SOFTWARE VALIDATION TEST REPORT (SVTR)

SVTR#: ***TaskID- Report # 14***	**Project#: 20.06002.01.352**
Software Name: TPA	**Version: 5.0.1beta**
Test ID: ***Module abbr Test# NRUTIL***	**Test Series Name:**

Test Method

- ■ **code inspection**
- □ **output inspection**
- □ **hand calculation**
- □ **spreadsheet**
- □ **graphical**
- □ **comparison with external code results**

Test Objective: Verify that functions in the module NRUTIL are consistent with equations in paper

S. Mohanty and G. Adams, *A Model For Estimating Heat Transfer Through Drift Degradation Based Natural Backfill Materials*, 40th U.S. Rock Mechanics Symposium in Anchorage, Alaska, June 25-29, 2005.

Test Environment Setup

Hardware (platform, peripherals): Pentium 4, 3 Ghz and 2 GB of RAM.

Software (OS, compiler, libraries, auxiliary codes or scripts): Microsoft Windows XP, auxiliary scripts programmed in Mathematica 5.1.

Input Data (files, data base, mode settings): The code NRUTIL.f was directly inspected.

Assumptions, constraints, and/or scope of test: The test focused on detecting typographical errors in equations in NRUTIL. The functions in NRUTIL are passed as arguments to numerical routines in the NUMRECIP module (routines zbrak and zbrent) to find roots. These root finding routines will be tested as part of Task 12.

Test Procedure: Equations in Fortran format were copied into Mathematica to be translated into standard mathematical notation. Translated equations were visually compared to equations in Mohanty and Adams (2005). Equations in this paper are intended to estimate temperatures in a system with natural backfill on top of drip shields. Equations in NRUTIL consider all possible cases, such as no-backfill and no-drip shield. Equations for these cases can be inferred from equations in Mohanty and Adams applying symmetry arguments. The attachment includes a comparison of the original Fortran lines, the Mathematica 5.1 translation, and the corresponding equation in Mohanty and Adams (2005), or derived equations from symmetry arguments for the cases not explicitly considered in the paper.
Test Results **Location:** The file *translations.nb* is a Mathematica 5.1 notebook including the translations from Fortran to standard mathematical notation. **Test Criterion or Expected Results:** Equations in NRUTIL must be consistent with equations in Mohanty and Adams (2005) or with inferred equations. **Test Evaluation (Pass/Fail): PASS**
Notes:

Tester: Osvaldo Pensado	**Date:** May 25, 2005

NOTE: Notation in NRUTIL closely follows the Mohanty and Adams (2005) parameter. Consult this paper for meaning of parameters and coefficients.

function fTargetBackfillOuter(temperatureBFO)

Fortran form

```
      d_kcv2 = 0.386d0 * D_kair * ((D_pr / (0.861d0 + D_pr))**0.25d0) *
     &  (
     &    (
     &      (dlog(radius_rw/radius_bfo)**4.0d0) *
     &      D_g * D_beta * (temperatureBFO - D_Temperature_RW) /
     &      (
     &        D_nu * D_alpha *
     &        (
     &          ((2.0d0 * radius_bfo)**(-3.0d0/5.0d0)) +
     &          ((2.0d0 * radius_rw)**(-3.0d0/5.0d0))
     &        )**5.0d0
     &      )
     &    )
     &  **0.25d0)
```

Mathematica translation

$$d\_kcv2 = 0.386\, D\_kair \left(\frac{D\_pr}{0.861 + D\_pr}\right)^{0.25} \left(\frac{\mathrm{Log}\left[\frac{radius\_rw}{radius\_bfo}\right]^4 D\_g\, D\_beta\,(temperatureBFO - D\_Temperature\_RW)}{D\_nu\, D\_alpha\,((2\, radius\_bfo)^{-3/5} + (2\, radius\_rw)^{-3/5})^5}\right)^{0.25}$$

Mohanty and Adams (2005)

$$k_{cv2} = 0.386 k_{air}\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}\left[\frac{\left(\ln\frac{D_{rw}}{D_{bfo}}\right)^4 g\beta\left(T_{bfo}-T_{rw}\right)}{\nu\alpha\left(D_{bfo}^{-3/5}+D_{rw}^{-3/5}\right)^5}\right]^{1/4} \qquad (12)$$

Conclusion: Equations match

Fortran form

```
      d_kr2 =
     &  dlog(radius_rw / radius_bfo) * D_SBolt * radius_bfo *
     &  (temperatureBFO**2 + D_Temperature_RW**2) *
     &  (temperatureBFO + D_Temperature_RW) /
     &  (
     &    (1.0d0 / D_EmissBF) +
     &    ((1.0d0 - D_EmissRW) / D_EmissRW) *
     &    (radius_bfo / radius_rw)
     &  )
```

Mathematica translation

$$d\_kr2 = \frac{\mathrm{Log}\left[\frac{radius\_rw}{radius\_bfo}\right] D\_SBolt\, radius\_bfo\,(temperatureBFO^2 + D\_Temperature\_RW^2)\,(temperatureBFO + D\_Temperature\_RW)}{\frac{1}{D\_EmissBF} + \frac{(1 - D\_EmissRW)\, radius\_bfo}{D\_EmissRW\, radius\_rw}}$$

