

ANALYSIS OF THE SUBSTITUTION TECHNIQUE FOR THE
DETERMINATION OF D₂O LATTICE BUCKLINGS

by

William E. Graves
Experimental Physics Division

Work done by

F. D. Benton	W. E. Graves
A. E. Dunklee	M. B. Stroud
H. R. Fike	S. V. Topp

June 1963

FOR RETENTION

E. I. DU PONT DE NEMOURS & COMPANY
EXPLOSIVES DEPARTMENT - ATOMIC ENERGY DIVISION
TECHNICAL DIVISION - SAVANNAH RIVER LABORATORY
AIKEN, SOUTH CAROLINA

CONTRACT AT (07-2) - 1 WITH THE
UNITED STATES ATOMIC ENERGY COMMISSION

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

Printed in USA. Price \$0.75
Available from the Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

ABSTRACT

Three methods of analysis for the substitution experiments were tested against experiment: a one-group, three-region perturbation method; a two-group, two-region diffusion theory method; and a two-group, three-region diffusion theory method. It was found that the one-group perturbation method worked well over the full range of the experiments to which it was applied. The two-group, two-region method worked well when diffusion coefficient mismatches were the main problem, but broke down when there was a bad mismatch in resonance escape probability. The two-group, three-region method worked well, but did not offer any advantage over the one-group perturbation method to justify its greater complexity.

CONTENTS

	<u>Page</u>
List of Tables and Figures	4
Introduction	5
Discussion	5
Methods of Analysis	5
One-Group Perturbation Method	5
Two-Region, Two-Group Method	11
Three-Region, Two-Group Method	12
Experimental Procedure	14
Results	18
$\delta D/D$ Measurements	18
Substitution Measurements	20
Acknowledgment	29
Bibliography	30

LIST OF TABLES AND FIGURES

<u>Table</u>		<u>Page</u>
I	Fuel Assembly Details	14
II	Changes in Neutron Diffusion Coefficient on D ₂ O Removal from UO ₂ Rod Cluster Fuel Assemblies	18
III	Basic Data from Substitution Measurements	21
IV	Two-Group Parameters	21
V	Results of One-Group Perturbation Analysis	25
VI	Results of Two-Region, Two-Group Analysis	25
VII	Results of Three-Region, Two-Group Analysis	26
VIII	Results of One-Region Measurements	26
IX	Comparison of Corrected Results	28

<u>Figure</u>		
1	File Loading Diagram - One Test Assembly - Persson Unit Cell Definition	7
2	File Loading Diagram - Seven Test Assemblies - Persson Unit Cell Definition	7
3	File Loading Diagram - Seven Test Assemblies - Cylindricized Unit Cell Definition	11
4	Cross Sections of Fuel Assemblies	15
5	Critical Vertical Buckling of Reference Loading vs Run Number	17
6	Example of Measurement of $\delta D_z/D_{z1}$	19
7	Example of Measurement of $\delta D_r/D_{z1}$	19
8	One-Group Perturbation Analysis Plots	22

ANALYSIS OF THE SUBSTITUTION TECHNIQUE FOR THE DETERMINATION OF D₂O LATTICE BUCKLINGS

INTRODUCTION

A typical substitution measurement of a lattice buckling has three steps: (1) a reference measurement in a one-region lattice; (2) the substitution of one or more test fuel assemblies for reference fuel assemblies; and (3) a remeasurement in the mixed lattice. The resultant data are then combined with subsidiary information from calculations and experiments and analyzed to give the buckling of the test lattice. In critical substitution measurements on D₂O-moderated lattices, the measured physical property is the moderator height at criticality.

Various straightforward methods of analyzing the data have been used in the past^(1,2) to obtain the material buckling of the test lattice. There has not, however, been a very extensive attempt to verify these methods of analysis empirically over a wide range of lattice parameters. The present report makes such an effort by comparing experimental results of substitution measurements with test lattices having bucklings known from previous one-region pile measurements. A number of the lattices contained air-filled fuel assembly housings so as to include the effects of neutron streaming and bad diffusion coefficient mismatches.

DISCUSSION

METHODS OF ANALYSIS

ONE-GROUP PERTURBATION METHOD

This method of analysis was suggested and described by Persson⁽¹⁾. It is a "successive substitution" method; i.e., the measurements are performed with at least two different sized test regions. These results are then used to extrapolate to the condition where the test lattice extends throughout the pile.

