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ASEISMIC DESIGN OF ELASTIC STRUCTURES FOUNDED ON FIRM GROUND

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ASEISMIC DESIGN OF ELASTIC STRUCTURES FOUNDED ON FIRM GROUND

L. E. Goodman,[1] E. Rosenblueth,[2] and N. M. Newmark[3]

SYNOPSIS

A rational basis for the design of earthquake-resistant structures is developed. Structures designed on this basis will have uniform strength in the sense that each element will have an equal probability of successfully withstanding strong-motion earthquakes of the general type recorded by the U. S. Coast and Geodetic Survey on the North American continent. The theory is based on inferences drawn from accelerograms of strong-motion earthquakes and on the theory of probability. While the general method of approach is capable of wider extension, conclusions of the present study are limited to structures whose behavior is elastic and which are founded on ground of stiffness comparable to that of sites at which reliable earthquake accelerograms have been recorded.

INTRODUCTION

There is an important branch of engineering seismology concerned with the analysis of structures subjected to earth tremors. In recent years impetus has been given to this work by the availability of reliable accelerograms of some fourteen strong-motion earthquakes recorded by the U. S. Coast and Geodetic Survey. The complexity of the recorded ground motions, however, has been a handicap to analysis and (even more) to design. Details of future ground motions can hardly be anticipated by the designer. Progress in analysis has leaned heavily on the construction by means of the torsion pendulum (1)[4] or digital and electronic analogy computing machinery (2) of 'response spectrums' giving the maximum response (stress, velocity, or displacement) of a single-degree-of-freedom structure as a function of its natural period. More or less idealized spectrum diagrams have been proposed as a basis for structural design (3, 4).

An entirely different approach to the design problem is suggested by the discovery of G. W. Housner A.M. ASCE (5) that, so far as their effects on structures are concerned, destructive earthquakes may be regarded as successions of perfectly random motions. Reference here is to local effects at surface sites near the epicenter and not to disturbances recorded seismographically after having been propagated over long distances. The latter have, of course, been studied extensively. Local effects, on the other hand, determine whether a given structure will or will not fail. They are influenced in a complex way by local geological and topographical conditions. The discovery

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that perfectly random pulse sequences produce spectrums which do not differ significantly from those of recorded accelerograms opens the way to a systematic analysis of the probable effects of such motions on structures.

The chief advantage of the approach advanced here lies in its ability to establish a basis for design in which every part of a structure will have an equal probability of successfully withstanding ground motion.[5] What that probability ought to be—or in other words, the intensity of earthquake for which a structure should be designed—is a question the answer to which will always depend upon engineering judgment based on accumulated experience of seismic destructiveness in various regions of the world, and on the importance of structural integrity to public safety, as well as on economic considerations. In this respect aseismic design resembles the design of dams to resist flood contingencies. The present approach based on the theory of probability also makes possible the prediction, in a statistical sense, of the relative magnitudes of earthquake effects in different existing structures and in different parts of any one structure.

Consideration has been limited in the present work to structures whose behavior can be idealized as elastic or which are subject to viscous damping. Conclusions presented are applicable only to structures that rest on formations having a considerable stiffness, comparable to that of the ground where reliable accelerograms have been recorded. While the method of analysis has been extended to structures resting on soft formations, the somewhat different response of such structures and the additional assumptions which are required indicate the desirability of a separate treatment.

STRUCTURAL RESPONSES

For convenience of reference certain relationships of use in connection with the motion of structures are stated in this section. Developments of the theorems are available in texts dealing with the theory of structural vibrations (6, 7).

Figure 1 is a schematic representation of any linearly elastic structure having a simple degree of freedom. If the system starts from rest at time $t = 0$ and the support undergoes a motion $x_g(t)$, the displacement, x, of the mass relative to the support is, at time t,

$$x = x_s - x_g = \frac{-1}{p}\int_0^t \ddot{x}_g(\tau) \sin p(t-\tau)\, d\tau . \qquad (1)$$

Here superscript dots indicate differentiations with respect to t.

x_s = absolute displacement of the mass

$p = \sqrt{k/m}$ = natural (circular) frequency of system

k = static force required to produce unit relative displacement of the mass and its support

m = mass of system

The natural period of the system is

$$T = 2\pi/p . \qquad (1a)$$

5. Uniform-Safety design has also been advanced in a manuscript "Analysis of Seismic Forces" 1950, by G. W. Housner, A.M. ASCE.

The dimensional units in which these quantities are measured must, of course, be mutually consistent. In the English system of units, for example, x_s, x_g and x would be measured in inches, p in radians per second, k in lb. per foot, m in lb. sec² per inch, $\ddot{x}_g$ in inches per second per second, t and T in seconds. The dummy variable t, which disappears on integration, stands for time.

Equation (1) can be used for a study of actual structures if the effects of damping, ground coupling, and higher modes of vibration can be neglected. Under these circumstances the support in Fig. 1 represents the foundation of the structure; the spring represents a group of flexible elements and the concentrated mass stands for the mass of the structure and of the loads which it supports.

Ideal elastic systems having a single degree of freedom and no energy losses are referred to as simple structures throughout this analysis.

Engineers are agreed that in general several modes of vibration of structures are transiently excited by earthquakes (8). To take account of additional degrees of freedom, the natural modes can be treated separately as individual simple structures. Total responses—by which is meant either stresses or deformations at any point—are, at any given time, the algebraic sums of those associated with the several natural modes considered. For example, the displacement x, relative to the ground, at any point of a structure, can be computed by means of the expression

$$x = \sum_{i=1}^{N} -\frac{a_i}{p_i} \int_0^t \ddot{x}_g(\tau) \sin p_i (t-\tau)\, d\tau \qquad (2)$$

where N = the number of degrees of freedom of the structure = number of modes considered.

p_i = natural (circular) frequency of the ith mode

a_i is a dimensionless number, the excitation coefficient of the mode

and the symbol $\sum_{i=1}^{N}$ indicates that the sum of terms of this kind for all the modes i = 1,2,...N is to be taken. This equation can be written in the form

$$x = \int_0^t \ddot{x}_g(\tau)\, \varphi(t-\tau)\, d\tau \qquad (3)$$

where ϕ is a convenient shorthand symbol for the sum of a number of sine functions

$$\varphi(t-\tau) = \sum_{i=1}^{N} -\frac{a_i}{p_i} \sin p_i (t-\tau)\, d\tau. \qquad (3a)$$

It may be noted that the first part of the integrand of Equation (3) $\ddot{x}_g$, involves only the nature of the ground motion and is independent of the structure in question, whereas the second part, ϕ(t-t) depends only on the properties of the structure and not on the particulars of the earthquake.

The function ϕ may or may not be periodic. In some instances, e.g., the shear deformations of tall uniform structures, the frequencies of all higher modes are integral multiples of the fundamental frequency. In this case ϕ is periodic and its period is then the same as the fundamental period of the structure.

When $\ddot{x}_g(t) = 0$ for all values of t greater than, say, t', Equation (3) can be written

$$x(t) = \int_0^{t'} \ddot{x}_g(\tau)\ \varphi(t-\tau)\ d\tau\ , \qquad t \geqslant t' \tag{3b}$$

whereupon, integrating by parts and recalling that the structure starts from rest, one finds, for $t > t'$

$$x(t) = \dot{x}_g(t')\ \varphi(t-t') - \int_0^{t'} \dot{x}_g(\tau)\ \frac{d\varphi}{d\tau}\ d\tau \tag{3c}$$

Setting $\dot{x}_g(t') = 1$ and letting t' approach zero, one can see that the displacement response x(t) to an instantaneous unit increment in ground velocity occurring at $t = 0$ is numerically identical with $\phi(t)$. Sudden changes of ground velocity will be referred to as acceleration pulses.

