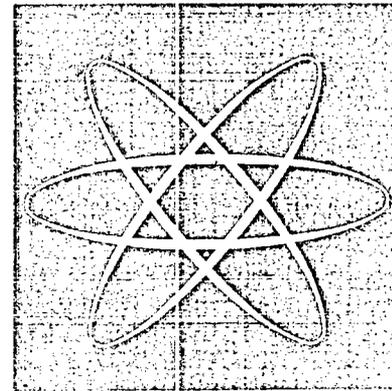


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Combination of Modal Responses

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In the response spectrum method of analyzing structures under seismic loads, values of the maximum probable responses are calculated in each mode of vibration. These responses can be combined using a double sum equation if the correlation between the modal responses is known. An approximate expression for the correlation coefficient given by Rosenblueth and Elorduy[1] does not apply in the low and high frequency ranges. It is shown that the modal responses can be assumed to consist of two components, a rigid component, and a damped periodic component. Various modal rigid components are perfectly correlated. An expression for the correlation between the damped periodic components is presented which is similar to that presented by Rosenblueth and Elorduy[1]. An equation is derived for the correlation between modal responses on this basis, which is shown to be in good agreement with the numerically obtained values.

1. Introduction

In the response spectrum method of analyzing structures under seismic loads, the values of the maximum probable responses are calculated in each mode of vibration. The variation of responses with time is not known, therefore, there is no "exact" way of combining the maximum modal responses. If the maximum response in mode i is R_i , it can be shown that the combined maximum response, R , would satisfy the following inequality

$$\sum_i |R_i| \geq R \geq \sqrt{\sum_i R_i^2} \quad (1)$$

the left hand side being an upper bound, and the right hand side a lower bound. In many cases, when the modal frequencies are sufficiently far apart, the right hand side, commonly known as the SRSS (square root of the sum of the squares) combination, gives reasonably accurate values. Statistically, in these cases, the modal responses can be considered to be independent, or uncorrelated. When the modal frequencies are relatively close, however, the modal responses no longer remain independent, and then the combined response can be significantly greater than the SRSS value. In fact, when two of the modes have identical frequencies, it stands reasons that their responses should be directly added in which cases the two responses are perfectly correlated.

In general, for combining responses from two modes one can write

$$R^2 = R_1^2 + R_2^2 + 2\epsilon_{12}R_1R_2 \quad (2)$$

in which

$$1 \geq \epsilon_{12} \geq -1$$

which depends on the correlation between the modal responses. Theoretically, one could think of a negative correlation also, or when $\epsilon_{12} < 0$, but as will be seen later, it is not very likely. Assuming that the earthquake motion can be represented by a finite duration segment of the white noise, Rosenblueth and Elorduy[1] have obtained an approximate equation similar to Eq(2). Based on their work, the value of ϵ_{12} can be written as

$$\epsilon_{12} = \left\{ 1 + \left[\frac{f_2 - f_1}{\zeta(f_2 + f_1) + 2/(\pi t_d)} \right]^2 \right\}^{-1} \quad (3)$$

where

f_1, f_2 = modal frequencies, Hz

ζ = critical damping ratio

t_d = duration of the earthquake motion, secs

It can be seen from Eq(3) that $\epsilon_{12} = 1$ for $f_2 = f_1$, and $\epsilon_{12} \rightarrow 0$ for $f_2 \gg f_1$. Although, Eq(3) is approximate it works well for a range of frequencies. Some of the problems are discussed below.

One of the problems with Eq(3) is that it is not clear as to what value of t_d should be used. When the frequencies, f_1 and f_2 , are sufficiently large, and for relatively large values of the critical damping ratio, the term consisting of t_d does not play a significant role. In those cases, it does not matter what value of t_d is used in the equation as long as it is reasonable. However, when the frequencies are small, the $2/(\pi t_d)$ term increases the effective value of the damping, as intended, thus giving a larger value of the ϵ_{12} . It is in this case the value of ϵ_{12} is quite sensitive to the value of t_d . Using the complete duration of the ground motion for t_d does not appear to be rational.

There are two other potential problems with modal frequencies are higher than the ground motion frequencies. In fact, the response time histories would be almost perfectly correlated, sufficiently apart. The writers found that for frequencies more than 1Hz, significant correlation between the two responses was discussed further in the subsequent section. Kennedy[2] is the following. When at a certain point, the correlation between the two responses decreases as predicted by Eq(3). It is quite likely that the high frequency response when the low frequency response reaches its peak. In the present study, however, no such trend was observed. In different segments of the ground motion, it is unlikely that the same segment of the ground motion has the same frequencies. Thus, the second potential problem is of a lesser degree.