Since one-group theory cannot properly take into account the interaction between the test and reference regions, a third region, called the transition or intermediate region, is introduced. In order to define the three regions, the lattices (in this discussion assumed to have triangular pitches) are broken up into unit cells consisting of rhomboids with the two vertices at the acute (60°) angles terminating on fuel assemblies.

Rhomboids with both acute vertices terminating on test fuel were assigned to the test region, those with one vertex on a test fuel assembly and the other on a reference fuel assembly were assigned to the intermediate region, and those with both vertices on reference fuel to the reference region. Examples of this choice of regions, for one and seven test fuel assemblies, are shown in Figures 1 and 2, respectively.

The final equation used in the analysis, derived by Persson⁽¹⁾, is given below. (In a paper presented by Persson at a meeting of the European-American Committee on Reactor Physics in Zurich in February 1963, a slightly different final equation was given. The numerical differences resulting from the change are quite negligible.)

$$\frac{(\alpha^2 - \alpha_1^2)(1 + \xi)}{\bar{w}_r} - \beta^2 \frac{\sum_1 (W_{r_1} - U_{r_1}) \delta D_{r_1} / D_{z_1}}{\bar{w}_r} = (\alpha_3^2 - \alpha_1^2) + \delta \alpha_2^2 \frac{\bar{w}_r}{w_r} \quad (1)$$

In equation (1), the subscript 1 refers to the reference region, 2 refers to the intermediate region, and 3 refers to the test region. The quantity α^2 is the vertical buckling of the pile with test assemblies in place; α_1^2 is the vertical buckling of the pile when all fuel assemblies are of the reference type; α_3^2 is the vertical buckling that would be obtained if all of the reference fuel were replaced by test fuel and if the radial buckling remained constant; $\delta \alpha_2^2 = \alpha_2^2 - (\alpha_3^2 + \alpha_1^2)/2$; and β^2 is the radial buckling of the pile. D refers to the one-group diffusion coefficient, with the subscripts r and z referring to radial and vertical directions, respectively, and $\delta D_1 = D_1 - D_1$. Values of W_{r_1} and U_{r_1} are given by

$$W_{r_1} = \frac{\int_1 \phi \phi' dv}{\sum_1 \int_1 \phi \phi' dv} \quad (2)$$

and

$$U_{r_1} = \frac{1}{\beta \beta_1'} \frac{\int_1 (\nabla_r \phi)(\nabla_r \phi') dv}{\sum_1 \int_1 \phi \phi' dv} \quad (3)$$

