# Deterministic Algebraic Sum (AS) Approach for Incoherent SSI Analysis


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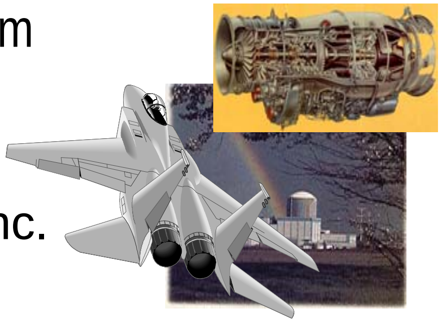


**NEI/NRC Meeting**

**EPRI, Palo Alto, July 23-24, 2007**

# Stochastic Field Models Using Spectral Expansion of Covariance Matrix

Covariance matrix is positive definite. It can be decomposed:

1. $\sum_{\mathbf{u},\mathbf{u}'} = \mathbf{\Phi\Lambda\Phi}^{\mathrm{T}} = (\mathbf{\Phi}\sqrt{\mathbf{\Lambda}})(\mathbf{\Phi}\sqrt{\mathbf{\Lambda}})^{\mathrm{T}}$ Spectral Decomposition (KL/PCA/POD)

2. $\sum_{\mathbf{u},\mathbf{u}'} = \mathbf{LL}^{\mathrm{T}}$ - Choleski Decomposition (SS)

**Wiener-Fourier Series**

Simulation of stochastic field:

$\mathbf{u}(\mathbf{x},\boldsymbol{\theta}) = \mathbf{u}(\mathbf{x})^{\mathrm{T}}\mathbf{z}(\boldsymbol{\theta})$ (or for each point j $\mathrm{u}_{\mathrm{j}}(\mathbf{x},\theta) = \sum \mathrm{u}_{\mathrm{j,i}}(\mathbf{x})\mathrm{z}_{\mathrm{i}}(\theta)$)

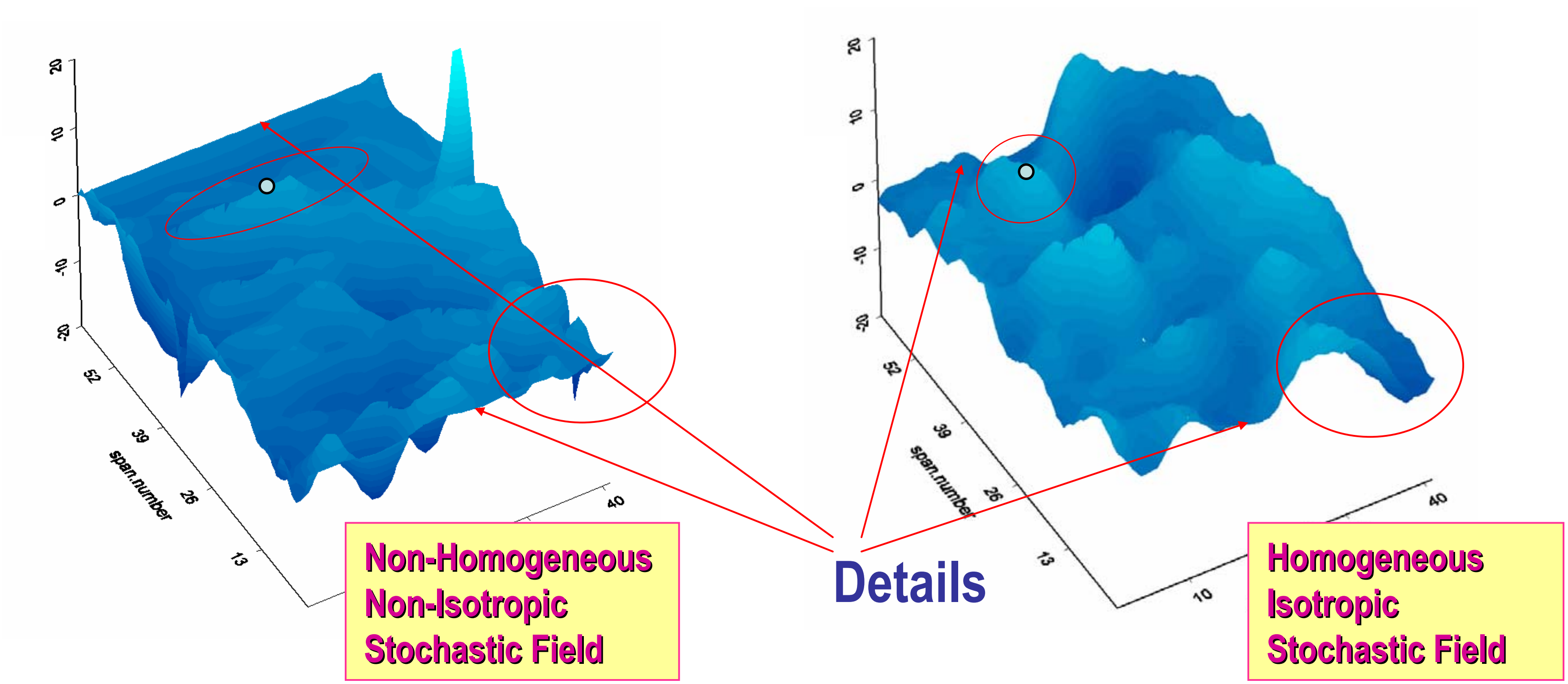
1. $\mathbf{u} = \mathbf{\Phi}\sqrt{\mathbf{\Lambda}}$ (or for each point j $\mathrm{u}_{\mathrm{j}}(\mathbf{x},\theta) = \sum_{\mathrm{i}=0}^{\mathrm{n}} \sqrt{\lambda_{\mathrm{i}}}\,\Phi_{\mathrm{j,i}}(\mathbf{x})\mathrm{z}_{\mathrm{i}}(\theta)$) - *can be truncated*

2. $\mathbf{u} = \mathbf{L}$ (or for each point j $\mathrm{u}_{\mathrm{j}}(\mathbf{x},\theta) = \sum_{\mathrm{i}=0}^{\mathrm{n}} \mathrm{l}_{\mathrm{j,i}}(\mathbf{x})\mathrm{z}_{\mathrm{i}}(\theta)$ ) - *no truncation*

