ANALYTICAL AND NUMERICAL SOLUTIONS OF THE EXPECTED NUMBER OF OCCURRENCES FOR COMBINATIONS OF EVENT SEQUENCES DUE TO VARIABILITY

R. R. Benke

Center for Nuclear Waste Regulatory Analyses
Southwest Research Institute®
6220 Culebra Road, San Antonio, TX, 78238, USA

ABSTRACT

This paper investigates variability in the occurrence of different event sequences on an annual basis during the operation of a proposed nuclear facility. During the operational period of a nuclear facility, the annual radiological dose received by workers or members of the public depends on the number of event sequence occurrences. Based on the facility design, some combinations of event sequences will be expected to occur at least once during the operational period, and some combinations will not. In these simpler terms, a combination of event sequences can be designated as either "expected" or "not expected" to occur during the operational life of the proposed facility. This paper provides analytical solutions for calculating the expected number of combinations of independent event sequences. These analytical solutions agree with numerical solutions for an example problem. Although uncertainties can be incorporated into the method, only point-estimate parameter values are used in the example problem presented. The main objective of this paper is to present calculational approaches to identify which combinations of event sequences are expected to occur during the operational life of a proposed facility. Facility performance based on some proposed design is evaluated against the operational dose limits. Because the operational dose limits tend to be annual quantities that may not be exceeded in any year of operation, calculation of the doses resulting from those expected combinations of event sequences can provide insight on the maximum annual dose expected during the operation of a proposed facility.

Keywords: Variability, aleatory uncertainty, annual dose, event sequence, combination of events, facility design

INTRODUCTION

The term "expected" is used in two ways throughout this paper. In its standard usage, the term "expected" relates to using a Poisson distribution and the event sequence frequency (i.e., probability theory) to calculate the expected number of occurrences of the event sequence during the entire operational period. The additional usage of the term "expected" as an adjective in this paper builds on its standard usage and allows for the differentiation of particular event sequences or combinations of event sequences. In this additional usage, the phrase "those expected combinations" is equivalent to "those combinations of event sequences with an expected number of occurrences during the operational period of greater than or equal to one." Simplification of the terminology is the compelling reason for incorporating the additional usage of "expected" into this paper.

The annual radiological dose received by workers or members of the public depends on the number and type of event sequence occurrences within a single year of operation. Considering only the radiological doses from event sequences and not those from routine operations, some years will result in no radiological doses, some years will result in radiological doses from a single event sequence; and still other years will result in radiological doses from multiple event sequences. Most operational dose limits specified in the U.S. Nuclear Regulatory Commission regulations (e.g., 10 CFR Parts 20 and 63) are annual quantities that may not be exceeded in any year of operation. For example, the dose limits for Category 1 event sequences are specified as annual quantities in 10 CFR 63.111(a) and 63.311. As specified in 10 CFR 63.111(b)(2), the dose limits for Category 2 event sequences are not annual quantities, and, therefore, the methodology for combinations of event sequences presented in this paper is not applicable to Category 2 event sequences. For operational facilities, compliance is determined from radiation monitoring and record keeping. For facilities not yet built, the performance provided by the facility and operational design with regard to safety can be evaluated by calculating consequences resulting from those event sequences expected to occur at least once during the operation of the proposed facility.

10 CFR Part 63 is the governing regulation for a potential geologic repository of high-level waste at Yucca Mountain, Nevada. In 10 CFR 63.2, an event sequence is defined as a series of actions, occurrences, or both within the natural and engineered components that could potentially lead to exposure of individuals to radiation. An event sequence includes one or more initiating events and associated combinations of repository system component failures, including those produced by the actions or inaction of operating personnel. Category 1 event sequences are defined as those event sequences expected to occur one or more times before permanent closure of the geologic repository operations area. As part of any potential license application for a high-level waste repository, one may expect multiple Category 1 event sequences would be identified by the preclosure safety analysis. It is also plausible that more than one Category 1 event sequence could occur during a single year of operation. Because of the large number of repetitions and low component failure rates for the fuel handling operations, Poisson distributions are used to describe the occurrence of event sequences. This paper presents analytical and numerical solutions for the expected number of occurrences for combinations of independent Category 1 event sequences. All event sequences referred to hereafter are Category 1 event sequences.

A simple example is presented to illustrate the variability of nine different event sequences occurring independently during 20 years of facility operations. The occurrence of an event sequence in any year of operation is described by a Poisson distribution with a recurrence of λ, which equals the average number of occurrences of the event sequence in some time period (e.g., 1 year). Table 1 displays the recurrence and consequence given as a radiological dose for each of the nine event sequences. Random number sampling was used to simulate the behavior of this system. Nine random numbers uniformly distributed between 0 and 1 are sampled for each year (one random number for each event sequence per year), such that the occurrence of event sequences in 1 year is independent of the occurrence of event sequences in any other year during the operational period. The value of the random number indicates how many times the event sequence occurred during that year of operation. Specifically, the cumulative distribution function for each event sequence is assembled from the probability that the event sequence will not occur, will occur once, will occur twice, and so on, within 1 year. The number of occurrences of the event sequence in that year is determined from the location of the value of the random number on the cumulative distribution function.