Stresses and bending moments, as well as structural deformations, can be obtained, in the usual way in which displacements and stresses are related, in terms of linear combinations of displacements measured relative to the ground. Expressions similar to Equation (3) can be established for any of these types of response or for their first derivatives with respect to time. The functions ϕ which appear in these expressions will, of course, differ for different kinds of response.

Similar relationships can be derived for systems subject to viscous damping. For example, the function ϕ governing relative displacement in a structure having a single degree of freedom is, under these conditions

$$\varphi(t-\tau) = \frac{-1}{p'}\, e^{-c(t-\tau)} \sin p'(t-\tau) \tag{3d}$$

where

$$p' = \sqrt{(k/m) - c^2} \tag{3e}$$

and the damping force is $2mc\dot{x}$. Expressions of the form of Equation (3) also hold for elastic structures coupled to the ground, provided the ground can be treated as an elastic body (which may have internal viscous damping).

In all cases the amount that any individual acceleration pulse, $\ddot{x}_g dt$, contributes to the relative displacement, x(t), is independent of the remainder of the ground motion. For a ground motion consisting of a sequence of such pulses, x(t) can therefore be obtained by adding the products of all the pulses multiplied by a function ϕ whose value is independent of the ground motion. The conclusion follows from Equation (3) and it is true also as regards stresses, bending moments, and shears in the general types of structure under consideration.

RESPONSE OF STRUCTURES TO IDEALLY RANDOM GROUND MOTIONS

A) Prediction of Response at Any Time

As a first approximation to true earthquakes we consider motions consisting of random arrays of concentrated acceleration pulses. There are a number of aspects of the pulses which may be made random. Many of these different ways of specifying random pulses lead to substantially the same final results and at least one of them has been studied previously (9). In view of the fact that the final velocity of the ground after an earthquake is zero, it is reasonable to idealize earthquake motions as sets of pulses selected subject to the condition that their sum be zero. The pulses are taken to be distributed in time in a random order, either at closely and uniformly spaced intervals, or at randomly spaced instants of time. In either case the number of pulses is large and the magnitude of the largest pulse is small in comparison with the sum of the absolute magnitudes of all the pulses. This appears to be true for all reliable strong-motion accelerograms of destructive earthquakes. Therefore, the probability theory of large numbers may be applied to structures affected by them without introducing excessive errors.

Displacements relative to the ground are given by Equation (3). Therefore, when the ground motion consists of a series of pulses of magnitude

$$u_i = \int_{t_i-\epsilon}^{t_i+\epsilon} \ddot{x}_g \, d\tau \tag{4}$$

the relative displacement at time t is

$$x = \sum_i \varphi(t-t_i)\, u_i \tag{4a}$$

Here t_i denotes the instant at which the pulse u_i occurs. The theorem follows from Equation (3c) and the linear relationship between static forces and displacements assumed to be valid for the class of structures under discussion.

All individual terms of the type $\phi(t-t_i)u_i$ are small in comparison with $\sum_i |\phi(t-t_i)u_i|$.[6] If the fundamental period of the structure is small compared with t, every such element has practically random sign. The elements $\phi(t-t_i)u_i$ can be likened to the random errors which arise in a series of independent measurements. The response x would correspond to the sum of a large number of such errors.

According to the theory of probability (10,11), the probability distribution of a quantity x determined by a series of random events is given by

6. Vertical lines indicate that the quantity enclosed is to be taken as positive, only its magnitude being of interest, e.g., $|-3| = 3$.

$$P(x_1,x_2) = \int_{x_1}^{x_2} \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\, dx \tag{5}$$

where $P(x_1,x_2)$ is the probability that x lies between the values x_1 and x_2 and σ is the standard deviation of values of x from the mean. In this case the mean is zero; motions in either direction being equally probable. The quantity σ is a measure of the dispersion or variability of the quantity x from its mean value.

From a structural viewpoint, however, one is more interested in the average value of $|x|$ resulting from a very large number of determinations of x. This quantity is designated as the 'expected' value of $|x|$, $E|x|$. In a similar way we may also think of the expected value of x^2, $E(x^2)$. This last is related to the standard deviation, σ.

$$\sigma^2 = E(x^2). \tag{5a}$$

It can also be shown (12,13) that

$$E(x) = \int_{-\infty}^{\infty} \frac{x\, e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\, dx \tag{5b}$$

Since $E(-x) = E(+x)$ it follows that

$$E|x| = 2E(+x) = \sqrt{\frac{2}{\pi\sigma^2}} \int_0^{\infty} x\, e^{-\frac{x^2}{2\sigma^2}}\, dx \tag{5c}$$

$$= \sigma\sqrt{2/\pi}\,, \tag{5d}$$

because the integral is equal to σ^2. And in view of Equations (5a) and (5d)

$$E\,|x| = \sqrt{2\,E(x^2)/\pi}\,. \tag{6}$$

A quantity which is the product of independent variables can be shown to have an expected value equal to the product of the expected values of the individual variables (14). Since the acceleration pulses that constitute the motions under examination are independent of ϕ, Equation (3) and (4) imply

$$E(x^2) = E\left(\sum_1 u_i^2\right) E(\varphi^2) \tag{7}$$

The number of pulses which have occurred at time t is independent of u_i. Denoting this number by N_t, a further separation of variables is possible, yielding

$$E(x^2) = E(N_t)\,E(u^2)\,E(\varphi^2). \tag{7a}$$

If there are a great many of these pulses, or if they occur at randomly spaced instants, one can write

$$E(\varphi^2) = \frac{1}{t}\int_0^t \varphi^2(t-\tau)\,d\tau . \tag{8}$$

It follows from Equations (6) and (7) that

$$E\,|x| = \left[\frac{2E(N_t)\,E(u^2)}{\pi t}\int_0^t \varphi^2(t-\tau)\,d\tau\right]^{1/2}. \tag{9}$$

This formula expresses the expected magnitude of the displacement as a function of time.

The quantity $2E(N_t)E(u^2)/\pi$ does not depend on the structure but on the nature of the earthquake motion. This quantity may be denoted by the symbol K^2. Equation (9) then becomes

$$E\,|x| = \left[\frac{K^2}{t}\int_0^t \varphi^2(t-\tau)\,d\tau\right]^{1/2}. \tag{9a}$$

The term K^2/t appearing in Equation (9a) may be seen to be directly related to the average number of pulses occurring in time t. Since the pulses u_i occur at closely spaced instants, this average number is relatively constant and independent of time t. Therefore, the factor K^2/t appearing in Equation (9a) is dependent solely on the magnitudes of pulses that comprise the motion. It can be evaluated, for example, by computing the responses of a structure subjected to a number of motions consisting of a set of acceleration pulses rearranged in different orders and then taking the average of the (slightly) different responses.

In particular, if the structure is such that the function ϕ is periodic and its period, T, is small compared with the times t in which we are interested, Equation (9a) can be put in the form

$$E\,|x| = \left[\frac{K^2}{T}\int_0^T \varphi^2(\tau)\,d\tau\right]^{\frac{1}{2}}. \tag{9c}$$

The value of the expected magnitude of displacement is useful in establishing a basis for a uniformly safe design. Knowledge of the deviation from the "expected" value which may be anticipated is also useful in this connection as well as for comparing the effects of random motions with those of true strong-motion earthquakes. The dispersion (squared standard deviation) of $|x|$ canbe obtained by means of the relationship (15)

$$\sigma^2(|x|) = E(x^2) - (E|x|)^2. \tag{10}$$

From Equation (6) it follows that

$$\sigma^2(|x|) = [(\pi/2) - 1](E|x|)^2 \tag{11}$$

$$= 0.571\ (E|x|)^2. \tag{11a}$$

Up to this point the structural response—primarily the expected magnitude of the displacement—has been considered as a function of time. That is, formulas such as Equations (7, 8, 9) refer to the response at a specified instant. These quantities are of no interest in themselves for design purposes. What is wanted is the probable maximum numerical value reached by these responses. The determination of these maxima is treated in the following section.

