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RESPONSES OF LINEAR SYSTEMS TO CERTAIN TRANSIENT DISTURBANCES

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Synopsis. Earthquakes are idealized as segments of modified stationary gaussian processes. Methods are developed for calculating responses of linear systems including systems that do not have classical modes. These methods are more accurate and, for some systems, much more so than the root of the sum of squared modal responses.

It is shown that many current code provisions for torsion in buildings and for overturning-moment reduction in buildings and chimneys are inadequate. More realistic criteria are proposed.

Introduction. Modal analysis of systems that behave linearly requires knowledge of the response spectra of the family of disturbances for which design is intended and a criterion for combining the modal responses. Such criteria have thus far been confined to identifying the total response with that in the fundamental mode of vibration, with the sum of numerical values of modal responses, or with the root of the sum of these responses squared.(ii) Yet there are structures for which even the last method is grossly inadequate. This is usually the case when two or more natural frequencies of vibration are approximately equal to each other. And many systems do not have classical modes.(iii)

The present paper develops methods of analysis more accurate and general than the ones used up to now. The new methods are based on an idealization of earthquakes as stationary gaussian processes, but results are modified to recognize the transient character of actual disturbances.

Rather than treating the probability distribution of a response to a family of motions we deal mostly with the expectation of the response or with a close approximation thereto, as though this were a deterministic variable. This is justified to some extent because there is vast uncertainty about the characteristic parameters of future earthquakes, and this overshadows the variability of responses to individual earthquakes about the mean.[2] It is still desirable to have an approximate description of the distribution of deviations about the mean; the question is touched upon in this paper.

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(ii) Linear combinations of the last two have been proposed but they lack generality and have almost the same limitations as the third criterion.

(iii) Caughey[1] has given a necessary and sufficient condition for a linear system to have natural modes in the classical sense. This condition is not met in general. Still, the assumption that a system has such modes is acceptable for sufficiently small damping.

The methods developed here allow arriving at interesting conclusions concerning the seismic behavior of several types of structures.

Transfer Functions. We define the transfer function $\psi_q(t)$, as a system's response, $q(t)$, to a Dirac-delta accelerogram, $\ddot{x} = \delta(t)$. (Here t is time and x is the ground motion.) Linear behavior insures that, given an arbitrary disturbance,

$$q(t) = \int_0^t \ddot{x}(\tau)\ \psi_q(t-\tau)\ d\tau \tag{1}$$

For systems having classical modes of vibration we can write

$$q = \sum_i q_i \qquad \psi_q = \sum_i \psi_{qi} \tag{2}$$

where q_i is the response in the ith natural mode and ψ_{qi} is the modal transfer function for q_i.

In conservative systems the modal transfer functions are periodic. Functions ψ_q are also periodic when all the natural periods are submultiples of a multiple of the system's fundamental period of vibration, T_1. The smallest such multiple is the period of the transfer functions.

In subcritically damped structures the modal transfer functions are damped sine functions of time.

For damped systems having classical modes the ψ's are damped periodic waves (they change in amplitude but not in shape from one cycle to another) if the damped natural periods meet the foregoing condition and the damping ratios of the natural modes are proportional to the corresponding natural periods.

Responses to White Noise. Consider a single-degree linear system potentially subjected to a family of accelerograms which are a segment of white noise, of duration s. Let $Q = \max_t |q(t)|$, let subscript 0 refer to an undamped system, E denote expectation, and T_1 the system's undamped natural period. An approximate solution is available[3] for the distribution of $Q/E(Q_0)$ which is valid when $s \gg T_1$ (or even when $s \geq T_1$).

The probability, P, that $|q(t)|$ exceed a given response, Q, may be called the probability of failure. In a given system, Q can be expressed as a function of P. It is found that

$$\lim_{P \to 0} \frac{Q}{Q_0} = \left[\frac{1 - e^{-2\zeta_1\omega_1 s}}{2\zeta_1\omega_1 s}\right]^{1/2} \tag{3}$$

where ζ_1 and ω_1 are the system's coefficient of damping and natural circular frequency respectively.