For example, on an annual basis, the probability that event sequence A occurs 0 times is 0.9512294, occurs 1 time is 0.0475615, occurs 2 times is 0.0011890, and occurs 3 times is 0.0000198. The probability that event sequence A occurs 4 or more times within 1 year is 0.00000025. Therefore, a random number value between 0 and 0.9512294 indicates that event sequence A did not occur within that year. Likewise, a random number value between 0.9512294 and 0.9987909 (0.9512294 + 0.0475615) indicates that event sequence A occurred once within that year.

Four complete sets of random numbers were generated to simulate four realizations of faciltiy operation, and the outcomes are displayed in Figures 1 and 2. Figure 1 shows the event sequences that occurred in each year of operation. The realization in the upper left hand corner of figure is used as an example for further discussion. None of the event sequences occurred in 4 of the 20 years. A single event sequence occurred in 10 of the 20 years, and two or more event sequences occurred in 6 of the 20 years. Figure 2 displays the annual doses for each year of operation. Based on the event sequences with the same frequencies in Table 1 and the 20-year operational period, 3 occurrences (3×0.05/year $\times$ 20 years = 3) are expected of either event sequences A, D, or G (3 occurred in the small sample set); 6 occurrences of either event sequences B, E, or H (6 occurred in the small sample set); and 15 occurrences of either event sequences C, F, or I (17 occurred in the small sample set). These numbers of occurrences from the small sample set match remarkably well with the long-term behavior of the system. Other 20-year realizations of facility operation in Figure 1, however, can exhibit much larger differences with the long-term behavior (where number of occurrences of event sequences A, D, or G range from none to six, instead of three, for example).

By summing the product of frequency and dose for each event sequence in Table 1, a frequency-weighted dose of 0.515 mrem/yr is obtained for this system (which can be interpreted as a long-term average). Based on the single simulated outcome shown in the upper left hand corner of Figure 2, the sum of all doses divided by the 20 years of facility operation equals an average annual dose of 0.495 mrem. The simulated average is slightly smaller than the frequency-weighted result for this outcome, but could just as well be larger in another outcome (i.e., from another set of random numbers). The fact that 20 percent of the annual doses (4 of the 20 years) exceeded the frequency-weighted dose by factors of approximately 2 to more than 4 is more important. For this example system, most years will contain 0 or one event sequence occurrences, and some years will contain two or more event sequence occurrences (i.e., combinations of event sequences). Also, generally speaking, most years will result in annual doses close to or less than the frequency-weighted dose; yet some years will have annual doses that exceed significantly the frequency-weighted dose mainly because of combinations of event sequences. It is known that combinations can occur, but which specific event sequence combinations can be expected to occur during the facility operation? What are the annual doses for those expected combinations?

Ultimately, what is the maximum annual dose expected during operation? To answer these questions, we need to apply probability theory.

THEORY

The event sequences represent the occurrence of a set of discrete events, and, thus, probabilities of event sequences are expressed as sums over probability mass functions (related to the probability density functions for continuous random variables). In other words, the probability mass function is the set of probabilities corresponding to the discrete values that may be taken by the random variable. Joint probability mass functions are used to represent the set of discrete combinations, which are composed of multiple random variables.

As a general example for two random variables, X and Y (referred to later as event sequences), the joint probability mass function takes the following form:

$$p_{X,Y}(x_r, y_s) = P[\{X = x_r\} \cap \{Y = y_s\}] \quad \text{for all } r \text{ and } s. \qquad (1)$$

The subscripts r and s denote that the random numbers, X and Y, can take numerous values. Particular values are represented by the terms x_r and y_s. From probability theory, the joint probability mass function can be determined from the product of a conditional probability and marginal probability[(1)]:

$$p_{X,Y}(x_r, y_s) = p_Y(y_s | x_r) p_X(x_r) \qquad \text{for all } r \text{ and } s \qquad (2)$$

$$= p_X(x_r) p_Y(y_s) \qquad \text{for all } r \text{ and } s \text{ when X and Y are independent.}$$

Interest will be in the probability that Y is in the domain A, which is a subset of the entire sample space (e.g., formed from pairs of the variables with certain values):

$$\begin{aligned} & E[\text{some combination of event sequences X and Y}] \\ &= \sum_{(\text{all } r)} \sum_{(\text{all } s)} j(x_r, y_s) p_{X,Y}(x_r, y_s) \\ &= \sum_{(\text{all } r)} \sum_{(\text{all } s)} j(x_r y_s) p_X(x_r) p_Y(y_s) \end{aligned} \qquad (3)$$

where X and Y are independent and $j\,(x_r, y_s)$ represents the number of that combination, which is a function of x_r and y_s. For our problem, the summations are performed for the number of times a given event sequence occurs represented by the terms x_r and y_s.

