LaSalle UHS Temperature Limit Modification Request -- TAC MD0336 / 7

Measurement uncertainty review and alternative calculation

- Uncertainties are presented in units of degrees Fahrenheit unless otherwise noted.
- Uncertainties are stored as elements of vectors named for the physical component to which they apply. Vector element numbers are related to the names of the various uncertainties. Element zero is not used.
- "Calculation" refers to licensee calculation L-003230 revision 000, transmitted via letter RS-06-106, (August 4, 2006)
- Prefixes "c" and "a" indicate licensee and alternative values, respectively
- Suffixes "r" and "n" indicate random and non-random uncertainties, respectively.
- If the calculation assigns a value of zero to an uncertainty and the alternative calculation does not assign a different value, then the uncertainty may not be explicitly presented below.
- The symbol ":=" indicates definition: $x := 2$ assigns the value of "2" to the variable "x."
- The symbol "=" reports the value of a variable or computation: $10 \bullet x = 20.0000$

SRSS function definition: SRSS(x) :=

```
"x must be a vector; element zero is ignored"
u2 ← 0
for r ∈ 1..rows(x) − 1
    u2 ← u2 + (x_r)^2
√u2
```

demonstration of the function:

$$SRSS\left(\begin{pmatrix}100\\1\\2\end{pmatrix}\right) \bullet \frac{1}{\sqrt{5}} = 1.00000000000000000$$

RTD Errors (Module 1)

random errors:

Elements:	Values:	Alternative Values:	
$RA1 := 1$	$cRTDr_{RA1} := 0.1$	$aRTDr_{RA1} := 0.2$	arbitrary factor of 1/2 is removed in the alternative calculation.
$CAL1 := 2$	$cRTDr_{CAL1} := 0$	$aRTDr_{CAL1} := 0$	
$ST1 := 3$	$cRTDr_{ST1} := 0$	$aRTDr_{ST1} := 0$	
$\sigma 1in := 4$	$cRTDr_{\sigma 1in} := 0$	$aRTDr_{\sigma 1in} := 0$	

$D1 := 5$

The calculation applies SRSS to time-based drift. This is not valid, since drift is not random over time and the interval over which drift is assumed is arbitrary. Nevertheless, the alternative calculation accepts the SRSS assertion pending a more complete justification from the licensee.

- ***The alternative calculation is therefore nonconservative in this regard.***

The calculation also applies an arbitrary factor of 1/2, which is omitted in the alternative calculation. The drift rate is presented in the calculation as 0.1F per year.

$cRTDr_{D1} := 0.112$ $aRTDr_{D1} := 2 \bullet cRTDr_{D1}$ $aRTDr_{D1} = 0.2240$

$c\_\sigma 1 := SRSS(cRTDr)$ $c\_\sigma 1 = 0.1501$ $a\_\sigma 1 := SRSS(aRTDr)$ $a\_\sigma 1 = 0.3003$

non-random errors

$eIR1 := 1$	$cRTDn_{eIR1} := 0$	$aRTDn_{eIR1} := 0$
$eRD1 := 2$	$cRTDn_{eRD1} := 0$	$aRTDn_{eRD1} := 0$
$eT1 := 3$	$cRTDn_{eT1} := 0$	$aRTDn_{eT1} := 0$
$e1in := 4$	$cRTDn_{e1in} := 0$	$aRTDn_{e1in} := 0$

$c\Sigma e1 := \sum cRTDn$ $c\Sigma e1 = 0.0000$ $a\Sigma e1 := \sum aRTDn$ $a\Sigma e1 = 0.0000$

Transmitter Errors (Module 2)

random errors:

Elements:	Values:	Alternative Values:	

$RA2 := 1$

The calculation applies a divisor of 3 based upon an assumption that the vendor data are based upon 3σ, and uses SRSS to accommodate time-based drift. The alternative calculation again accepts the SRSS assertion pending a more complete justification from the licensee, and applies the extention only to the estimated drift portion of the overall accuracy specification. The basis for the 3σ assumption and the factor of 1/3 do not seem reasonable, and these are omitted in the alternative calculation.

