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UNSTEADY FLOW SIMULATION IN RIVERS AND RESERVOIRS[a]

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INTRODUCTION

Most mathematical modeling of unsteady flow phenomena has been limited to prismatic channels and somewhat idealized conditions. The Tennessee Valley Authority (TVA) using the numerical methods developed by Stoker (10),[4] has successfully applied them to a number of complex unsteady flow conditions which have occurred or are expected to occur in some of the Authority's reservoirs and natural river channels.

Using the varying geometry of these rivers and reservoirs at finite stations located from 0.3 miles to 2.3 miles apart and iteration periods of from 18 sec to 3 min, accuracies of from 0.1 ft to 0.3 ft in stage for depths ranging from 10 ft to 100 ft have been obtained. Computed velocities have checked measured quantities closely over a velocity range of - 0.5 fps to + 5.0 fps.

The results show the advantages and applicability of the digital computer over quasisteady flow methods of handling unsteady flow problems. Also pointed out are areas in the mathematical model in which difficulties have been encountered along with the methods required to overcome them. These are described to assist others in using the model.

Prior to the advent of the digital computer, the complexities introduced by attempting to determine flow conditions from unsteady flow principles were so severe that steady or quasisteady flow methods had to be used. Although the steady flow approach, in many cases, produced valuable and usable results

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[4] Numerals in parentheses refer to corresponding items in the Appendix I.—References.

when tempered with sound engineering judgment, the steady flow approach has often been inadequate because important transient effects cannot be directly determined. For example, flow reversals in a reservoir created by hydroplant operations at one or both ends of the reservoir cannot be determined from steady flow principles. Also, the steady flow approach has led to a number of popular misconceptions about actual flows in real systems, involving such things as downstream flows under upstream sloping water surfaces, upstream flows under downstream sloping water surfaces, flow reversals, wave travel vs. water travel, and water movements induced by passage of long waves.

Great progress has been made in using unsteady flow principles to determine transient flow conditions in reservoirs and rivers with the digital computer (1,2,6,7,11). The state of the art has now reached the point where mean velocity, discharge and depth variations can be accurately predicted at any point along a stream or reservoir, given appropriate boundary and initial conditions. As a consequence, many conditions of flow previously poorly understood or even thought unrealistic are now understood and can be physically explained.

A digital computer program for solving the basic equations of unsteady flows in reservoirs and natural rivers is now being used by the TVA to solve a variety of open-channel flow problems. The program, which has been supported by field measurements, has recently been used: (1) To investigate flow conditions at the cooling water facilities of a nuclear power plant to be located along the Wheeler Reservoir resulting from hydroplant operations at both ends of the reservoir (3); (2) to determine velocity and stage variations in a narrow winding river below an existing hydroelectric station used for power peaking operations; (3) to determine the time-space variations in discharge, velocity, and water surface elevation in a reservoir subjected to operations of a proposed pumped-storage plant; (4) to determine unsteady flow conditions in a system of two large reservoirs connected by a 1.2-mile long navigable canal; and (5) to investigate the passage of a flood wave through a possible future reservoir. Computer results and field measurements were in remarkable agreement in the three cases where field data were taken, including reverse flows in two reservoirs and in the canal and locking operations in one of the river reaches. Results from these studies are described herein.

SUMMARY OF MATHEMATICAL MODEL

The mathematical model for unsteady flows in open channels is assumed to be one-dimensional in the sense that the flow characteristics such as depth and velocity are considered to vary only in the longitudinal (x) direction and with time. The channel geometry is three-dimensional.

The two equations of unsteady flow, the continuity equation and the equation of motion, (as used in Ref. 9) are

$$\frac{\partial(AV)}{\partial x} + B\frac{\partial h}{\partial t} - q = 0 \qquad (1)$$

$$g\frac{\partial h}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + g\,S_f + \frac{q}{A}\,V = 0 \qquad (2)$$

in which A = flow area; V = mean velocity; x = distance; B = surface width;

h = water surface elevation; t = time; q = lateral local inflow per unit distance and time; g = the gravitational constant; and S_f = the energy gradient given by

$$S_f = \frac{n^2 V|V|}{2.21 R^{4/3}} \qquad (3)$$

in which n = Manning's resistance coefficient and R = the hydraulic radius. The terms $\partial A/\partial x$ in the expanded form of Eq. 1 can be expressed as a function of $\partial h/\partial x$. Therefore, these equations make up a system of two nonlinear, first order, first degree partial differential equations with two independent variables x and t, and with two unknowns h and V. No analytical solutions to this system of equations exist. They may be solved numerically, however, by writing them in finite difference form for digitial computer manipulation.

The boundary conditions may be given as discharge or water surface elevation versus time, or as a stage-discharge relationship. A steady-flow profile, a flat pool-zero flow profile or a transient flow profile from previous computations may be used as the initial conditions.

In addition to boundary and initial conditions, input data on local inflows, channel geometry, and boundary resistance must be prescribed. From these input data the computer determines flows, mean velocities, and water surface elevations at any number of desired locations and times for the channel reach under study.

In finite difference methods the differential equation is replaced by an approximating difference equation, and the continuous region in which the solution is desired is replaced by a set of discrete points called a net. A variety of net schemes for approximating the differential equations of unsteady flow have been studied by various investigators (6, 7, 10). A characteristic computation net has several apparent advantages over other schemes, particularly in the stability and convergence of the solution and in optimization of net size. However, this computation scheme has the disadvantage that the net of points in the x-t plane is determined as the computation proceeds. It is therefore necessary to compute x and t in addition to h and B, and to use an interpolation procedure if it is desired to obtain results at regular or specific distance and time intervals. The main advantage of fixed net schemes is that the net of points in the x-t plane can be selected prior to computation, so that only h and V need to be computed. The major disadvantage of most explicit fixed-net computation schemes is the difficulty in finding a net size that will give a stable and convergent solution. Based on basic studies of many different explicit fixed-net schemes, a centered difference scheme proposed by Stoker (10) was found sufficiently stable and convergent for the unsteady flow computations if the relation

$$\left(V + \sqrt{g\frac{A}{B}}\right)\frac{\Delta t}{\Delta x} \le 1 - \frac{gn^2|V|}{2.21 R^{4/3}}\Delta t \qquad (4)$$

is satisfied, in which Δt = the time interval and Δx = the distance interval (8). This scheme is now (1969) being used to compute transient flows.

