0.287375
11500	0.017206	-0.047594	-0.119988	-0.201393	-0.293605
11000	0.024591	-0.042454	-0.117773	-0.202997	-0.300218
10500	0.032724	-0.036691	-0.11514	-0.20451	-0.307252
10000	0.041702	-0.030211	-0.112015	-0.205898	-0.314751
9500	0.051643	-0.022905	-0.108307	-0.207118	-0.322766
9000	0.062682	-0.014641	-0.103906	-0.208113	-0.331353
8500	0.07498	-0.005259	-0.098677	-0.208808	-0.34058
8000	0.088728	0.005431	-0.092449	-0.209107	-0.350523
7500	0.104154	0.017664	-0.08501	-0.20888	-0.361267
7000	0.121531	0.031729	-0.07609	-0.207954	-0.372912
6500	0.141192	0.047981	-0.065346	-0.206091	-0.385571
6000	0.163538	0.066866	-0.052331	-0.202967	-0.399367
5500	0.189063	0.088946	-0.03646	-0.198125	-0.414431
5000	0.218373	0.114936	-0.016957	-0.190916	-0.430889
4500	0.252222	0.145752	0.007227	-0.180394	-0.448832
4000	0.291546	0.182586	0.037523	-0.165148	-0.468243
3500	0.337518	0.226993	0.075922	-0.143023	-0.488834
3000	0.391601	0.281025	0.125235	-0.110626	-0.509639
2500	0.45562	0.347386	0.189486	-0.062439	-0.527983
2000	0.531806	0.429624	0.274481	0.0108	-0.536594
1500	0.62277	0.53226	0.38852	0.125028	-0.514715
1000	0.73122	0.660568	0.542841	0.307485	-0.397798
500	0.858801	0.818849	0.749384	0.598556	0.021366
400	0.886469	0.854046	0.797357	0.672867	0.178002
300	0.914693	0.890195	0.847219	0.752192	0.365398
200	0.943309	0.927034	0.89849	0.835407	0.579084
100	0.972014	0.964072	0.950241	0.920112	0.803559

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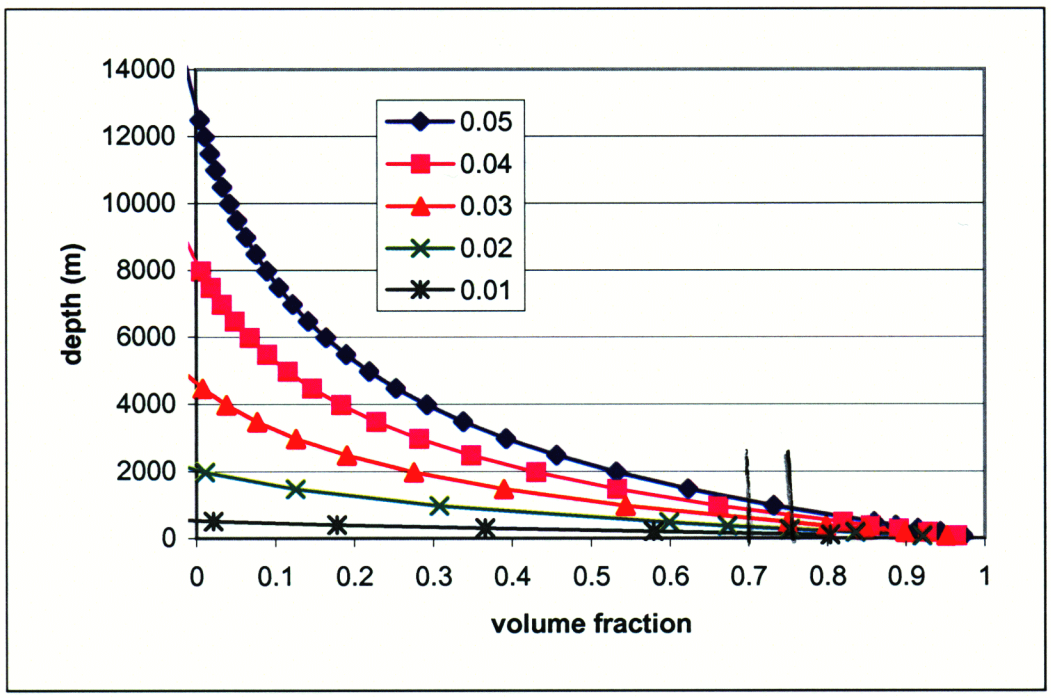
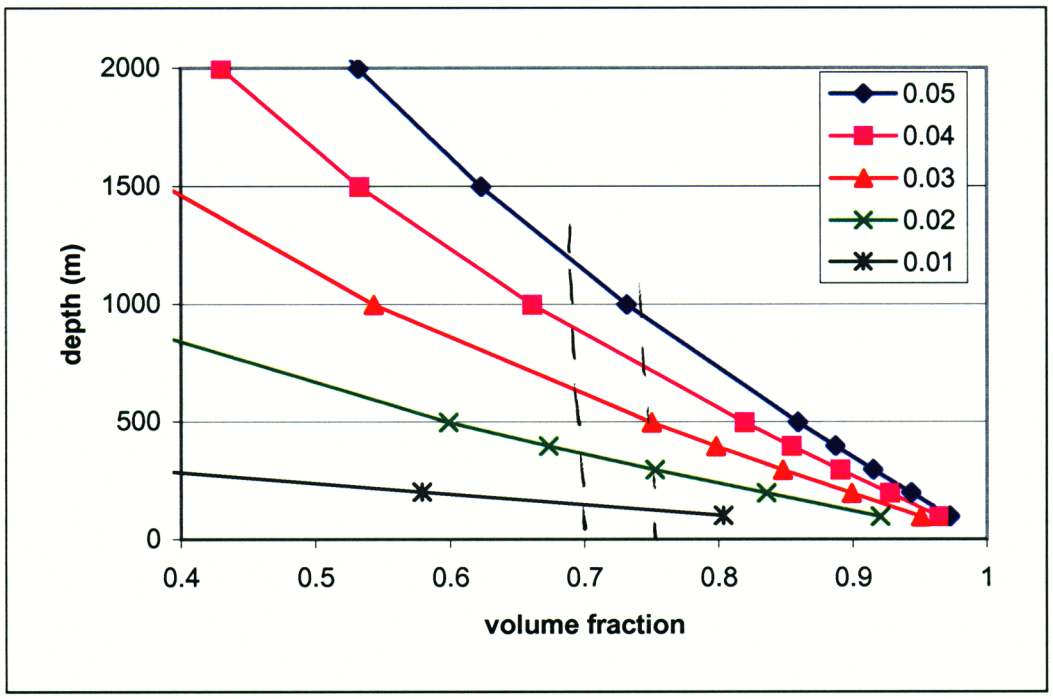
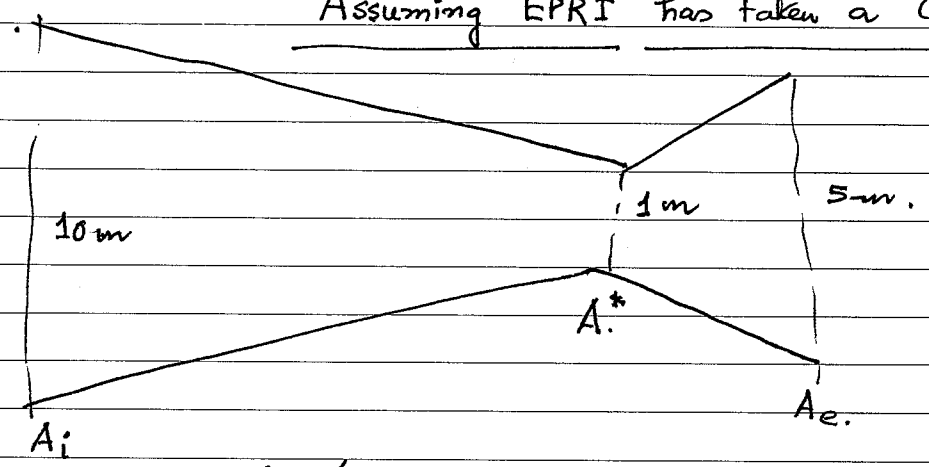