The ratio $E(Q)/E(Q_0)$ is available in graphical form. Approximately,[4]

$$E(Q)/E(Q_0) = (1 + \zeta_1\omega_1 s/2)^{-1/2} \tag{4}$$

This solution assumes that a structure survives as long as $|q|$ does not exceed a critical value. Problems of this nature are far more difficult than those of finding the distribution function of the response at a given instant. Indeed, the solution we have quoted for single-degree systems is only approximate[(iv)] and no analytical solution is known for other Ψ functions. Consequently, attempts have been made to estimate the distribution of maximum responses, or their expectations, with basis on the responses at a specified instant to stationary processes or to finite segments thereof.

Stationary processes are attractive because of the simplicity of their mathematical treatment. However, we cannot speak of the maximum responses of a linear system to a white noise or stationary gaussian process because such a response is infinite with probability one. Adjustments are necessary to convert the responses at an arbitrary instant to a stationary process into the maximum responses to transient disturbances such as earthquakes. A more direct approach will base the desired solution on that for the responses at a specified instant to a transient disturbance.

Consider the class of transient disturbances $\ddot{x} = f(t)\, w(t)$ where f is a deterministic function and w is white noise. The distribution of responses of every linear system at any specified instant to such a disturbance is gaussian, with expectation zero. Consider now the class of systems with damped periodic transfer functions. If, for a given f, the damped period of Ψ_q, T', is sufficiently short, the ratio of successive maxima of $|q|$ will have a distribution practically independent of the shape of Ψ_q. Therefore, the distribution of $Q/E(Q_0)$ (and, hence, the curves for $E(Q)/E(Q_0)$ for single-degree systems will be directly applicable if we replace $\zeta_1\omega_1$ with $\zeta_i\omega_i$, which is a constant for each system.

It is shown from eq 1 that, for the systems at hand,

$$Q^2 \propto \int_{t_1}^{t_1+T'} \Psi_q^2 \, dt \tag{5}$$

where Q is the response associated with any given probability of failure and t_1 is such that $\Psi_q(t)$ is damped periodic for $t \geq t_1$.

When q is the pseudovelocity (ω_1 times the displacement relative to the ground) of a conservative single-degree system, Ψ_q is $\sin \omega_1 t$. The integral in eq 5 is then $T'/2$. Hence,

$$Q = V_0 \left(\frac{2}{T'} \int_{t_1}^{t_1+T'} \Psi_q^2 \, dt\right)^{1/2} \tag{6}$$

where V_0 is the undamped spectral pseudovelocity associated with the given probability of failure, that is, the <u>design pseudovelocity</u>.

Now, if Ψ_q is periodic, Ψ_{qi} are orthogonal in every interval of

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(iv) There us room for doubt[5] concerning the validity of some intermediate steps in the derivation of the distribution of $Q/E(Q_0)$ but the results have been amply confirmed.

duration equal to the period of ψ_q. It follows from eq 5 that, for systems having periodic transfer functions of sufficiently short period,

$$Q^2 = \sum_i Q_i^2 \tag{7}$$

where Q_i is the design response in the ith natural mode. And since the distribution of $Q/E(Q_0)$ is practically independent of ψ_q when T' is sufficiently short, we conclude that eq 7 also applies to the class of systems that we are considering.

The solution that we have mentioned for single-degree systems corresponds to responses to finite segments of white noise with $T' \ll s$ but the foregoing conclusions apply to solutions for any functions f satisfying the restrictions in question.

So as to widen the type of linear system that we can treat, direct the attention now to white noise proper ($f = 1$). Since the distribution of $q(t)$ is gaussian, $E|q(t)| \propto \{E[q^2(t)]\}^{1/2}$. The value of $|q(t)|$ associated with a given probability that it be exceeded is also proportional to $\{E[q^2(t)]\}^{1/2}$. This suggests assuming that the design value of the maximum response to a transient disturbance of form $\ddot{x} = f(t)\, w(t)$ is proportional to $\{E[q^2(t)]\}^{1/2}$, where this q is the response to white noise at an arbitrary instant. From the same reasoning which led to eq 5 we conclude that

$$E[q^2(t)] \propto \int_{-\infty}^{\infty} \psi_q^2(t-\tau)\, d\tau = \int_0^{\infty} \psi_q^2(t)\, dt$$