Flows, stages or rating curves are prescribed at the upstream or downstream boundaries by means of tables. Rating curves are most successfully applied when a downstream control exists, i.e., a pool, weir, or other device. They may also be used without a downstream control; however, the rating is only an approximation in this case because of the unknown loop nature of the actual curve. Off-channel storage in a reach is accounted for in the mathematical model by adjusting width B in the equation of continuity to give the correct volume in the reach. Storage width is, generally, different from the flow width upon which the hydraulic radius is based. Channel geometry data

TABLE 1.—TYPICAL COMPUTING TIME REQUIREMENTS FOR THE TVA PROGRAM

Project or study (1)	Time interval Δt, in seconds (2)	Distance interval, Δx, in miles (3)	Number of reaches (4)	Number of cross sections (5)	Total length, in miles (6)	Depth range, in feet (7)	Width range, in feet (8)	Computer time per day of routing, in minutes (9)
Browns Ferry	90	2.0	36	37	72.0	20-60	1,500-20,000	2.0
Raccoon Mountain	90	2.1	22	23	46.0	10-50	700-7,000	3.0
Cumberland River	60	0.5	56	57	28.0	20-50	500	7.0
Barkley Reservoir	180	2.3	51	52	118.0	20-80	500-10,000	2.0
Barkley Canal	15	0.3	4	5	1.2	20	500	2.5
Columbia	90	2.0	28	29	56.0	5-100	50-3,000	2.0

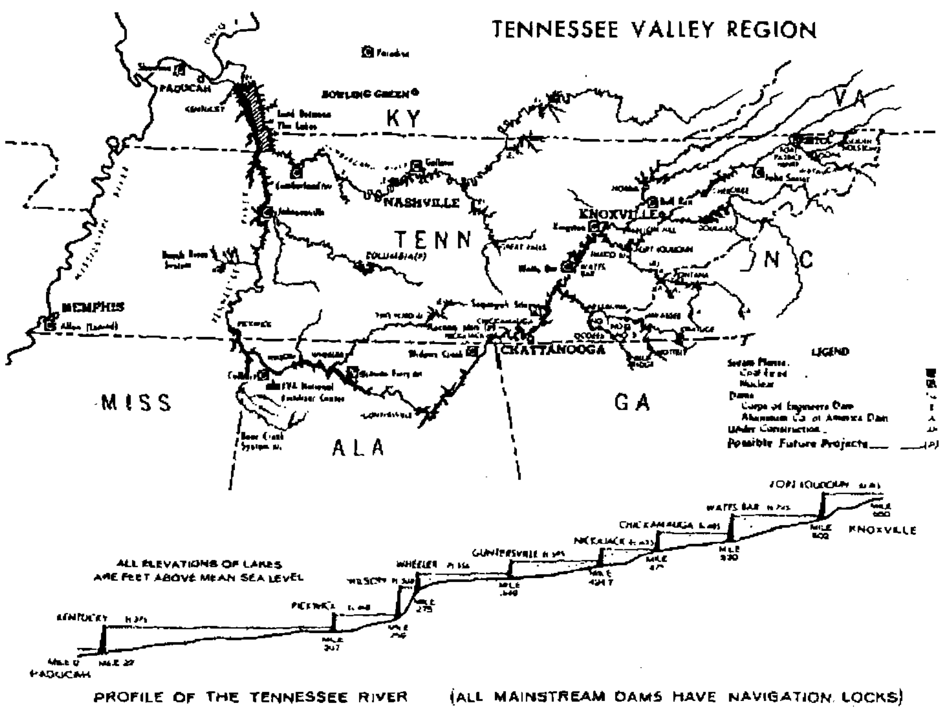

FIG. 1.—LOCATION MAP OF TVA PROJECTS

are determined from interpolations from tabular values of area, (hydraulic radius)$^{2/3}$, and storage width vs. water surface elevation for each station. Tributary inflows are distributed over a reach of length Δx. This causes no problems because flow conditions near a junction can be studied in sufficient detail by reducing Δx in successive runs. An IBM System/360, model 50, or an equivalent system is required to accommodate the program. Typical computing time requirements for the program are given in Table 1. The location of the projects in Table 1 are shown in Fig. 1.

APPLICATIONS

Browns Ferry Study.—The transient flow conditions in the Wheeler Reservoir are caused primarily by the intermittent hydropower operations of the turbines located at the upstream and downstream boundaries of the reservoir. Water required for condenser cooling purposes at the Browns Ferry Nuclear

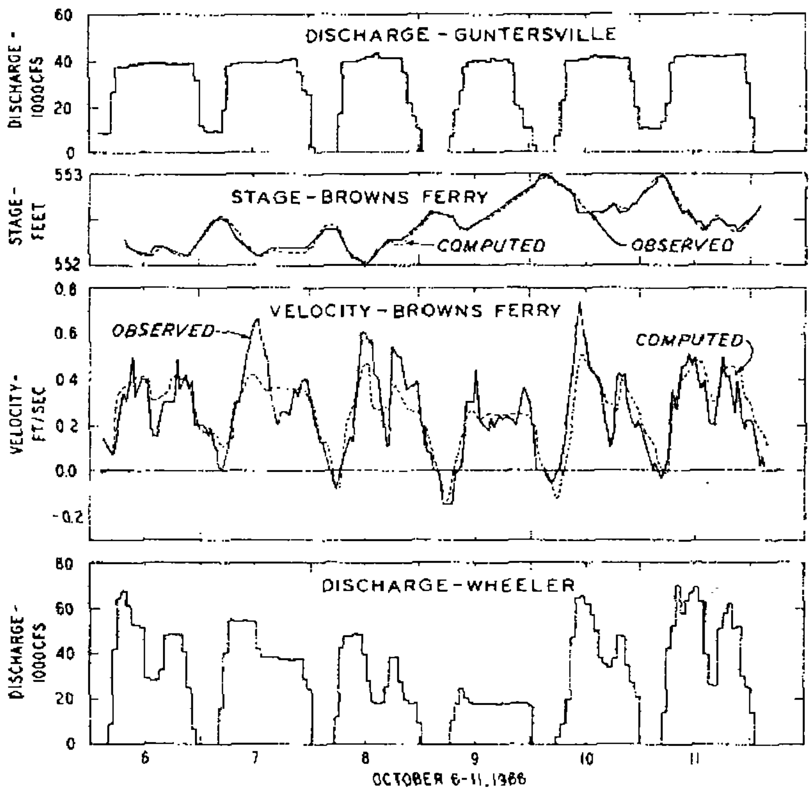

FIG. 2.—TYPICAL RESULTS FOR BROWNS FERRY STUDY

generating plant now under construction along the Wheeler reservoir will be withdrawn from and returned with an elevated temperature to this reservoir in which flow conditions are continuously changing. These cooling water facilities must therefore be so designed as to meet the established water quality standards for the area and to prevent recirculation of these heated waters back through the cooling system. Consequently, a thorough knowledge of the Wheeler Reservoir transient flow conditions is one basic need for effective design of these cooling water facilities.

