

INSUFFICIENT CONSIDERATION OF SCALE EFFECT IN ROCK STRENGTH

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Prepared by Sui-Min (Simon) Hsiung

Center for Nuclear Waste Regulatory Analyses  
San Antonio, Texas

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## 1. INTRODUCTION

Rock strength can be determined by laboratory and in situ tests. Laboratory tests have been carried out on cylindrical or cubic specimens 2 to 6 in. in diameter and of various heights. The disadvantage of laboratory testing is that small specimens seldom represent the in situ conditions. For one thing, the heterogeneity of and numerous cracks and/or fractures in a rock mass can not be truly reproduced in the laboratory. Therefore, smaller specimens contain fewer defects and usually exhibit greater strength under both uniaxial and triaxial conditions. Moreover, the specimens available for laboratory testing are those which survive rigorous preparation.

Rock strength is also found to depend on specimen geometry - that is, the ratio of diameter-to-height or width-to-height of the specimens (Evans et al., 1961). Size and geometrical (shape) effects are called scale effect.

Application of the laboratory results to underground structure design has long been the effort of mining engineers or rock mechanists and there have been many efforts attempting to predict in situ rock strength from laboratory test results. It is the author's opinion that, although considerable progress has been achieved, the scale effect has not been fully accounted for. This is especially true for the prediction of dynamic behavior of in situ rock joints, which is an important subject in the design of an underground repository in a jointed rock mass. A detail discussion regarding this position is presented in the following sections. Suggestions for further improvement are also submitted at the end of this position paper.

## 2. ROCK MASS STRENGTH

In ground control, pillar design and excavation stability are two areas of interest.

### 2.1 Scale Effect on Pillar Design

For the past few decades, it has been a common practice to design a pillar considering only uniaxial compressive strength. Application of yield criteria to pillar design is fairly new. The uniaxial compressive strength is apparently scale (including size and shape) dependent.

## 2.1.1 Size Effect

It was generally true that coal strength is higher for smaller specimens and decreases exponentially as the size of the specimens increase. The relationship between the size and the strength of the specimen can be generalized by the equation (Evans et al., 1961):

$$\sigma_1 = k_1 d_s^{-a} \quad (1)$$

where  $\sigma_1$  is the uniaxial compressive strength of the cubical coal specimens,  $d_s$  is the side length of the specimen, and  $k_1$  and  $a$  are constants. Values of "a" range from 0.38 and 0.66 (Peng, 1978) with 0.5 being the average. Figure 1 shows effects of cubical specimen size on coal strength. Since the curves of strength versus size decay exponentially, the asymptotic value of the curve is the lower limit of the strength and may represent the strength of the in situ coal pillars. Bieniawski (1969) found that the asymptotic strength for South African coals can be obtained by testing coal cubes with side dimensions of 5 ft. or more.

Based on Eq. (1), the strength of a 5 ft. coal cube is roughly 1/8 of the strength of a 1 in. cube. (Note  $a = 0.5$  is used for calculation). Similar conclusions were reached through laboratory testing results by Greenwald (1941) and Bieniawski and Van Heerden (1975), which reported a ratio of 1/7. Wilson (1981), on the other hand, suggested that the ratio of the in situ coal strength to the laboratory value should be equal to 1/5. However, he did not specify the specimen size for laboratory tests.

## 2.1.2 Shape Effect

Many pillar strength formulas have been proposed recognizing the shape effect. Two types of expressions predominate:

$$\sigma_p = \sigma_1 \left( A + B \frac{W_p}{H} \right) \quad (2)$$

and

$$\sigma_p = K \frac{W_p^\alpha}{H^\beta} \tag{3}$$

where  $\sigma_p$  is the pillar strength,  $\sigma_1$  is the uniaxial compressive strength of a cubic specimen and can be calculated by Eq. (1), K is the strength of the cubic specimen representative of underground pillars, A, B,  $\alpha$ , and  $\beta$  are constants which depend on the characteristics of the coal seam,  $W_p$  is the pillar width, and H is the pillar height.