$vendorRA := 0.54$

vendorRA includes conventional RA, 0.1F resolution error, and drift. Assume 2-yr drift and conventional RA are equal, and that all are combined via SRSS: then (2-yr drift)^2 + RA^2 + 0.1^2 = vendorRA^2.

$$RA := \sqrt{\frac{vendorRA^2 - 0.1^2}{2}} \qquad RA = 0.3752$$

$$drift := RA \bullet \sqrt{\frac{5yr}{2yr}} \qquad \sqrt{RA^2 + RA^2 + 0.1^2} = 0.5400 \qquad drift = 0.5933$$

$cXMTRr_{RA2} := 0.285$ $\quad aXMTRr_{RA2} := \sqrt{RA^2 + drift^2 + 0.1^2}$ $\quad aXMTRr_{RA2} = 0.7091$

$CAL2 := 2$

- The calculation includes meter and calibration standard errors as random effects. The calculation indicates that alternate channels are calibrated using different equipment, but there are only two meters and there are four channels of which two are credited in the ultimate UHS temperature measurement. The alternative calculation therefore addresses meter error as a non-random uncertainty.
- In addition, the same standard would be used to calibrate both meters and is therefore common to all four channels. The calculation asserts that the calibration standard is sufficiently accurate to be ignored, but also indicates that it may be better than the meters themselves by no more than 4:1, which does not appear to be inconsequential. The alternative calculation includes the calibration standard error as a non-random uncertainty.
- Finally, the calculation makes an unsupported assumption that the calibration setting tolerance is a 3-sigma value rather than the more conventional 2-sigma, and applies an incorrect factor of 1/3 (rather than 2/3) to make the conversion. The alternative calculation accepts the stated value as 2-sigma.
- The alternative calculation retains only the setting tolerance as calibration uncertainty.

$cXMTRr_{CAL2} := 0.29$ $\quad aXMTRr_{CAL2} := 0.54$

$\sigma T2 := 3$

- The calculation makes an unsupported assumption that the ambient temperature effect is a 3-sigma value rather than the more conventional 2-sigma, and applies an incorrect factor of 1/3 (rather than 2/3) to make the conversion. The alternative calculation accepts the stated value as 2-sigma. The effect is 0.1% of range per 10 degrees C.
- The calculation is based upon a minimum ambient temperature is 75F. This seems unreasonably high for the indicated location. The alternative calculation does not alter this assumption, since no alternative value is available.
- The alternative calculation corrects a numerical error in the temperature unit conversion factor. The calculation uses a value of 5/8 rather than 5/9.

$cXMTRr_{\sigma T2} := 0.051$ $\quad aXMTRr_{\sigma T2} := \left(\frac{0.1\% \bullet 90}{10C}\right) \bullet \left(\frac{100C}{180F}\right) \bullet 27F$ $\quad aXMTRr_{\sigma T2} = 0.1350$

$\sigma 2in := 4$ $\quad cXMTRr_{\sigma 2in} := 0.150$ $\quad aXMTRr_{\sigma 2in} := a\_\sigma 1$ $\quad aXMTRr_{\sigma 2in} = 0.3003$

$\sigma 2PS := 5$ $\quad cXMTRr_{\sigma 2PS} := 0$ $\quad aXMTRr_{\sigma 2PS} := 0$

$c\_\sigma 2 := SRSS(cXMTRr)$ $\quad c\_\sigma 2 = 0.4364$ $\quad a\_\sigma 2 := SRSS(aXMTRr)$ $\quad a\_\sigma 2 = 0.9502$

non-random errors

The calculation indicates that all non-random uncertainty components for the transmitter are zero. The alternative calculation treats the M&TE uncertainties as non-random and applies calculation assumption 3.1 without dismissing the numerical result as insignificant. The alternative calculation also eliminates arbitrary factors of 1/2 that appear at various points in the calculation, and applies the ambient temperature variation which is addressed in the text of the calculation but omitted from the calculation itself. As indicated elsewhere, the lower ambient temperature limit seems unexpectedly high, but is not altered in the alternative calculation because no credible alternative estimate is available.
The RTD data show 0.214 ohms per degree F.

$MTE2 := 1$ HP meter uncertainties

$cRAMTE2hp := 0.050$

$$aRAMTE2hp := \frac{.01\% \bullet 115.013 + .001\% \bullet 1000}{0.214}$$

$aRAMTE2hp = 0.1005$

$cTEMTE2hp := 0.00395$

$$aTEMTE2hp := \frac{(.0006\% \bullet 115.013 + .0001\% \bullet 1000) \bullet 10.9}{0.214}$$

$aTEMTE2hp = 0.0861$

$$cMTE2hp := \sqrt{cRAMTE2hp^2 + cTEMTE2hp^2} \qquad aMTE2hp := \sqrt{aRAMTE2hp^2 + aTEMTE2hp^2}$$

$cMTE2hp = 0.0502$ $aMTE2hp = 0.1323$

Fluke meter uncertainties:

$cMTE2f := 0.228$

$$aMTE2f := \frac{0.05\% \bullet 115 + 2 \bullet 0.01 + 0.02}{0.214}$$

$aMTE2f = 0.4556$

conservative selection:

$cXMTRn_{MTE2} := \max(cMTE2hp, cMTE2f)$ $aXMTRn_{MTE2} := \max(aMTE2hp, aMTE2f)$