B) Maximum Response

At present no completely general solution is available for calculating the value of maximum response as a function of ϕ. Approximate conclusions are developed here for ϕ functions which characterize two classes of structures. Qualitative conclusions are reached for other cases.

When ϕ is periodic the process of selecting the maximum $|x|$ in response to a given ground motion can be done in two operations. The first consists in dividing the time interval between the first and the last acceleration pulse into portions of duration T, the period of ϕ,—at most the last portion need have a duration different from T—then finding the response at all division points and selecting the maximum numerical value. This maximum may be denoted by $\max|x_T|$. The ratio $E[\max|x_T|]/E|x(s)|$ where $x(s)$ is the response that occurs at the end of the ground motion, $t = t_s$, is designated by the symbol C_1.

$$C_1 = \frac{E(\max|x_T|)}{E|x(s)|} \tag{12}$$

The second operation consists in introducing a phase shift, say θ, smaller than T and then selecting that value of θ which maximizes $\max|x_T|$. The largest $\max|x_T|$, found in this operation is evidently the absolute maximum, $\max|x|$, of response to the motion. For simplicity it will be designated by the symbol X. We further define the symbols C_2 and C by Equations (13) and (14)

$$C_2 = \frac{E(X)}{E(\max|x_T|)} \tag{13}$$

$$C = C_1 C_2 = \frac{E(X)}{E\,|x(s)|} \tag{14}$$

As noted in the Introduction, a graph giving values of X as ordinates vs. the period T as abscissa can be designated a 'displacement spectrum'. Instead of plotting values of $\max|x|$ as ordinates, one can plot $\max|\dot{x}|$. The resulting graph is known as a 'velocity spectrum'. It can be shown that the ordinates in a graph of pX vs. T have the same probability distributions as those in a velocity spectrum. This relationship is useful in studying the characteristics of destructive earthquakes since the important characteristics of the graph are more noticeable in velocity than in displacement spectra.

When $|\phi|$ has a period T/2, or is symmetric at intervals of T/2, the first operation may consist of dividing the earthquake duration s into intervals of duration T/2 instead of T. Under these circumstances the phase shift θ introduced in the second step need only vary between zero and T/2. This is the case for the sinusoidal and square-wave functions ϕ considered below.

The quantity C_1 can be determined by reference to the theory of probability. It is analogous to the probable maximum amount held by a player in a game of chance at any one of a number of instants during the game, divided by his probable final holdings. Proceeding along the lines suggested by the theory it is possible to show that for structural periods small compared with the duration of the temblor ($T \ll s$), C_1 is nearly constant, independent of ϕ and equal to $\pi/2$. The dispersion of $\max|x_T|$ is found to be $0.17[E(\max|x_T|)]^2$.

The value of C_2 can be calculated for sinusoidal functions ϕ by means of a simple trigonometric transformation. Equation (1) can be put in the form (16, 17)

$$x = \frac{1}{p}\left\{\left[\int_0^t \ddot{x}_g(\tau)\sin p\tau\, d\tau\right]^2 + \left[\int_0^t \ddot{x}_g(\tau)\cos p\tau\, d\tau\right]^2\right\}^{\frac{1}{2}} \sin(pt-\alpha) \tag{15}$$

in which α is a phase angle. If $\ddot{x}_g$ is assumed to be zero in the interval from t to t + T/2, the maximum x during this interval is obtained when $\sin(pt-\alpha) = 1$. When the period of the structure is small compared with the duration of the temblor ($T \ll s$), neglect of the acceleration pulses falling within any one interval of duration T/2 does not introduce appreciable error. Confining attention

In a single interval it can be seen that C_2 is very nearly equal to the ratio between the expected value of |x| computed from Equation (15a)

$$x = \frac{1}{p}\left\{\left[\int_0^t \ddot{x}_g(\tau)\sin p\tau\, d\tau\right]^2 + \left[\int_0^t \ddot{x}_g(\tau)\cos p\tau\, d\tau\right]^2\right\}^{\frac{1}{2}} \qquad (15a)$$

and the value computed from Equation (9a).
From Equation (6)

$$E\,|x| = \sqrt{2E(x^2)/\pi} \qquad (16)$$

and from Equation (15a)

$$E\,|x| = \frac{1}{p}\left[\frac{K^2}{T}\left(\int_0^T \sin^2 p\tau\, d\tau + \int_0^T \cos^2 p\tau\, d\tau\right)\right]^{\frac{1}{2}} = \frac{K}{p} \qquad (16a)$$

The denominator as obtained from Equation (9c) is

$$E\,|x| = \frac{1}{p}\left[\frac{K^2}{T}\int_0^T \sin^2 p\tau\, d\tau\right]^{1/2} = \frac{K}{p\sqrt{2}}\,. \qquad (16b)$$

Taking the ratio of the expressions (16a) and (16b) we have

$$C_2 = (K/p)/(K/p\sqrt{2}) = \sqrt{2} \qquad (17)$$

whence

$$C = \pi/\sqrt{2} = 2.22. \qquad (18)$$

In connection with tall buildings of relatively uniform cross section, functions ϕ having a square-wave shape are of special interest (Fig. 2a). For these ϕ functions a somewhat greater value of C_1 is found than in the case of sinusoidal ϕ functions. Therefore, in this case $C_1 = C_2 = \pi/2$ and $C = \pi^2/4 = 2.47$.

Another ϕ function which occurs in the analysis of tall buildings is shown in Fig. (2b). It coincides with a square-wave over some range, say t_1, of its period, and is equal to zero elsewhere. Although in this case C_1 is still approximately equal to $\pi/2$, C_2 may be considerably greater than in the case of the functions previously considered. The difference in C_2 is caused by the fact that, ϕ being zero part of the time, certain pulses affect the x_T while others do not. The phase shift θ, which is greater than t_1, may introduce sets of pulses which had not previously entered the computation. The sharper the function ϕ, the more pronounced will be the effect of the phase shift operation. In other words, C_2 and hence C increase as the interval of time during which ϕ differs from zero decreases.

The estimates of C, and hence of the ratio of the probable absolute maximum displacement or stress to the probable final displacement or stress, derived from theoretical considerations based on the theory of probability have been confirmed by empirical studies. For this purpose a number of motions having pulses of random sign were constructed. The results of these studies were in good agreement with the conclusions presented in the previous part of this section. However, to interpret the results of this and other investigations of a like nature it is necessary to take account of the limited number of acceleration pulses employed. It is generally impractical to investigate fictitious motions having a large number of acceleration pulses; the computational labor is too great, even for a structure of only moderate complexity. Values of C_1, C_2, and therefore C, determined in this way are and ought to be slightly lower than the predicted values given above which are based on a theory which presumes a very large number of closely spaced pulses. Actually, within a considerable range of moderately large numbers of pulses, C_1 was found to be approximately 1.41 for both sine and square-wave ϕ functions as compared with the theoretical estimate of 1.57 in both these cases. The ratio C_2 was also found to be about 1.41 as compared with theoretical estimates of $\sqrt{2}$ in the case of sine-wave and a slightly greater value in the case of square-wave functions. The value of C found in this way was 2.00 for both cases as compared with estimates of 2.22 and a somewhat greater value. The check on the estimates of maximum response obtained in this way appears to be satisfactory. It should be remembered that what are being obtained are essentially estimates of the probable maximum response and it must not be expected that these will agree precisely with calculations based on a particular motion or motions.