To achieve this basic understanding of the Wheeler Reservoir transients, two paths of endeavor were undertaken almost simultaneously: (1) A mathematical model of the Wheeler Reservoir was constructed; and (2) field mea-

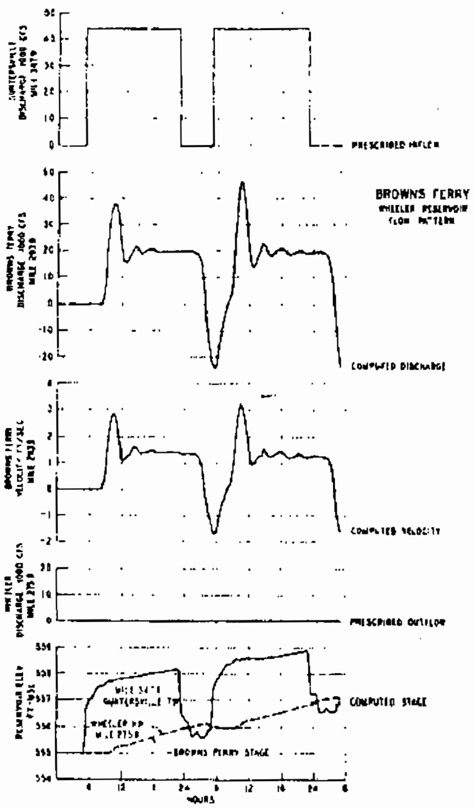

FIG. 3.—FLOW CONDITIONS AT BROWNS FERRY DUE TO PULSE FLOW AT UPSTREAM END OF WHEELER RESERVOIR

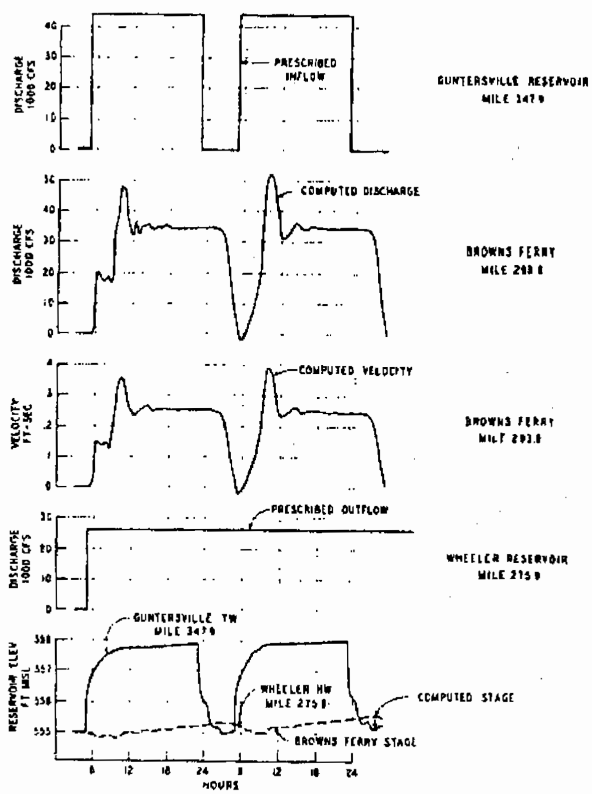

FIG. 4.—FLOW CONDITIONS AT BROWNS FERRY DUE TO PULSE FLOW UPSTREAM AND STEADY FLOW DOWNSTREAM

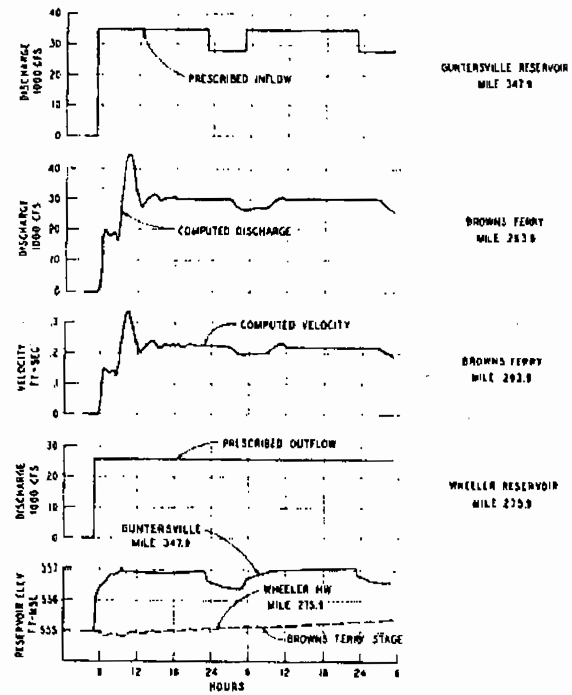

FIG. 5.—FLOW CONDITIONS AT BROWNS FERRY DUE TO MODIFIED PULSE FLOW UPSTREAM AND STEADY FLOW DOWNSTREAM

surements of velocity and stage were undertaken at the plant site beginning September 20, 1966.

Using the actual turbine discharges of the Guntersville and Wheeler turbines as boundary conditions along with the known reservoir profile as the initial conditions, the equations of the mathematical model were solved on the computer. The calculated velocities and elevations in the reservoir at the plant site, as well as the upstream and downstream turbine releases producing these changes for a 6-day portion of the field measurement period, are shown in Fig. 2. These are compared with the actual velocities and elevations measured at the site during the field tests. The elevations match perfectly. The calculated average velocities for the entire cross section are reasonably close to the measured average velocity taken along a single vertical location within this section. By successfully reproducing known reservoir transient conditions, the mathematical model was verified and could then be used with confidence to obtain the transient flow conditions accompanying any set of boundary conditions.

Applications of the calibrated mathematical model used in this manner are demonstrated in Figs. 3 through 5.