$cXMTRn_{MTE2} = 0.2280$ $aXMTRn_{MTE2} = 0.4556$

$STD2 := 2$ $cXMTRn_{STD2} := 0$ $aXMTRn_{STD2} := \frac{1}{4} \bullet aXMTRn_{MTE2}$

$aXMTRn_{STD2} = 0.1139$

$e2in := 3$ $cXMTRn_{e2in} := c\Sigma e1$ $cXMTRn_{e2in} = 0.0000$ $aXMTRn_{e2in} := a\Sigma e1$ $aXMTRn_{e2in} = 0.0000$

NOTE: The calculation treats these values as random, but the alternative calculation treats them as non-random. The calculation values are presented here for comparison with the corresponding values in the alternative calculation. These values are included in the calculation as components of CAL2, presented in the "random" section as

$cXMTRr_{CAL2} = 0.2900$ Also note that $\sum cXMTRn = 0.2280$ The cXMTERn vector is not used in the computation of results in this derivation. Instead, the net non-random uncertainty used herein is forced to the calculation final value.

$c\Sigma e2 := 0$ $c\Sigma e2 = 0.0000$ $a\Sigma e2 := \sum aXMTRn$ $a\Sigma e2 = 0.5695$

I/O Module Errors (Module 3)

random errors:

Elements:	*Values:*	*Alternative Values:*
$RA3 := 1$	$clOMr_{RA3} := 0.0113$	$alOMr_{RA3} := 0.0113$
$CAL3 := 2$	$clOMr_{CAL3} := 0$	$alOMr_{CAL3} := 0$
$D3 := 3$	$clOMr_{D3} := 0$	$alOMr_{D3} := 0$
$\sigma 3in := 4$	$clOMr_{\sigma 3in} := c\_\sigma 2$	$alOMr_{\sigma 3in} := a\_\sigma 2$
	$clOMr_{\sigma 3in} = 0.4364$	$alOMr_{\sigma 3in} = 0.9502$

The calculation asserts that there is no calibration error or drift. That does not seem a reasonable assumption, but there are no data available upon which an alternative estimate could be based.

The dropping resistor at the input to the I/O module is treated as a non-random error in the calculation. In most installations of this type, such a component would exist independently in each channel and the associated errors would therefore be considered to be random. The alternative calculation does not alter this treatment because there is insufficient detailed design information available to support an alternative treatment.

$c\_\sigma 3 := SRSS(clOMr)$ $c\_\sigma 3 = 0.4365$ $a\_\sigma 3 := SRSS(alOMr)$ $a\_\sigma 3 = 0.9502$

non-random errors

The calculation shows most non-random uncertainties as zero. It shows the resistor error as non-random, although it would seem reasonable to expect such a term to be random. The alternative calculation does not alter this approach.

$e3SR := 1$	$clOMn_{e3SR} := 0.018$	$alOMn_{e3SR} := 0.02\% \bullet 90$	$alOMn_{e3SR} = 0.0180$
$e3in := 2$	$clOMn_{e3in} := c\Sigma e2$	$alOMn_{e3in} := a\Sigma e2$	

$c\Sigma e3 := \sum clOMn$ $c\Sigma e3 = 0.0180$ $a\Sigma e3 := \sum alOMn$ $a\Sigma e3 = 0.5875$

Total Uncertainty Estimate

Since each module uncertainty estimate incorporates the uncertainty from the preceding module, the uncertainty estimate for the last module in the sequence includes all uncertainties in the channel. The total error is the sum of the random and non-random errors. Thus, for a single channel:

$cTE := c\_\sigma 3 + c\Sigma e3$ $\boxed{cTE = 0.4545}$ $aTE := a\_\sigma 3 + a\Sigma e3$ $\boxed{aTE = 1.5377}$

If two, three, or four readings are averaged, then the uncertainty of the average is reduced:

$c\_\sigma Avg2 := \frac{c\_\sigma 3}{\sqrt{2}} + c\Sigma e3$ $\boxed{c\_\sigma Avg2 = 0.3267}$ $a\_\sigma Avg2 := \frac{a\_\sigma 3}{\sqrt{2}} + a\Sigma e3$ $\boxed{a\_\sigma Avg2 = 1.2594}$

$c\_\sigma Avg3 := \frac{c\_\sigma 3}{\sqrt{3}} + c\Sigma e3$ $c\_\sigma Avg3 = 0.2700$ $a\_\sigma Avg3 := \frac{a\_\sigma 3}{\sqrt{3}} + a\Sigma e3$ $a\_\sigma Avg3 = 1.1361$

$c\_\sigma Avg4 := \frac{c\_\sigma 3}{\sqrt{4}} + c\Sigma e3$ $c\_\sigma Avg4 = 0.2363$ $a\_\sigma Avg4 := \frac{a\_\sigma 3}{\sqrt{4}} + a\Sigma e3$ $a\_\sigma Avg4 = 1.0626$