Mohanty and Adams (2005)

$$k_{r2} = \frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right) r_{bfo}\left(T_{bfo}^2 + T_{rw}^2\right)\left(T_{bfo} + T_{rw}\right)}{\frac{1}{\epsilon_{bfo}} + \frac{1 - \epsilon_{rw}}{\epsilon_{rw}}\left(\frac{r_{bfo}}{r_{rw}}\right)} \qquad (14)$$

Conclusion: An extra factor D_SBolt, representing the Stefan-Boltzman constant (5.67×10^{-8} W/(cm^2 K^4)), appears in the Fortran equations. This extra factor is not an error. There is a typographical error in Equation (14) in Mohanty and Adams (2005). The radiative thermal conductivity, k_{r2}, must include the Stefan-Boltzman constant. Equation (11-14) in the Risk Analysis for Risk Insights Progress Report (Mohanty el al., 2005, Doc Q200505230001) does include the Stefan-Boltzman factor. Equation (11-14) in the RARI report is reproduced below:

$$k_{r2} = \frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right) \sigma r_{bfo}\left(T_{bfo}^2 + T_{rw}^2\right)\left(T_{bfo} + T_{rw}\right)}{\frac{1}{\epsilon_{bfo}} + \frac{1 - \epsilon_{rw}}{\epsilon_{rw}}\left(\frac{r_{bfo}}{r_{rw}}\right)}$$

Fortran form

```
     fTargetBackfillOuter = D_Temperature_RW - temperatureBFO +
    &  (
    &    (D_Thermal_Load_Above / (2.0d0 * 3.14159d0 * D_WPSpace)) *
    &    (
    &      dlog(radius_rw / radius_bfo) /
    &      (d_kcv2 + d_kr2)
    &    )
    &  )
```

Mathematica translation

$$\text{fTargetBackfillOuter} = \text{D\_Temperature\_RW} - \text{temperatureBFO} + \frac{(\text{D\_Thermal\_Load\_Above})\ \text{Log}\left[\frac{\text{radius\_rw}}{\text{radius\_bfo}}\right]}{(2\ 3.14159\ \text{D\_WPSpace})\ (\text{d\_kcv2} + \text{d\_kr2})}$$

Equation (10) in Mohanty and Adams (2005)

$$T_{wp} - T_{rw} = \frac{q}{2\pi wpl}\left[\frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2} + k_{r2}} + \frac{ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1} + k_{r1}}\right] (10)$$

can be rewritten as

$T_{wp} - T_{rw} = \left(T_{bfo} - T_{rw}\right) + \left(T_{dso} - T_{bfo}\right) + \left(T_{dsi} - T_{dso}\right) + \left(T_{wp} - T_{dsi}\right)$, where, for example

$$T_{bfo} - T_{rw} = \frac{q}{2\pi\, wpl} \frac{\ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2} + k_{r2}}$$

NRUTIL defines the following function

$$\Delta T_{bfo} = T_{rw} - T_{bfo} + \frac{q}{2\pi wpl} \frac{\ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2} + k_{r2}}$$

The TPA code iteratively finds the root of the equation $\Delta T_{bfo} = 0$; thus allowing to define the temperature T_{bfo}.

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

function fTargetThermalLoad(thermaload)

Fortran form

```
      gcondK = (2.0d0 * 3.14159d0 * D_WPSpace * (1.0d0 - D_FracInv) *
     &  D_CondFloor) / dlog(D_DriftDiaFloor/D_WPDia)
```

Mathematica translation

$$gcondK = \frac{2\ 3.14159\ D\_WPSpace\ (1 - D\_FracInv)\ D\_CondFloor}{Log[\frac{D\_DriftDiaFloor}{D\_WPDia}]}$$

This equation does not explicitly appear in Mohanty and Adams (2005). This equation is used to estimate the thermal conductance of the invert. In Figure 1 in Mohanty and Adams the invert is modeled as a cylindrical segment spanning from the waste package to the drift wall. The fraction of the drift not covered by the invert is represented by D_FracInv.

Equation (10) in Mohanty and Adams (2005):

$$T_{wp} - T_{rw} = \frac{q}{2\pi wpl}\left[\frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2}+k_{r2}} + \frac{ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}\right] \quad (10)$$

Equation (10) can be rewritten as

$$T_{wp} - T_{rw} = \left(T_{bfo} - T_{rw}\right) + \left(T_{dso} - T_{bfo}\right) + \left(T_{dsi} - T_{dso}\right) + \left(T_{wp} - T_{dsi}\right),$$ where, for example

$$T_{bfo} - T_{rw} = \frac{q}{2\pi\ wpl}\frac{\ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2} + k_{r2}}$$

Using a symmetry argument, the conductance between two concentric cylinders of radius r_1 and r_2 ($r_1 < r_2$), is therefore computed as

$$gcondK = \frac{q}{T_1 - T_2} = 2\pi\ wpl\frac{k}{\ln\left(\frac{r_2}{r_1}\right)}$$

It is assumed that a medium of thermal conductivity k fills the space between r_1 and r_2. If the space is partially filled, by a fraction f, the equation can be corrected as

$$gcondK = \frac{q}{T_1 - T_2} = 2\pi\ wpl\frac{k\ f}{\ln\left(\frac{r_2}{r_1}\right)}$$

From the previous equation and the notation in Mohanty and Adams (2005), the following equation is derived for the thermal conductance of the invert:

$$gcondK = 2\pi\, wpl \frac{k_{invert}\,(1 - D\_FracInv)}{\ln\left(\frac{r_{rw}^{o}}{r_{wp}}\right)}$$

In the previous equation r_{rw}^{o} represents the initial rock wall diameter. As time elapses and drift degradation proceeds, the drift wall radius grows. Thus, in general, $r_w \geq r_w^o$. If $r_w > r_w^o$ an additional contribution to the conductance must be computed. The following statements in NRUTIL are intended to define that case.