Therefore,

$$Q^2 \propto \int_0^{\infty} \psi_q^2\, dt \tag{8}$$

When q is the pseudovelocity of a single-degree system, the second member in eq 8 gives $1/2\zeta_1\omega_1$. The result differs appreciably from that for the square of the maximum response to a finite segment of white noise, which depends on the duration of the motion and on the probability of failure. Let us adjust the percentage of damping coincide for the expected response. To this end we shall make use of eq 4. We seek, then, the "equivalent" percentage of damping, ζ_i', which will make $2\zeta_i'\omega_i \propto 1 + \zeta_i\,\omega_i\, s/2$, where we have replaced subscript 1 with i. The answer must be such that $\zeta_i' \doteq \zeta_i$ when s tends to infinity. We find

$$\zeta_i' = \zeta_i + 2/\omega_i s \tag{9}$$

Thus, we may use eq 8 with the increased damping ratios given by eq 9 in the system's natural modes of vibration. This will be correct for single-degree systems and can be expected to be satisfactory for a wide class of multidegree systems.

If we assume that the modal transfer functions are damped trigonometric functions and if all the damping ratios are small compared with unity, eqs 2 and 8 lead to the approximate relation

$$Q^2 = \sum_i Q_i^2 + \sum_{i \neq j}\sum Q_i Q_j /(1 + \varepsilon_{ij}^2) \tag{10}$$

where $\varepsilon_{ij} = |\omega_i' - \omega_j'|/(\zeta_i'\omega_i + \zeta_j'\omega_j)$, ω_i' is the ith damped natural circular frequency, and Q_i is to be taken with the sign that $\Psi_{qi}(t)$ has when it attains its maximum numerical value.

Equation 10 improves over eq 7 when ζ_i is not inversely proportional to ω_i (usually it is not). The difference between eqs 7 and 10 tends to zero when the natural frequencies are well differentiated, the damping ratios are small, and the ratios s/T_i' are large. To illustrate the influence of the second term in eq 10, fig 1 has been prepared for a two-degree system having $\omega_1 = \omega_2$ and $\zeta_1' = \zeta_2'$.

When Ψ_q is not damped periodic, the distribution of Q may be assumed to lie between the one for damped periodic Ψ_q and the one which results from assigning each Q_i the gaussian distribution associated with the corresponding Ψ_{qi}.

Although derivation of eq 10 has been quite heuristic, we notice that it cannot give very large errors in two-degree systems when $\varepsilon_{12} << 1$; when $\varepsilon_{12} >> 1$ and $|Q_1| \doteq |Q_2|$; nor when $|Q_1| >> |Q_2|$ whatever the value of ε_{12}. Also that it satisfies the obvious restrictions $Q \doteq |Q_1 - Q_2|$ in these systems and $Q \leq \sum_i |Q_i|$ in all cases.

This expression is limited to systems that have classical modes of vibration. Besides, it is easy to imagine Ψ functions having a very long period and small degrees of damping, for which eq 10 would give poor results. Further to widen the range of applicability of our results we shall take as basis for estimating Q the responses at a specified instant to a transient disturbance of the type $\ddot{x} = f(t)w(t)$. Proceeding as for stationary disturbances we anticipate that $Q^2 \propto \int_{-\infty}^{t} f^2(t-\tau)\, \Psi_q^2(t-\tau)\, d\tau$. If we take $t = 0$ and $f = \exp(-t/s)$, this expression can be written in the form $Q^2 \propto \int_0^{\infty} e^{-2t/s}\, \Psi_q^2(t)\, dt$. In this manner we obtain answers that practically coincide with those of eq 4 for single-degree systems having $T_1 << s$ and $\zeta_1 << 1$. Better results can be expected by taking

$$Q^2 \propto \max_t \int_0^t f^2(\tau)\, \Psi_q^2(t-\tau)\, d\tau \tag{11}$$

with $f = \exp(-ct)$. For single-degree systems this gives $Q^2 \propto \max_t \int_0^t e^{-2c\tau}\, \Psi_q^2(t-\tau)\, d\tau$ which approximates the "exact" solution for finite-segment disturbances if we make $c = 2/s$. In fact, because of the variability of intensity per unit time this is very likely a better idealization of earthquakes than the finite segment. The matter will, however, not be explored further here because eq 11 does not lend itself to simple modal analysis, which we wish to use in the applications.