A practical method of calculating the combined response, as presented in the following, is based on the method presented in the following section.

2. Proposed Method of Calculating

Based on the observation of the above, the following assumption is made: Any modal response, R_i^P , which has characteristics of the white noise[1], and a rigid input ground motion. It is further assumed that the combined response is given by

$$R_i^2 = R_i^P^2 + R_i^R^2$$

Thus we can write

$$\begin{aligned} R_i^R &= \alpha_i R_i \\ \text{and} \\ R_i^P &= \sqrt{1 - \alpha_i^2} R_i \end{aligned}$$

When two modal responses R_1 and R_2 combine, the combined response is given by

$$R^2 = R^R^2 + R^P^2$$

in which

$$\begin{aligned} R^R &= \alpha_1 R_1 + \alpha_2 R_2 \quad (\text{perfectly correlated}) \\ R^P^2 &= R_1^P^2 + R_2^P^2 + 2\epsilon_{12} R_1^P R_2^P \end{aligned}$$

In Eq(7) ϵ_{12}^P is the correlation coefficient and is defined by an equation similar to Eq(3)

$$\epsilon_{12}^P = \left\{ 1 + \left[\frac{f_2 - f_1}{\zeta(f_2 + f_1) + c_{12}} \right]^2 \right\}^{-1}$$

under seismic loads, the values of vibration. The variation in the "exact" way of combining the modes, R_1 , it can be shown that the quality

(1)

provide a lower bound. In many cases, the right hand side, commonly known as the combination, gives reasonably good responses can be considered to be relatively close, however, a combined response can be obtained if the modes have identical frequencies directly added in which cases

can write

(2)

Theoretically, one could assume that the response will be seen later, it is not represented by a finite duration. The obtained approximate equation can be written as

(3)

for $f_2 \gg f_1$. Although, Eq(3) is one of the problems are discussed

as to what value of t_d should be chosen, and for relatively large t_d does not play a significant role. Used in the equation as long as t_d , the $2/(\pi t_d)$ term increases with a larger value of the ϵ_{12} . The value of t_d . Using the value of t_d to be rational.

There are two other potential problems pointed out by Kennedy[2]. First, when the modal frequencies are higher than the maximum ground motion frequency, Eq(3), does not hold. In fact, the response time histories will be practically scaled input time histories, and would be almost perfectly correlated, in which case $\epsilon_{12} = 1.0$, even when f_1 and f_2 are sufficiently apart. The writers found that even at other frequencies in the range greater than 1Hz, significant correlation between modes existed. This particular aspect is discussed further in the subsequent sections. The other potential problem pointed out by Kennedy[2] is the following. When the modal frequencies are sufficiently apart, beyond a certain point, the correlation between the modal responses may start increasing, rather than decrease as predicted by Eq(3). Heuristically, the reason is simply that it would be quite likely that the high frequency response, can easily be maximum about the same time when the low frequency response reaches the maximum. In the numerical work done for the present study, however, no such trend was evident. The explanation for that is perhaps that different segments of the ground motion have different frequency contents. As such, it is unlikely that the same segment of the motion would excite two modes with widely disparate frequencies. Thus, the second potential problem was not found to exist to any discernable degree.

A practical method of calculating the correlation factor ϵ_{12} , which has a wide range of applicability, is presented in the following section. The numerical work on which this method is based, is presented in the subsequent section.

2. Proposed Method of Calculating ϵ_{12}

Based on the observation of the modal responses and their combinations, a heuristic assumption is made: Any modal response R_1 consists of two parts, a damped periodic response, R_1^p , which has characteristics similar to that obtained by using a finite segment of the white noise[1], and a rigid response, R_1^r , which is perfectly correlated with the input ground motion. It is further assumed that the two parts are mutually uncorrelated, i.e.

$$R_1^2 = R_1^p{}^2 + R_1^r{}^2 \quad (4)$$

Thus we can write

$$\begin{aligned} R_1^r &= \alpha_1 R_1 \\ \text{and} \quad R_1^p &= \sqrt{1-\alpha_1^2} R_1 \end{aligned} \quad (5)$$