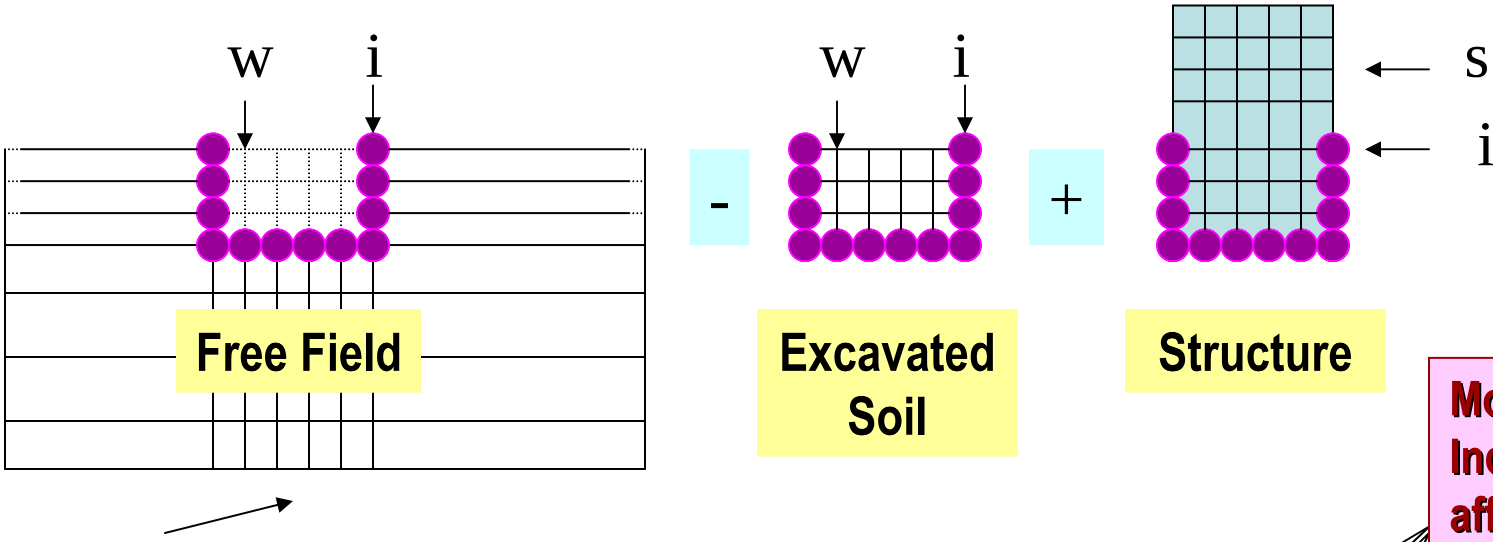
Compute uncorrelated random variables: $\mathrm{z}_{\mathrm{i,k}}(\theta) = \int_{\mathrm{D}} \mathrm{u}_{\mathrm{i}}(\mathrm{x})\,\mathrm{u}_{\mathrm{k}}(\mathrm{x},\theta)\,\mathrm{dx}$

**Remarks:** KL, PCA and POD are mathematically equivalent!

# Simulated Spatial Variations Using KL Expansion



# Implementation in the SASSI Framework




Motion Incoherency affects free-field motion at interaction nodes

**Flexible Volume Method** (all excavated volume nodes)

$$\begin{bmatrix} \mathbf{C}^e_{ii} - \mathbf{C}^e_{ii} + \mathbf{X}_{ii} & -\mathbf{C}^e_{iw} - \mathbf{X}_{iw} & \mathbf{C}^s_{is} \\ -\mathbf{C}^e_{wi} + \mathbf{X}_{wi} & -\mathbf{C}^e_{ww} + \mathbf{X}_{ww} & \mathbf{0} \\ \mathbf{C}^s_{si} & \mathbf{0} & \mathbf{C}^s_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_i \\ \mathbf{U}_w \\ \mathbf{U}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{X}_{ii}\mathbf{U'}_i + \mathbf{X}_{iw}\mathbf{U'}_w \\ \mathbf{X}_{wi}\mathbf{U'}_i + \mathbf{X}_{ww}\mathbf{U'}_w \\ \mathbf{0} \end{Bmatrix}$$

**Flexible Interface or Subtraction Method** (only interface nodes)

$$\begin{bmatrix} \mathbf{C}^e_{ii} - \mathbf{C}^e_{ii} + \mathbf{X}_{ii} & -\mathbf{C}^e_{iw} & \mathbf{C}^s_{is} \\ -\mathbf{C}^e_{wi} & -\mathbf{C}^e_{ww} & \mathbf{0} \\ \mathbf{C}^s_{si} & \mathbf{0} & \mathbf{C}^s_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_i \\ \mathbf{U}_w \\ \mathbf{U}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{X}_{ii}\mathbf{U'}_i \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

$$\boxed{\mathbf{C}(\omega)\mathbf{U}(\omega) = \mathbf{Q}(\omega)}$$

where $\mathbf{C}(\omega) = \mathbf{K} - \omega^2\mathbf{M}$

# Incoherent Seismic SSI Response

The complex frequency response is computed as follows:

- Coherent SSI response:

Structural transfer function given input at interaction nodes

Coherent ground transfer function at interface nodes given control motion

$$U_s(\omega) = H_s(\omega) * H_g^c(\omega) * U_{g,0}(\omega)$$

Complex Fourier transform of control motion

- Incoherent SSI response:

Incoherent ground transfer function given coherent ground motion and coherency model (random spatial variation in horizontal plane)

$$U_s(\omega) = H_s(\omega) * S_g^i(\omega) * H_g^c(\omega) * U_{g,0}(\omega)$$

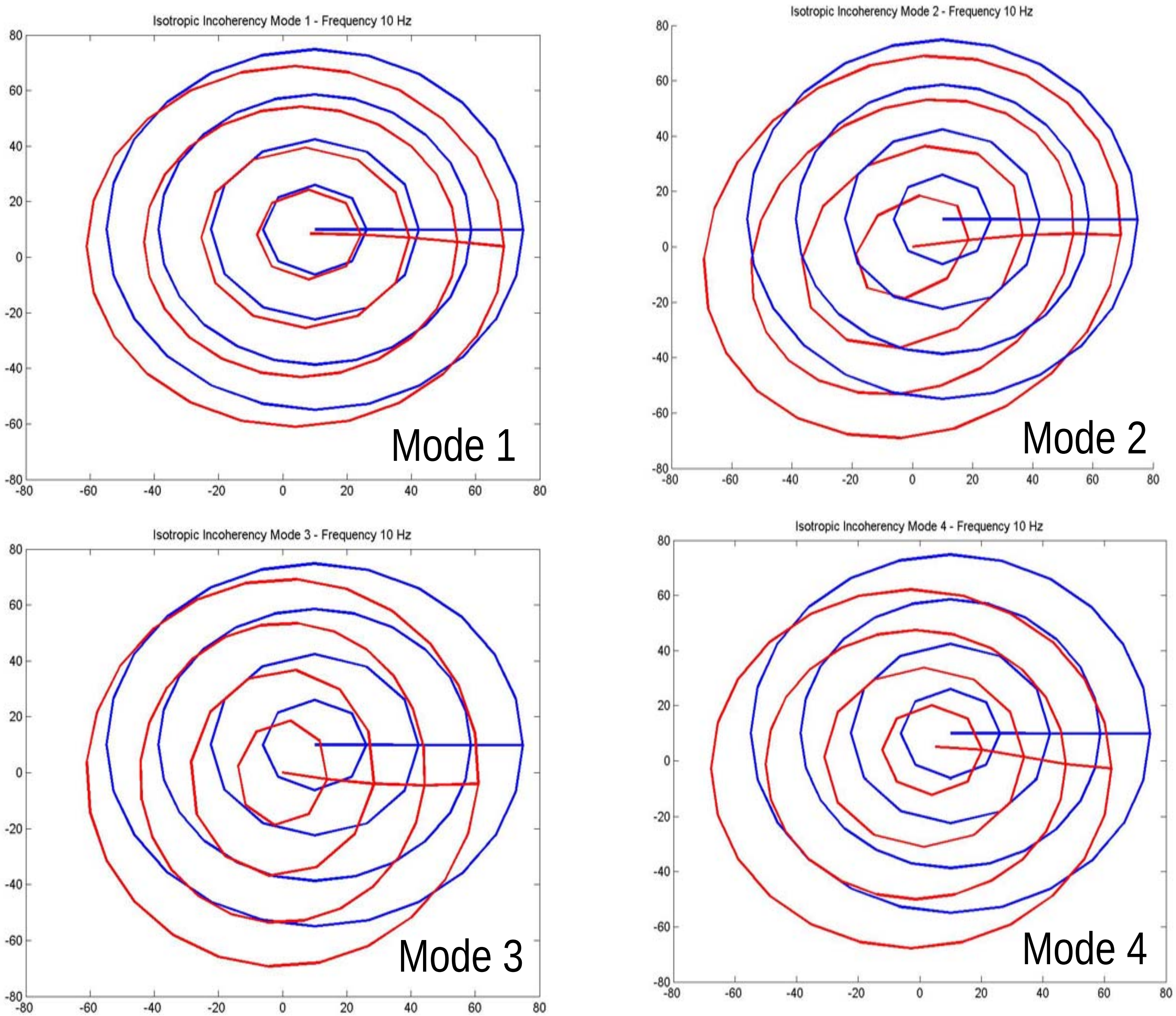
$$S_g(\omega) = [\Phi(\omega)]\mathrm{diag}[\lambda(\omega)]\{\eta_\theta\}$$