The expression we quoted for the limit of $E(Q)/E(Q_0)$ as $P \rightarrow 0$ (eq 3) is also valid for $\{E[q^2(s)]/E[q_0^2(s)]\}^{1/2}$ where q is the response of a single-degree system to a finite segment of duration s of white noise. This suggests taking the responses at the end of such a segment as bases for computing the maximum responses to earthquakes. We would make the segment's duration such as to give the desired results. The approach lends itself both to analysis in terms of transfer functions and to modal analysis. Letting s' denote the segment's duration we would have $Q^2 \propto \max_t \int_0^t \Psi_q^2\, dt$ with $t \doteq s'$. If $T << s'$, there is little error in taking $t = s'$. The same argument applies to the integrals of the squared modal Ψ functions. However, in taking the second member of eq 3 proportional to that of eq 4 we cannot find a unique relation between s' and s. If $\zeta_i\omega_i s$ is known to lie within a certain range, an approximate s' can be found to satisfy this relation. Thus, when $\zeta_i\omega_i s \gg 1$ we find $s' \doteq s/4$. Even for a system without classical modes, its Ψ functions may be approximated by the combination of damped trigonometric functions to conclude that this approximation may be satisfactory.

Responses to Gaussian Processes and to Earthquakes. Consider a single-degree system of period T', subjected to a family of disturbances which are segments of duration s of a stationary gaussian process. If the motion's power spectral density varies smoothly with period in the neighborhood of T', the distribution of the system's maximum responses to this family of disturbances lies close to the distribution of its maximum responses to a segment of white noise of duration s and whose power spectral density equals that which the segment of gaussian process has at T'.[6] Therefore the expressions we have derived for combining modal responses apply when the disturbances are a segment of a stationary gaussian process, with essentially the same limitations, provided the power spectral density, or the spectral pseudovelocity, is sufficiently smooth in the neighborhood of the pertinent natural period.

Particularly for systems without classical modes we may use an approach parallel to the one which assumes that maximum responses to the disturbance are proportional to the root of expected squared responses at a specified instant to $x = f(t)\, w(t)$, except that we now replace $w(t)$ with the corresponding stationary gaussian process. The use of Fourier transforms in the manner of Tajimi[7] is then indicated.

Extremely rigid systems require a special treatment, as the pseudovelocity spectrum is not smooth at the origin. We shall not delve into this matter, as solutions to the examples we will examine are not strongly affected thereby.

In extrapolating to real earthquakes the conclusions derived for gaussian processes, several differences between the two must be borne in mind. Some important earthquakes are too short and their accelerograms too simple to admit idealization as gaussian processes. Nonetheless, with due caution the present results should be useful in calculating the responses of multidegree systems.

Application to Uniform Shear-Beams. The responses of shear beams having constant stiffness and mass per unit height are often used as indicative of the behavior of tall buildings. The transfer functions for shear and overturning moment at various elevations of an undamped beam are depicted in fig 2, where k and m denote the stiffness and mass of a unit length of beam and h is the beam's height.

Using eqs 10 and 7, the responses associated both with a hyperbolic and with a flat acceleration spectrum have been computed. The hyperbolic spectrum has a cutoff that makes A = constant for $T \leq 0.1T_1$. It has been assumed that ζ_i = 5 percent in all the natural modes and that s = 20 T_1 and that the spectra already include effects of damping. Results appear in figs 3 and 4 including a comparison with the shears and moments computed from statically applied accelerations that vary in proportion to height above ground. The base shears for the two design spectra and the static method have all been computed for the same spectral acceleration, A_1, associated with fundamental mode.

These results lead to the following conclusions:[8] The base shear is from 0 877 (for a hyperbolic spectrum) to 0.816 (for a flat spectrum) times $(A_1/g)W$, where g is the acceleration of gravity and W is the building's weight. (Shear beams are a special case of structures in whose ground story the deformation is proportional to the base shear. In such structures the design base shear does not exceed $(A_1'/g)W$, where A_1' is the acceleration for the structure's fundamental period in a hyperbolic spectrum whose ordinates, at the structure's natural periods, are not smaller than those of the design spectrum. In the cases considered we may take $A_1' = A_1$. Thus it is not surprising to find the design shear smaller than $(A_1/g)W$.)