Initial conditions used to obtain the computed results in these figures were a flat reservoir pool at El. 555 and zero flow everywhere. The figures show computed flows and velocities at Browns Ferry site for three different sets of boundary flow conditions at Guntersville and Wheeler. Computed variations in water surface elevations at Guntersville, Browns Ferry, and Wheeler are also shown in the figures. Elevations at Browns Ferry and Wheeler are essentially the same in the flat section of the reservoir. In Fig. 3 the boundary conditions consist of a periodic pulse flow at Guntersville and zero flow at Wheeler. The influence of the shutoff at Guntersville on flow conditions at Browns Ferry is shown. The Guntersville influence has been established as the primary cause of flow reversals at the Browns Ferry site such as the one shown in the diagram. In Fig. 4 the boundary conditions consist of the same periodic pulse flow at Guntersville but with a steady flow at Wheeler. It can be seen that this steady outflow at Wheeler is not sufficient to fully counteract the flow reversal resulting from the Guntersville shutdown. In Fig. 5 the boundary conditions consist of a modified Guntersville periodic pulse flow and steady flow at Wheeler. In this case, the Wheeler steady flow is sufficient to overcome the reduced Guntersville shutoff effect. Using the model in this fashion, sets of operating boundary conditions can be found which will produce satisfactory flow conditions at the plant site.

Cumberland River Below Barkley Dam.—Stage measurements in reservoirs and rivers can be made with sufficient accuracy by means of mechanical recording devices. However, accurate measurements of velocity and discharge are difficult and expensive to make. To date no economical or practical system exists for making simultaneous and continuous measurements of velocity at a number of points. This is where a mathematical model capable of computing stage, velocity, and discharge at any time and location can be used to great advantage to complement and extend field measurements. Such a use is demonstrated by the following example. Navigation on the Cumberland River from Barkley Dam to the Ohio River sometimes experiences difficulty in negotiating this narrow, winding channel. This is particularly true when the Ohio River is at low stage and power peaking operations are being conducted at the Barkley project. The river in this reach is about 500 ft wide, 10 ft to 40 ft deep, and follows an irregular and winding path (see Fig. 6). The Corps of Engineers, as a first step in finding a solution to this problem, decided to conduct a series of field test at specific locations in a 28-mile reach below Barkley. The tests, made in August, 1967, were a cooperative effort among the Corps, USGS, and TVA. In these tests, a number of different turbine operating patterns were scheduled at Barkley, and during these scheduled releases measurements were made in the study reach to ascertain the flow conditions. Measurements of velocity and stage were taken at selected stations in the reach. Velocity mea-

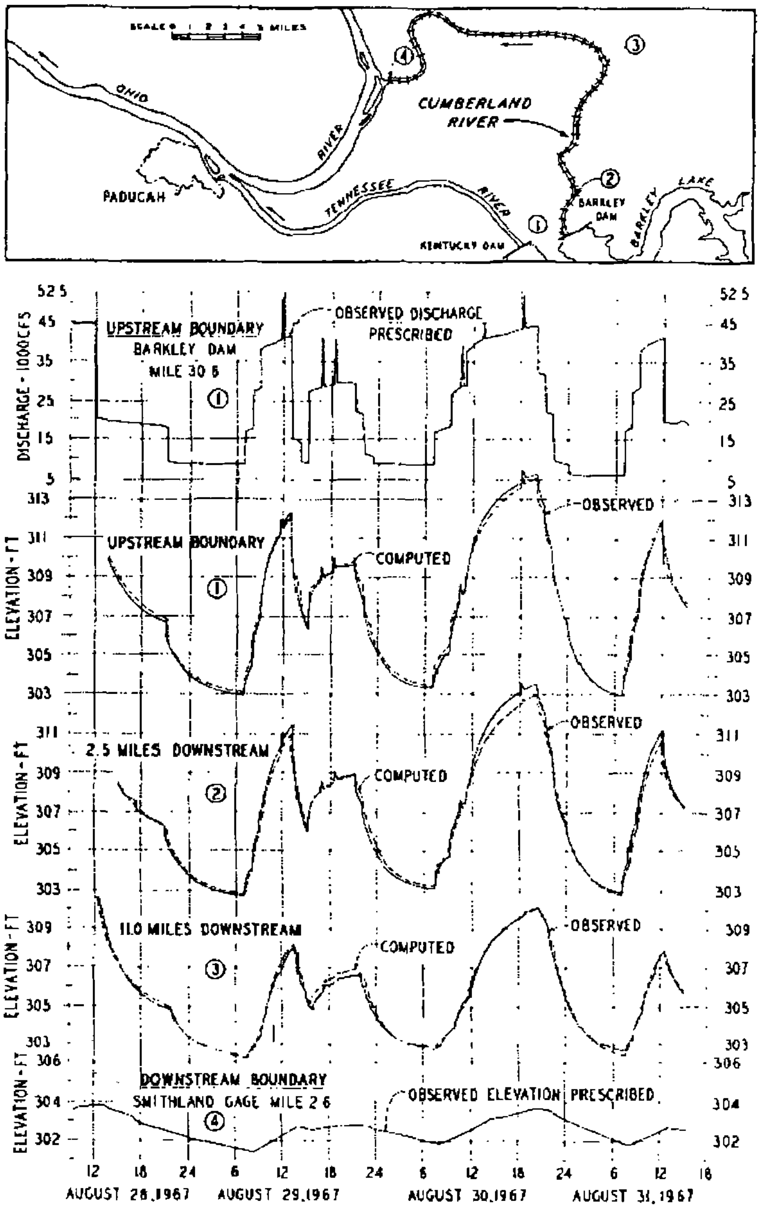

FIG. 6.—TYPICAL STAGE RESULTS FOR CUMBERLAND RIVER BELOW BARKLEY DAM STUDY

surements were taken at depths of 0.5 ft, 4.0 ft, 7.5 ft, and 11.0 ft at midstream to define the velocity in the navigation channel. Stage was automatically recorded at each selected station.

Prior to the field tests, a mathematical model of the river reach using known stages at permanent gage locations and the corresponding discharges from Barkley Dam and stage at gage 28 miles down the rivers, had been established. Then, by prescribing the turbine operations scheduled for the tests and the estimated stage at the downstream end of the reach, the variations in stage, velocity and discharge at the selected stations were predicted using the computer model. The results permitted the field crews to know in advance when velocity and stage changes would occur at measuring stations and, therefore, when to make their measurements. After the tests were completed, comparisons were made between the measured and predicted values of stage

