Statistical Comparison of CROSSFLOW Test Data to Other High Precision Flow Instrumentation

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Westinghouse Electric Company LLC
P.O. Box 355
Pittsburgh, PA 15230-0355

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1.0 OBJECTIVE AND SUMMARY

The objective of this report is to document a statistical comparison of CROSSFLOW meter flow measurements versus other high precision flow instrument measurements that shows there is reasonable assurance that the CROSSFLOW meter is capable of delivering the nominal flow measurement accuracy stated in the Topical Report, Reference 4.3 (i.e., ≤0.50%).

The report concludes that this objective is achieved. [

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2.0 ANALYSIS

2.1 Selection of Data

The selection of data involved gathering high precision flow measurement comparison data for CROSSFLOW installations that had independent measurements with a level of precision that was similar to, or better than, the accuracy of the CROSSFLOW meter. The independent measuring devices included ASME flow sections, recently calibrated venturis and chemical tracer tests. [

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2.2 Normalization of Data Points

In order to allow the comparison of differences of widely varying nominal flows, each data point was normalized to a percentage difference between the CROSSFLOW and the independent measurement. That is, the difference (CROSSFLOW minus independent measurement) was divided by the independent measurement and this value was converted to a percentage.

2.3 Removal of Outlier Data Points

Once the data had been normalized, a statistical test was applied to determine if any of the data points were outliers. [ ]a, c

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]a, c

The test analysis indicated that three (3) data points were potential outliers. [

]a, c Normally, once a data point is identified as a potential outlier, it is considered good practice to also determine if there is a physical reason why the point should be removed. [

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Figure 1 provides bar graph and histogram plots of the complete data set and the data set without the outliers. The impact of removing the outliers can be seen by examining the two histograms.

The data is skewed to the right for both data sets, indicating that the CROSSFLOW meters tended to read higher than the independent measurement of the same flow.

2.4 Check for Data Set Normality

Both data sets, the complete set and the set with the outliers removed, were checked for normality using the American National Standard Institute (ANSI) recommended method, Reference 4.1. This standard specifies that the Wilk's method be used for data sets containing 50 or less data points. If it cannot be shown that the data points are normally distributed, the estimation of the population statistics needs to be performed using methods defined for the particular distributions.

The Wilk's test for normality confirmed that both sets of data passed the ANSI criteria for normality at the confidence level of 95%. Hence, both sets of data should provide a reliable estimate of differences between CROSSFLOW and high precision independent measurements when analyzed with methods that rely on the normality of sample distribution.

2.5 Statistical Comparison

Once it is determined that a data set is normally distributed, it is possible to determine whether the estimated uncertainty of the CROSSFLOW meter envelops the difference between the meter and the independent measurement using the methods described below.

2.5.1 Calculation of the Standard Deviation

The first step in this process is to calculate the distribution parameters of the data set, the mean and the standard deviation. The sample mean, $\overline{X}$, is the algebraic average of the differences. The standard deviation of the sample is calculated using Equation 1.

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} \qquad \text{Equation 1}$$

where:

n = the number of data points.

X_i = the i[th] data point defined as the CROSSFLOW reading minus the independent reading, all divided by the independent reading and then converted to a percentage.

$\overline{X}$ = the average of the n, X_i data points.

[$\overline{X}$

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2.5.2 Calculation of the Standard Error of the Mean

The standard error of the mean is calculated using Equation 2, taken from Reference 4.2, Section 9.2. It is defined as the ratio of the standard deviation from Equation 1 divided by the square root of the number of data points.

$$Standard\ Error = \frac{s}{\sqrt{n}} \qquad \text{Equation 2}$$

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2.5.3 Test for High or Low Data Set Bias

Using the standard error of the mean, a test can then be performed to determine if there is reasonable assurance that the data is biased or not. This can be done using Equation 3, which includes the sample mean, $\overline{X}$, and the uncertainty of the differences. This equation is taken from Reference 4.2, Section 15.3, equation 15.3.1:

$$\mu = \overline{X} \pm t_{(n-1,0.975)} \frac{s}{\sqrt{n}} \qquad \text{Equation 3}$$

where: [μ] = the 95% confidence interval for the mean of the population

$t_{(n-1,0.975)}$ = the Student-t coefficient for n-1 degrees of freedom with a two sided significance of 5%.