The dispersion of $\dot{x}$ in the case of sinusoidal function is of some interest in connection with comparisons of structural effects of true earthquakes with those predicted by possible 'substitute' motions, including the class examined herein. Since a complete analytical solution is not available, the dispersion was investigated by computing the response of a structure having a sinusoidal ϕ function ($\phi = -p^{-1}\sin pt$) to motions of the type shown in Fig. 3. This motion consists of 48 unit pulses, equally spaced at intervals of 0.25 sec., and having random sign. Responses to this motion were calculated by means of a graphical procedure (18). The maximum numerical values of these responses (pX and $\dot{X}$ spectra) are shown in Fig. 4. The dispersion was found to be $\sigma^2(p^2X^2) = 0.043p^2\bar{X}^2$ where $\bar{X}$ is the average value of X for all periods of structures investigated.

RESPONSE OF STRUCTURES TO TRUE EARTHQUAKES

The random motions treated in the foregoing sections differ from true earthquakes in several respects. The principal differences are considered below.

(A) In reality acceleration pulses cannot be instantaneous. This would imply accelerations of infinite magnitude lasting for infinitesimal intervals of time. Actually ground accelerations are finite as are also the durations of seismic waves. A method of correcting the computed expected spectrum ordinates to account for this effect is presented in the next section. It may be said here that the correction is not large, in general.

(B) The various kinds of seismic waves have different velocities. This situation gives rise to three distinct phases or segments in recorded accelerograms. The first or 'primary' phase contains the waves of shortest period. The secondary phase includes the waves having the largest average amplitude. Finally the tertiary or standing-wave phase includes the ground motions having the longest period and smallest average amplitude of seismic waves. As an approximation to the variation of seismic intensity with time, it is reasonable to assume a sequence of three regimes each having uniform intensity, as shown schematically in Fig. 5.

A quantity which is the sum of variables having a Gaussian probability distribution with respect to time will itself have a Gaussian distribution with respect to time. Conclusions drawn in previous sections referring to responses at specified instants have been inferred from the Gaussian distribution of these responses. Since the contribution of any one phase to the total structural response has this distribution, these conclusions are not affected by the existence of distinct phases.

(C) The accelerograms of strong-motion earthquakes published by the U. S. Coast and Geodetic Survey indicate that in destructive earthquakes the ground acceleration oscillates rapidly from positive to negative values. If ground motions were truly random, one would be entitled to expect intervals of single-signed pulses. The perfectly random motion theory is therefore not completely justified even within each phase of an earthquake. The importance of this and other departures from perfect randomness, as they affect structural responses, can be assessed by comparing predicted with calculated structural responses in actual earthquakes.

The ten velocity spectra studied by G. W. Housner A.M. ASCE (19) have been selected for comparison with the spectra predicted by the foregoing random motion theory. According to Equation (16b), for simple structures,

$$E(X) = E\,|x(s)|\,C_1C_2 = \frac{C_1C_2}{p}\,\frac{K}{\sqrt{2}} \qquad (19)$$

It follows that $pE(X)$ and consequently $E(\dot{X})$ ought to show no consistent variation with T, at least so long as T/s is small. This is indeed the case for the spectra of actual earthquakes. Furthermore, if the ten spectra computed by Housner, from actual earthquake accelerograms are averaged, the mean is very nearly a horizontal line within the range of structural periods for which the spectra have been obtained (approximately 0.2 to 3.0 seconds). This average should exhibit, approximately, the characteristics of the expected or probable spectrum ordinates. The agreement between spectra of actual earthquakes and 'expected' spectra generated by random motions is very satisfactory.

On the other hand, the dispersions in velocity spectrums of actual earthquakes have been found to be approximately $0.125p^2\overline{X}^2$, compared with a value of $0.043p^2\overline{X}^2$ for purely random motions, as discussed in the previous section. Figure 6 shows the way in which the relative dispersion, $\sigma^2(p^2X^2)/E(p^2X^2)$, varies with the sharpness of the variation in X (or in ϕ, or in the product C_1C_2). The comparison between responses to actual recorded earthquakes and to random acceleration pulses has indicated a good agreement in expected responses, but a discrepancy in expected deviations from the average. From this it seems justified to conclude that the expected responses at specified instants are practically unaffected by the lack of perfect randomness in actual earthquakes, but that this lack does have an influence on the ratio C, tending to reduce it. Since any increase in C is attributable to the possibility of selecting between independent sets of pulses, it may be concluded that C is actually more nearly a constant for true earthquakes than it is when the pulses have perfect randomness.

CORRECTION FOR FINITE WAVE DURATION

The velocity spectra of true earthquakes computed by G. W. Housner (19) permit an estimate of the effect that the finite duration of seismic waves has upon structural responses. From what has been said about the agreement between these computations and the spectra predicted by a random motion theory employing infinitely sharp pulses, it may be concluded that the effect, in the case of simple structures, is negligible, at least down to natural structural periods of 0.2 secs. Below this natural period there is not sufficient information available to permit definite conclusions. Such small fundamental periods, however, are not of much structural interest. In the case of structures which are not simple it is of some interest to correct for the effect of finite duration of seismic waves.

It has been suggested (20) that, as an approximation to true earthquakes, concentrated pulses be replaced by half-sine waves. The general appearance of accelerograms discloses important deviations from sinusoidal shape. For this reason other wave shapes such as square and triangular have been investigated. In this way it has been possible to assess the importance of the wave shape as well as the duration.

The effect of changing concentrated pulses into finite pulses can be taken into account by replacing the function ϕ with a 'corrected' function ϕ_c. The latter must be such that the effects of a concentrated pulse on a structure characterized by the function ϕ_c are the same as the effects of a finite pulse on the structure characterized by ϕ. The area under the finite pulse acceleration-time graph must, of course, be equal to the magnitude of the instantaneous pulse.

In accordance with the foregoing specification, the corrected function for half-sine waves of duration t_0 ($\ddot{x}_g = (\pi/2\,t_0)\cos \pi t/t_0$ for $-t_0/2 < t < t_0/2$) is

$$\varphi_c(t) = \frac{p_0}{2}\int_{-\frac{t_0}{2}}^{+\frac{t_0}{2}} \varphi(t-\tau)\cos p_0\tau\, d\tau, \qquad (20)$$

where $p_0 = \pi/t_0$ is the circular frequency of the seismic wave. For simple structures, ϕ is also sinusoidal and Equation (20) reduces to

$$\varphi_c = \frac{\cos\left[(\pi/2)(p/p_0)\right]}{1-(p/p_0)^2}\,\varphi. \tag{21}$$

Consequently, if X_0 denotes the uncorrected maximum relative displacement of a simple structure,

$$\frac{E(X)}{E(X_0)} = \left|\frac{\cos\left[(\pi/2)(p/p_0)\right]}{1-(p/p_0)^2}\right|. \tag{22}$$

Proceeding in a like manner for triangular shaped waves,

$$\frac{E(X)}{E(X_0)} = \frac{8}{\pi^2}\left(\frac{p_0}{p}\right)^2\left|1-\cos\left(\frac{\pi}{2}\frac{p}{p_0}\right)\right|. \tag{23}$$

Similarly, square-shaped waves give, for simple structures

$$\frac{E(X)}{E(X_0)} = \frac{2}{\pi}\,\frac{p_0}{p}\left|\sin\left(\frac{\pi}{2}\frac{p}{p_0}\right)\right|. \tag{24}$$

The principal error associated with Equations (22, 23, 24) arises from the tacit assumption that t_0 is a constant. Alternatively, a frequency distribution of t_0 such as that shown in Fig. 7 could be investigated. In this Figure t_0' represents the duration of any seismic wave, the ordinates F are proportional to the corresponding amplitudes multiplied by the number of times that a wave of duration t_0' occurs before the maximum response takes place, and t_0 is now the weighted average of t_0', using the ordinates F as weighting factors. Since, for any t_0', the response at a specified instant, $|x(t)|$, has a Gaussian probability distribution, the square of $E|x|$ is proportional to $E(x^2)$. Therefore, in the case of variable t_0', $E|x|$ is equal to the square root of the weighted average of the squares of $E|x|$ that correspond to each t_0'. The weighting factors are again the ordinates F in the frequency curve.