Appendix: Verification of RTD Temperature/Resistance Factor

$$T := \begin{pmatrix} 100.5 & 114.799 \\ 100.6 & 114.820 \\ 100.7 & 114.842 \\ 100.8 & 114.863 \\ 100.9 & 114.884 \\ 101.0 & 114.906 \\ 101.1 & 114.927 \\ 101.2 & 114.949 \\ 101.3 & 114.970 \\ 101.4 & 114.992 \\ 101.5 & 115.013 \\ 101.6 & 115.034 \\ 101.7 & 115.056 \\ 101.8 & 115.077 \\ 101.9 & 115.099 \\ 102.0 & 115.120 \\ 102.1 & 115.142 \\ 102.2 & 115.163 \\ 102.3 & 115.184 \\ 102.4 & 115.206 \\ 102.5 & 115.227 \end{pmatrix}$$

T =

	0	1
0	100.500	114.799
1	100.600	114.820
2	100.700	114.842
3	100.800	114.863
4	100.900	114.884
5	101.000	114.906
6	101.100	114.927
7	101.200	114.949
8	101.300	114.970
9	101.400	114.992
10	101.500	115.013
11	101.600	115.034
12	101.700	115.056
13	101.800	115.077
14	101.900	115.099
15	102.000	115.120
16	102.100	115.142
17	102.200	115.163
18	102.300	115.184
19	102.400	115.206
20	102.500	115.227

d(T) =

	0
0	0.210000
1	0.220000
2	0.210000
3	0.210000
4	0.220000
5	0.210000
6	0.220000
7	0.210000
8	0.220000
9	0.210000
10	0.210000
11	0.220000
12	0.210000
13	0.220000
14	0.210000
15	0.220000
16	0.210000
17	0.210000
18	0.220000
19	0.210000

$$d(F) := \left| \begin{array}{l} \text{for } n \in 0..\text{rows}(F) - 2 \\ \quad d_n \leftarrow \dfrac{F_{n+1,1} - F_{n,1}}{F_{n+1,0} - F_{n,0}} \\ d \end{array} \right.$$

$\text{rows}(T) = 21.0000$

$T_{10,0} = 101.5000 \quad T_{10,1} = 115.0130$

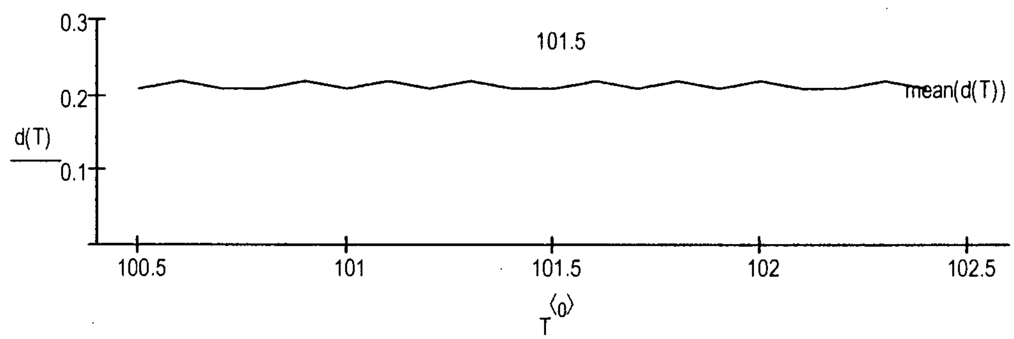
$T_{20,0} = 102.5000 \quad T_{20,1} = 115.2270$

$$\frac{T_{20,1} - T_{10,1}}{T_{20,0} - T_{10,0}} = 0.214000$$

$\text{mean}(d(T)) = 0.214000$

$F := 1$

$C := \dfrac{100}{180}$



Calculation ref 5.4.1 lists the resistance temperature coeficient as 0.00385 Ω/Ω/C: $Rt := 0.00385 \cdot T_{10,1} \cdot C$ $\quad Rt = 0.2460$

The discrepancy between the quoted value and the data is: $\dfrac{Rt - \text{mean}(d(T))}{\text{mean}(d(T))} = 14.9533\%$

Calculation ref 5.4.1 lists repeatability as 0.2F, but ref 5.4.3 lists accuracy as 0.06% at 0C. The ref 5.4.3 specification results in a net accuracy estimate of: $\dfrac{0.06\% \cdot T_{10,1}}{\text{mean}(d(T))} = 0.3225$ using calibration R/T data

This discrepancy may be due to differing definitions of "accuracy" and "repeatability" and so is not addressed in the alternative calculation. $\dfrac{0.06\% \cdot T_{10,1}}{Rt} = 0.2805$ using specification R/T data