Fortran form

```
        gcondKip = (2.0d0 * 3.14159d0 * D_WPSpace *
     &    (1.0d0 - D_FracInv) *
     &    D_CondRW) / dlog(D_RWDia / D_DriftDiaFloor)
        gcondKeq =
     &    1.0d0 / ((1.0d0 / gcondK) + (1.0d0 / gcondKip))
```

Mathematica translation

$$gcondKip = \frac{2\; 3.14159\; D\_WPSpace\,(1 - D\_FracInv)\; D\_CondRW}{Log\left[\frac{D\_RWDia}{D\_DriftDiaFloor}\right]};$$

$$gcondKeq = \frac{1}{\frac{1}{gcondK} + \frac{1}{gcondKip}}$$

Applying the same approach to derive the invert conductance and the notation in Mohanty and Adams (2005), the following equation is derived for the conductance of the cylindrical segment or rock material below the invert:

$$gcondK = 2\pi\, wpl \frac{k_{rw}\,(1 - D\_FracInv)}{\ln\left(\frac{r_w}{r_{rw}^{o}}\right)}$$

The equivalent conductance is computed as the conductance of two elements in series:

$$gcondKeq = \frac{1}{\frac{1}{gcondK} + \frac{1}{gcondKip}}$$

Thus, equations in NRUTIL are consistent with equations in Mohanty and Adams (2005).

Fortran form

```
      fTargetThermalLoad = D_Thermal_Load - thermalLoad -
     &  (D_Temperature_WP - D_Temperature_RW) * gcondKeq
```

Translation

fTargetThermalLoad = D_Thermal_Load - thermalLoad - (D_Temperature_WP - D_Temperature_RW) * gcondKeq

By an iterative approach, the TPA code finds the root to the equation fTargetThermalLoad = 0. This is equivalent to requiring

$$q_{wp} = q_{above} + \left(T_{wp} - T_{rw}\right) k_{eq}$$

where

q_{wp} = D_Thermal_Load : heat output from the waste package
q_{above} = thermal load above the drift wall
$(T_{wp} - T_{rw})\ k_{eq}$ = heat loss between the waste package and drift wall

The unknown variable in the equation is q_{above}.

Conclusion: the format of the NRUTIL equations is consistent with equations and concepts in Mohanty and Adams (2005).

function fTargetDripShieldOuter(temperatureDSO)

Fortran form

```
        calc_mphi = -1.0d0 * DLOG(D_di * 1000.0d0) /
     &    DLOG(2.0d0)
        calc_sigmaphi = (calc_mphi * D_sort**2 - calc_mphi) /
     &    (0.675d0 + 0.675d0 * D_sort**2)
        calc_mdp = DEXP(
     &            -1.0d0 * (calc_mphi * DLOG(2.0d0) + 2.5 *
     &            calc_sigmaphi**2 * (DLOG(2.0d0))**2)
     &            )
```

Mathematica translation

$$\text{calc\_mphi} = -\frac{\text{Log}[\text{D\_di}\ 1000]}{\text{Log}[2]};$$

$$\text{calc\_sigmaphi} = \frac{\text{calc\_mphi D\_sort}^2 - \text{calc\_mphi}}{0.675 + 0.675\,\text{D\_sort}^2};$$

$$\text{calc\_mdp} = e^{-(\text{calc\_mphi Log}[2] + 2.5\,\text{calc\_sigmaphi}^2\,\text{Log}[2]^2)};$$

Mohanty and Adams (2005) equations

$$\overline{\Phi} = -\log_2 D_{P_{med}}$$

$$\sigma_\Phi = \frac{\overline{\Phi} S_0^2 - \overline{\Phi}}{0.675 + 0.675\,S_0^2} \qquad (6)$$

$$\overline{D}_p = \exp\left\{-\left[\overline{\Phi}(ln\,2) + 2.5\sigma_\Phi^2 (ln\,2)^2\right]\right\} \qquad (4)$$

Conclusion: The factor 1000 next to D_di is to transform a particle diameter from meters to milimeters. Thus, NRUTIL equations are consistent with equations in Mohanty and Adams (2005).

Fortran form

```
      calc_sigmadp = DSQRT(
     &                 DEXP(
     &                       -1.0d0 * (
     &                                  2.0d0 * calc_mphi * DLOG(2.0d0) +
     &                                  4.0d0 * calc_sigmaphi**2 *
     &                                    (DLOG(2.0d0))**2
     &                                  )
     &                      )
     &                     )
```

Mathematica translation

$$calc\_sigmadp = \sqrt{e^{-\left(2\, calc\_mphi\, Log[2] + 4\, calc\_sigmaphi^2\, Log[2]^2\right)}}$$

Mohanty and Adams equation

$$\sigma_{D_p}^2 = \exp\left\{-\left[2\overline{\Phi}\left(ln 2\right) + 4\sigma_{\Phi}^2\left(ln 2\right)^2\right]\right\} \qquad (5)$$