A combination of event sequences involves the occurrence of one or more event sequences. Each event sequence is represented by a Poisson random variable, thus, that the probability that event sequence X occurs k times within 1 year can be written

$$p_X(k) = \frac{(\lambda_X)^k}{k!} e^{-(\lambda_X)} \quad (4)$$

where λ_X (unitless) represents the product of the frequency of the event sequence X (1/year) and a time period of 1 year.

ANALYTICAL SOLUTIONS

In this section, analytical solutions for the expected number of combinations of event sequences are presented by increasing number of event sequences that compose the combination, m. By definition, m must be less than or equal to the number of event sequences.

Combinations of a Single Event Sequence ($m = 1$)

The solution for combinations of a single event sequence is straightforward but included here for completeness. Although the solution is well known, the expected number of occurrences of a single event sequence in 1 year is determined from the following expression:

$$E[X]_1 = \sum_{j=1}^{\infty} j \cdot p_X(j) = 1_X \quad (5)$$

Obviously, the calculation in Eq. (5) should be repeated for each of the other event sequences.

Combinations of Two Event Sequences ($m = 2$)

The solution for combinations of two event sequences is divided into two cases: (a) combinations of the same event sequence and (b) pair combinations of different and independent event sequences. The two cases are described separately next.

Case a

The probability of the same event sequence occurring multiple times is dictated by its Poisson distribution. For combinations of the same event sequence, the expected number of combinations is determined from the following expression:

$$E[2X]_2 = \sum_{j=1}^{\infty} j \cdot [p_X(2j) + p_X(2j+1)] \quad (6)$$

where j denotes multiples of the combination (in this case, combinations of two, $m = 2$). Table 2 also shows the relationship between the occurrence of individual event sequences within the same year and the number of combinations. The terms in Eq. (6) parallel the structure of Table 2. The number of combinations in the second column of Table 2 relates to the value of j in Eq. (6). The number of event sequence occurrences in the first column of Table 2 relates to the quantities in parentheses from Eq. (6), namely $2j$ and $2j + 1$.

From Eq. (6) and Table 2, two options qualify for each multiple of combinations of two: the number of occurrences of the event sequence can equal the multiple of the combination (i.e., equal $2j$) or the number of occurrences can be one greater (i.e., exceed $2j$ by one). Note, $2j + 2$ occurrences of the event sequence corresponds to the next higher multiple of the combination. Furthermore, an event sequence not occurring or occurring once does not qualify as a combination of two. An event sequence occurring twice or three times qualifies as a single combination of two (j=1). An event sequence occurring four or five times qualifies as a double combination of two (j=2). An event sequence occurring six or seven times qualifies as a triple combination of two (j=3). And, so forth. The calculation in Eq. (6) should be repeated for each of the other event sequences (i.e., for event sequences Y, Z, and so on.).

Case b
For the combination of two different and independent event sequences, the expected number of combinations is determined from the following expression:

$$E[X,Y]_2 = \sum_{j=1}^{\infty} j\left\{ p_X(j)p_Y(j) + p_X(j)\left[1-\sum_{x=0}^{j} p_Y(x)\right] + \left[1-\sum_{x=0}^{j} p_X(x)\right]p_Y(j)\right\} \qquad (7)$$

for X ≠ Y. Table 3 also shows the relationship between the occurrence of individual event sequences within the same year and the number of combinations. The terms in Eq. (7) parallel the structure of Table 3. The number of combinations in the third column of Table 3 relates to value of j in Eq.(7). For a given number of combinations of two, the numbers of occurrences of event sequence X and Y in the first and second columns of Table 3 relate to the p_X and p_Y terms in Eq. (7).

From Eq. (7), three options qualify as a single combination of different event sequences (j=1): (1) each event sequence occurs once, (2) event sequence X occurs once and event sequence Y occurs more than once, or (3) event sequence Y occurs once and event sequence X occurs more than once.

For a double combination of different event sequences (j=2), the three options are (1) each event sequence occurs twice, (2) event sequence X occurs twice and event sequence Y occurs more than twice, or (3) event sequence Y occurs twice and event sequence X occurs more than twice. The calculation in Eq. (7) should be repeated for the other combinations of two event sequences (e.g., for event sequences X and Z, for event sequences Y and Z, and so on.).

Combinations of Three Event Sequences (m = 3)

The solution for combinations of three event sequences is divided into three cases:
(a) combinations of the same event sequence occurring three times, (b) triplet combinations of different and independent event sequences occurring once, and (c) combinations of two different and independent event sequences with one event sequence occurring twice and the other occurring once. The three cases are described separately next.