As will be seen subsequently, it is justifiable to assume that $E(X)$ is proportional to $E|x|$. Therefore the square of the ratio $E(X)/E(X_0)$ can be obtained by averaging the squares of the ratios given by Equation (22) and using as weighting factors the ordinates in Fig. 7.

In Fig. 8 is shown the value of $E(X)/E(X_0)$ as a function of the ratio of the period of the structure to the duration of seismic waves, T/t_0. Curves have been plotted for half sine-wave, triangular-shaped, and square-wave pulses of constant duration as well as for half sine-wave pulses having a distribution of durations as shown in Fig. 7. For structures having a fundamental period greater than six times the duration of the seismic wave the ratio between corrected and uncorrected expected maximum responses is practically unity for all the wave shapes investigated.

Judging from published strong-motion accelerograms and velocity spectra a reasonable design value of t_0 would lie between 0.05 and 0.10 sec. The former value is conservative and may be adopted in the absence of accurate computations based on measured accelerograms. Preferably t_0 should be deduced from the ordinates of velocity spectra within the range 0 - 0.20 sec., when these are available. The value of t_0 to be used in design can then be obtained by adjusting the time scale in Fig. 8 so as to get the best possible agreement with average spectra.

As may be inferred from Fig. 8, the wave shape selected for this purpose is not critical. If the time scales of the curves of Fig. 8 are properly adjusted, all the curves can be made to agree within three percent with the one corresponding to half-sine waves of constant duration down to natural periods $T = t_0$, or about .05 sec. Shorter natural periods have little structural significance. The assumption of half-sine wave shapes of constant duration is satisfactory for practical applications.

APPLICATION TO DESIGN

From the point of view of the designer, interest centers in the displacements or forces which the structure under consideration must withstand. For the present, the point of view adopted is that of design for certain displacements, but if the structure behaves elastically, displacements and stresses are proportional and can be converted to equivalent static forces. Denoting by X_D the displacement which a point of the structure should be designed to withstand, it follows from Equations (9) and (9a) that X_D is given by the right-hand sides of these equations when t is chosen so as to make these terms as large as possible. That is,

$$X_D = \max_t \left\{ K \sqrt{\frac{1}{t}\int_0^t \varphi_c^2(t-\tau)\,d\tau} \right\}. \tag{25}$$

The corrected function, ϕ_c, is used in place of ϕ to take account of the finite duration of seismic waves. This function is defined in terms of ϕ by Equation (21). When ϕ (and hence ϕ_c) is periodic and, as is often the case, the fundamental period T is small compared with the duration of the earthquake, the criterion expressed by Equation (25) can be written in a simpler form. Following Equation (9c)

$$X_D = K \sqrt{\frac{1}{T}\int_0^T \varphi_c^2(\tau)\,d\tau} \; . \tag{26}$$

Here K is defined as in Equation (9b)

$$K^2 = E(N_t)\,\frac{2}{\pi}\,E(a^2). \tag{27}$$

The quantity K (ft. per sec.) is a measure of the earthquake intensity. It can and should be determined from accelerogram records of strong-motion earthquakes such as those obtained by the U. S. Coast and Geodetic Survey.

It may be noted that the function ϕ depends upon the natural frequencies and mode shapes of the structure. In practice this means that the preliminary design must be carried to a point at which the frequencies of the structure can be estimated before the design deflections, X_D, are determined. This awkwardness is common to structural design in general. Even a simple truss subjected to static loads cannot be designed until its dead weight is known. This information in turn can be obtained only from a preliminary design, or from the examination of similar structures.

It is, of course, to be understood that in designing for the displacement X_D or for the corresponding equivalent static forces, a factor of safety appropriate to the uncertainty in material properties and static loadings is to be included. In addition, the earthquake intensity factor, K, ought also to contain a further factor of safety. As noted in the Introduction, the intensity of earthquake for which a structure should be designed is dependent upon the seismic record of its site, the public importance of the structure, and on economic considerations. When this choice has been made, however, design by means of Equations (25) and (26) insures a uniform strength in the sense that no part of the structure is overdesigned or under-designed. All have an equal probability of successfully resisting ground motion. By selecting a design intensity, K, corresponding to an intense temblor that probability may be made very high.

The sine functions whose sum gives the ϕ_c of Equation (26) may be regarded as the Fourier expansion of ϕ_c. The integrals of the products of different terms cancel over an interval T. Equation (27) can therefore be written in the form

$$X_D = \sqrt{\sum_{i=1}^{N} X_{Di}^2} \qquad (28)$$

where X_{Di}^2 denotes the square of the design displacement that the point under consideration would have, with respect to the ground, if the structure vibrated in the i'th mode alone. The symbol $\sum_{i=1}^{N}$ indicates that these quantities are to be summed for all of the modes of the structure which are to be considered.

Equations (26) and (28) are applicable to structures whose fundamental periods are short compared with the earthquake duration. Although this restriction still includes the majority of structural cases, the use of these equations when T is large or ϕ is markedly aperiodic may result in error on the unsafe side. In this case, Equation (25) must be used. The use of this equation is somewhat hampered by the present lack of quantitative data on the time-variation of $E(u^2)$ which enters into K. It is possible, however, to find an upper limit for the design response. As was noted, Fig. 5 constitutes a good approximation to the variation of $E(u^2)$ with time. It is conservative to neglect the first and third phases of earthquakes and to assume a constant intensity for the secondary phase which is most severe. This assumption tends to predict proportionately greater responses for structures in which T/s is large. If the duration of the secondary phase in destructive earthquakes is designated t_s[7], these simplifications give as an estimate for design purposes,

7. This duration may be estimated to be 10 sec.

$$X_D = \max_t \left\{ K \sqrt{\frac{1}{t_2}\int_0^{t_2} \varphi_c(t-\tau)\,d\tau} \right\} \tag{29}$$

where t, in seconds, is selected so as to maximize the radical.

Equations (25, 26, 28, 29) have been derived from Equation (1) for displacements relative to the ground. Since expressions similar to Equation (1) can be established for stresses, bending moments, and equivalent statical forces, it is clear that the design values of these types of response can be obtained from Equations (25, 26, 28, 29) by substituting the response in question for X_D. It is, of course, necessary to substitute the corresponding ϕ_c in the right hand members.

CONCLUSIONS

The present work has led to a basis for the design of earthquake-resistant structures. The scope of the material developed here is limited to cases in which the design distortions are small enough so that the behavior of the structure is essentially elastic and to structures founded on firm ground. The structures may have any number of degrees of freedom. Both structure and ground may have internal viscous damping. Under these conditions structural responses, at any time t, can be obtained from expressions of the form

$$R(t) = \int_0^t \ddot{x}_g(\tau)\,\varphi(t-\tau)\,d\tau \tag{30}$$

in which the response R can be a displacement relative to the ground, a stress, or a bending moment; x_g is the ground motion; ϕ is a function of the structure, independent of the ground motion and equal to the response generated by a sudden unit change of ground velocity.