When two modal responses R_1 and R_2 with frequencies f_1 and f_2 are to be combined, then the combined response is given by

$$R^2 = R^r{}^2 + R^p{}^2 \quad (6)$$

in which

$$\begin{aligned} R^r &= \alpha_1 R_1 + \alpha_2 R_2 \quad (\text{perfectly correlated}) \\ R^p{}^2 &= R_1^p{}^2 + R_2^p{}^2 + 2\epsilon_{12}^p R_1^p R_2^p \end{aligned} \quad (7)$$

In Eq(7) ϵ_{12}^p is the correlation coefficient for the damped periodic part of the responses, and is defined by an equation similar to Eq(3)

$$\epsilon_{12}^p = \left\{ 1 + \left[\frac{f_2 - f_1}{\zeta(f_2 + f_1) + c_{12}} \right]^2 \right\}^{-1} \quad (8)$$

(9)

The term c_{12} here replaces to be used with Eq(3) varies equations (2), (4) - (7) yield

(10)

close to the numerically high frequencies.

in the present study. The action of the critical damping values of α , which was found to values

(12)

should be taken equal to 1.0, could be set equal to -0.1. ratio and are given by

(13)

two single degree of freedom subjected to the same earthquake covariance σ_{12} are defined as

(14)

(15)

to the value of the duration statistically independent of the are covered in the assumed is not pursued any further here.

The intent of the present study is to arrive at the values of ϵ_{12} which can be used in conjunction with the response spectrum method of analysis. The response spectra which are commonly used in design, such as in ref[4] are based on a number of measured earthquake ground motions. Similarly, the present study is performed using ten such motion time histories, which are listed below

- Parkfield, 1966, N50E and N40W
- El Centro, 1940, NS and EW
- El Centro, 1934, NS and EW
- Taft, 1952, N21E and S69E
- Olympia, 1949, S04E and S86W

For any pair of frequencies f_1 and f_2 , the mean and the standard deviation of ϵ_{12} were calculated using the responses from the ten time histories.

When f_2 is infinite, $\bar{R}_2(t)$ is equal to scaled input motion time history. For any frequency f_1 , the factor α_1 is then given by ϵ_{12} as stated in Eq(11). The mean values of α were calculated for various frequencies and damping ratios, and were found to be in good agreement with Eq(12).

Fig 1 shows the comparison of ϵ_{12} values calculated using Eq(10), and those obtained from the average of the values calculated using the ten motion time histories, for the critical damping ratio of 1%. In Eq(10), actually calculated mean values of α 's were used. As shown, the agreement between the calculated values and those given by Eq(10) is good. The agreements for other damping ratios (2, 4 and 7%) were also found to be good. This in effect verifies the assumptions made in this study, viz, a modal response have two components: rigid and damped periodic and that they are uncorrelated. When the frequency f_1 is small, it is seen in Fig 2 that for certain range of f_2 values the correlation coefficients have a small negative value. The writers feel that perhaps these negative values can be attributed to a limited data in terms of using a small number of earthquake time histories. In any case, since these values are numerically quite small, writers did not attach much significance to them, and decided not to investigate them further.

Next, the effect of any error in the value of ϵ_{12} is investigated. It is noted that the value of the combined response would be most sensitive to any errors in the values of the correlation coefficients, when there are only two significant modal responses and when they are also equal in magnitude. The value of the combined response is then given by

$$R^2 = 2R_1^2 \times (1 + \epsilon_{12}), R_1 = R_2 \quad (16)$$

If a different value of the correlation coefficient, ϵ_{12}' , were used, a new value of the combined response, R' , would be obtained. The percentage change in the value of the combined response is given by

$$\Delta R\% = \left(\sqrt{\frac{1 + \epsilon_{12}'}{1 + \epsilon_{12}}} - 1 \right) \times 100 \quad (17)$$

There are two types of variations in ϵ_{12} of interest here. One is the dispersion in the value of ϵ_{12} represented by the standard deviation. The other is the error introduced because of using the idealized equations such as Eqs(10) and (12), which attempt to calculate the mean value of ϵ_{12} . In order to investigate the accuracy of Eqs(10) and (12), and the effect of dispersion in the value of ϵ_{12} , Eq(17) is used to calculate the maximum error in the response value by substituting in the denominator the value of ϵ_{12} from

Eqs (10) and (12), and in the numerator substituting for ϵ_{12} the calculated mean + one standard deviation. The maximum of all ΔR values for $f_2/f_1 = 1$ to 10 for four f_1 frequencies are given below.