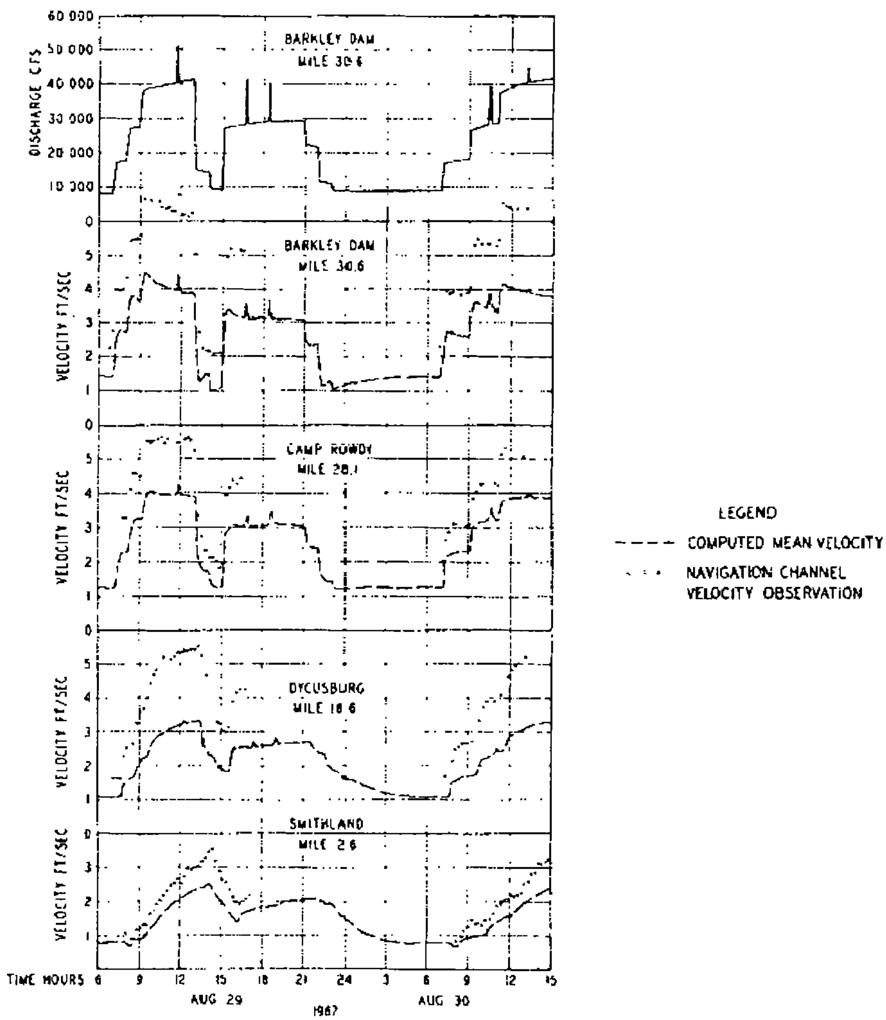

FIG. 7.—TYPICAL VELOCITY RESULTS FOR CUMBERLAND RIVER BELOW BARKLEY DAM STUDY

and velocity. Although the actual turbine discharge and estimated downstream stage were slightly different from the values used in the prediction, there was still good agreement between observed and predicted values. Timing of stage and velocity agreed perfectly with the predictions. The magnitude of the measured velocities was slightly higher than the predicted values, because the measured quantities were discrete values measured in the upper central region of the navigation channel, while the predicted values were mean values for the cross section.

After completion of the tests the actual conditions that occurred during the tests were programmed to check the calibration of the model. Even the discharges due to lock operations at Barkley Dam were included. The comparison between the computations and measurements was excellent, as shown in Fig. 6. The computed stage is generally within 0.1 ft of the observed one; and timing of all events, including lockages, agrees perfectly. The velocity comparison is shown in Fig. 7.

Raccoon Mountain Pumped Storage Study.—The TVA is studying a pumped storage project, the intake-outlet structure of which would be located on the navigable Nickajack Reservoir at Tennessee River mile 444.6. At this location the reservoir is narrow and sinuous. The purpose of this study is to determine the effect of pumped withdrawals and turbine releases on stages and velocities in the vicinity of the intake-outlet structure.

The use of the mathematical model for this study is straightforward and,

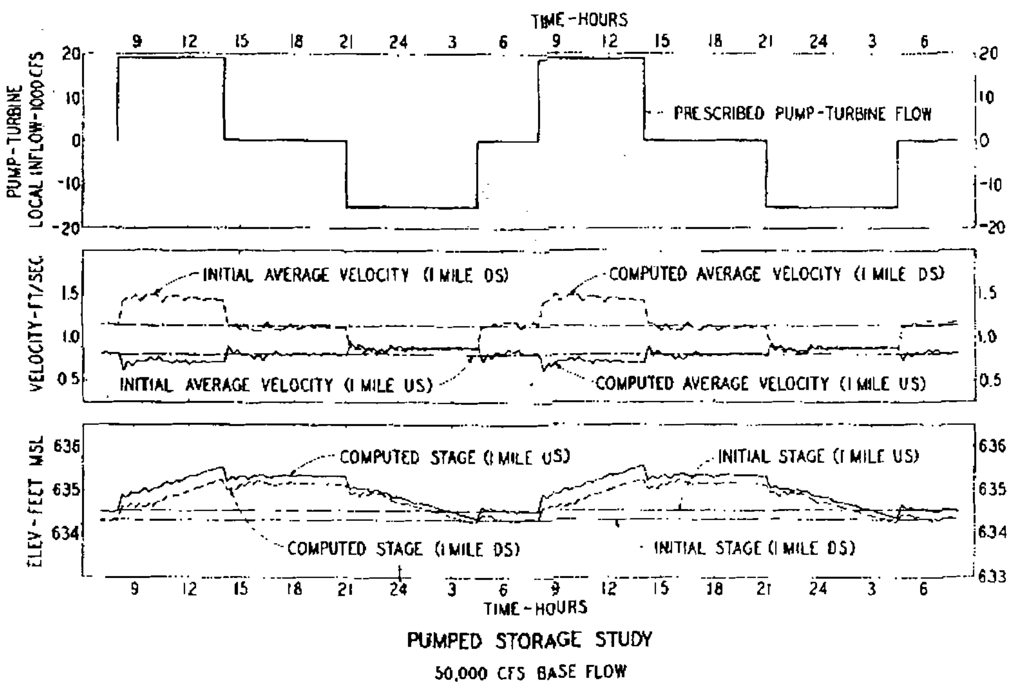

FIG. 8.—TYPICAL RESULTS FOR RACCOON MOUNTAIN STUDY

although no direct verification of the results can be made because the project is not yet built, confidence can be placed in the results because of past experience gained from the Browns Ferry and Cumberland River studies in which verification was achieved.

The Nickajack Reservoir is mathematically described using a Δx of 2.1 miles. The upstream boundary is Chickamauga Dam and the downstream boundary is Nickajack Dam. The Raccoon Mountain plant inflow or outflow is introduced uniformly over the 2.1-mile computation reach as either a positive or negative local inflow, depending upon the mode of plant operation.