Complex Fourier transform of relative spatial variations of motion at interaction nodes that is stochastic by nature

Spectral factorization of coherency matrix

Random phases (stochastic part)

# Motion Incoherency Modes of Basemat at 10 Hz




**REMARKS:**

**1) For low frequencies only a number of few incoherency modes are sufficient.**

**2) Motion incoherency modes are stochastically combined. We try to use simple mode superposition rules – single SSI run.**

# Key Features of Deterministic AS Approach

**BASIC IDEA:** Develop an approach based on spectral factorization of coherency matrix to Approximate mean response in a *single* SSI run.

This approximate approach is based on the assumption that "Mean Incoherency Input provides Mean SSI Response". *Median Input – Median Output* rule (used in SPRA reviews)

1) Assumes that spatial modes (wavelength components) scaled with their sqrt eigenvalues (equal to the standard deviations) are combined using zero phases instead of random phases (zero phases are mean phase values). AS reproduces exactly the free-field coherency function if all modes are used (can be checked).

2) Assumes that the Fourier amplitude variations due to spatial motion variation are uncorrelated random variables for all selected frequencies (similar to the time domain random variability). *The single set of* ITF amplitudes for all frequencies are obtained by superposition of scaled spatial modes. A revised ITF interpolation scheme was implemented to avoid spurious peaks and valleys.

3) *Single* set of ITF phases computed at all frequencies from SSI solution. Phases are partially random due to stochastic nature of incoherency and partially deterministic due to SSI physics. Avoid phases that produce canceling effects. Phase angles limited to Q1-2 in phase space.

# Computational Steps:

1) Compute free-field coherency matrix
2) Perform spectral factorization of coherency matrix (check)
3) Use linear superposition of scaled spatial modes at each selected frequency assuming zero phases
4) Computed ITF including smoothing to avoid spurious peaks
5) Adjust ITF phases for Q1-2 to avoid canceling phase effects
6) Perform convolution of ITF with input FFT (control motion)
7) Compute time histories by inverse FFT
8) Compute RS at selected nodes

# AS Approach As Described in EPRI Report

“To compute the incoherent free field motion at interaction nodes the AS approach uses the algebraic sum of the spatial modes scaled by the square root of their eigenvalues. It should be noted that the square roots of the mode eigenvalues are equal to the standard deviations of the random mode contribution factors to the total stochastic spatial variability of the input motion.combination is applicable.

Since this approach is applied directly in the context of the complex Fourier transform representation of the input motion in SASSI and assumes that the Fourier amplitudes of two neighbor frequencies are statistically uncorrelated. As a result of the high frequency resolution of the incoherent transfer function amplitudes (obtained by convolution in frequency domain of complex structural transfer functions with complex Fourier spatial free-field input variation at interaction nodes), the original SASSI interpolation scheme (Tajirian, 1981) could produce spurious spectral peaks and valleys.

To avoid these spurious peaks and valleys in the interpolated transfer functions, an improved interpolation scheme that bases on the original SASSI interpolation scheme but includes a parametric spectral smoothing capability of the interpolation error was implemented.

More specifically, first, the differences between the original computed complex transfer functions at the selected frequencies and the interpolated transfer function at all Fourier frequencies are computed at all Fourier frequencies (separate for the Fourier amplitudes and phases) and then, these amplitude differences are smoothed using the Parzen windowing technique with a user specified smoothing parameter (Parzen, 1962).