Case a
For combinations of the same event sequence occurring three times, the expected number of combinations is determined from the following expression:

$$E[3X]_3 = \sum_{j=1}^{\infty} j \cdot [p_X(3j) + p_X(3j+1) + p_X(3j+2)] \tag{8}$$

where j denotes multiples of the combination (in this case, combinations of three, because $m = 3$). Table 4 also shows the relationship between the occurrence of individual event sequences within the same year and the number of combinations. The terms in Eq. (8) parallel the structure of Table 4.

From Eq. (8) and Table 4, three options qualify for each multiple of combinations of three: the number of occurrences of the event sequence can equal the multiple of the combination (i.e., equal $3j$), can be one greater (i.e., exceed $3j$ by one), or can be two greater (i.e., exceed $3j$ by two). Note, $3j + 3$ occurrences of the event sequence correspond to the next higher multiple of the combination. Furthermore, an event sequence occurring less than three times does not qualify as a combination of three. An event sequence occurring three, four, or five times qualifies as a single combination of three (j=1). An event sequence occurring six, seven, or eight times qualifies as a double combination of three (j=2). An event sequence occurring 9, 10, or 11 times qualifies as a triple combination of three (j=3) and so forth. The calculation in Eq. (8) should be repeated for other event sequences (e.g., for event sequences Y, Z, and so on.).

Case b
For the combination of three different and independent event sequences, the expected number of combinations is determined from the following expression in condensed notation:

$$E[X,Y,Z]_3 = \sum_{j=1}^{\infty} j \begin{pmatrix} X_j Y_j Z_j + X_j Y_{>j} Z_{>j} + X_{>j} Y_j Z_{>j} + X_{>j} Y_{>j} Z_j \\ + X_{>j} Y_j Z_j + X_j Y_{>j} Z_j + X_j Y_j Z_{>j} \end{pmatrix} \quad \text{for } X \neq Y \neq Z \tag{9}$$

where

$$X_j = p_X(j) \qquad Y_j = p_Y(j) \qquad Z_j = p_Z(j)$$

$$X_{>j} = \left[1 - \sum_{x=0}^{j} p_X(x)\right] \quad Y_{>j} = \left[1 - \sum_{x=0}^{j} p_Y(x)\right] \quad Z_{>j} = \left[1 - \sum_{x=0}^{j} p_Z(x)\right]$$

Table 5 also shows the relationship between the occurrence of individual event sequences within the same year and the number of combinations. The terms in Eq. (9) parallel the structure of Table 5.

From Eq. (9) and Table 5, several options qualify as a single combination (j=1) of three different event sequences:

- each event sequence occurs j times;

- one event sequence occurs j times and the two other event sequences occur more than j times, which correspond to the $X_jY_{>j}Z_{>j}$, $X_{>j}Y_jZ_{>j}$, and $X_{>j}Y_{>j}Z_j$ terms in Eq. (9); or
- one event sequence occurs more than j times and the two other event sequences occur j times, which correspond to the $X_{>j}Y_jZ_j$, $X_jY_{>j}Z_j$, and $X_jY_jZ_{>j}$ terms in Eq (9).

The calculation in Eq. (9) should be repeated for the other combinations of three different event sequences.

Case c
For mixed combinations of three consisting of two different and independent event sequences (e.g., one event sequence occurring twice and the other occurring once), the expected number of combinations is determined from the following expression:

$$E[2X,Y]_3 = \sum_{j=1}^{\infty} j[p_X(2j) + p_X(2j+1)]\left[1 - \sum_{x=0}^{j-1} p_Y(x)\right] \qquad \text{for X} \neq \text{Y}. \qquad (10)$$

Table 6 also shows the relationship between the occurrence of individual event sequences within the same year and the number of combinations. The terms in Eq. (10) parallel the structure of Table 6. From Eq. (10) and Table 6, two options qualify as a mixed combination of three: the number of occurrences of event sequence X can equal $2j$ or $2j + 1$ when event sequence Y occurs at least j times. For example, event sequence X occurring twice or three times when event sequence Y occurs at least once qualifies as a single combination ($j = 1$). Event sequence X occurring four or five times when event sequence Y occurs at least twice qualifies as a double combination (j=2). Event sequence X occurring six or seven times when event sequence Y occurs at least three times qualifies as a triple combination (j=3) and so forth. The calculation in Eq. (10) should be repeated for the other mixed combinations of three consisting of two different event sequences {e.g., [2Y,X], [2X,Z], [2Z,X], and so on}.

Generalized Solutions for Combinations of m Event Sequences

The generalized solutions for the combinations of m event sequences are presented next for the three cases which correspond to $m \leq 3$: (a) combinations of the same event sequence occurring m times, (b) combinations of m different event sequences each occurring once, and (c) mixed combinations of three consisting of two different event sequences where at least one event sequence occurs more than once. It is important to note the number of mixed combinations for Case c increases with the value of m. When $m = 3$, there is a single mixed combination subcase for Case c, namely, when one event sequence occurs twice and another different event sequence occurs once {e.g., [2X,Y]}.