The static method is satisfactory for computing the distribution of shears in buildings idealized as uniform shear-beams except near the top of very tall buildings on firm ground, subjected to nearby earthquakes (this condition approaches hyperbolic acceleration spectra). (The same conclusion does not apply to nonuniform buildings in general.)

The overturning moments computed dynamically are smaller at all elevations than the corresponding integrals of the shear diagrams. The reduction factor ranges between 0.886 and 0.989 if we take for comparison the integral of the dynamically computed shears,[v] and between 0.773 and 0.779 if we take that of the shears computed statically.

A slightly more drastic reduction can be defended on the basis of inelastic behavior. This justifies the rule[9] that the moment at any elevation be taken equal to the shear at that section times the distance to the centroid of the building's portion lying above the elevation considered, which for uniform shear-beams gives a reduction factor varying between 0.75 at the base and 1.0 at the building's top. Certainly the reduction factor[10] $J = 0.5T_1^{-2/3}$, but ≥ 0.33, cannot be justified. Such drastic reduction comes about from an overoptimistic interpretation of structural behavior and from an analysis that takes much lower spectral accelerations for moment than for shear.

Behavior has been misinterpreted because of the overconservativeness that has prevailed in the design of columns under axial force;

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[v] Theoretically this factor cannot be smaller than the value we would obtain if the overturning moment were due only to the fundamental mode, which would give 0.775 for a hyperbolic spectrum.

also out of semantic reasons: "overturning" seems to imply toppling in the style of some buildings in Niigata, while conditions leading to such occurrences are rarely met; most failures due to high overturning moment consist in column fracture[11] (whose direct cause is difficult to diagnose when combined with flexure and shear) and in damage to various structural members because of stress redistribution traceable to foundation deformations.

Application to Chimneys. The same method of analysis has been applied to cylindrical chimneys idealized as uniform flexural beams fixed at their base.[12] Gravity effects, shear deformations, and rotational inertia were neglected. (These matters are often important and should be incorporated in future studies.) The transfer functions are not periodic. Yet there is little difference in the shears and moments computed from eqs 7 and 10 (less than 1.5 percent of the base shear and less than 0.4 percent of the base moment).

Again two acceleration spectra were considered: hyperbolic (with a cutoff at $T = 0.1T_1$) and flat. All modes were assumed to have 5 percent damping. Earthquakes were idealized as finite segments of stationary gaussian processes of duration $s = 4.78T_1$.

Computed shears and moments are shown in figs 5 and 6. Simple expressions can be used to approximate them.[12] Again we find that the code formula for reduction of the integral of the shear envelope to obtain the base overturning moment ($J = 0.6T^{-1/2}$ but ≥ 0.4) often errs seriously on the unsafe side (the computed J's are 0.587 with hyperbolic spectra and 0.972 with flat spectra) and for the same reason as in buildings. Besides, the analyses of chimneys[10,13] which serve as basis for design compute shears by the adding the numerical values of shear in the first three natural modes, and this series diverges. Results of such analyses are not defensible.

For spectra of arbitrary shape, one need not take design shears greater than those for either a hyperbolic or a flat spectrum that constitute upper bounds to the design spectrum in the range of periods equal to or shorter than the chimney's fundamental period. This consideration leads to a simple method for specifying a conservative shear envelope for an arbitrary spectrum (fig 7). The same criterion may be used for the design moments.

Analysis of several tapered chimneys[14] subjected to records of actual earthquakes confirms that the root of the sum of squared modal responses gives a satisfactory approximation to the total response. The base shear coefficients are given with adequate accuracy by the same expression as for cylindrical chimneys, and again the moment reduction coefficients appreciably exceed the code J's.[15]

Actually there is no need for moment reduction coefficients in chimneys, as simple "static" rules have not been developed for computing the shear envelopes.

Application to Single-Story Buildings with Torsion. The methods of analysis we have presented are particularly useful in connection with buildings subjected to torsion, since dynamic and static torques may differ greatly and since pairs of natural frequencies may lie very close to each other, so that eq 7 may entail large errors. The problem has been dealt with for single-story buildings with eccentricity only in the direction perpendicular to the ground motion, so that only two natural modes are excited by the earthquake.[15]

Responses to the hyperbolic and flat spectra were studied. The two natural modes were assumed to have 10 percent of equivalent damping.