Considering the origin and properties of earthquake motions it has been concluded that they can be replaced for purposes of structural design by nearly random motions. This conclusion has been substantiated by comparison of the spectra predicted by a random motion theory with those of actual destructive strong-motion earthquakes recorded by the U. S. Coast and Geodetic Survey.

By means of the theory of probability, a method for computing design values of responses has been developed. Structures designed to withstand these responses will be of uniform strength in the sense that all parts will have equal probability of successfully withstanding a temblor. Different parts of a structure will not be relatively over- or under-designed. When the ratio T/s (T = fundamental period of structure, s = earthquake duration) is small, the design value of R is

$$R_D = K \sqrt{\frac{1}{T}\int_0^T \varphi_c^2(t)\,dt} \tag{31}$$

where K (ft. per sec.) is a measure of the earthquake intensity and ϕ_c is given, approximately, by

$$\varphi_c(t) = \frac{1}{2} p_0 \int_{-\frac{t_0}{2}}^{\frac{t_0}{2}} \varphi(t-\tau) \cos p_0 \tau \, d\tau \quad . \tag{32}$$

In this equation t_0 is the average seismic wave duration, and $p_0 = \pi / t_0$.

The design value given above for R can be put in the form

$$R_D = \sqrt{\sum_{i=1}^{N} R_{D_i}^2} \tag{33}$$

in which R_{Di} is the design response associated with the i 'th natural mode and N is the number of degrees of freedom of the structure.

These expressions usually give satisfactory results. However, when T/a is not small they err on the unsafe side. An upper limit to the design value can be computed from the expression

$$R_D = \max_t \left\{ K \sqrt{\frac{1}{t_2} \int_0^{t_2} \varphi_c(t-\tau) \, d\tau} \right\} \tag{34}$$

in which t_2 (= approx. 10 sec.) is the duration of the second phase of the earthquake motion and t is selected so as to maximize the right hand side.

ACKNOWLEDGEMENT

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BIBLIOGRAPHY

(1) "A Mechanical Analyzer for the Prediction of Earthquake Stresses", Biot, M. A., Bulletin, Seis. Soc. Am., Vol. 31 (1941), pp. 151-171.

(2) "Computations on the Response of One-Mass Undamped Structures to the El Centro Accelerograph Record", Robison, E. C., Watson Scientific Computing Laboratory, Columbia University, New York, New York.

(3) "The Physical Evaluation of Seismic Destructiveness", Bulletin Seismological Society of America, Vol. 24 (1934), pp. 398-403.

(4) "Lateral Forces of Earthquake and Wind", Joint Committee Report, San Francisco, Cal. Section ASCE and Structural Engineers Ass'n of Northern Cal., Proceedings, ASCE, Vol. 77 (1951), Separate No. 66.

(5) "Characteristics of Strong-Motion Earthquakes", Housner, G. W., Bulletin Seismological Society of America, Vol. 37 (1947), pp. 19-31.

(6) "Vibration Problems in Engineering", by Timoshenko, S., D. Van Nostrand Company, New York, New York, 1928.

(7) "Theory of Sound", Lord Rayleigh, 2nd ed., Macmillan, London.

(8) Discussion by N. J. Hoff of "Analytical and Experimental Methods in Engineering Seismology" by M. A. Biot, Trans. ASCE, Vol. 108 (1943), pp. 389-390.

(9) Reference (5), p. 20.

(10) "Probability and Its Engineering Uses", by T. C. Fry, D. Van Nostrand Company, New York, New York (1928) pp. 206-213.

(11) "Introduction to Mathematical Probability", by J. V. Uspensky, McGraw-Hill Book Company, New York, New York (1937) pp. 243-245.

(12) Reference (10), pp. 184-185.

(13) Reference (11), pp. 161-162.

(14) Reference (11), pp. 170-171.

(15) Reference (11), pp. 172-173.

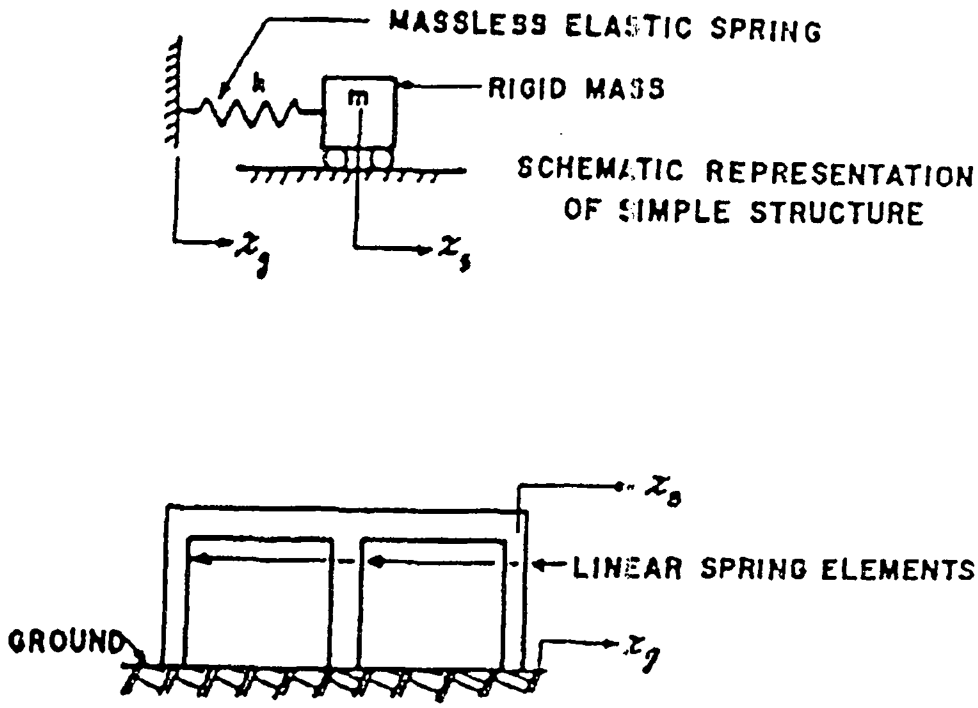
(16) "A Method of Calculating the Dynamic Force on a Building During An Earthquake", by M. P. White, Bulletin, Seismological Society of America, Vol. 32, (July 1942) pp. 194-195.

(17) Reference (5), p. 21.

(18) "Influence of Ductility on the Response of Simple Structures To Earthquake Motions", by S. L. Pan, L. E. Goodman, N. M. Newmark, Technical Report to the Office of Naval Research (1951), Structural Research Series, No. 11, Dept. of Civil Engineering, University of Illinois, Urbana, Illinois.

(19) Reference (5), pp. 23-28.

(20) Reference (5), pp. 22, 30.