Conclusion: equations are consistent

Fortran form

```
      calc_cdp = (calc_sigmadp / calc_mdp)
      calc_gama =  (DEXP(
     &                  -1.0d0 * (
     &                             3.0d0 * calc_mphi * DLOG(2.0d0) +
     &                             4.5d0 * calc_sigmaphi**2 *
     &                             (DLOG(2.0d0))**2
     &                           )
     &                 ) -
     &               3.0d0 * calc_mdp * calc_sigmadp**2 -
     &               calc_mdp**3
     &             ) / calc_sigmadp**3

      calc_perm0 = calc_mdp**2 * D_phi**3 /
     &   (72.0d0 * D_tau * (1.0d0 - D_phi)**2)
      calc_perm = calc_perm0 * (calc_gama * calc_cdp**3 +
     &   3 * calc_cdp**2 + 1.0d0)**2 / (1.0d0 + calc_cdp**2)**2
      Perm = calc_perm / 1.0d6
```

Mathematica translation

$$calc\_cdp = \frac{calc\_sigmadp}{calc\_mdp};$$

$$calc\_gama = \frac{e^{-(3\,calc\_mphi\,Log[2]+4.5\,calc\_sigmaphi^2\,Log[2]^2)} - 3\,calc\_mdp\,calc\_sigmadp^2 - calc\_mdp^3}{calc\_sigmadp^3};$$

$$calc\_perm0 = \frac{calc\_mdp^2\,D\_phi^3}{72\,D\_tau\,(1 - D\_phi)^2};$$

$$calc\_perm = \frac{calc\_perm0\,(calc\_gama\,calc\_cdp^3 + 3\,calc\_cdp^2 + 1)^2}{(1 + calc\_cdp^2)^2};$$

$$Perm = \frac{calc\_perm}{10^6};$$

Mohanty and Adams equations

C_{D_p} — Coefficient of variation of the particle size distribution = $\sigma_{D_p} / \overline{D}_p$ [unitless]

$$K = \frac{\overline{D}_p^2 \phi^3}{72\tau(1-\phi)^2}\left[\frac{\left(\gamma C_{D_p}^3 + 3C_{D_p}^2 + 1\right)^2}{\left(1 + C_{D_p}^2\right)^2}\right] \qquad (3)$$

γ — Skewness of a particle size distribution [unitless]

The equations in Mohanty and Adams (2005) do not include the skewness equation. The skewness is defined as

$$\gamma = \frac{\mu_3}{\sigma_{D_p}^{\ 3}} = \frac{\left\langle D_p^{\ 3}\right\rangle - 3\overline{D}_p \sigma_{D_p}^{\ 2} - \overline{D}_p^{\ 3}}{\sigma_{D_p}^{\ 3}}$$

where $\left\langle D_p^{\ 3}\right\rangle$ is the third moment of the particle size distribution. It is therefore inferred that

$$\left\langle D_p^{\ 3}\right\rangle = \exp\left\{-\left[3\overline{\phi}\ln(2) + 4.5\,\sigma_\phi^{\ 2}\ln(2)^2\right]\right\}$$

The above equation is not included in Mohanty and Adams (2005). However, that equation was published as Equation (17) by Manmath N. Panda and Larry W. Lake, Estimation of Single-Phase Permeability from Parameters of Particle-Size Distribution. AAPG Bulletin, V. 78, No. 7 (July 1994). p. 1028-1039

Conclusion: NRUTIL equations are consistent with Mohanty and Adams (2005) equations and the paper by Panda and Lake (1994).

Fortran form

```
      calc_perm0 = D_di**2 * D_phi**3 /
     &   (72.0d0 * D_tau * (1.0d0 - D_phi)**2)
      calc_perm = calc_perm0 * (D_gama * D_cdp**3 +
     &   3 * D_cdp**2 + 1.0d0)**2 / (1.0d0 + D_cdp**2)**2
      Perm = calc_perm
```

Mathematica translation

$$\text{calc\_perm0} = \frac{\text{D\_di}^2\ \text{D\_phi}^3}{72\ \text{D\_tau}\ (1 - \text{D\_phi})^2};$$

$$\text{calc\_perm} = \frac{\text{calc\_perm0}\ (\text{D\_gama}\ \text{D\_cdp}^3 + 3\ \text{D\_cdp}^2 + 1)^2}{(1 + \text{D\_cdp}^2)^2};$$

Mohanty and Adams (2005)

$$K = \frac{\overline{D}_p^2 \phi^3}{72\tau\left(1-\phi\right)^2}\left[\frac{\left(\gamma C_{D_p}^3 + 3C_{D_p}^2 + 1\right)^2}{\left(1 + C_{D_p}^2\right)^2}\right] \qquad (3)$$