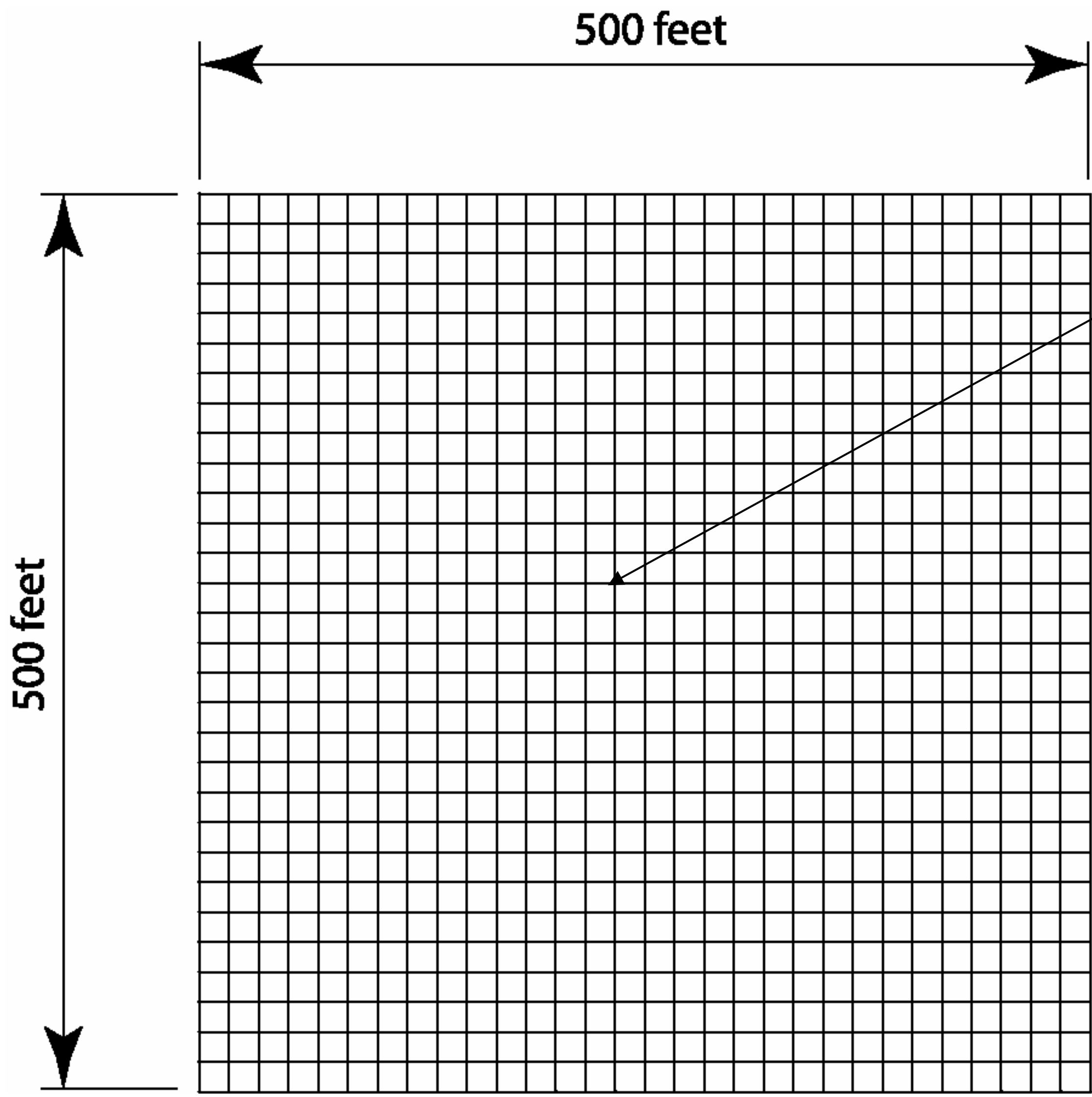
The smoothing parameter value represents the number of Fourier frequency steps that is used to define the bandwidth of the Parzen spectral window (equal to the standard deviation of the Gaussian function used).Advanced numerical techniques are applied to account for the appropriate phase relationship between spatial modes.

For small smoothing parameter values, the final complex transfer functions are closer to the interpolated transfer functions using original SASSI interpolation scheme, while for large smoothing parameter,  they are closer to the original computed transfer functions..

To include the effects of motion incoherency stochasticity of complex response phasing, the SASSI-AS approach assumes that complex response phase angles are limited to a variation range from –pi/2 to pi/2 that ensures that the phase angle cosines for all frequency components are positive. This phase angle variation limitation avoids the random occurrence of components with canceling effects due to motion incoherency and by this generates higher energy response time histories for given Fourier amplitude spectra.

The Linear Algebraic Sum approach incorporates smoothing and the above phase adjustment to linear mode combination to form an approximate approach for computing the mean incoherent response that produces similar results with the SASSI-SRSS and SASSI Simulation approaches as shown in this report."