From all the available pillar strength formulas, Bieniawski (1983) found the following five expressions are relevant for application to U. S. coal mines:

1. Obert-Duvall formula

$$\sigma_p = \sigma_1 \left( 0.778 + 0.222 \frac{W_p}{H} \right) \tag{4}$$

2. Holland formula

$$\sigma_p = \sigma_1 \sqrt{\frac{W_p}{H}} \tag{5}$$

3. Holland-Gaddy formula

$$\sigma_p = k_2 \frac{\sqrt{W_p}}{H} \tag{6}$$

4. Salamon-Munro formula

$$\sigma_p = \frac{k_2 W_p^{0.46}}{12 H^{0.66}} \quad (7)$$

5. Bieniawski formula

$$\sigma_p = \sigma_1 \left( 0.64 + 0.36 \frac{W_p}{H} \right) \quad (8)$$

Using the Pittsburgh coal seam as an example, the results from various formulas may be compared as shown in Fig. 2 (Bieniawski, 1983). The strengths predicted by Holland, Salamon-Munro, and Obert-Duvall formulas are fairly close within the range of width-to-height ratio shown in the figure. The strength predictions from Bieniawski, Obert-Duvall, and Holland-Gaddy formulas are substantially different. However, there is not sufficient evidence to accept or reject the application of any of the formulas. Successful attempts of using these formulas for pillar design were documented in various literature. Examples are Choi and McCain, 1979, and Skelly et al., 1977.

Figure 3 offers another angle of examining the potential impact of using different pillar strength formulas (Hsiung, 1984; Peng, 1986). Other things being equal, different pillar widths were recommended by different strength formulas. Under shallow depth (approximately under 1,000 ft), the difference among the predictions are relatively small. Most of the coal seams currently under production are generally less than 1,000 ft deep, especially for the coal seams in the eastern U. S. This may explain why most formulas are successfully applied. Even for the incidences where there were bad roof conditions, they can very easily be attributed to other causes. As overburden becomes deeper, the difference in predictions becomes larger. The question of how much confidence do mining engineers have in using these strength formulas for pillar design purposes obviously can not be answered without further study on and understanding of scale effect.

The major reason responsible for substantial difference in predictions is a lack of a systematic approach for studying scale effect. Large amount of experimental data are available only for coal specimen size ranges from 2 to 6 in. in diameter or cube while significantly less data are available for large size specimens. This is especially true for studying shape effect with relatively larger size specimens. Those empirical strength formulas were developed based

on experimental data of small size specimens without or with little benefit of the data from large size specimens. Therefore, their application should be limited to the range from which they were derived. Any attempts for extrapolating outside the range will result in unreliable predictions.

## 2.2 Scale Effect on Excavation Stability

Analysis of the performance of rock around excavations requires prior determination of the stress levels at which yield, fracture or slip occurs within the rock mass. There are several yield or fracture criteria available for this type of analysis. Depending on types of yield criteria, the scale effect discussed in the previous section also applies here to the extent of determination of strength related parameters such as uniaxial compressive strength, cohesion, and perhaps internal friction angle. It is not clear whether specimen size will have any effect on the relationship among principal stresses which determine the yield surface. To the author's knowledge, no literature is available on this subject.

The following discussion concerns not the scale effect but some fundamental aspects of yield criteria currently used in the rock mechanics field. It is believed that scale effect may be important to the aspects that are considered important.

To express failure conditions in the most general way, it is usual practice to seek an appropriate function, having the three principal stresses,  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  as variables. Those principal stresses take on a certain fixed value for any combination of the principal stresses at which fracture occurs (Paterson, 1978). The current practice assumes that the intermediate principal stress,  $\sigma_2$ , has no effect on rock strength. There are data for rock in support of this assumption (e.g., Brown and Gonano, 1975). There are also conclusions made from experimental studies against this assumption. Mogi (1971) reported that the ratio of the  $\sigma_2$  dependency to the  $\sigma_3$  dependency of the failure stress,  $\sigma_1$ , for Dunham dolomite was estimated to be about 20%, except for very low differential stress, but the  $\sigma_2$  effect on Solenhofen limestone is appreciably smaller and even smaller on the Westerly granite (Mogi, 1967). It seems that the effect of the intermediate principal stress is rock type dependent. A further study of this apparent effect is warranted.

Another point of interest may be that the intermediate principal stress should not affect crack initiation, which will occur in a plane parallel to the  $\sigma_2$  axis. Rather, it has impact on the propagation of the fracture initiated (Murrell and Bigby, 1970). The influence of  $\sigma_2$  can be neglected only when the propagation is also parallel to the  $\sigma_2$  axis, as predicted by the Coulomb criterion.

It does not seem to be appropriate to assume that a particular yield criterion will apply to more than one particular mode of fracture; in particular, it is possible that extension and shear fractures may be controlled by different criteria of failure (Paterson, 1978). Previous studies on rock strength were focused on shear-mode failure but only limited work has been done to understand extension failure. Consequently, there are no valid yield criteria for extension. It is not uncommon for an extension condition to occur around an excavation. There is little experimental information on the influence of  $\sigma_2$  on extension fracturing.