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      D_Perm = Perm
      K = Perm
      alfam = D_alpha
      Rari = D_g * D_beta * K * radius_dso *
     &  (temperatureDSO - D_Temperature_BFO) / (D_nu * alfam)
      qc1 = 2.0d0 * 3.14159d0 * D_km *
     &  (temperatureDSO - D_Temperature_BFO) /
     &  dlog(radius_bfo/radius_dso)
      NCapu = 0.44d0 * (Rari**0.5) * dlog(radius_bfo/radius_dso) /
     &  (1.0d0 + 0.916d0 * ((radius_dso / radius_bfo)**0.5))
      D_Nassult = NCapu
      D_Rari = Rari
```

Mathematica translation

$$\text{D\_Perm} = \text{Perm};$$

$$\text{K} = \text{Perm};$$

$$\text{alfam} = \text{D\_alpha};$$

$$\text{Rari} = \frac{\text{D\_g D\_beta K radius\_dso (temperatureDSO - D\_Temperature\_BFO)}}{\text{D\_nu alfam}};$$

$$\text{qc1} = \frac{2\ 3.14159\ \text{D\_km (temperatureDSO - D\_Temperature\_BFO)}}{\text{Log}\left[\frac{\text{radius\_bfo}}{\text{radius\_dso}}\right]};$$

$$\text{NCapu} = \frac{0.44\ \text{Rari}^{0.5}\ \text{Log}\left[\frac{\text{radius\_bfo}}{\text{radius\_dso}}\right]}{1 + 0.916\left(\frac{\text{radius\_dso}}{\text{radius\_bfo}}\right)^{0.5}};$$

$$\text{D\_Nassult} = \text{NCapu};$$

$$\text{D\_Rari} = \text{Rari};$$

Mohanty and Adams (2005) equations

$$\mathrm{Ra}_{r_i} = g\beta K r_i \frac{T_{do} - T_{bfo}}{\nu\alpha_m} \qquad (2)$$

$$q_c = 2\pi k_m \frac{T_{do} - T_{bfo}}{ln\left(\frac{r_o}{r_i}\right)} \qquad (17)$$

$$\mathrm{Nu} \cong 0.44\,\mathrm{Ra}_{r_i}^{1/2} \frac{ln(r_o / r_i)}{1 + 0.916(r_i / r_o)^{1/2}} \qquad (9)$$

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
        q1 = NCapu * qc1

        keff = q1 * dlog(radius_bfo/radius_dso) /
     &    (2.0d0 * 3.14159d0 * (temperatureDSO - D_Temperature_BFO))

        kbf = keff
```

Mathematica translation

```
q1 = NCapu qc1;
```

$$\text{keff} = \frac{\text{q1 Log}\left[\frac{\text{radius\_bfo}}{\text{radius\_dso}}\right]}{2\ 3.14159\ (\text{temperatureDSO} - \text{D\_Temperature\_BFO})};$$

```
kbf = keff;
```

Mohanty and Adams equations

$$q' = \text{Nu} \cdot q_c \tag{16}$$

$$k_{bf} = k_{eff} = \frac{q'}{2\pi} \frac{ln\left(\frac{r_o}{r_i}\right)}{T_{do} - T_{bfo}} \tag{15}$$

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      D_gcond_bf3 = kbf

      fTargetDripShieldOuter = D_Temperature_BFO - temperatureDSO +
     &  (D_Thermal_Load_Above * dlog(radius_bfo/radius_dso) /
     &  (2.0d0 * 3.14159d0 * kbf * D_WPSpace))
```

Mathematica translation

D_gcond_bf3 = kbf;

$$\text{fTargetDripShieldOuter} = \text{D\_Temperature\_BFO} - \text{temperatureDSO} + \frac{\text{D\_Thermal\_Load\_Above Log}[\frac{\text{radius\_bfo}}{\text{radius\_dso}}]}{2\ 3.14159\ \text{kbf D\_WPSpace}};$$

Equation (10) in Mohanty and Adams (2005):

$$T_{wp} - T_{rw} = \frac{q}{2\pi wpl}\left[\frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2}+k_{r2}} + \frac{ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}\right] (10)$$

Equation (10) can be rewritten as

$T_{wp} - T_{rw} = (T_{bfo} - T_{rw}) + (T_{dso} - T_{bfo}) + (T_{dsi} - T_{dso}) + (T_{wp} - T_{dsi})$, where, for example

$$T_{dso} - T_{bfo} = \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}}$$

In NRUTIL, the objective function, ΔT_{bfo}, is defined as

$$\Delta T_{bfo} = T_{bfo} - T_{dso} + \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}}$$

The TPA code solves the equation $\Delta T_{bfo} = 0$; thus defining temperatures satisfying

$$T_{dso} - T_{bfo} = \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}}$$

in consistency with equations in Mohanty and Adams (2005).

Conclusion: equations in NRUTIL are consistent with equations in Mohanty and Adams (2005).

function fTargetDripShieldOuterNoBackfill(temperatureDSO)

Fortran form

```
     kr1 = dlog(radius_rw/radius_dso) * D_SBolt * radius_dso *
    &  (temperatureDSO**2 + D_Temperature_RW**2) *
    &  (temperatureDSO + D_Temperature_RW) /
    &  (
    &    (1.0d0 / D_EmissDS) +
    &    ((1.0d0 - D_EmissRW) / D_EmissRW) *
    &    (radius_dso / radius_rw)
    &  )
```

Mathematica translation

$$\text{kr1} = \frac{\text{Log}[\frac{\text{radius\_rw}}{\text{radius\_dso}}]\ \text{D\_SBolt radius\_dso}\ (\text{temperatureDSO}^2 + \text{D\_Temperature\_RW}^2)\ (\text{temperatureDSO} + \text{D\_Temperature\_RW})}{\frac{1}{\text{D\_EmissDS}} + \frac{(1 - \text{D\_EmissRW})\ \text{radius\_dso}}{\text{D\_EmissRW radius\_rw}}}$$

Mohanty and Adams equation

$$k_{r1} = \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)\sigma r_{wp}\left(T_{wp}^2 + T_{dsi}^2\right)\left(T_{wp} + T_{dsi}\right)}{\frac{1}{\epsilon_{wp}} + \frac{1-\epsilon_{dsi}}{\epsilon_{dsi}}\left(\frac{r_{wp}}{r_{dsi}}\right)} \qquad (13)$$

Note: equations are not identical; however, the equation in NRUTIL can be derived by replacing the wp subscript by dso and dsi by rw. The conductance equation is symmetrical. It applies to two concentric cylinders, the cylinder with the larger (smaller) radius is represented by r_{dsi} (r_{wp}) in Equation (13). Since radius_rw > radius_dso, the replacement is valid.

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005).

Fortran form