# Incoherent Free-Field Motion Simulation Via Complex Fourier Representation

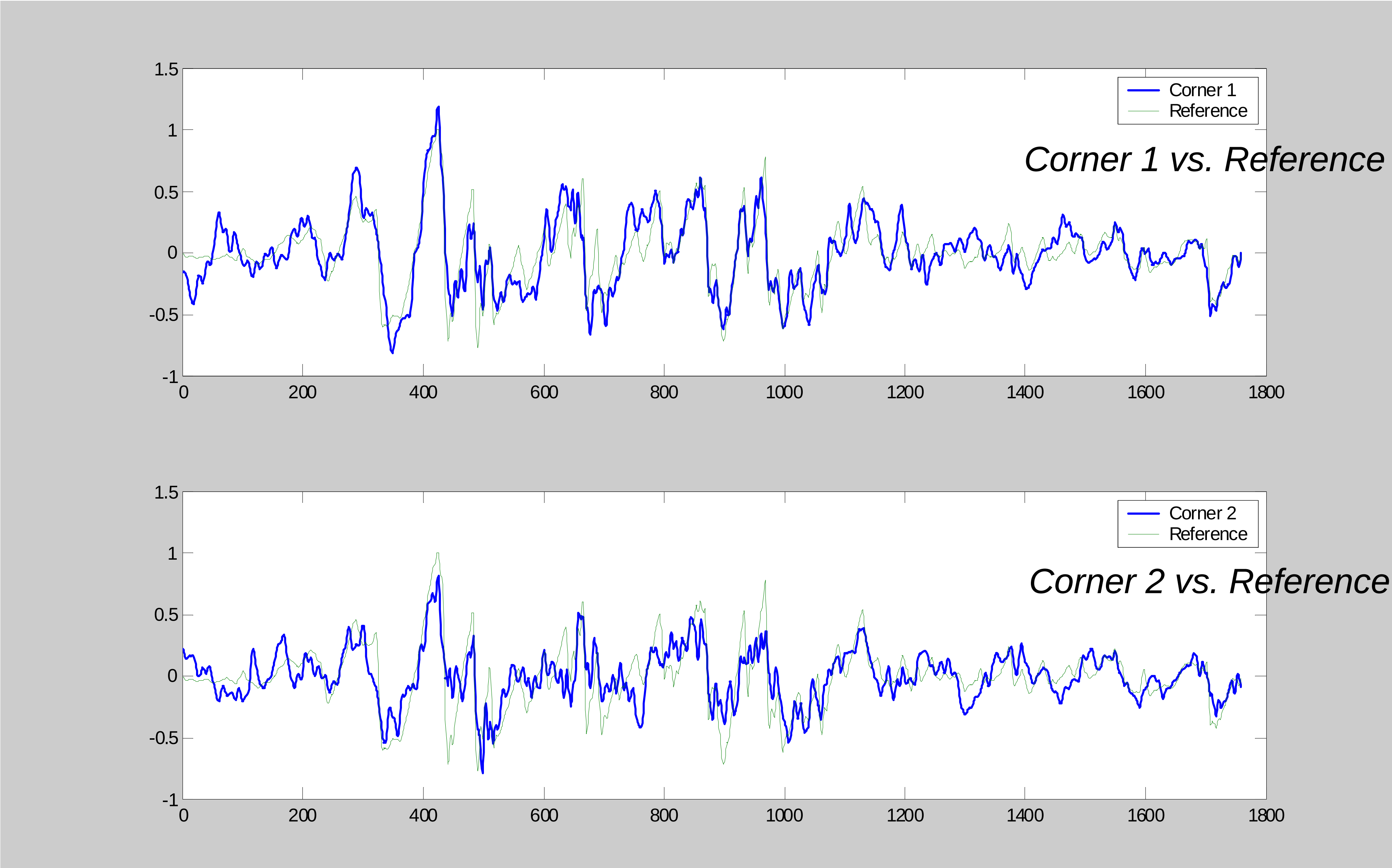



Free-Field Surface Grid

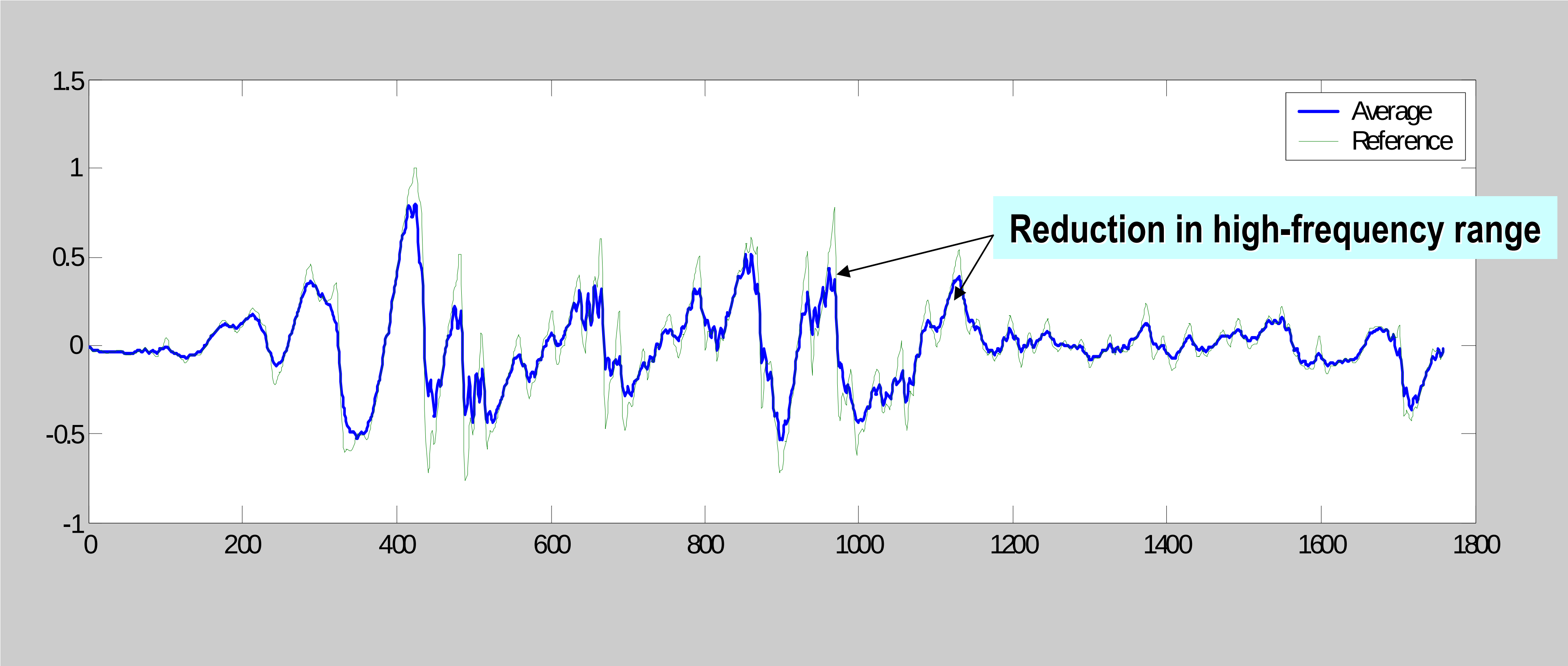
Dimension: 500ftx500ft

Mesh: 31 x 31 nodes

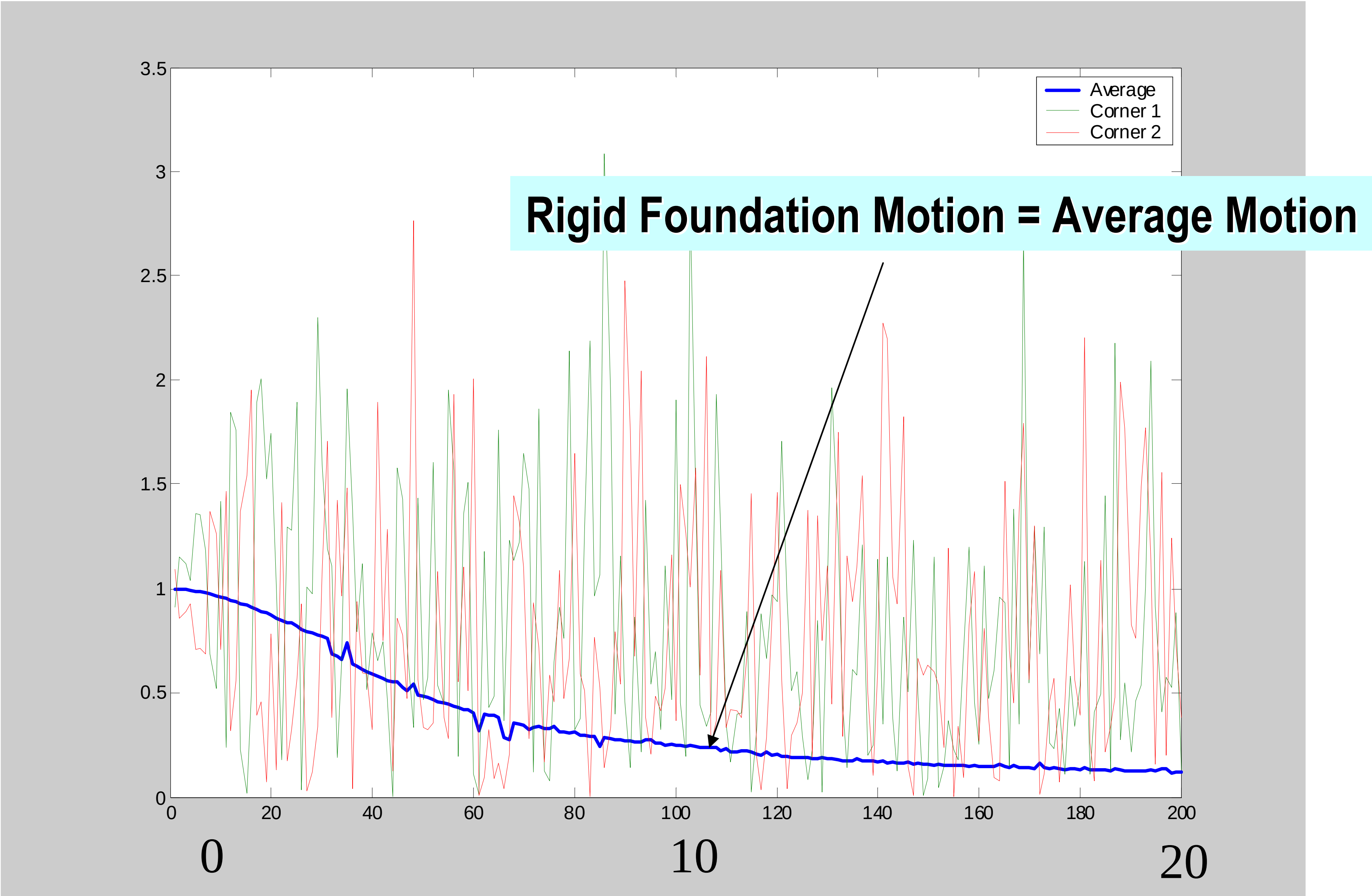
# Ground Motion at Two Points Separate by 700 ft (two corners on the grid, for γ=0.15)