Base reservoir flows of 0 cfs, 50,000 cfs, and 100,000 cfs with the pump-turbine discharge of 20,000 cfs introduced as a local inflow are prescribed at the upstream and downstream boundaries, corresponding to the practical range of flows under which navigation occurs.

Fig. 8 shows the effect of the project on water surface elevations and velocities for the 50,000 cfs base flow case. The computed quantities in this figure are those at the ends of a computation reach—about 1 mile upstream and downstream from the plant. The initial upstream and downstream stages and velocities are slightly different because of different cross-sectional areas and their 2-mile separation.

When the Raccoon Mountain turbine operation starts, the stage rises while the velocity increases downstream and decreases upstream, due to the vector addition of velocity components, as shown in Fig. 9. After the first 20 min the stage continues to rise, but at a slower rate, and the velocity oscillates around a new steady-state value. When the turbine operation stops the velocity returns to the original steady flow value in about 20 min and oscillates around this value before steadying out. During the pumping cycle, the conditions described previously are more or less reversed. The results of the study show that the proposed normal operation of the pumped-storage plant will not cause severe changes in stage and velocity and, therefore, will not adversely affect navigation conditions in Nickajack Reservoir.

Cumberland Steam Plant Studies.—The TVA is constructing, on the shores of the Corps of Engineers Barkley lake, the world's largest conventional thermal generating plant. Transient flow studies similar to those performed for the Browns Ferry plant were again necessary to provide data for design of the plants cooling water facilities. In this reservoir the studies were complicated by the fact that Barkley Lake is directly coupled to The TVA's Kentucky Lake by a large navigation canal.

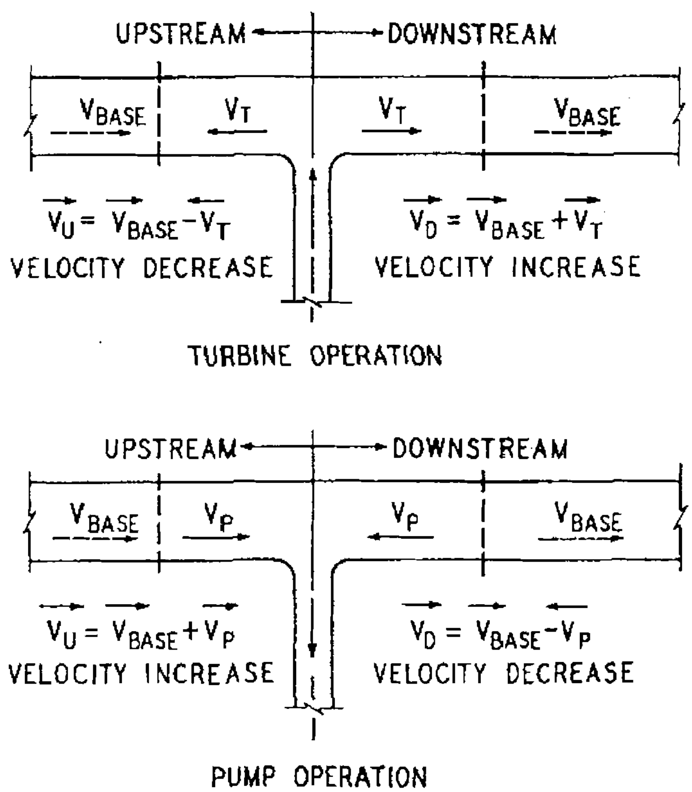

FIG. 9.—SCHEMATIC REPRESENTATION OF PUMP-TURBINE OPERATIONS, RACCOON MOUNTAIN STUDY

Flow can be either into or from Barkley Lake depending upon the operating levels of these two reservoirs. In a 400-ft wide, 20-ft deep canal these flows are large even for small head differences between the canal ends. Consequently, the transient canal behavior had to be determined before the transient conditions in the reservoir proper could be adequately assessed.

Stages at the ends of the canal have been recorded since the canal was first opened in 1966. In addition to these continuous stage records periodic canal discharge measurements were also available. Using the best discharge measurements, i.e., those taken when nearly a constant head differential existed between the canal ends for a period sufficiently long to obtain a good discharge measurement, the mathematical model was calibrated. Once calibrated, the transient canal behavior for any period of time could be determined

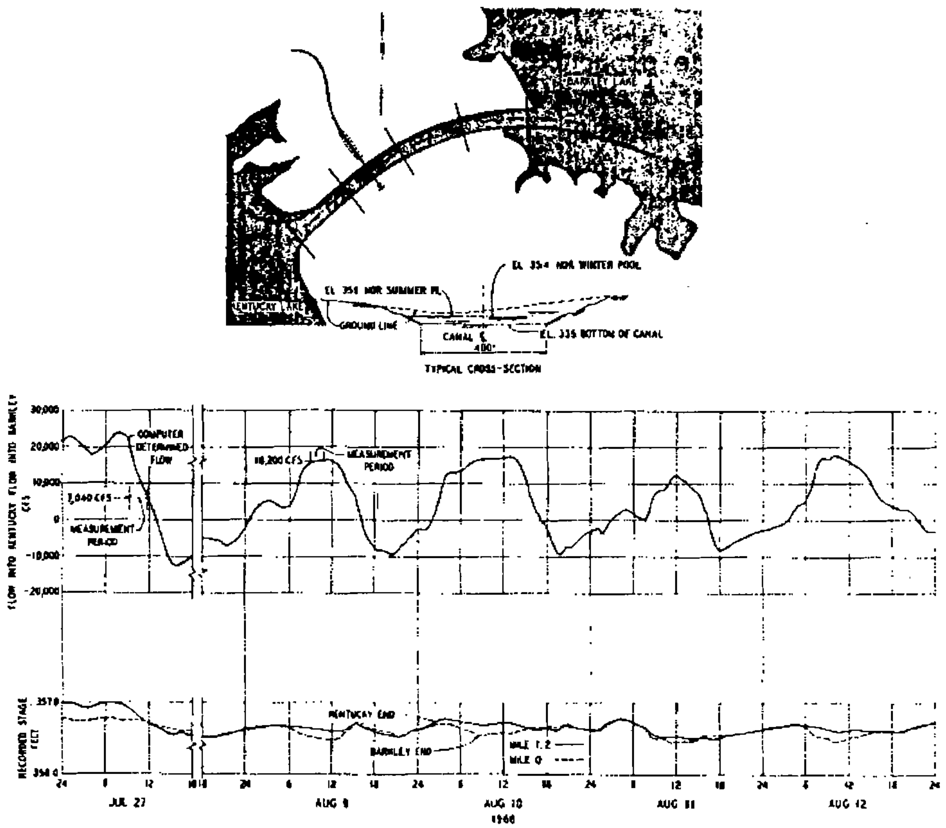

FIG. 10.—TYPICAL RESULTS FOR KENTUCKY-BARKLEY STUDY

by prescribing the water surface elevations which had prevailed at the canal ends. Typical calculated results are shown in Fig. 10. This figure also contrasts two field discharge measurements with the calculated results. One taken during a period of nearly constant head difference is good. The other obtained during a period of stage reversal between the canal ends is poor. This figure also shows that up to 2 hr is required for the flow in the canal to reverse after a reversal in stage difference, depending upon the initial rate of flow. This performance could not be represented by steady flow functions. It was also found that there is little variation in flow along the length of the canal, because of its relatively short length. The model could be easily adapted to a routine operation to provide daily or other periodic values of canal flow at minimal cost. The results in Fig. 10 show that flow measurements should be made only during carefully determined time intervals.