```
     kcv1 = 0.386d0 * D_kair * ((D_pr / (0.861d0 + D_pr))**0.25d0) *
    &  (
    &    (
    &      (dlog(radius_rw/radius_dso)**4.0d0) *
    &      D_g * D_beta * (temperatureDSO - D_Temperature_RW) /
    &      (
    &        D_nu * D_alpha *
    &        (
    &          ((2.0d0 * radius_dso)**(-3.0d0/5.0d0)) +
    &          ((2.0d0 * radius_rw)**(-3.0d0/5.0d0))
    &        )**5.0d0
    &      )
    &    )
    &  **0.25d0)
```

Mathematica translation

$$\text{kcv1} = 0.386\ \text{D\_kair}\left(\frac{\text{D\_pr}}{0.861 + \text{D\_pr}}\right)^{0.25}\left(\frac{\text{Log}[\frac{\text{radius\_rw}}{\text{radius\_dso}}]^4\ \text{D\_g D\_beta}\ (\text{temperatureDSO} - \text{D\_Temperature\_RW})}{\text{D\_nu D\_alpha}\ ((2\ \text{radius\_dso})^{-3/5} + (2\ \text{radius\_rw})^{-3/5})^5}\right)^{0.25}$$

Mohanty and Adams equation

$$k_{cv1} = 0.386 k_{air} \left(\frac{\Pr}{0.861 + \Pr} \right)^{1/4} \left[\frac{\left(\ln \frac{D_{dsi}}{D_{wp}} \right)^4 g \beta (T_{wp} - T_{dsi})}{\nu \alpha \left(D_{wp}^{-3/5} + D_{dsi}^{-3/5} \right)^5} \right]^{1/4} \quad (11)$$

Note: equations are not identical; however, the equation in NRUTIL can be derived by replacing T_{wp} by temperatureDSO and T_{dsi} by D_Temperature_RW. The thermal conductivity equation is symmetrical. It applies to two concentric cylinders, the cylinder with the larger (smaller) radius is represented by r_{dsi} (r_{wp}) in Equation (11). Since radius_rw > radius_dso, the replacement is valid. In Mohanty and Adams (2005), the equations presented correspond to the backfilled case. NRUTIL equations address general cases, including the no-backfilled case. Equations describing specific cases can be derived from equations in Mohanty and Adams (2005) by applying symmetry arguments.

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      fTargetDripShieldOuterNoBackfill =
     &  D_Temperature_RW - temperatureDSO +
     &  (D_Thermal_Load_Above / (2.0d0 * 3.14159 * D_WPSpace)) *
     &  (
     &    dlog(radius_rw / radius_dso) /
     &    (kcv1 + kr1)
     &  )

      D_gconv_2dw = kcv1
      D_grad_2dw = kr1
```

Mathematica translation

$$\text{fTargetDripShieldOuterNoBackfill} = \text{D\_Temperature\_RW} - \text{temperatureDSO} + \frac{(\text{D\_Thermal\_Load\_Above})\ \text{Log}\left[\frac{\text{radius\_rw}}{\text{radius\_dso}}\right]}{(2\ 3.14159\ \text{D\_WPSpace})\ (\text{kcv1} + \text{kr1})};$$

$$\text{D\_gconv\_2dw} = \text{kcv1};$$

$$\text{D\_grad\_2dw} = \text{kr1};$$

Equation (10) in Mohanty and Adams (2005) for the backfilled case:

$$T_{wp} - T_{rw} = \frac{q}{2\pi w p l} \left[\frac{\ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2} + k_{r2}} + \frac{\ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{\ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{\ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1} + k_{r1}} \right] (10)$$

For the no-backfilled case, applying symmetry arguments, the following relationship can be derived:

$$T_{wp} - T_{rw} = (T_{dso} - T_{rw}) + (T_{dsi} - T_{dso}) + (T_{wp} - T_{dsi}) = \frac{q}{2\pi\, wpl}\left[\frac{\ln\left(\frac{r_{rw}}{r_{dso}}\right)}{k_{cv2} + k_{r2}} + \frac{\ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{\ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1} + k_{r1}}\right]$$

where

$$T_{dso} - T_{rw} = \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{rw}}{r_{dso}}\right)}{k_{cv2} + k_{r2}}$$

In NRUTIL, the objective function, ΔT_{dso}, is defined as

$$\Delta T_{dso} = T_{rw} - T_{dso} + \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{brw}}{r_{dso}}\right)}{k_{cv2} + k_{r2}}$$

The TPA code solves the equation $\Delta T_{dso} = 0$ to define temperatures consistent with the equations in Mohanty and Adams (2005). There is a minor difference in the NRUTIL equations, that include a suffix 1 for the effective thermal conductivities: kcv1 and kr1. Such difference in the suffix is irrelevant. It is only a name change.