FRAME IDEALIZABLE AS A SIMPLE STRUCTURE

FIG. 1

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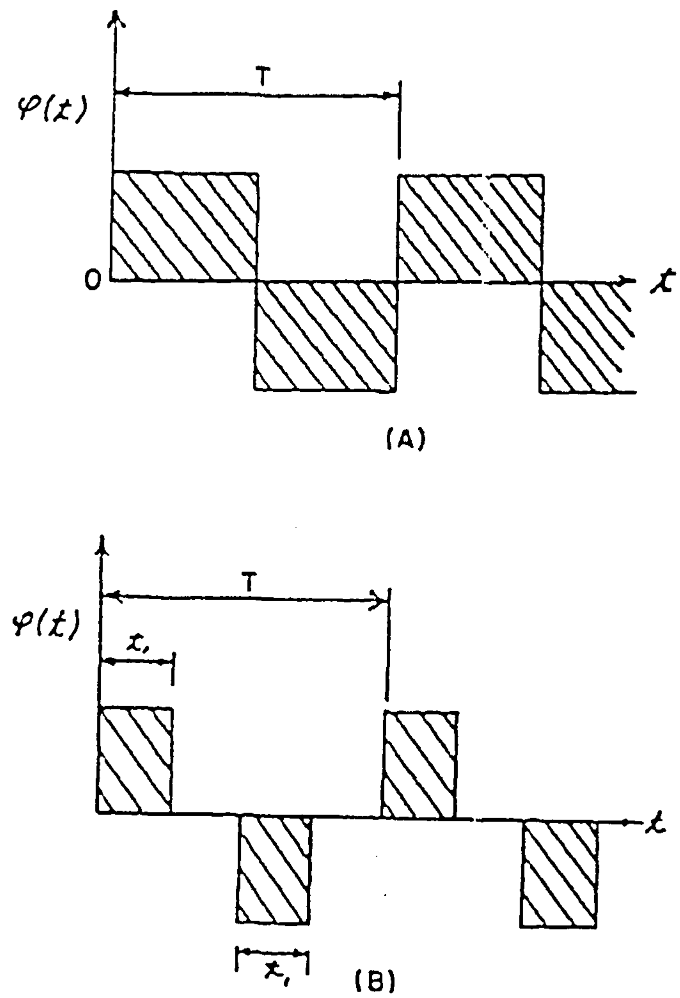
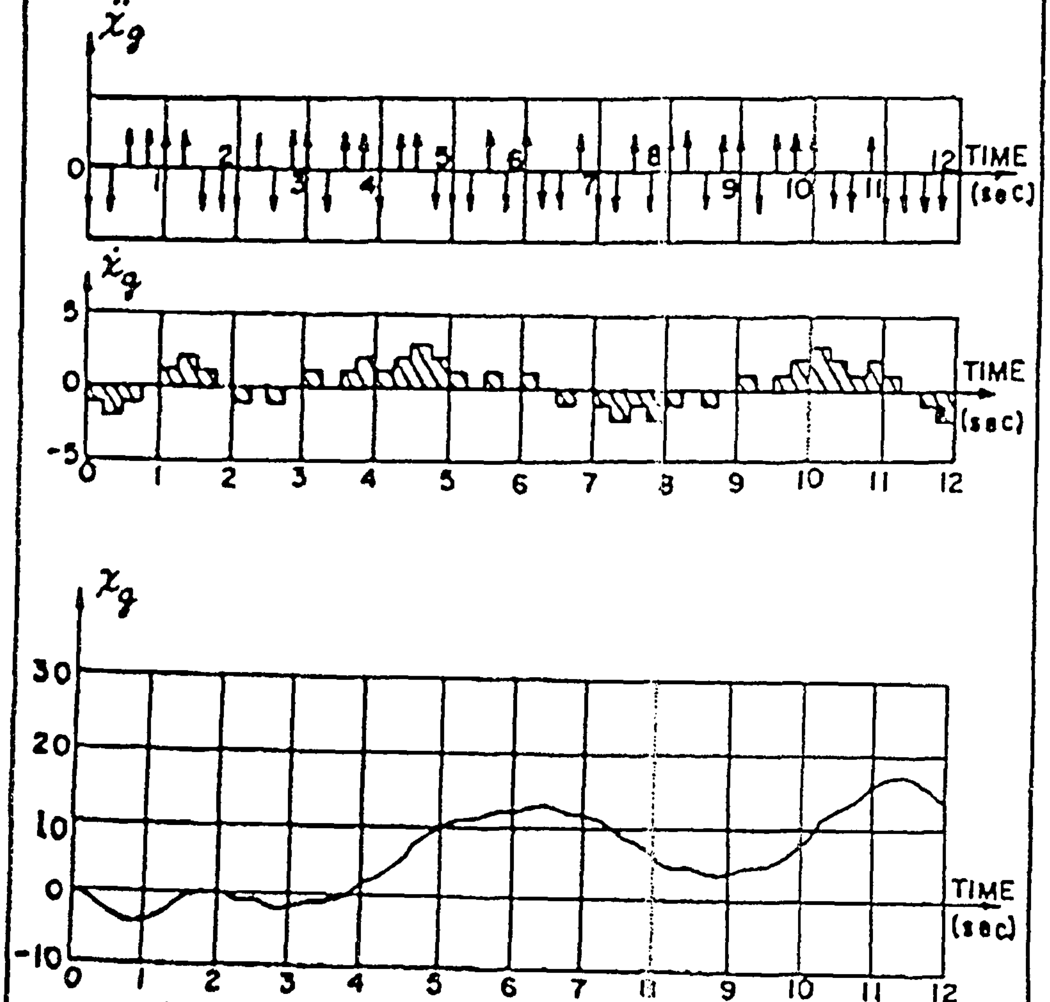


STRUCTURAL-RESPONSE FUNCTIONS FOR TALL UNIFORM BUILDINGS

(A) AT BASE OF STRUCTURE (SCHEMATIC)

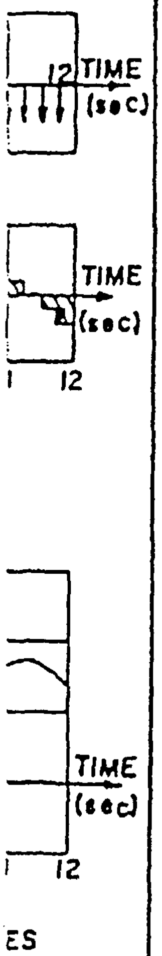
(B) INTERMEDIATE HEIGHT (SCHEMATIC)

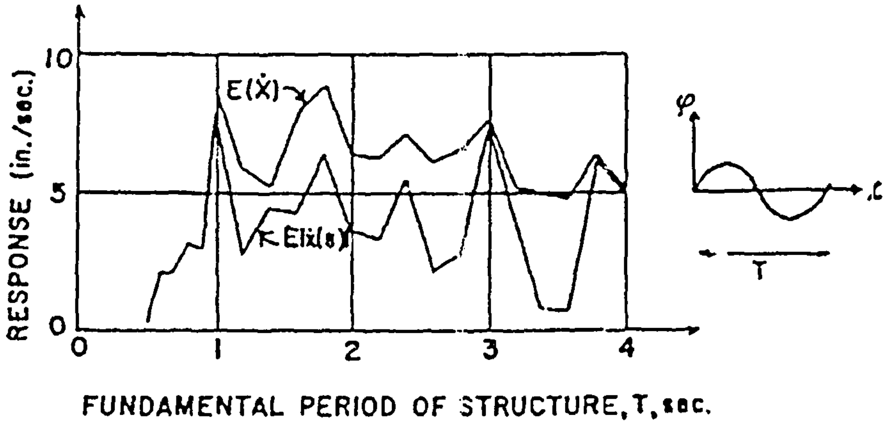
FIG. 2



MOTION GENERATED BY 48 UNIT ACCELERATION PULSES OF RANDOM SIGN (MOTION I)