### 3. ROCK JOINT STRENGTH

In rock mechanics problems other than those involving only fracture of intact rock, the shear behavior of discontinuities, such as faults, joints, shear zones and bedding planes, will determine rock mass deformation. Conditions for slip on major pervasive features such as faults or for the sliding of individual blocks from the boundaries of excavations are governed by the shear strengths that can be developed by the discontinuities concerned. Characterization of the complete load-deformation behavior of a joint requires determination of a normal stress-normal closure relation, a shear stress-shear deformation relation, and the relation between shear strength and normal stress.

Studies have shown an important size effect on joint shear behavior (Bandis, 1980; Barton and Choubey, 1977). Increased joint size causes marked reduction in joint roughness coefficient and joint wall compressive strength and increases in displacement at peak shear stress. Extrapolation functions from laboratory scale to in situ scale for values of joint roughness coefficient and joint wall compressive strength were proposed by Barton and Bandis (1982). These relations were specially developed for the need of the Barton-Bandis joint deformation model. Other joint deformation models have not yet taken into consideration the size effect. It should be noted that those extrapolation functions may be valid only for static or quasi-static loading conditions. Similar relations under dynamic loading condition have not yet been established.

Dynamic rock joint behavior is an obvious concern for underground excavations at locations where repetitive episodes of dynamic loadings may be possible. Studies (Brown and Hudson, 1974; Dowding et al., 1983) indicate that repetitive cyclic loading of jointed rock causes fatigue failure through progressive accumulation of shear deformation at the joints. However, there has been only minimal investigation and analysis of such rock mass behavior. This raises a concern as to whether there exists a rock joint model that can predict dynamic rock mass behavior. There is an apparent need for this determination, at least in the U. S. nuclear waste disposal program.

#### 4. FUTURE WORK

To alleviate the concerns discussed above, further study appears to be needed. Further work in the following areas would be useful.

- (1) To study shape effect under different size conditions, particularly at the higher end of the size spectrum,
- (2) To study extensional fracturing with a focus on development of yield criteria,
- (3) To study the potential impact of the intermediate principal stress on rock strength (under both shear and extensional modes),
- (4) To study and formulate the size effect on rock joint behavior (static and dynamic), and
- (5) To develop a rock joint model which is capable of predicting rock mass behavior under repetitive dynamic loading.

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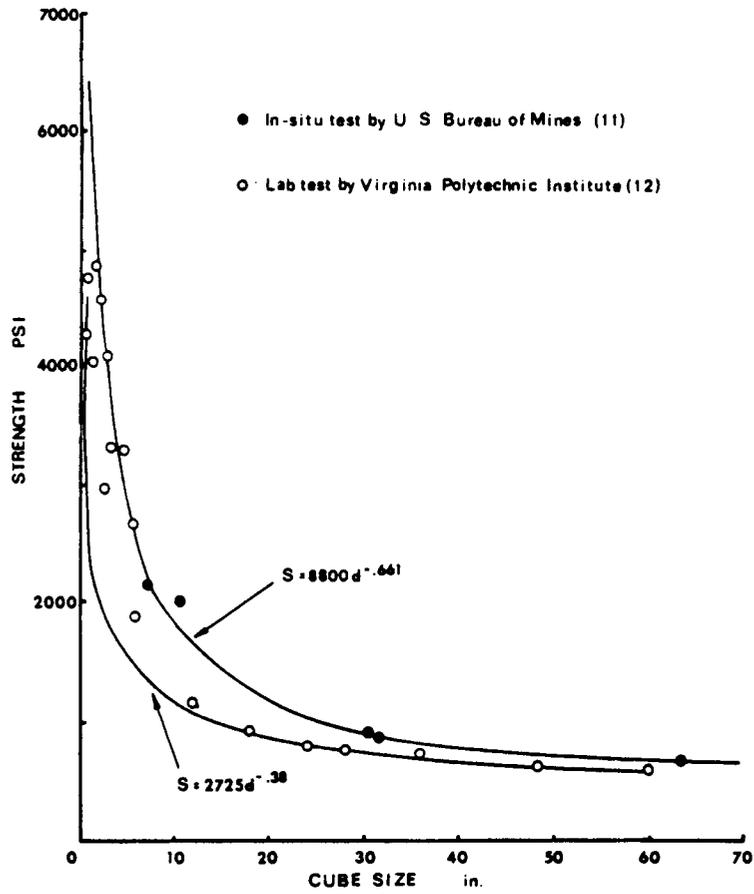


Fig. 1 Effect of cubical specimen size on coal strength (Peng, 1978).