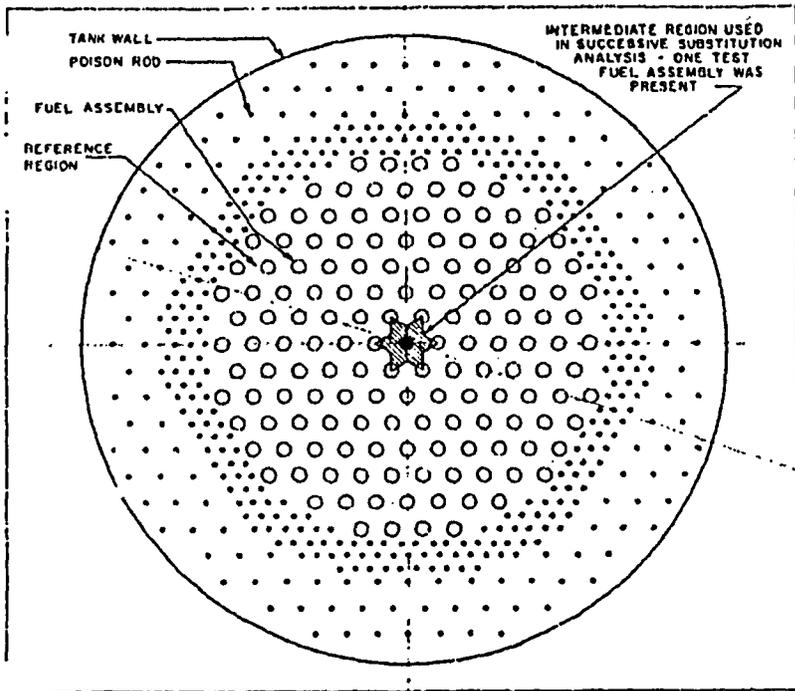


FIG. 1 PILE LOADING DIAGRAM - ONE TEST ASSEMBLY -
PERSSON UNIT CELL DEFINITION

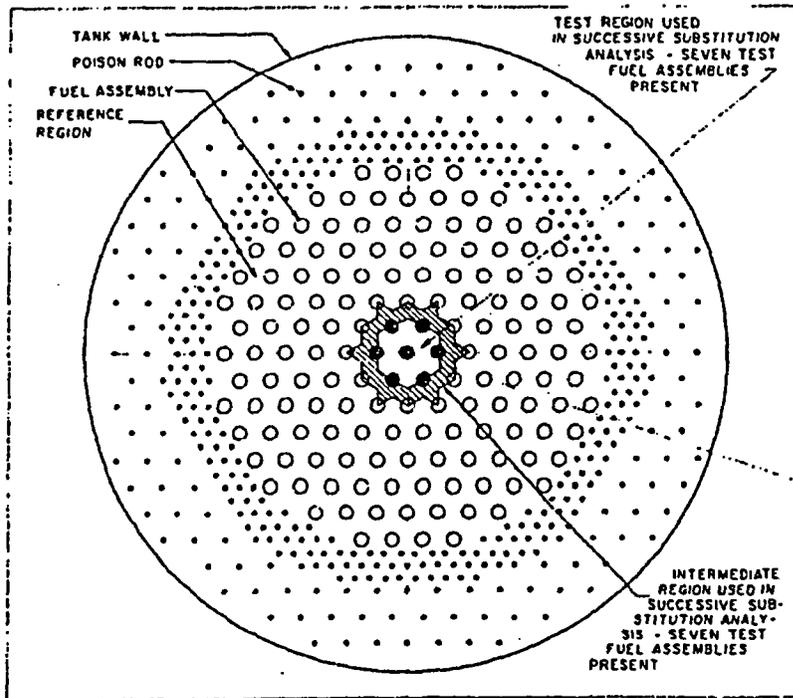


FIG. 2 PILE LOADING DIAGRAM - SEVEN TEST ASSEMBLIES -
PERSSON UNIT CELL DEFINITION

In the above equations, ϕ is the unperturbed flux ($= J_0(\beta r)$), ϕ' is the perturbed flux, and β_1^1 is the radial buckling in the 1th region. The perturbed fluxes were calculated by one-group theory. The remaining quantities in equation (1) are given by

$$\xi = \sum_1 W_{r_1} \frac{\delta D_{z_1}}{D_{z_1}} + \sum_1 U_{r_1} \frac{\delta D_{r_1}}{2D_{z_1}} \quad (4)$$

$$\bar{W}_r = W_{r_3} \frac{D_{z_3}}{D_{z_1}} + \frac{1}{2} W_{r_2} \frac{D_{z_2}}{D_{z_1}} - \frac{1}{2} U_{r_3} \frac{\delta D_{r_3}}{D_{z_1}} - \frac{1}{4} U_{r_2} \frac{\delta D_{r_2}}{D_{z_1}} \quad (5)$$

$$\bar{W}_{r_2} = W_{r_2} \frac{D_{z_2}}{D_{z_1}} - \frac{1}{2} U_{r_2} \frac{\delta D_{r_2}}{D_{z_1}} \quad (6)$$

If the left-hand side of equation (1) is plotted against \bar{W}_{r_2}/\bar{W}_r , a straight line with an intercept of $\alpha_3^2 - \alpha_1^2$ should result. Each point on the line would correspond to a different number of test fuel assemblies. If there were no anisotropy, the value of $\alpha_3^2 - \alpha_1^2$ would also be the difference between the material bucklings of the test and reference lattices.

The values of $\delta D_{r_1}/D_{z_1}$ and $\delta D_{z_1}/D_{z_1}$ used in the analysis can be calculated, but it is preferable to measure them in separate experiments with a single fuel assembly. In this case unperturbed flux shapes are used.

