The Effect of Model Conservatism on Identifying Influential Parameters

by

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Outline

• Background

• Objective

• Computing Sensitivity

• Example Problems and Results

• Conclusions

• Acknowledgments
What Is Sensitivity Analysis?

• Sensitivity analysis:
  – an important component of quantitative risk assessment
  – a computational step used in any risk-informed, performance-based approach, conducted for making decisions
  – identifies and ranks influential models, parameters, and components of the model

• Sensitivity analysis results are used to derive the risk significance of various aspects of the system

• General assumption: the model is realistic (i.e. neither overly-optimistic nor overly-pessimistic)

• Conservative assumptions (i.e., underestimating the performance of the system) in modeling large and complex systems are often used
Sources of Conservatism in Models

- Analysts make simplifying assumptions because of:
  - Paucity of data
  - Complexity of the processes modeled
  - Early stage of modeling
  - Limited resources and time

- Analysts may have simplified the model without any specific attention to conservatism or realism

- In some cases what is considered to be conservative may not be truly conservative

- Simplifying assumptions may be deliberately biased toward conservatism
Complex System Models

- A multidisciplinary system model may have different degrees of conservatism in its components

- It integrates multiple abstracted conceptual and mathematical models
  - from analysts with different levels of expertise
  - analysts have subject-specific biases

- Sensitivity analysis on balanced models (neither conservative nor optimistic) can give meaningful results
Objective

- Investigate the effect of conservatism on the ranking of influential parameters
- Illustrate the effects at various levels of conservatism using simple, nonlinear, stochastic examples
Approach

• Specify uncertainty range for each parameter in the response function

• Compute sensitivity coefficients corresponding to each input parameter

• Change the range of the specific input parameter whose effect on conservatism is investigated by
  – keeping the conservative end of the distribution fixed
  – shifting the non-conservative end of the distribution toward conservative values (i.e., the mean shifts conservatively)

• Repeat the computation of sensitivity
Approach: Generation of Conservative Cases

- Direct variation of $y$ with $w$
- $P(w)$ = probability distribution function for $w$

- Inverse variation of $y$ with $w$

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Diagram:

1. $(1) < (2) < (3) \
   \text{Increased Conservatism}$

2. $\Delta W_1 \
   \Delta W_2 \
   \Delta W_3$

3. $w_{\text{min}} \
   w_{\text{max}} \
   (\text{assumed realistic})$
Computation of Sensitivity

- Performance function:
  \[ y = f(x_1, x_2, \ldots, x_i, \ldots, x_N; a_1, a_2, \ldots, a_M) \]

  - \( y \): model response
  - \( x_i \): parameters
  - \( a_k \): assumptions

- Normalized first-order local sensitivity of \( y \) to \( x_i \)
  \[ S(y, x_i) = \frac{x_i}{y} \frac{\partial y}{\partial x_i} \]

- Partial derivative is calculated at sampled values of \( x_i \)
Computation of Sensitivity (cont’d)

• Sensitivity coefficient in a probabilistic model:

\[
S_{x_i} = \frac{1}{n} \sum_{j=1}^{n} \frac{x_{ij}}{y_j} \frac{\partial y_i}{\partial x_{ij}}
\]

\(y_j\): model response at the \(j\)-th Monte Carlo realization
\(x_{ij}\): value of \(x_i\) in the \(j\)-th realization
\(n\): number of realizations

• Analytically computed at each sampled point \(j\) in the multi-dimensional sample space
Example Problems

- Three examples: non-linear and analytic functions allowing analytical computation of sensitivity coefficients

- Example 1: a generic four-parameter function

- Example 2: radiation dose from drinking water. Model output is inversely proportional to the parameter of interest (e.g., distribution coefficient).

- Example 3: External radiation dose from a contaminated layer of soil. Model output directly proportional to the parameter of interest (e.g., layer thickness).
Example 1: Generic Function

- Non-linear function of stochastic input parameters
- Uniform distribution functions assigned to all parameters for simplicity
- Function sensitivity plotted with respect to $x$ and $w$ when $\Delta w$ is varied
Example 1: Generic Function (cont’d)

• Sensitivity changes non-linearly with conservatism in $w$ ($\Delta w$); $x$ and $w$ alternate as influential parameters

• $x$- and $w$-sensitivity are obtained with respect to variations of the conservatism in $w$ (i.e., varying $\Delta w$)

• Sensitivity plots:

$S_x(\Delta w) :$ sensitivity of $y$ to $x$ with respect to $\Delta w$

$S_w(\Delta w) :$ sensitivity of $y$ to $w$ with respect to $\Delta w$
Example 2: Drinking Water Pathway

- Six-parameter model
  
  \( n_e \): effective porosity  
  \( I \): infiltration rate  
  \( S \): radionuclide concentration  
  \( \rho \): soil density  
  \( n \): total porosity  
  \( K_d \): distribution coefficient  

- \( S_{ne}, S_I, S_S, S_\rho, S_n, S_{Kd} \) investigated while varying conservatism in \( K_d \) (i.e., by changing the range of \( K_d \) toward conservative values)
Example 2: Drinking Water Pathway (cont’d)

- Model response varies inversely with $K_d$ and $n$
- Toward the conservative end of $K_d$, $n$ ranks more influential than $K_d$
- Toward the non-conservative end (i.e., high $\Delta K_d$), $K_d$ ranks as more influential than $n$
Example 3: External Exposure

- Three-parameter model
- Model response varies directly with $d$
- For conservatively biased range of $d$'s (i.e. low $\Delta d$), $S_d$ is 40% higher than the non-conservative (i.e., realistic) case (high $\Delta d$), i.e., the sensitivity of the output function to $d$ clearly varies.
Conclusions

- The three examples presented illustrate that the conservatism in model parameters influences the sensitivity-based ranking of influential parameters.

- For the examples presented, depending on the level of conservatism assumed, the model output sensitivity may change non-linearly from insensitive to highly sensitive.

- Model output sensitivity changes non-linearly, with the degree of conservatism in input parameter, depending on the structure of the performance function.

- Sensitivity analyses results should be evaluated carefully in light of conservatism before being used in any resource allocation and decisions.
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