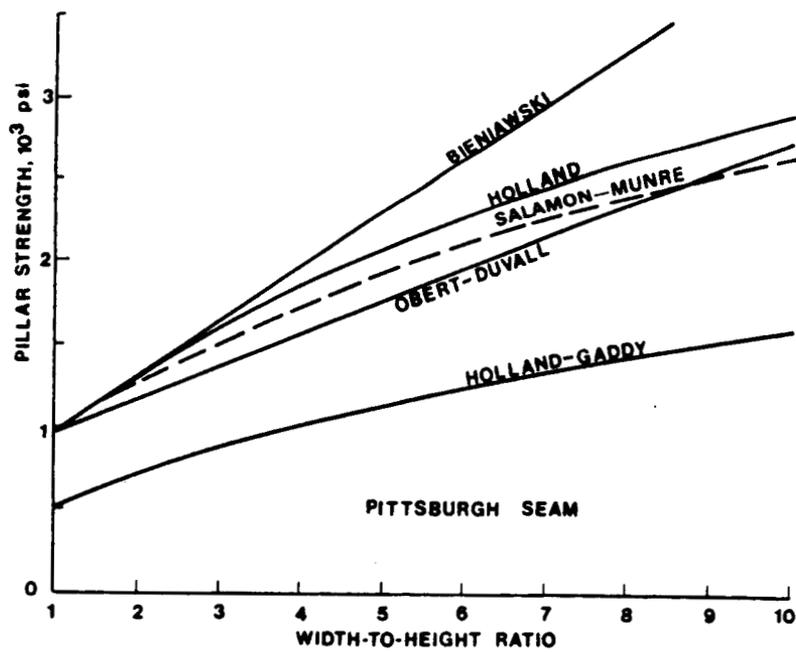


Fig. 2 Comparison of pillar strength formulas with respect to  $W_p/H$  ratio for Pittsburgh seam (Bieniawski, 1983).

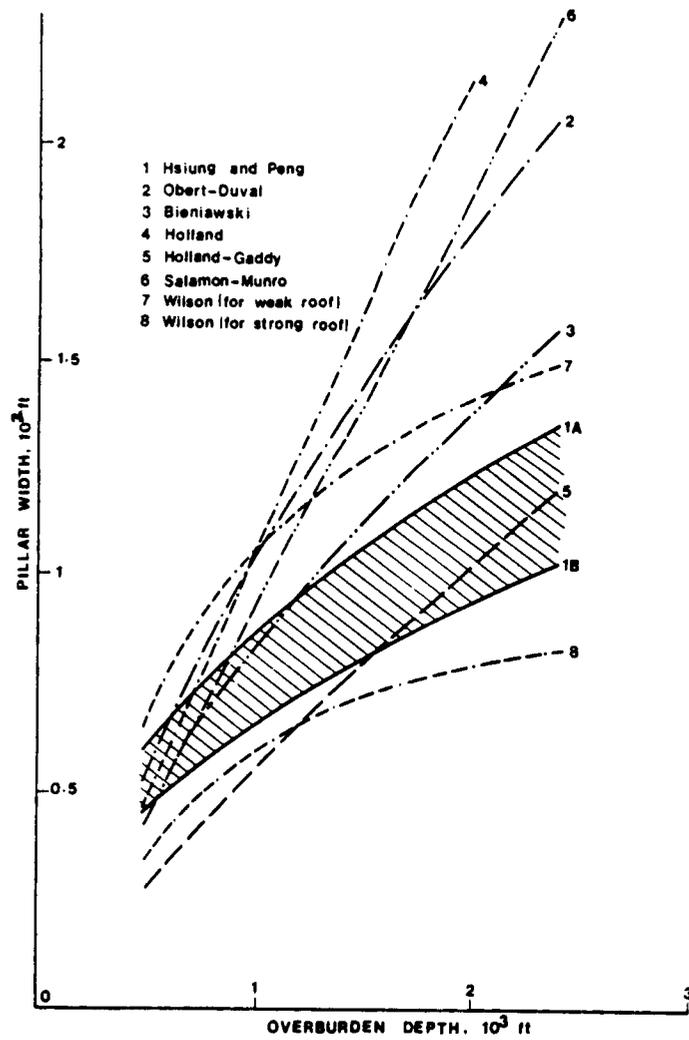


Fig. 3 Comparison of predicted pillar width by various formulas in terms of overburden depth for longwall mining (Peng, 1986).