Persson⁽¹⁾ treated the special case of obtaining $\delta D_{z_1}/D_{z_1}$ and $\delta D_{r_1}/D_{z_1}$ on voiding the housing of a reference fuel assembly. In this case 1 = 1 and 2 only. The result was

$$\begin{aligned} \frac{\alpha^2 - \alpha_1^2}{W_{r_2} W_{z_2}} &= (\alpha_2^2 - \alpha_1^2) \left(1 + \frac{\delta D_{z_2}}{D_{z_1}} \right) - \beta^2 \frac{\delta D_{r_2}}{D_{z_1}} \left(\frac{U_{r_2}}{W_{r_2}} - 1 \right) \\ &\quad - \alpha_1^2 \frac{\delta D_{z_2}}{D_{z_1}} \left(\frac{\alpha_1^2 U_{z_2}}{\alpha_1^2 W_{z_2}} - 1 \right) \end{aligned} \quad (7)$$

where

$$W_{z_1} = \int_1 \sin^2 \alpha z \, dz / (H/2) \quad (8)$$

and

$$U_{z_1} = \int_1 \cos^2 \alpha z \, dz / (H/2) \quad (9)$$

and the other symbols are as defined before. In the case in which the perturbation penetrates the whole core in the axial direction, $U_{z_2} = W_{z_2} = 1$ and equation (7) becomes

$$(\alpha^2 - \alpha_1^2) \left(\frac{1}{W_{r_2}} + \frac{\delta D_{z_2}}{D_{z_1}} \right) = (\alpha_2^2 - \alpha_1^2) \frac{D_{z_2}}{D_{z_1}} - \beta^2 \frac{\delta D_{r_2}}{D_{z_1}} \left(\frac{U_{r_2}}{W_{r_2}} - 1 \right) \quad (10)$$

The experimental procedure is to void the housing of the central fuel assembly in several steps, measuring the critical moderator height at each step. Then plotting the left-hand

side of equation (7) against $\left(\frac{\alpha^2}{\alpha_1^2} \frac{U_{z_2}}{W_{z_2}} - 1 \right)$ gives a straight line with slope equal to $-\alpha_1^2 (\delta D_{z_2} / D_{z_1})$. To obtain $\delta D_{r_2} / D_{z_1}$,

the critical moderator heights are measured on completely voiding the housings of single fuel assemblies at various radii in the pile. Then plotting the left-hand side of equation (10) against $\left(\frac{U_{r_2}}{W_{r_2}} - 1 \right)$ gives a straight line with slope equal to $-\beta^2 (\delta D_{r_2} / D_{z_1})$.

Of more general interest is the measurement of δD on voiding the housing of a single test assembly. In this case there are three types of regions in the pile: the reference region, denoted by subscript 1; the intermediate region between reference fuel and the D_2O -filled portion of the test assembly, denoted by subscript 2a; and the intermediate region between reference fuel and the voided portion of the test assembly, denoted by subscript 2b. It can easily be shown that the one-group perturbation result corresponding to equation (7) can be cast into the form

$$\begin{aligned}
& \left(1 + W_{r_2} \frac{\delta D_{z_{2a}}}{D_{z_1}} \right) \left(\frac{\alpha^2 - \alpha^{*2}}{W_{r_2} W_{z_{2b}}} \right) = \alpha_{2b}^2 \left(1 + \frac{\delta D_{z_{2b}}}{D_{z_1}} \right) - \alpha_{2a}^2 \left(1 + \frac{\delta D_{z_{2a}}}{D_{z_1}} \right) \\
& - \beta^2 \left(\frac{\delta D_{r_{2b}}}{D_{z_1}} - \frac{\delta D_{r_{2a}}}{D_{z_1}} \right) \left(\frac{U_{r_2}}{W_{r_2}} - 1 \right) - \alpha^{*2} \left(\frac{\delta D_{z_{2b}}}{D_{z_1}} - \frac{\delta D_{z_{2a}}}{D_{z_1}} \right) \left(\frac{\alpha^2}{\alpha^{*2}} \frac{U_{z_{2b}}}{W_{z_{2b}}} \right)
\end{aligned} \tag{11}$$

where α^2 is the vertical buckling when some or all of a single test assembly is voided, and α^{*2} is the vertical buckling prior to voiding but with the test assembly in place. This form of the equation is convenient, since $(\alpha^2 - \alpha^{*2})$ can be measured quite precisely. In practice, if the reference lattice is D_2O filled, $\delta D_{z_{2a}}$ can be set equal to zero, equation (11) becomes

$$\begin{aligned}
\frac{\alpha^2 - \alpha^{*2}}{W_{r_2} W_{z_{2b}}} &= \alpha_{2b}^2 \left(1 + \frac{\delta D_{z_{2b}}}{D_{z_1}} \right) - \alpha_{2a}^2 - \beta^2 \frac{\delta D_{r_{2b}}}{D_{z_1}} \left(\frac{U_{r_2}}{W_{r_2}} - 1 \right) \\
&- \alpha^{*2} \frac{\delta D_{z_{2b}}}{D_{z_1}} \left(\frac{\alpha^2}{\alpha^{*2}} \frac{U_{z_{2b}}}{W_{z_{2b}}} \right)
\end{aligned} \tag{12}$$