If the interval defined by Equation 3 includes zero, then one can conclude that there is a 95% probability that the data is not biased either high or low.

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2.5.4 Limiting Uncertainty for the Data Set Population

The bounding uncertainty for the population can be determined by calculating the standard deviation of the population and then multiplying it by 1.96 to obtain the two-sided tolerance band. The two-sided uncertainty tolerance band will envelope 95% of the difference, with a 95% confidence, between the CROSSFLOW measurement and the high precision independent measurement of the same flow.

The upper statistical limit for the population standard deviation is calculated using Equation 4, taken from Reference 4.2, Section 15.1, equation 15.1.4. This is a conservative estimate of what the standard deviation, σ, would be for all future comparisons of the CROSSFLOW meter with high precision independent measurements.

$$\sigma = S\sqrt{\frac{f}{\chi^2}} \qquad \text{Equation 4}$$

where:

f = the number of degrees of freedom, n-1 [

]a, c

χ^2 = the chi square value for the lower-tail probability and the number of degrees of freedom for a one-sided 95% probability. [

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The two-sided uncertainties represent conservative estimates of the maximum difference between the CROSSFLOW meter and an independent measurement of the same flow. Specifically, this statistic states that the difference between the two measurements would be less than or equal to these uncertainty bands 95% of the time for similar comparisons.

2.5.5 Adjustment of the Limiting Uncertainty to Compensate for Bias

The uncertainty band for the differences between the CROSSFLOW meter and independent flow measurements can be further adjusted to reflect the inherent biases. [

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2.5.6 Removal of the Temperature Uncertainty

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* It is important to note that the actual uncertainty of each meter is determined based on the accuracy of the measurements taken during the installation and the uncertainty of the inputs that will be used, such as the feedwater temperature. It is never assumed that the meter has an accuracy of 0.5% without regard to the circumstances of the installation.

Table 1

[ ]a, b, c

Figure 1

[ ] a, c

3.0 RESULTS AND CONCLUSIONS

Table 2 summarizes the results of the analysis. From Table 2, the Margin calculation shows that, for the expected conditions, [

]a, c the CROSSFLOW meter will support the nominal accuracy.

[

]a, c

Hence, one can conclude that there is a 95% probability that the CROSSFLOW measurement of the flow will be conservative regardless of the treatment of the outlier data points.

Table 2

Summary of Results

a, c

4.0 REFERENCES

4.1 "American National Standard Assessment of the Assumption of Normality (Employing Individual Observed Values)," ANSI N15-1974

4.2 "Statistical Theory with Engineering Applications," A. Hald, John Wiley & Sons, Inc., 1952

4.3 "Improved Flow Measurement Accuracy Using CROSSFLOW Ultrasonic Flow Measurement Technology," CENPD-397-P-A, May 2000

4.4 "Handbook of Statistical Methods for Engineers and Scientists," H.M. Wadsworth, McGraw-Hill, Inc. 1990

5.0 APPENDIX A - SELECTED HIGH PRECISION DATA SETS

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]a, c

Each of the data sets that met these criteria is presented below with a brief discussion concerning the qualifications of the specific data set:

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]a, c

a, b

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]a, c

[ ]a, b

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]a, c

[ ] a, b

[

]a, c

[ ] a, b

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]a, c

The CROSSFLOW data versus the sum of the two venturi readings is shown below:

[ ]a, b

[

]a, c

[

]a, c

a, b

[

]a, c

[

]a, c

[

]a, b

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]a, c