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

function fTargetWastePackage(temperatureWP)

Fortran form

```
      kr1 = dlog(radius_dsi/radius_wp) * D_SBolt * radius_wp *
     &  (temperatureWP**2 + D_Temperature_DSI**2) *
     &  (temperatureWP + D_Temperature_DSI) /
     &  (
     &    (1.0d0 / D_EmissWP) +
     &    ((1.0d0 - D_EmissDS) / D_EmissDS) *
     &    (radius_wp / radius_dsi)
     &  )
```

Mathematica translation

$$\mathrm{kr1} = \frac{\mathrm{Log}[\frac{\mathrm{radius\_dsi}}{\mathrm{radius\_wp}}]\ \mathrm{D\_SBolt\ radius\_wp}\ (\mathrm{temperatureWP}^2 + \mathrm{D\_Temperature\_DSI}^2)\ (\mathrm{temperatureWP} + \mathrm{D\_Temperature\_DSI})}{\frac{1}{\mathrm{D\_EmissWP}} + \frac{(1-\mathrm{D\_EmissDS})\ \mathrm{radius\_wp}}{\mathrm{D\_EmissDS\ radius\_dsi}}}$$

Mohanty and Adams (2005) equation

$$k_{r1} = \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)\sigma r_{wp}\left(T_{wp}^2 + T_{dsi}^2\right)\left(T_{wp} + T_{dsi}\right)}{\frac{1}{\epsilon_{wp}} + \frac{1-\epsilon_{dsi}}{\epsilon_{dsi}}\left(\frac{r_{wp}}{r_{dsi}}\right)} \qquad (13)$$

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      kcv1 = 0.386d0 * D_kair * ((D_pr / (0.861d0 + D_pr))**0.25d0) *
     &  (
     &    (
     &      (dlog(radius_dsi/radius_wp)**4.0d0) *
     &      D_g * D_beta * (temperatureWP - D_Temperature_DSI) /
     &      (
     &        D_nu * D_alpha *
     &        (
     &          ((2.0d0 * radius_wp)**(-3.0d0/5.0d0)) +
     &          ((2.0d0 * radius_dsi)**(-3.0d0/5.0d0))
     &        )**5.0d0
     &      )
     &    )
     &  **0.25d0)
```

Mathematica translation

$$\mathrm{kcv1} = 0.386\ \mathrm{D\_kair}\left(\frac{\mathrm{D\_pr}}{0.861 + \mathrm{D\_pr}}\right)^{0.25}\left(\frac{\mathrm{Log}[\frac{\mathrm{radius\_dsi}}{\mathrm{radius\_wp}}]^4\ \mathrm{D\_g\ D\_beta}\ (\mathrm{temperatureWP} - \mathrm{D\_Temperature\_DSI})}{\mathrm{D\_nu\ D\_alpha}\ ((2\ \mathrm{radius\_wp})^{-3/5} + (2\ \mathrm{radius\_dsi})^{-3/5})^5}\right)^{0.25}$$

Mohanty and Adams (2005) equation

$$k_{cv1} = 0.386 k_{air}\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left[\frac{\left(ln\frac{D_{dsi}}{D_{wp}}\right)^4 g\beta\left(T_{wp} - T_{dsi}\right)}{\nu\alpha\left(D_{wp}^{-3/5} + D_{dsi}^{-3/5}\right)^5}\right]^{1/4} \qquad (11)$$

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      ftargetWastePackage = D_Temperature_DSI - temperatureWP +
     &  (D_Thermal_Load_Above / (2.0d0 * 3.14159 * D_WPSpace)) *
     &  (
     &    dlog(radius_dsi / radius_wp) /
     &    (kcv1 + kr1)
     &  )
```

Mathematica translation

$$\text{ftargetWastePackage} = \text{D\_Temperature\_DSI} - \text{temperatureWP} + \frac{(\text{D\_Thermal\_Load\_Above})\ \text{Log}[\frac{\text{radius\_dsi}}{\text{radius\_wp}}]}{(2\ 3.14159\ \text{D\_WPSpace})\ (\text{kcv1} + \text{kr1})}$$

Equation (10) in Mohanty and Adams (2005):

$$T_{wp} - T_{rw} = \frac{q}{2\pi wpl}\left[\frac{ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2}+k_{r2}} + \frac{ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}\right] \quad (10)$$

Equation (10) can be rewritten as

$$T_{wp} - T_{rw} = \left(T_{bfo} - T_{rw}\right) + \left(T_{dso} - T_{bfo}\right) + \left(T_{dsi} - T_{dso}\right) + \left(T_{wp} - T_{dsi}\right),$$ where, for example

$$T_{wp} - T_{dsi} = \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1} + k_{r1}}$$

In NRUTIL, the objective function, ΔT_{wp}, is defined as

$$\Delta T_{wp} = T_{dsi} - T_{wp} + \frac{q}{2\pi\, wpl}\frac{\ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1} + k_{r1}}$$

The TPA code solves the equation $\Delta T_{wp} = 0$; thus defining temperatures consistent with equations in Mohanty and Adams (2005).