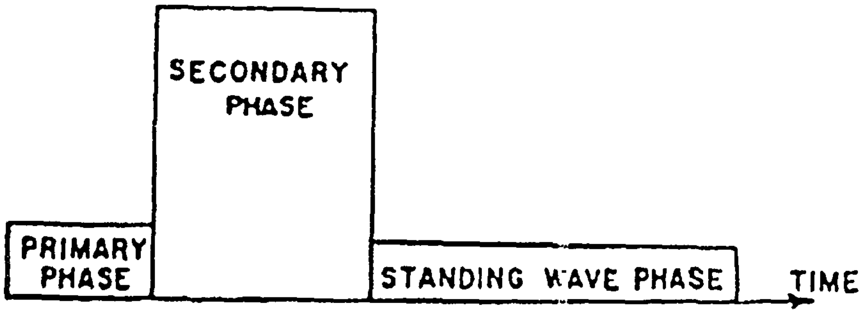
FIG. 3





VELOCITY SPECTRUM OF MOTION I

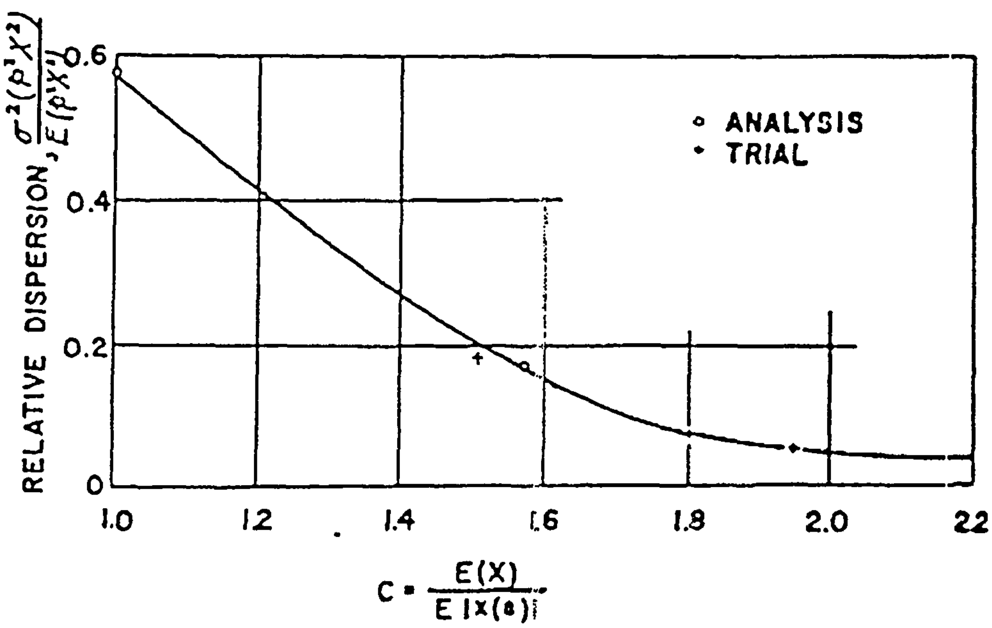
FIG. 4



IDEALIZATION OF SEISMIC INTENSITY

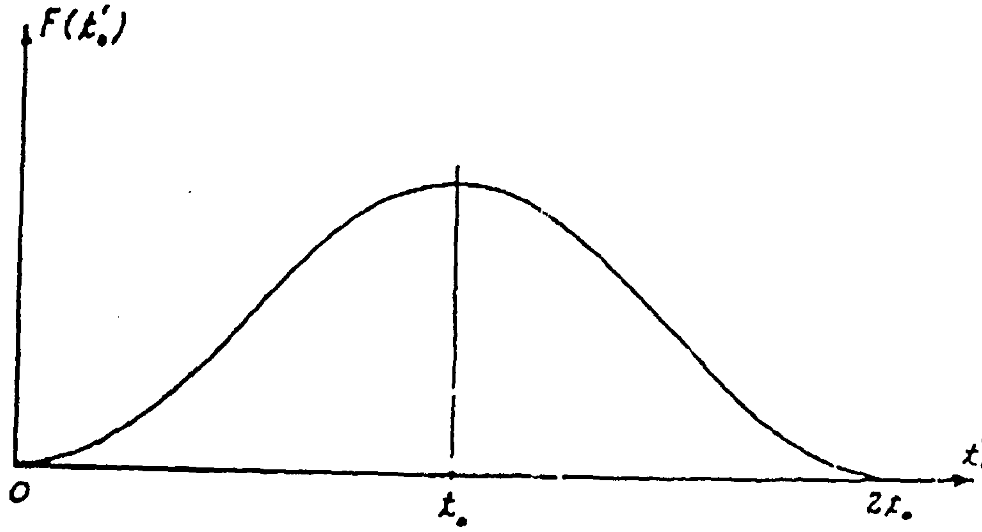
FIG. 5

TIME



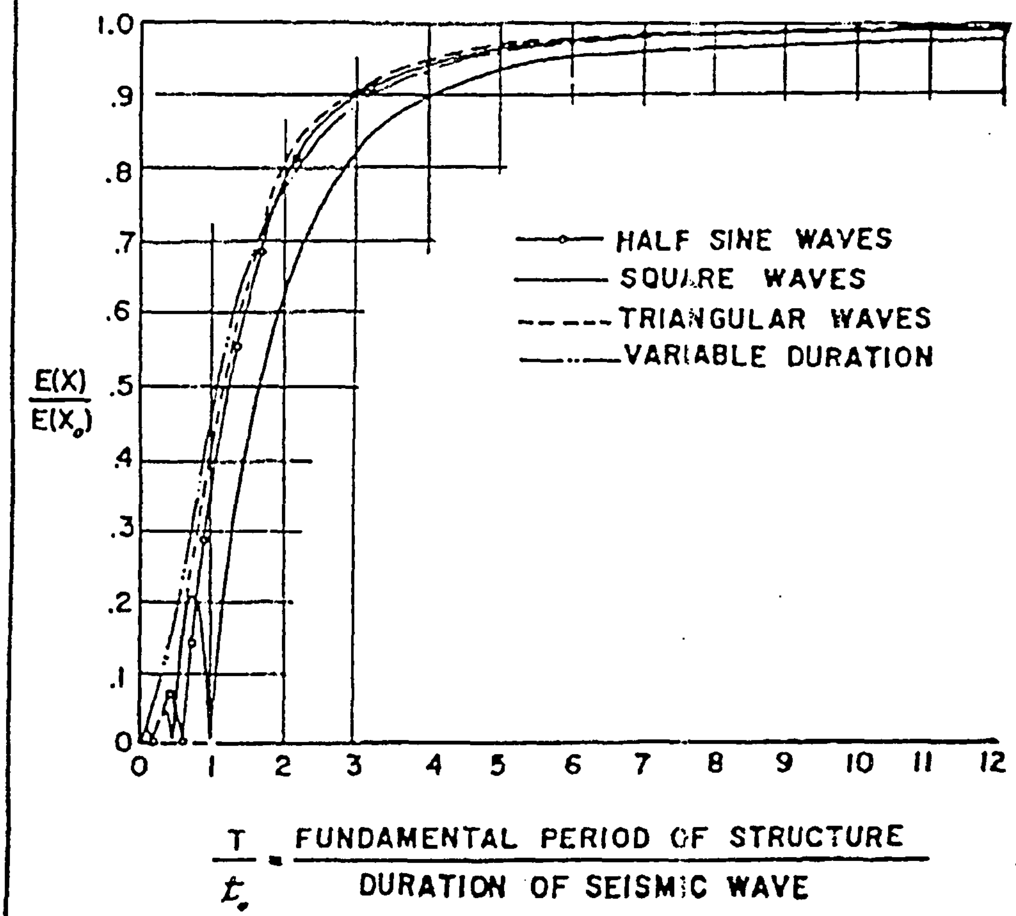
DEPENDENCE OF DISPERSION ON THE RATIO OF THE EXPECTED MAXIMUM RESPONSE TO THE EXPECTED FINAL RESPONSE

FIG. 6



ASSUMED DISTRIBUTION OF WAVE DURATIONS

FIG. 7



CORRECTION FACTOR FOR FINITE WAVE DURATION

FIG. 8