Case a
For combinations of the same event sequence occurring m times, the expected number of combinations is determined from the following expression:

$$E[mX]_m = \sum_{j=1}^{\infty} j \sum_{i=0}^{m-1} p_X(mj+i) \tag{11}$$

Case b
For combinations of m different and independent event sequences each occurring once, the expected number of combinations is determined from the following expression:

$$E[X_1, X_2, \ldots, X_m] = \sum_{j=1}^{\infty} j \left\{ \prod_{F \in S_m}^{\text{all } S_m} p_F(j) + \sum_{F \in S_m}^{\text{all } S_m} p_F(j) \prod_{\substack{G \in S_m, \\ G \notin F}}^{\text{all } S_m} \left[1 - \sum_{i=0}^{j} p_G(i) \right] + \sum_{F \in S_m}^{\text{all } S_m} \left[1 - \sum_{i=0}^{j} P_F(i) \right] \prod_{\substack{G \in S_m, \\ G \neq F}}^{\text{all } S_m} p_G(j) \right\}$$

$$\text{for } X_1 \neq X_2 \neq \ldots \neq X_m, \tag{12}$$

where S_m represents the set of m different and independent event sequences $[X_1, X_2, \ldots, X_m]$ and the notation X_m denotes m different event sequences, such that $X_1 = X$, $X_2 = Y$, and so forth.

Case c (for $m = 3$)
For mixed combinations of less than m different and independent event sequences where at least one event sequence occurs more than once, the expected number of combinations is determined from the following expression:

$$E[(m-1)X, (m-2)Y]_{m=3} = \sum_{i=1}^{\infty} i \left[1 - \sum_{j=0}^{i-1} p_Y(j) \right] \sum_{k=0}^{m-2} p_X[(m-1)i + k] \qquad \text{for } m = 3 \text{ and } X \neq Y. \tag{13}$$

Multiple subcases of mixed combinations exist for Case c for m values of 4 or greater. In symbolic notation, Cases a and b for $m = 4$ are [4X] and [X,Y,Z,W]. When $m = 4$, there are three subcases of mixed combinations for Case c, shown symbolically as [3X,Y], [2X,2Y], and [2X,Y,Z]. Generalized solutions for multiple subcases of mixed combinations when m exceeds a value of 3, however, are not presented here.

Calculation of Doses for Combinations

Thus far, the solutions have focused on determining the expected number of combinations. Once the expected number for a combination has been calculated, the expected number should be paired with the conditional dose for that combination of event sequences. Consistent with the concentration of this paper on the variability of discrete occurrences of event sequences within individual years of operation, interest is in the conditional dose for the combination rather than an expected annual dose quantity (because expected annual dose does not relate to a discrete occurrence of event sequences within an individual year). Specifically, the maximum annual dose from those combinations of event sequence expected to occur during the facility operation is of regulatory interest. The conditional dose for some combination of event sequences is merely the weighted sum of its corresponding event sequence doses, where the weighting factor for each event sequence is equal to the number of times the event sequence occurs in the definition of the combination. The weighting factors are provided by the notations in Eqs. (5) through (10). For example, the dose corresponding to the $[2X,Y]_3$ combination equals two times the dose from event sequence X plus one times the dose from event sequence Y. It is important to note, the multiple options (i.e., multiple probability terms) from Eqs. (6) through (10) are considered in the calculation of the expected number of occurrences for the defined combination because those extra event sequence occurrences still imply the original combination occurred. Those multiple options beyond the original combination definition, however, should not be factored into the calculation of the conditional dose for the combination because each of those options corresponds to a new combination definition that should be evaluated separately. To illustrate these points, [3X,Y] implies [2X,Y] occurred with an extra occurrence of event sequence X; yet, the extra occurrence of event sequence X does not factor into the conditional dose calculation for [2X,Y] because [3X,Y] should be considered as a separate combination.

ILLUSTRATIVE EXAMPLE AND NUMERICAL CORROBORATION

An example problem was created and solved using the analytical solutions presented previously. To demonstrate the analytical approach accurately calculates the expected number of occurrences of combinations, the same problem was solved numerically using Mathematica 4.1® and compared with the analytical solution. Table 7 defines the independent event sequences for the example problem where the occurrence of each event sequence is described by a Poisson distribution characterized by a single quantity, λ, representing the product of the event sequence frequency and time. In other words, a frequency of 0.1/year and a time of 1 year corresponds to a λ value of 0.1. To make the example more applicable to long-term operational facilities, an operational time period of 30 years was chosen. For determining whether a combination is "expected" or "not expected" to occur during the operational life of the facility, the result of interest is the expected number of occurrences for that combination in a single year for the entire operational period (i.e., 30 years). In summary, the combinations of event sequences must be defined to occur (or not to occur) in a 1-year time period. The expected number of occurrences of those combinations, however, was calculated for a 30-year operational period.