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

function fTargetWastePackageEarly(temperatureWP)

Fortran form

```
      kr1 = dlog(radius_rw/radius_wp) * D_SBolt * radius_wp *
     &  (temperatureWP**2 + D_Temperature_RW**2) *
     &  (temperatureWP + D_Temperature_RW) /
     &  (
     &    (1.0d0 / D_EmissWP) +
     &    ((1.0d0 - D_EmissRW) / D_EmissRW) *
     &    (radius_wp / radius_rw)
     &  )
```

Mathematica translation

$$kr1 = \frac{\text{Log}[\frac{\text{radius\_rw}}{\text{radius\_wp}}]\ \text{D\_SBolt radius\_wp (temperatureWP}^2 + \text{D\_Temperature\_RW}^2\text{) (temperatureWP + D\_Temperature\_RW)}}{\frac{1}{\text{D\_EmissWP}} + \frac{(1-\text{D\_EmissRW})\ \text{radius\_wp}}{\text{D\_EmissRW radius\_rw}}}$$

Mohanty and Adams equation

$$k_{r1} = \frac{ln\left(\frac{r_{dsi}}{r_{wp}}\right)\sigma r_{wp}\left(T_{wp}^2 + T_{dsi}^2\right)\left(T_{wp} + T_{dsi}\right)}{\frac{1}{\epsilon_{wp}} + \frac{1-\epsilon_{dsi}}{\epsilon_{dsi}}\left(\frac{r_{wp}}{r_{dsi}}\right)} \qquad (13)$$

Using symmetry arguments, the NRUTIL equation can be derived from Equation (13) by replacing T_{dsi} by D_Temperature_RW. This case is intended to represent the case when the drip shield is not present (preclosure).

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      kcv1 = 0.386d0 * D_kair * ((D_pr / (0.861d0 + D_pr))**0.25d0) *
     &  (
     &    (
     &      (dlog(radius_rw/radius_wp)**4.0d0) *
     &      D_g * D_beta * (temperatureWP - D_Temperature_RW) /
     &      (
     &        D_nu * D_alpha *
     &        (
     &          ((2.0d0 * radius_wp)**(-3.0d0/5.0d0)) +
     &          ((2.0d0 * radius_rw)**(-3.0d0/5.0d0))
     &        )**5.0d0
     &      )
     &    )
     &  **0.25d0)
```

Mathematica translation

$$kcv1 = 0.386\ \text{D\_kair}\left(\frac{\text{D\_pr}}{0.861 + \text{D\_pr}}\right)^{0.25}\left(\frac{\text{Log}[\frac{\text{radius\_rw}}{\text{radius\_wp}}]^4\ \text{D\_g D\_beta (temperatureWP - D\_Temperature\_RW)}}{\text{D\_nu D\_alpha}\left((2\ \text{radius\_wp})^{-3/5} + (2\ \text{radius\_rw})^{-3/5}\right)^5}\right)^{0.25}$$

Mohanty and Adams (2005) equation

$$k_{cv1} = 0.386 k_{air}\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}\left[\frac{\left(\ln\frac{D_{dsi}}{D_{wp}}\right)^4 g\beta\left(T_{wp}-T_{dsi}\right)}{\nu\alpha\left(D_{wp}^{-3/5}+D_{dsi}^{-3/5}\right)^5}\right]^{1/4} \quad (11)$$

Using symmetry arguments, the NRUTIL equation can be derived from Equation (11) by replacing T_{dsi} by D_Temperature_RW. This case is intended to represent the case when the drip shield is not present (preclosure).

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)

Fortran form

```
      ftargetWastePackageEarly = D_Temperature_RW - temperatureWP +
     &  (D_Thermal_Load_Above / (2.0d0 * 3.14159 * D_WPSpace)) *
     &  (
     &    dlog(radius_rw / radius_wp) /
     &    (kcv1 + kr1)
     &  )
```

Mathematica translation

$$\text{ftargetWastePackageEarly} = \text{D\_Temperature\_RW} - \text{temperatureWP} + \frac{(\text{D\_Thermal\_Load\_Above})\ \text{Log}\left[\frac{\text{radius\_rw}}{\text{radius\_wp}}\right]}{(2\ 3.14159\ \text{D\_WPSpace})\ (\text{kcv1} + \text{kr1})}$$

Equation (10) in Mohanty and Adams (2005) applies to the backfilled case with a drip shield present:

$$T_{wp} - T_{rw} = \frac{q}{2\pi wpl}\left[\frac{\ln\left(\frac{r_{rw}}{r_{bfo}}\right)}{k_{cv2}+k_{r2}} + \frac{\ln\left(\frac{r_{bfo}}{r_{dso}}\right)}{k_{bf}} + \frac{\ln\left(\frac{r_{dso}}{r_{dsi}}\right)}{k_{ds}} + \frac{\ln\left(\frac{r_{dsi}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}\right] \quad (10)$$

If the drifts are open and the drip shield is not present, equation (10) can be replaced by

$$T_{wp} - T_{rw} = \frac{q}{2\pi\, wpl}\,\frac{\ln\left(\frac{r_{rw}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}$$

In NRUTIL, the objective function, ΔT_{wp}, is defined as

$$\Delta T_{wp} = T_{rw} - T_{wp} + \frac{q}{2\pi\, wpl}\,\frac{\ln\left(\frac{r_{rw}}{r_{wp}}\right)}{k_{cv1}+k_{r1}}$$

The TPA code solves the equation $\Delta T_{wp} = 0$; thus defining temperatures consistent with equations in Mohanty and Adams (2005).

Conclusion: NRUTIL equations are consistent with equations in Mohanty and Adams (2005)