Frequency f_1 Hz	Maximum Error at ζ			
	.01	.02	.04	.07
0.1	11.0	10.5	9.8	8.8
1.0	3.4	3.9	3.5	3.4
10.0	5.0	6.8	7.1	5.6
21.5	4.6	2.0	0.5	0.0

The highest error of 11% is encountered at the lowest frequency and damping. Generally, as the frequency and the damping go up the error diminishes until it becomes zero. Since the worst cases are considered, in most practical situations the actual errors are likely to be much smaller.

4. Conclusions

A modal response can be assumed to consist of two components which are uncorrelated, a rigid component and a damped periodic component. Whereas the all the modal rigid components are perfectly correlated, the correlation between the modal damped periodic components is a function of the modal frequencies; an expression for the correlation coefficient ϵ_{12}^P , which is similar to one given in Ref[1], is given, Eq(8). On this basis then, the correlation coefficient for modal responses is evaluated which is given by Eq(10). It is shown that the theoretically calculated modal correlation coefficients are in good agreement with those calculated directly from the response time histories of single degree of freedom systems subjected to actual ground motion histories. This in effect verifies the assumptions made in the present study.

5. References

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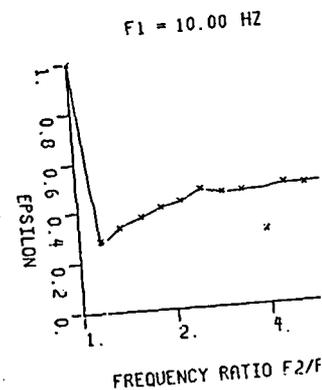
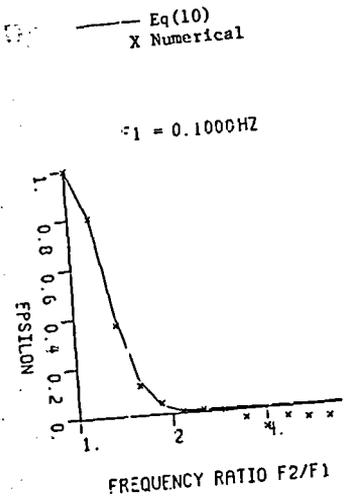


Figure 1 - Comparison of with Eq(10), $\zeta = 0.01$

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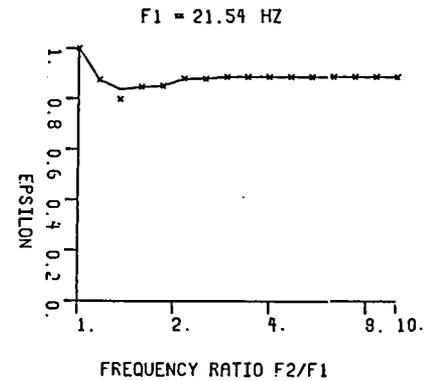
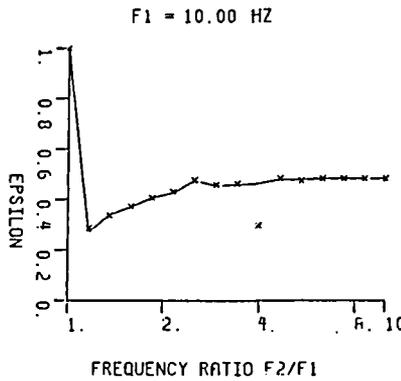
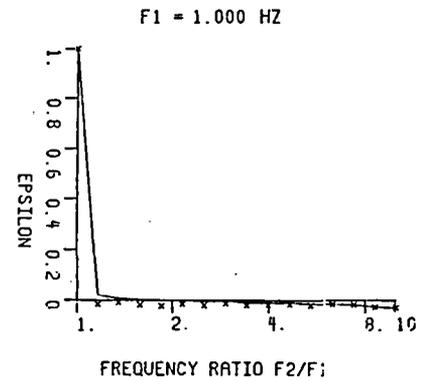
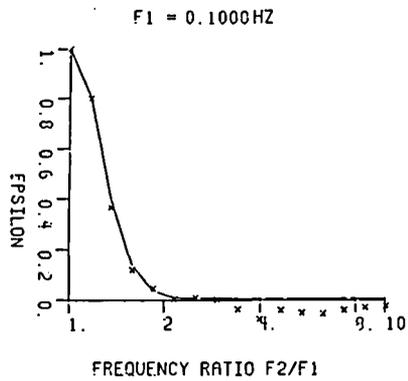


Figure 1 - Comparison of Numerically Calculated Correlation Coefficients, c
 with Eq(10), $\zeta = 0.01$