The analytical solution was calculated using Eqs. (5) through (10), which determine the expected number of combinations for a 1-year time period (based on the definition of the event sequences). To yield the analytical solution for the expected number of combinations, the results from Eqs. (5) through (10) must be scaled to account for the longer operational time period (i.e., multiplied by 30, in this case). The numerical solution simulated 1,000,000 time intervals of 1 year and counted the number for each type of combination occurring within the 1-year time periods. The numerical solution for the expected number of combinations in a single time interval was computed from dividing the summation of the number of recorded combinations by the total number of time intervals simulated. The numerical solution for the expected number of combinations in a single time interval was multiplied by 30 to yield the numerical solution for the expected number of combinations within the operational time period. Table 8 presents the analytical and numerical results for the expected number of combinations within the 30-year operational time period. Although the statistical error in the numerical results for simulating 1,000,000 annual time intervals was not calculated, the analytical and numerical results are in good agreement. In the context of this example, the combinations expected to occur at least once during the facility operational life are $E[A]_1$, $E[B]_1$, $E[C]_1$, $E[2C]_2$, and $E[B,C]_2$. The maximum dose expected within any year of operation is maximum dose from any "expected" combination plus the dose resulting from routine operations throughout the year (not from event sequences). For this example, the maximum dose expected in any year during the operational period is the dose resulting from routine operations throughout the year plus the maximum from the set (D_A, D_B, D_C, $2D_C$, $D_B + D_C$), where D_X represents the radiological dose resulting from a single occurrence of event sequence X.

CONCLUSIONS

This paper presented analytical solutions for calculating the expected number of occurrences for combinations of independent event sequences. The analytical solutions were found to be in agreement with numerical solutions for an example problem. Calculations of expected number of occurrences for combinations can determine which combinations are expected to occur during the operational life of the facility and which are not. Operational dose limits are annual quantities that may not be exceeded in any year of operation. Calculations of the radiological doses resulting from those expected combinations of event sequences can provide insight on the maximum dose expected within any year of operation. For proposed operations, this information is useful for regulatory safety evaluations of the facility and operational design.

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Table 1. Example list of event sequences associated with the operation of a hypothetical facility.

Event Sequence	λ = frequency × time, (unitless)	Dose (mrem)
A	0.05	1.0
B	0.10	0.5
C	0.25	0.5
D	0.05	1.0
E	0.10	0.3
F	0.25	0.3
G	0.05	2.0
H	0.10	0.1
I	0.25	0.1

Table 2. Relationship between individual event sequence occurrences and the number of combinations of two for the same event sequence, $[2X]_2$ ($m = 2$, Case a)

Number of Event Sequence Occurrences	Corresponding Number of Combinations of Two
2 or 3	1 combination of two
4 or 5	2 combinations of two
6 or 7	3 combinations of two
. . .	. . .
$2j$ or $2j + 1$ (where x is even)	j combinations of two

Table 3. Relationship between individual event sequence occurrences and the number of combinations of two for two different event sequences, $[X,Y]_2$ (m = 2, Case b)

Number of Occurrences of Event Sequence X	Number of Occurrences of Event Sequence Y	Corresponding Number of Combinations of Two
1	1	1
1	2, 3, 4, ...	1
2, 3, 4, ...	1	1
2	2	2
2	3, 4, 5, ...	2
3, 4, 5, ...	2	2
. . .	. . .	. . .
j	j	j
j	$j+1, j+2, j+3, \ldots$	j
$j+1, j+2, j+3, \ldots$	j	j

Table 4. Relationship between individual event sequence occurrences and the number of combinations of three for the same event sequence, $[3X]_3$ ($m = 3$, Case a)

Number of Event Sequence Occurrences	Corresponding Number of Combinations of Three
3 or 4 or 5	1 combination of three
6 or 7 or 8	2 combinations of three
9 or 10 or 11	3 combinations of three
. . .	. . .
$3j$ or $3j + 1$ or $3j + 2$	j combinations of three

Table 5. Relationship between individual event sequence occurrences and the number of combinations of three for three different event sequences, $[X,Y,Z]_3$ (m=3, Case c)

Number of Occurrences of Event Sequence X	Number of Occurrences of Event Sequence Y	Number of Occurrences of Event Sequence Z	Corresponding Number of Combinations of Three
1	1	1	1
1	2 or more	2 or more	1
2 or more	1	2 or more	1
2 or more	2 or more	1	1
2 or more	1	1	1
1	2 or more	1	1
1	1	2 or more	1
⋮	⋮	⋮	⋮
j	j	j	j
j	$j+1$ or more	$j+1$ or more	j
$j+1$ or more	j	$j+1$ or more	j
$j+1$ or more	$j+1$ or more	j	j
$j+1$ or more	j	j	j
j	j+1 or more	j	j
j	j	$j+1$ or more	j

Table 6. Relationship between individual event sequence occurrences and the number of combinations of three for two different event sequences, $[2X,Y]_3$ ($m = 3$, Case c)

Number of Occurrences of Event Sequence X	Number of Occurrences of Event Sequence Y	Corresponding Number of Combinations of Three
2	1, 2, 3, 4, ...	1
3	1, 2, 3, 4, ...	1
4	2, 3, 4, ...	2
5	2, 3, 4, ...	2
⋮	⋮	⋮
$2j$	j or more	j
$2j + 1$	j or more	j

Table 7. Definition of event sequences for a 1-year time period.