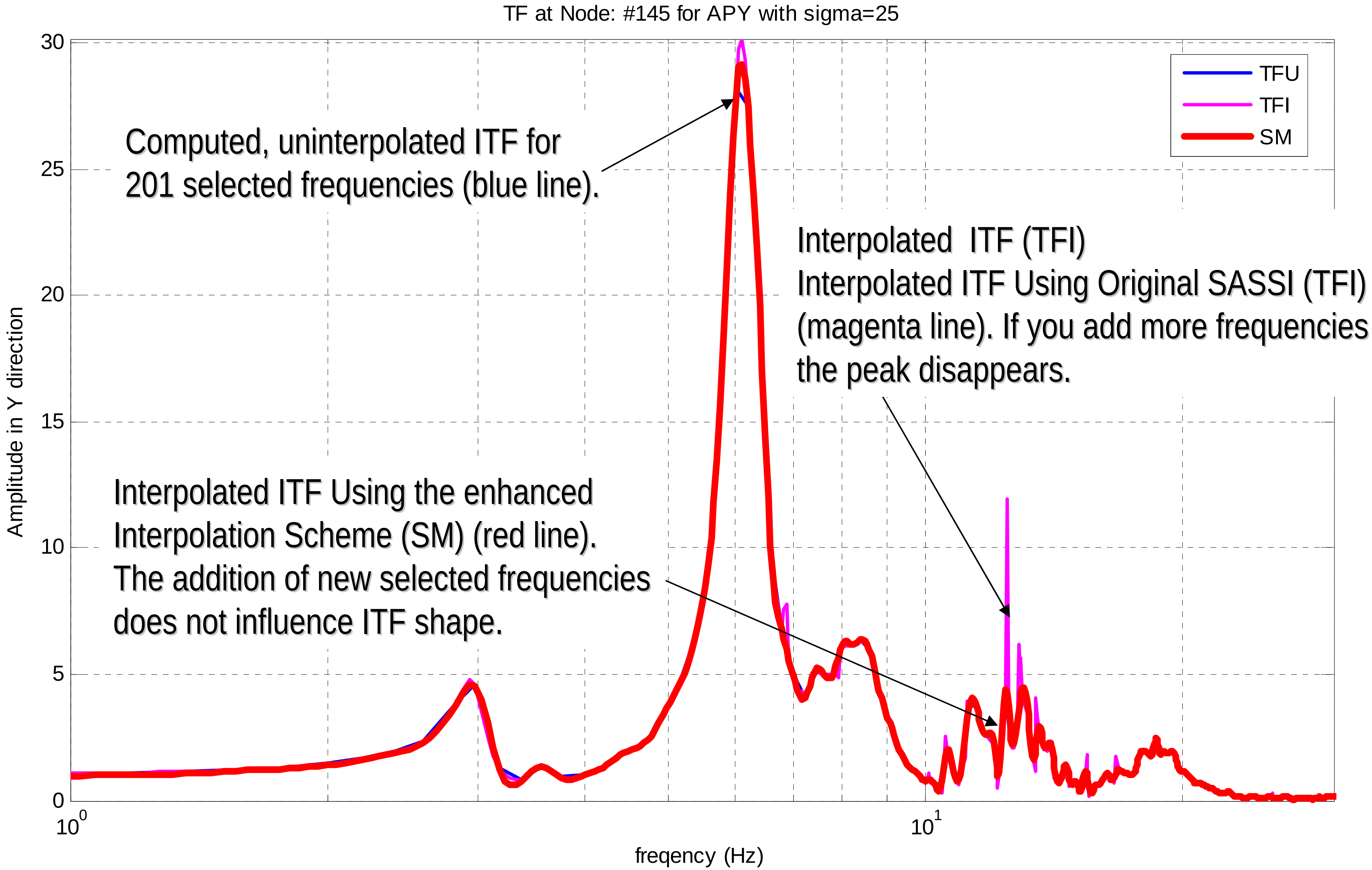
# Reference Input vs. Average Translation Motion (for γ=0.15)