In order to study separately the effects of shear and torsional moments structures like the one in fig 8 were analyzed. Results in figs 9 and 10 refer to the shear force and the magnification factor for eccentricity, relative to its "static" value, e_s (see fig 8), including a comparison with taking the total response equal to the root of the sum of the squared modal values responses for different values of the parameters.

This study shows that shear forces computed as the root of the sum of squared modal responses (eq 7) are practically equal to those obtained from the more accurate eq 10 and that it is always conservative to ignore torsion in computing base shear. However, eq 7 seriously overestimates torques in ranges where natural frequencies are nearly equal to each other, but in these same ranges even the more accurate criterion yields much higher torques than may be obtained from the product of e_s and shear. Results for buildings of arbitrary shape indicate that under some conditions much higher magnification factors may obtain for the design shear on certain walls or frames.

One way of overcoming these deficiencies of the static method consists in specifying that eccentricity be taken equal to e_s or $1.5e_s$, whichever is more severe, and that the method be applied only when $K/k \geq 36J/m - 25e_s^2 m/Jm$ where K = torsional stiffness about the center of mass, J = polar moment of inertia about the same center, k = translational stiffness, and m = mass.[15]

The present study is confined to structures having natural periods that are neither too short nor too long. When the fundamental period is extremely short, all maximum accelerations approach the ground's maximum acceleration, so that the static method of analysis yields satisfactory results; in other words, design responses approach the algebraic sum of the modal responses. When the periods of natural modes that contribute significantly to the overall responses are very long, the design responses approach the numerical sum of the modal responses.

Additional research should cover the matters mentioned in the foregoing paragraph as well as such questions as earthquake resistant design of chimneys including effects of shearing deformations, rotational inertia, gravity forces, foundation compliance, and inelastic behavior; torsion in multistory buildings with eccentricity in two directions including effects of inelastic behavior, torsional excitation, and randomness of mass and stiffness distributions; and behavior of appendixes on shear-type buildings.

Acknowledgment. Part of this paper is based on ref 16.

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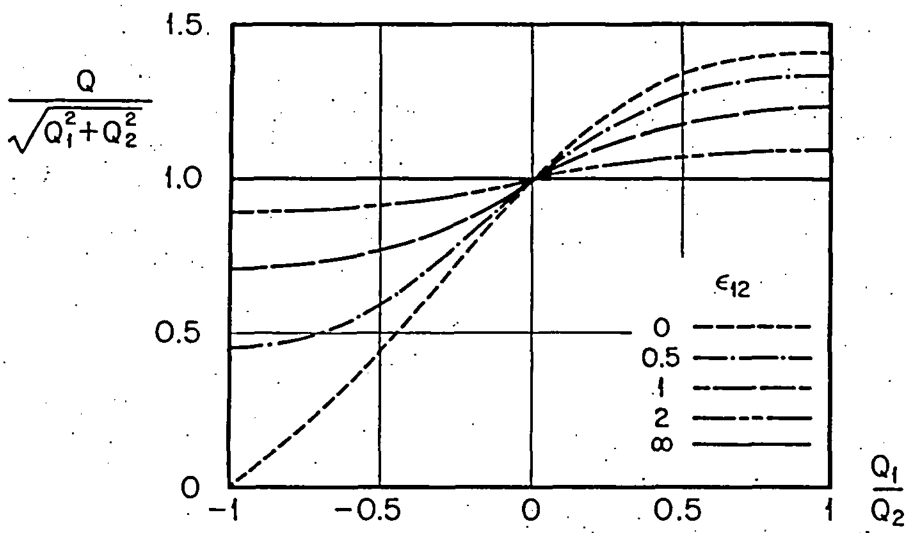