Using the computer-determined canal flows, known local inflows, and turbine releases at Barkley and Cheatham Dams, the model was calibrated against known stages at gage locations in the 118-mile long Barkley Reservoir. The calibrated model was then used to compute stages, velocities and flows throughout the reservoir for a selected month-long period of widely varying actual discharges at the two dams. A portion of the results is shown in Fig. 11.

Generally, agreement exists between computed and measured stages. The accuracy of the stage records themselves, as well as the turbine and local

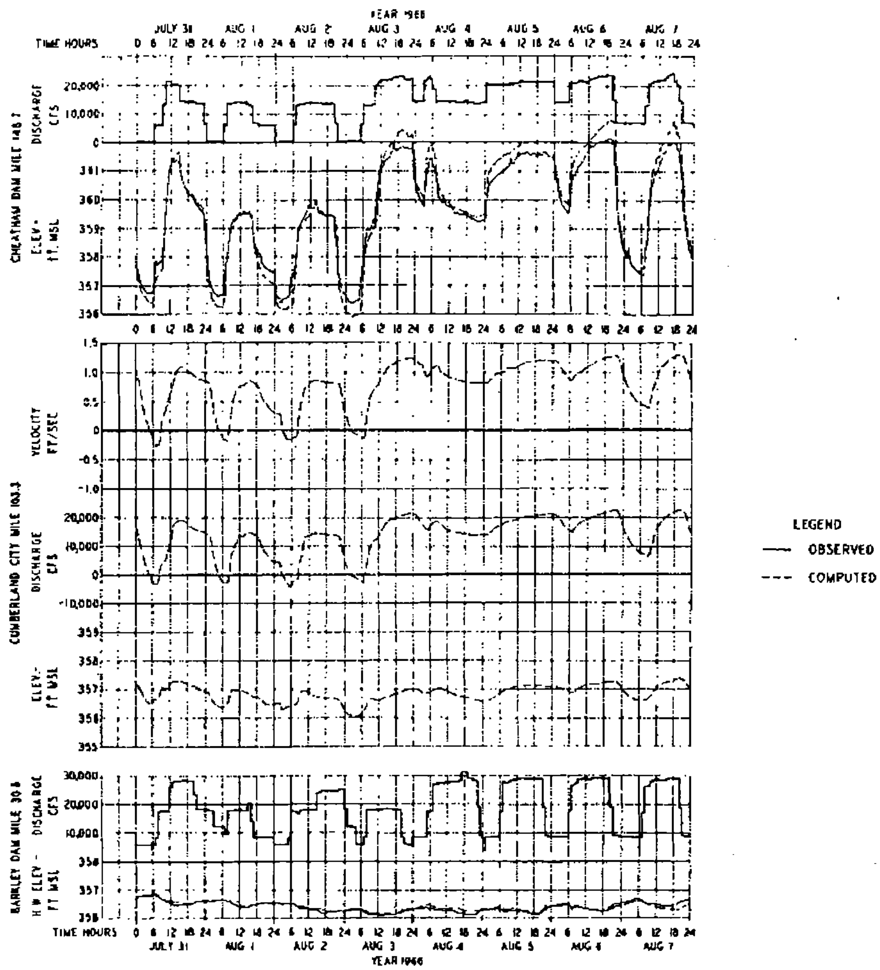

FIG. 11.—TYPICAL RESULTS FOR CUMBERLAND STEAM PLANT STUDY, BARKLEY RESERVOIR

flows for this particular reservoir, is not as well defined as it is for some of the other studies cited. Therefore, in view of past experience, it is likely that the computer results are as precise as the measured stages. From Fig. 11 it can be seen that shutoffs at the upstream Cheatham plant create low and even negative flows at the plant site.

Columbia Project Study.—The Duck River, a major tributary of the lower Tennessee, is subject to frequent and severe floods. The TVA is studying the feasibility of constructing a multipurpose dam and reservoir on this river a few miles upstream from the city of Columbia, Tennessee (see Fig. 1). This dam, the Columbia project, would create a reservoir 54 miles long and raise the water level about 80 ft at the dam site. Quoting directly from Ref. 4:

> "The formation of a long reservoir in a natural drainage basin may materially alter the regimen of flood runoff by synchronizing high rates of runoff originating above the head of the reservoir with maximum rates from areas contributing laterally to the reservoir. Under natural river conditions, runoff from the upper portion of a basin is retarded by valley storage and normal frictional resistance as it passes through the reservoir reach, the resultant velocity corresponding to that indicated by Manning's formula for flow in open channels. However after a deep reservoir has been formed by construction of a dam inflow near the upper end of the reservoir moves through the pool largely by a process of translation, with long-wave velocities subject to momentum control equal approximately to $\sqrt{gd}$ in which d is the depth of flow in feet and g is the acceleration due to gravity (32.2). Estimates of the time required for flood waves to traverse natural