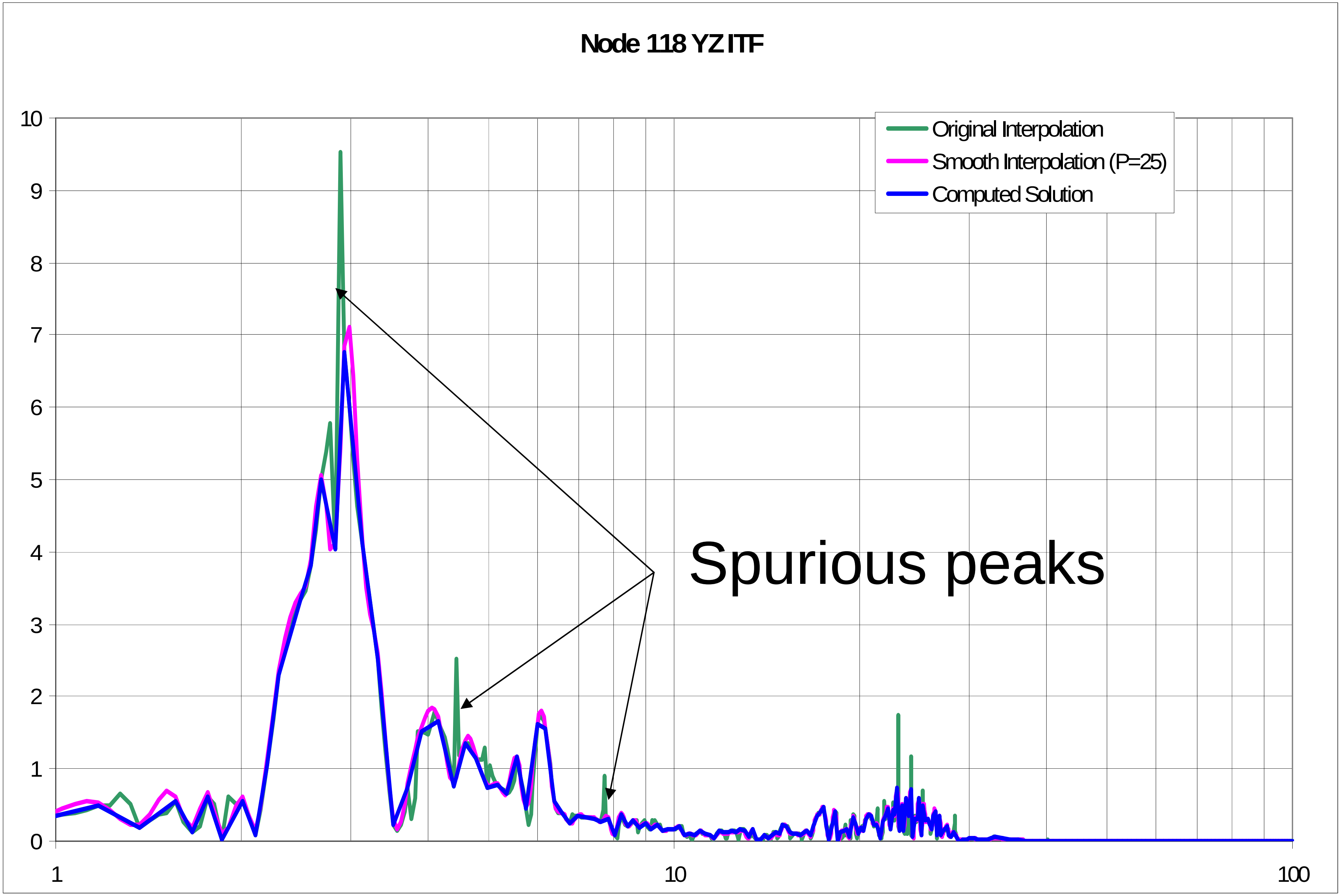


# Effects of Motion Incoherency at Two Corner Locations (in Average and Locally, for $\gamma$=0.15)



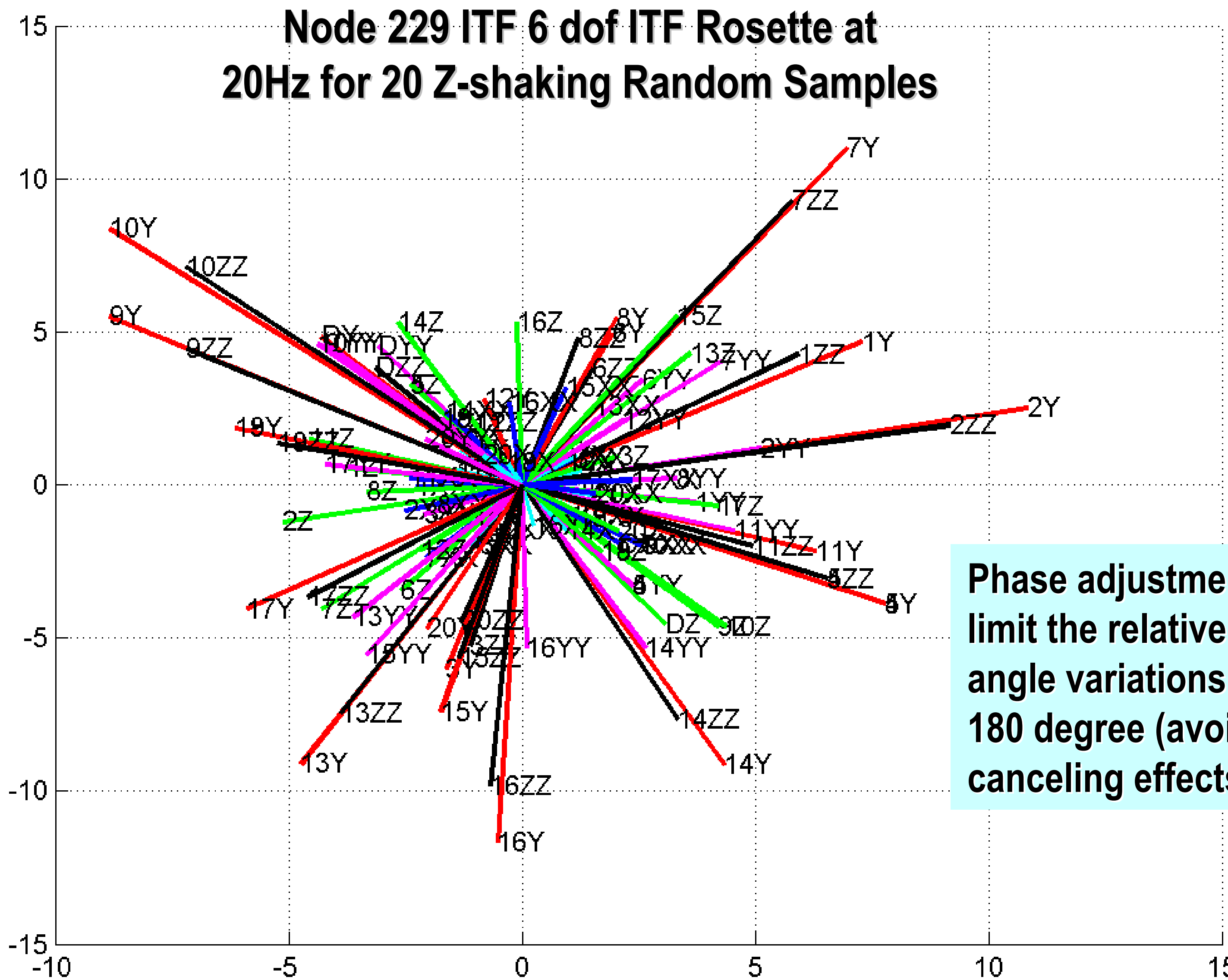
# Revised ITF Interpolation Scheme to Potential Remove Spurious Peaks and Valleys