Event Sequence	λ = (frequency)(time), unitless
A	0.1
B	0.2
C	0.3

Table 8. Analytical and numerical results for the expected number of combinations within a 30-year operational time period

Combination	Analytical, expected number	Numerical, expected number
$E[A]_1$	3.00	2.98
$E[B]_1$	6.00	6.00
$E[C]_1$	9.00	9.02
$E[2A]_2$	0.140	0.143
$E[2B]_2$	0.527	0.528
$E[2C]_2$	1.12	1.12
$E[A,B]_2$	0.520	0.523
$E[A,C]_2$	0.745	0.738
$E[B,C]_2$	1.43	1.43
$E[3A]_3$	0.0046	0.0042
$E[3B]_3$	0.034	0.036
$E[3C]_3$	0.108	0.107
$E[A,B,C]_3$	0.134	0.134
$E[2A,B]_3$	0.025	0.027
$E[2A,C]_3$	0.036	0.035
$E[2B,A]_3$	0.050	0.050
$E[2B,C]_3$	0.136	0.135
$E[2C,A]_3$	0.105	0.107
$E[2C,B]_3$	0.200	0.199

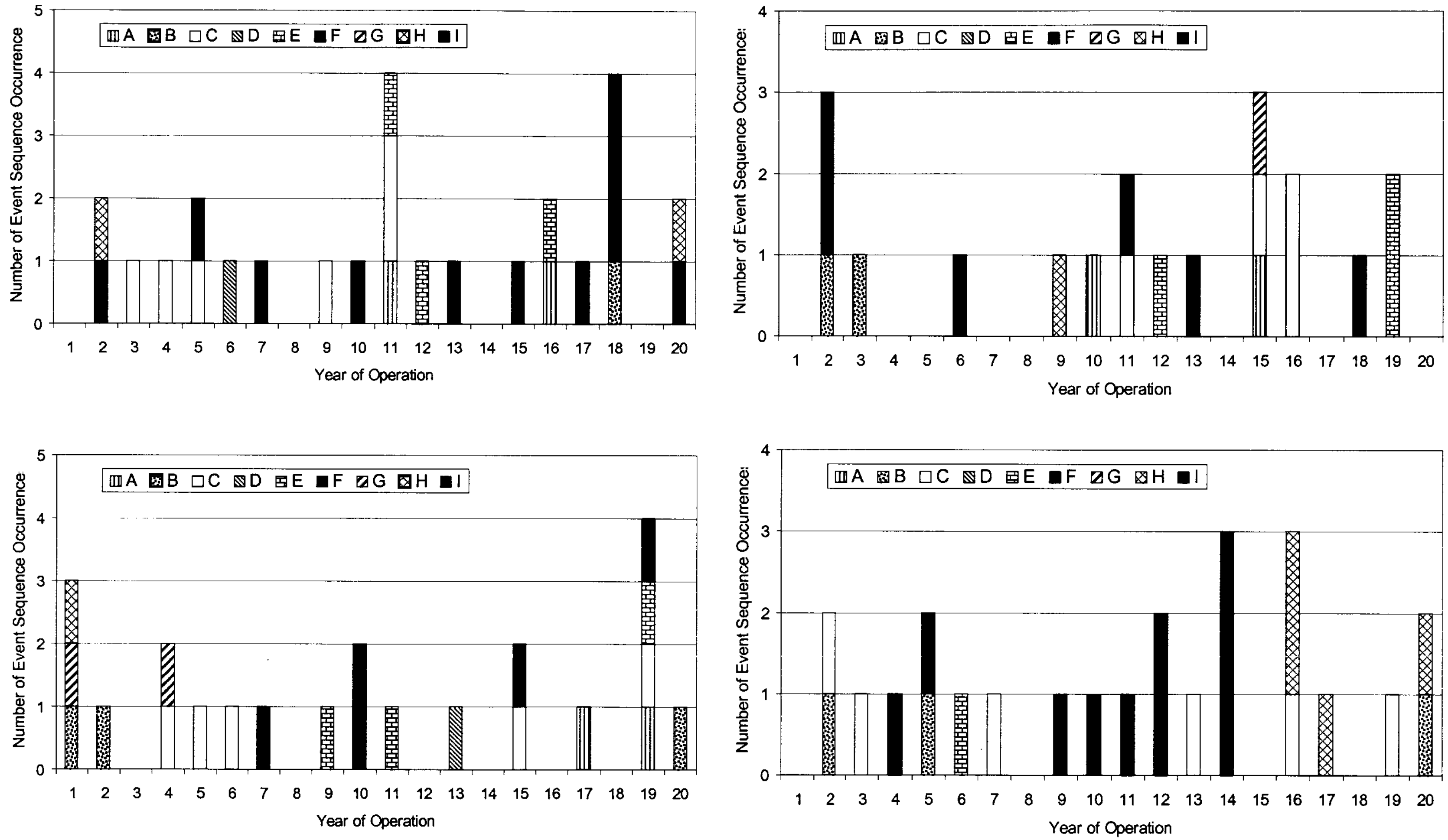

Figure 1. Four realizations simulating facility operation to highlight the variability of occurrences of different event sequences.

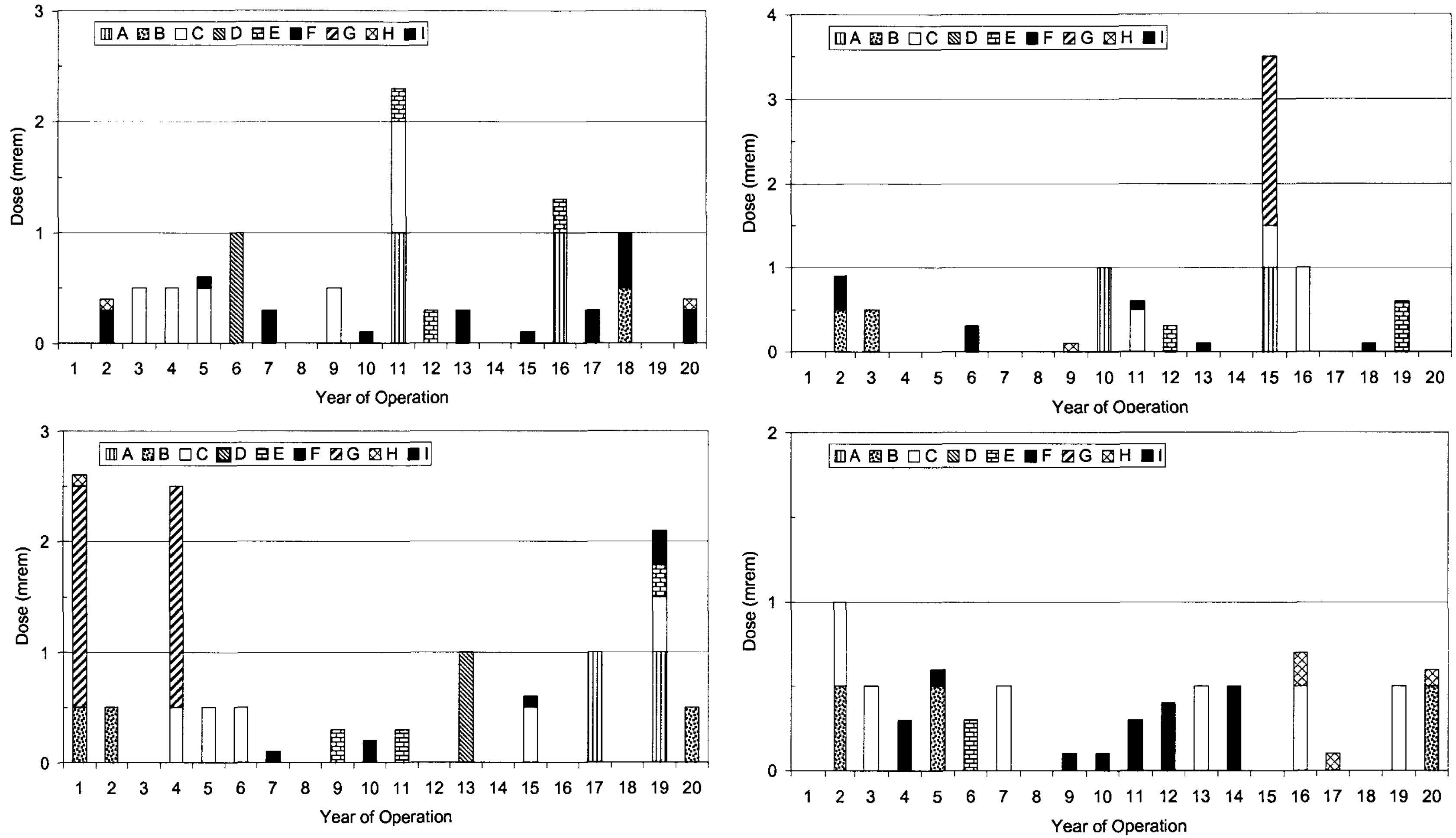

Figure 2. Corresponding annual doses from the four realizations simulating facility operation (shown in Figure 1) to highlight the variability of annual doses due to the simulated occurrences of different event sequences.