Fig I Responses of a system with two degrees of freedom

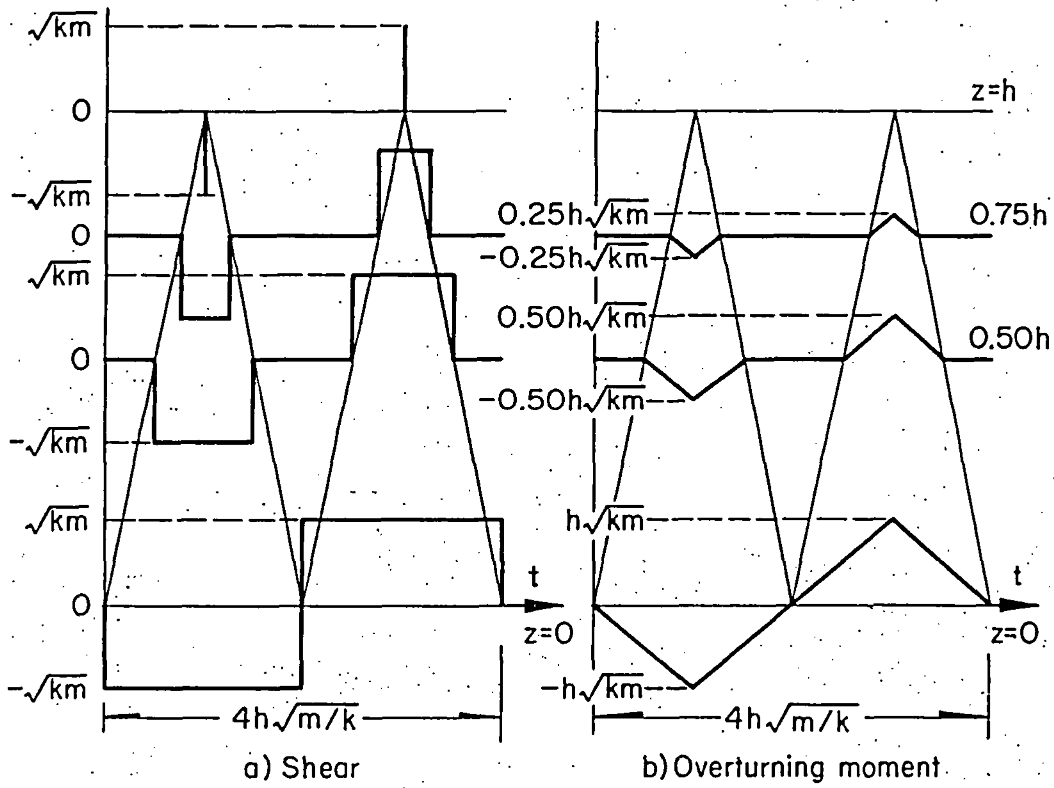

Fig 2 Transfer functions for uniform shear beams

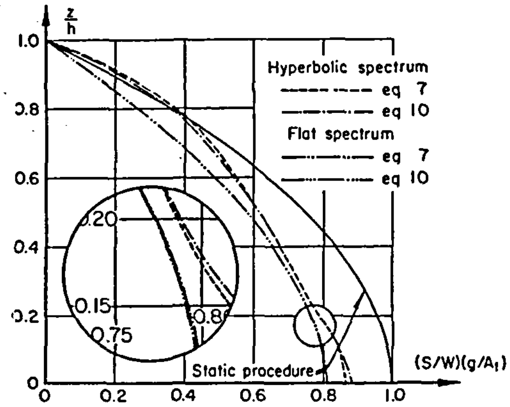

Fig 3 Distribution of shear forces

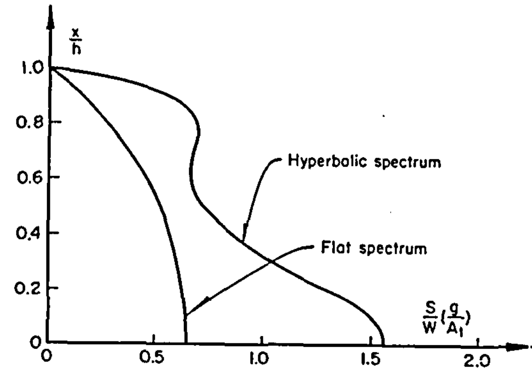

Fig 5 Distribution of shear forces

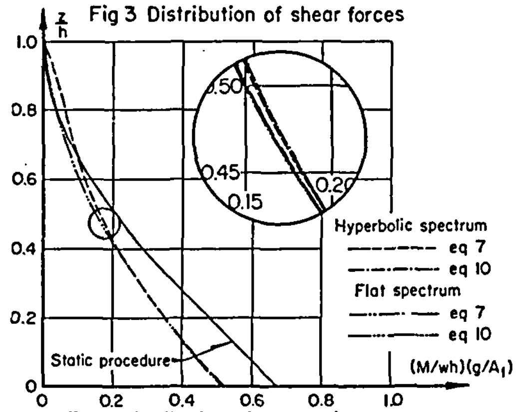

Fig 4 Distribution of overturning moments

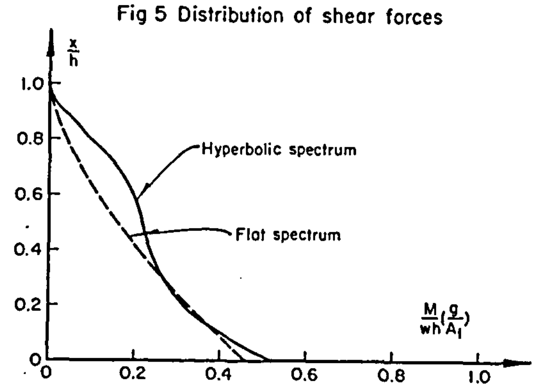