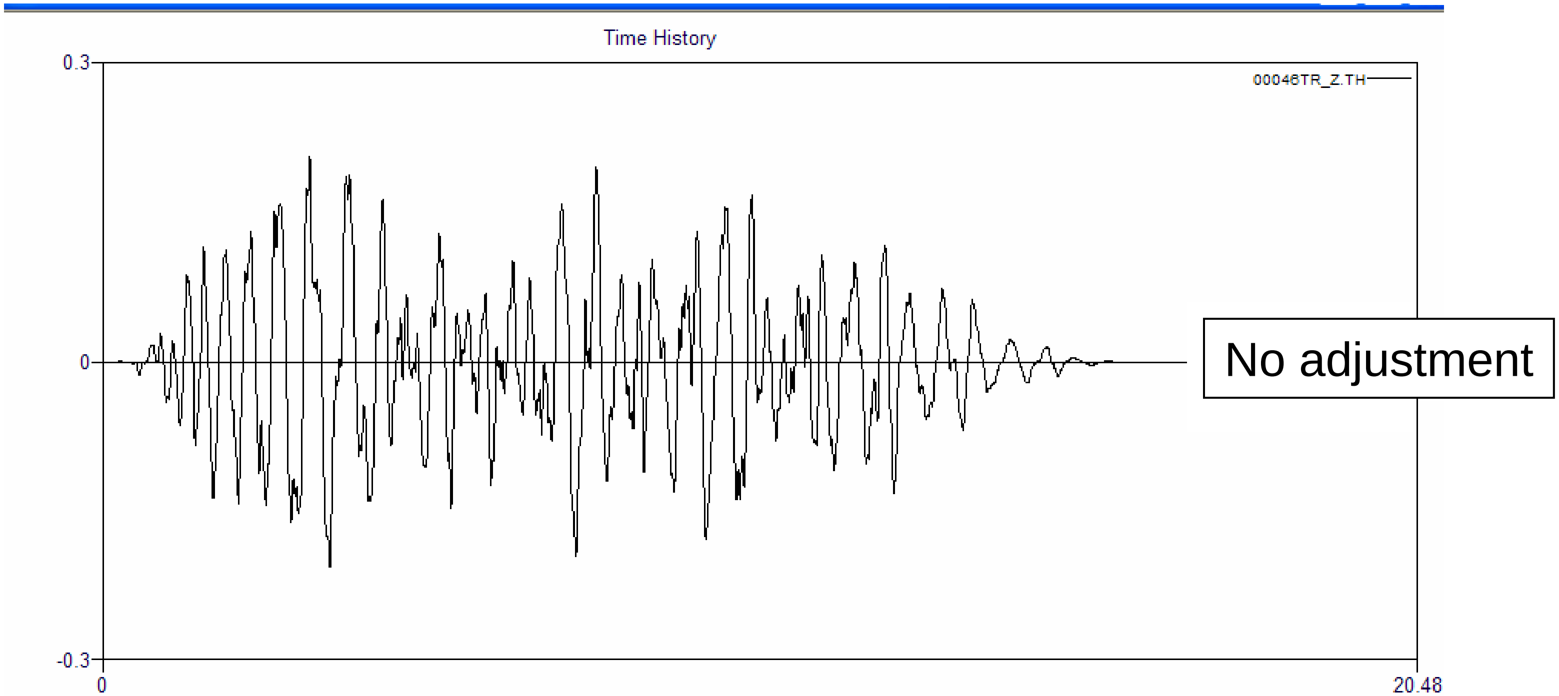


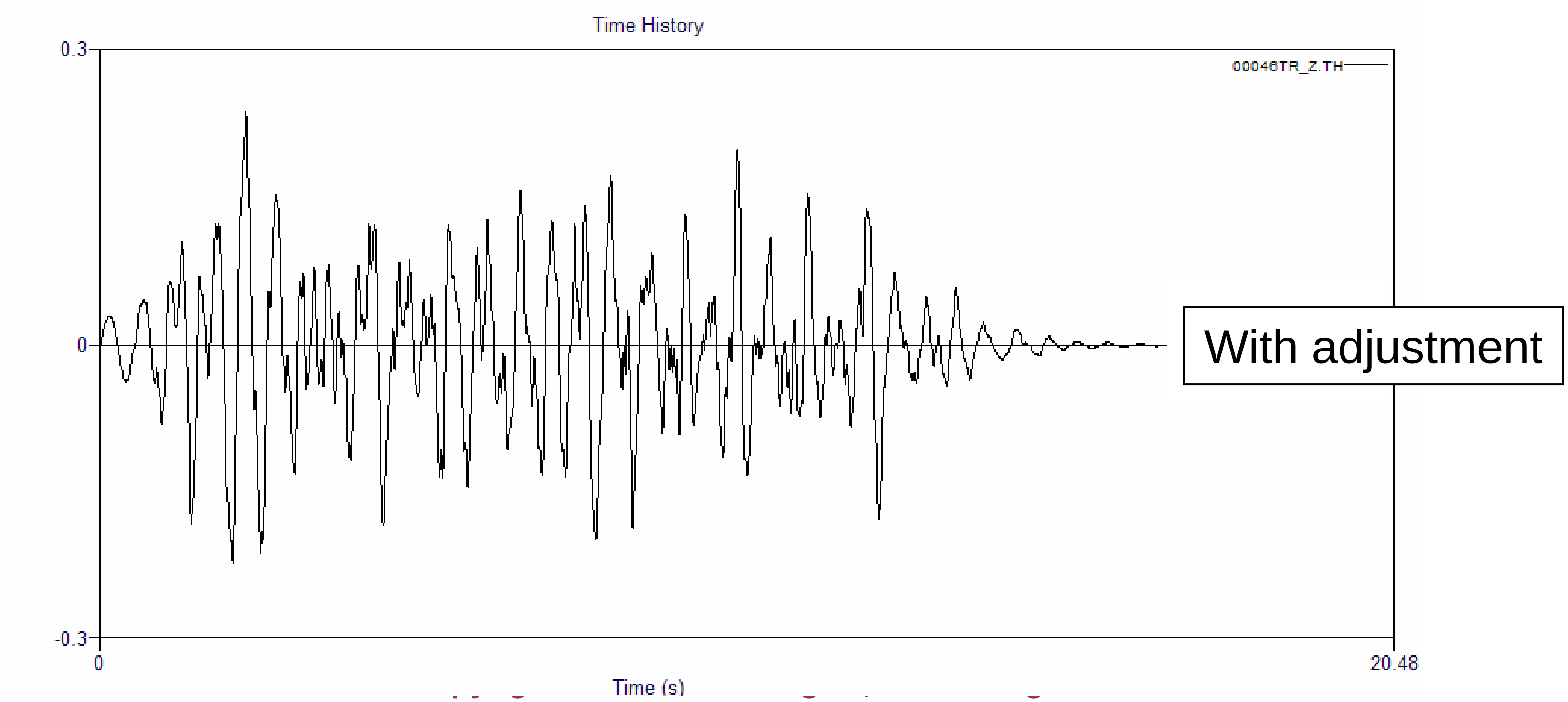
Node 118 YZ ITF
Original Interpolation
Smooth Interpolation (P=25)
Computed Solution
Spurious peaks
10
9
8
7
6
5
4
3
2
1
0
1
10
100

# ITF Phase Adjustment to Avoid Random Canceling Phase Effects




Phase adjustment limit the relative phase angle variations to 180 degree (avoid canceling effects)


Time History
00046TR_Z.TH
0.3
0
-0.3
0
20.48
No adjustment


Time History
00046TR_Z.TH
0.3
0
-0.3
0
20.48
Time (s)
With adjustment

# Concluding Remarks:

## Practicality of Deterministic AS Approach:

- Good theoretical basis (modeling of free-field incoherent motion)
- Fast, single SSI restart analysis (only 25% increase in SSI analysis run time)
- Good accuracy for rigid foundation applications as shown in ARES/EPRI report
    - selection of frequencies influences (add frequencies around peaks)
    - statistical scatter influences (mean approximation error is proportional)
    - existence of spurious peaks/valleys in ITF (could be improved by adding more frequencies around spectral peaks, easy plots for TFU, TFI)

**Personal Notes for Flexible Foundation Applications:**

- For flexible foundations the free-field motion local spatial variations at higher frequencies are not filtered by kinematic SSI interaction. A deterministic approach limits the modeling of incoherent SSI response to a single variation pattern. Stochastic simulation is suggested for flexible foundations applications.
- Non-uniformity of interaction nodes has larger effects for incoherent analysis than for coherent analysis (could create a non-uniform soil stiffness distribution).