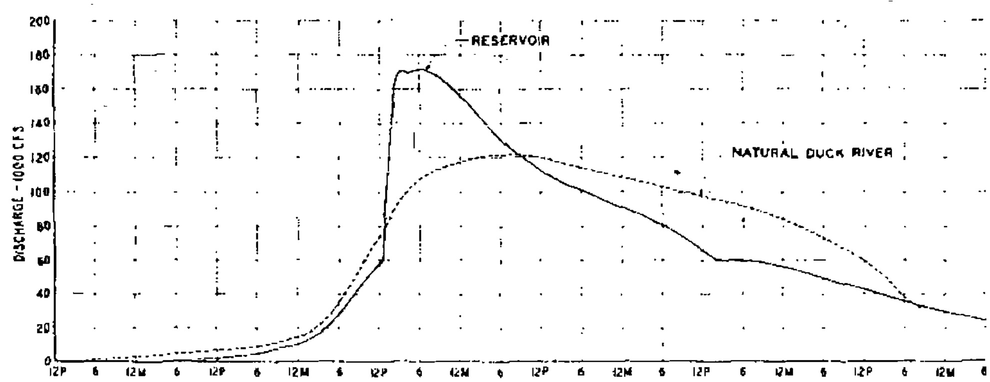

FIG. 12.—TYPICAL RESULTS FOR COLUMBIA RESERVOIR-DUCK RIVER STUDY, MAXIMUM PROBABLE FLOOD

> river channels within the limits of several proposed reservoirs have ranged from a few hours to approximately 1-1/2 days, whereas the time required for inflow into the upper end of the full reservoir to become effective at the point of reservoir outflow would range from practically zero to a few hours for comparable storms. Changes in the synchronization of runoff from various portions of a drainage basin may be such as to produce rates of inflow into a full reservoir that are substantially higher than would occur at the damsite under natural river conditions, although in some cases the differences may be negligible."

Quasisteady flow routings of the maximum probable flood through both the natural river and the proposed reservoir showed that "reservoir acceleration" could be a factor; therefore, studies were made using the mathematical model. The model was calibrated using the maximum known flood (1948) in the natural river. It was then used to route the same flood in the proposed reservoir. The maximum probable flood for the river and reservoir were also studied.

The hydraulic roughness calibration of the model for the natural river was one of the major problems in this study. Determination of the rating curve for the downstream boundary was also a major problem. The routing of the 1948 flood through the proposed reservoir showed an acceleration of the arrival of the peak as expected. The acceleration effect was much more evident in the case of the maximum probable flood. The spillway operation used with these routings called for storing the initial inflow until top of gates was reached, holding this elevation by means of gated operation until free discharge occurred, and then following the spillway rating. Results for the maximum probable case are shown in Fig. 12.

Maximum inflow hydrographs for the upstream end of the Duck River and for a number of tributaries and runoff reaches were prescribed. Tables of water surface elevation vs. discharge were prescribed at the downstream boundary (Columbia Dam site).

Although downstream flooding, in the case of extremely large floods of the magnitude of the maximum probable flood, would be worsened by the existence of the dam, damage for smaller floods, including the largest flood of record, would be eliminated or greatly reduced by the project. This study affords an excellent example of the use of the mathematical model as a planning tool, in which adequate spillway capacity is an important factor.

LIMITATIONS

From the studies described in this paper, from other studies of similar and specialized natures, and from the basic computation scheme employed, several limitations of the mathematical model, none of them too serious, have been found. These are:

1. Stability and convergence of the computation scheme. This has created some problems in the past, but these are now resolved (8).
2. The explicit scheme requires short time and distance steps when longer intervals are often desirable.
3. Tributary flows must be distributed uniformly over a minimum distance of one computation reach. This is no real problem, because Δx may be made smaller, or a short reach containing the tributary can be rerun with internal boundary conditions computed from a previous run.
4. Flows in dry or nearly dry channels cannot be computed.
5. Supercritical flows cannot be calculated.
6. Bores cannot be computed.
7. Branching flows or complex system flows can only be computed by successive applications of the model. No situation has developed to date, however, where this approach has not worked.
8. Stratified flows cannot be computed with the model described herein, but the model can be extended to cover these flows.
9. Some distortion of wave speed occurs in regions where rapid changes occur due to the nature of the computation scheme used. This has not been a problem of any consequence to date.
10. Slight differences in computed results occur between timelines with boundary points and those without boundary points, again due to the computation scheme employed. This has been no real problem. Work is underway

to overcome these limitations through different computation schemes, special subroutines, and generalized programs.

SUMMARY AND CONCLUSIONS

Mathematical models for solving the basic differential equations of unsteady flows in open channels are developed and used with the digital computer to solve a variety of flow problems in rivers and reservoirs.

The basic model has been verified against actual field measurements of stage and water velocity in both rivers and reservoirs. Other models are being developed to complement and extend the basic model and to handle special complex flow situations. By properly calibrating an appropriate model for a particular river or reservoir channel, time variations in stage, velocity, and water flow can be accurately predicted at any desired location along the channel from prescribed flow conditions at each end of the channel and through tributaries.

The examples described in this paper show the potential of the unsteady flow mathematical model in a wide variety of engineering problems. A few of its uses in other areas would be:

1. To complement present methods of measuring discharges in the field; the model would show when suitable measurement periods occur.
2. To aid in analysis and monitoring of water quality parameters in rivers and reservoirs. The model would show the actual movement of a mass of water in a flow system, giving accurate definitions of decay times, uptake times, exposure times, etc., regardless of the system unsteadiness. It could also be used as the basis for regulating industrial pollution loads on the basis of flow (dilution) and to maintain desired flows.
3. To select the best locations for future plant sites on the basis of the most desirable flow conditions.
4. To create any desired flow condition along regulated stream by programing flows at the upstream or downstream ends, or both, of the channel.
5. To aid in system operation of a network of reservoirs where certain portions of the flow are unregulated, such as in canals connecting reservoirs, large tributaries, etc.
6. To route large floods through a reservoir or river reach, thereby predicting stages and flows all along the reach.
7. To study dam failure problems.
8. To establish accurate resistance coefficients in open channels.
9. To obtain steady-flow backwater curves precisely.
10. To separate and analyze storm hydrographs.

It is believed that this type of approach to the more complex flow problems is one that holds promise. With this tool, if the flow situation can be described with a differential equation, then the problem can be solved by digital computer. This new approach yields accurate and flexible results for a minimal cost once the model is developed. It also promotes a thorough understanding of the dynamics of real flows and should bring about new trends in field data collection for direct input to the computer model. Once experience with this approach has been obtained, return to the older, conventional methods is not expected.

APPENDIX I.—REFERENCES

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

A = cross-sectional flow area, or region of x-t plane;
B = weighted width of channel cross section, or region of x-t plane;
g = acceleration of gravity;
h = water surface elevation above mean sea level;
n = Manning's resistance coefficient;
Q = discharge given by mean velocity times cross-sectional flow area;
q = lateral inflow per unit distance along channel and per unit time;

R = hydraulic radius given by ratio of cross-sectional flow area to wetted perimeter;
S_f = energy gradient;
S_0 = channel bottom slope;
t = time;
V = mean flow velocity;
x = distance along channel;
Δt = time increment;
Δx = distance increment;
$\partial/\partial t$ = rate of change with respect to time; and
$\partial/\partial x$ = rate of change with respect to distance.