Fig 6 Distribution of overturning moments

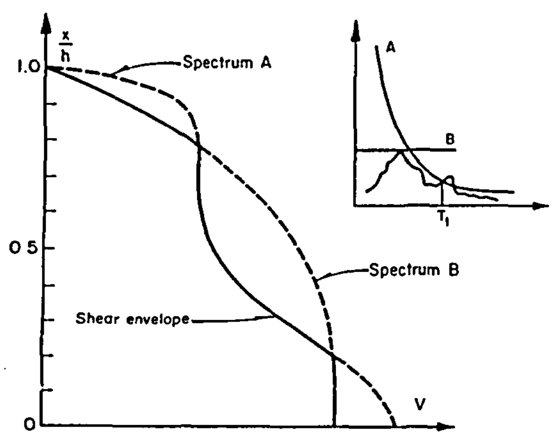

Fig 7 Shear envelope for arbitrary spectrum

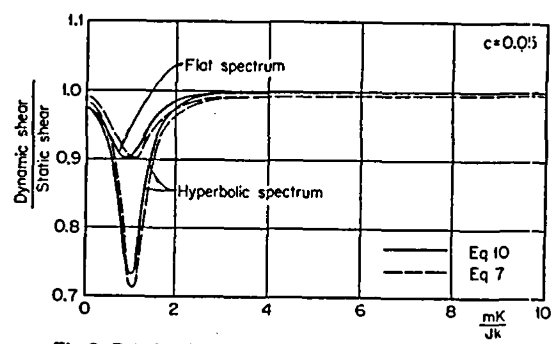

Fig.9 Relation between the dynamic and the static shears
Lenght/width = 0.5

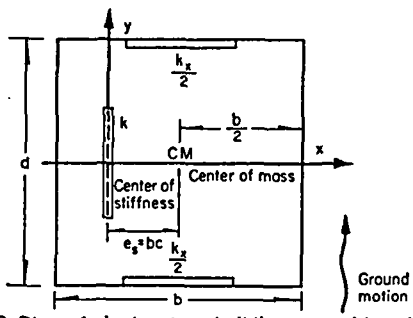

Fig 8 Plan of single-story buildings considered

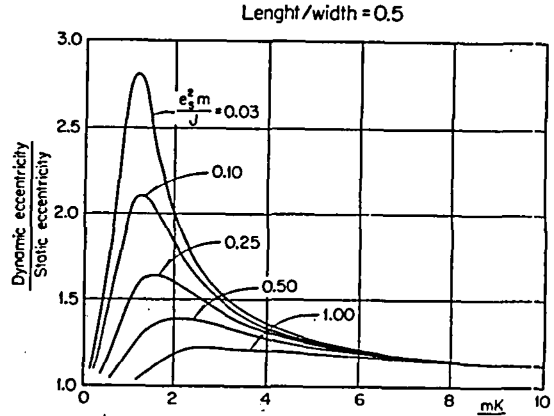

Fig.10 Magnification factor for eccentricity