14000
12000
10000
8000
6000
4000
2000
0
depth (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
volume fraction
0.05
0.04
0.03
0.02
0.01


2000
1500
1000
500
0
depth (m)
0.4
0.5
0.6
0.7
0.8
0.9
1
volume fraction
0.05
0.04
0.03
0.02
0.01

Assuming EPRI has taken a C-D nozzle calculation



$$A_e/A^* = 25.$$

Corresponding to $A/A^* = 25$, $P_0/P = 529.1.$

$$T_0/T = 6.0$$

$$\rho_0/\rho = 88.18.$$

$$M = 5.$$

Given $P_0 = 10\ MPa$, $P \Rightarrow \dfrac{10 \times 10^3}{529.1}$

$$\approx 1.89\ Pa. > 1 \text{ atmosphere.}$$

$T_0 = 1150$, $T \Rightarrow 1150/6 \approx 192\ °C.$

$$\approx 465.15\ °K$$

$$\therefore a = \sqrt{\gamma . R . T} = \sqrt{1.4 * 287 * 465.15}$$

$$= 432.316169\ m/s.$$

$$\therefore V = M . a = 5 * 432.316169 = 2161\ m/s.$$

As $P_e > 1$ atmosphere $\Rightarrow$ under expanded nozzle.

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∴ There will be expansion fan initially, so that the pressure can reduce.

Flow is definitely Supersonic. Shock → Subsonic.

- As magma fills up, pressure rises.
  - Reflected shock waves !!

* Shock waves will happen only when exit pressure > back pressure.

Flow will try to equilibriate through shocks.

$$\frac{C}{5} = \frac{F-32}{9}$$

$C = 1150$

$$230 = \frac{F-32}{9} \Rightarrow F \Rightarrow 2102°F$$

REFERENCES:

1. Andersom, John D., "Modern Compressible Flow: with historical perspective", McGraw-Hill Inc., 1990.

2. Woods, A. W., and Huppert, Hurbert, E., "On magma chamber evolution during slow eruptive eruptions", Journal of Geophysical Research, Vol. 108, No. B8, 2403, doi:10.1029/2002JB002019, 2003.

3. Sparks, R. S. J., "The dynamics of bubble formation and the growth in magmas-A review and analysis, Journal of Geophysical Research, Vol. 83, pp. 1-37, 1978.

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