The use of equation (11) and its specialized form when $U_{z_{2b}} = W_{z_{2b}} = 1$ are the same as that of equations (7) and (10) in determining $\left(\frac{\delta D_{z_{2b}}}{D_{z_1}} - \frac{\delta D_{z_{2a}}}{D_{z_1}} \right)$ and $\left(\frac{\delta D_{r_{2b}}}{D_{z_1}} - \frac{\delta D_{r_{2a}}}{D_{z_1}} \right)$.

Persson⁽¹⁾ used statistical weights corresponding to the cell definition of Figure 1 and later multiplied the $\delta D_{z_2}/D_{z_1}$ and $\delta D_{r_2}/D_{z_1}$ values by 2 to obtain test lattice parameters. In the present report, statistical weights corresponding to the conventional unit cell definition were used in the analysis. The slopes of the curves then should give test lattice parameters directly without multiplying by 2. There seems to be little preference between the two procedures.

To summarize, equation (11) is used to obtain diffusion coefficient differences when voids are present. After the successive substitution measurements, equation (1) is used to obtain $(\alpha_3^2 - \alpha_1^2)$. Additional corrections necessary to convert this to the material buckling of the test lattice will be discussed in a later section.

Two-Region, Two-Group Method

This method of analysis has a long history of use at the Savannah River Laboratory (SRL). It requires only a single measurement of critical moderator height with test assemblies present, in addition to the measurement of the critical moderator height with the one-region reference loading. With triangular lattice pitches, the commonest number of test assemblies is seven. As in all the methods of analysis, it is necessary to know the vertical extrapolation distance and the extrapolated radius of the pile. These are determined from flux profile measurements in one-region reference loadings. The test region is defined in terms of conventional unit cells, rather than the cells described in the previous section. An example of the test region according to this definition is shown in Figure 3. The boundary between the test and reference

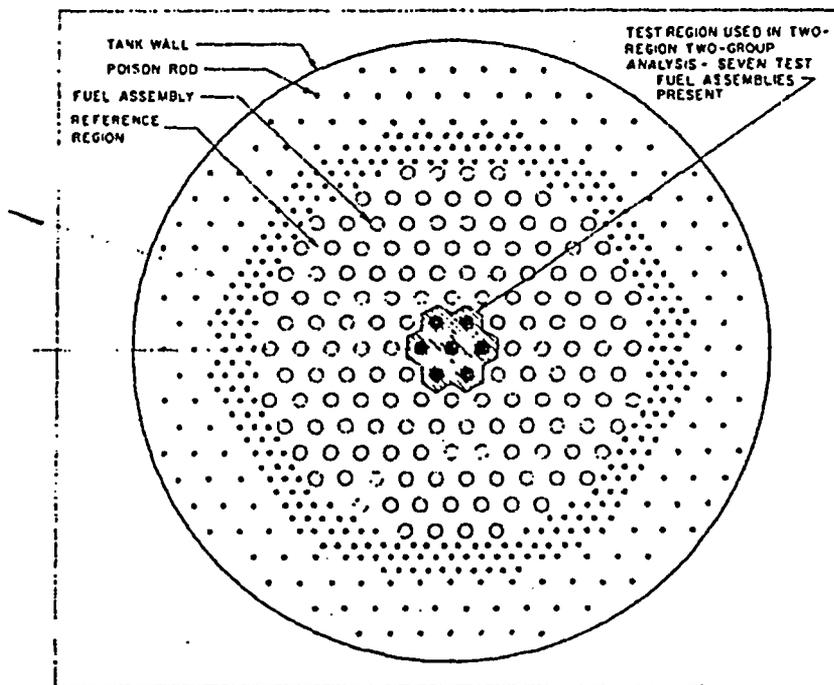


FIG. 3 PILE LOADING DIAGRAM - SEVEN TEST ASSEMBLIES -
CYLINDRICIZED UNIT CELL DEFINITION

loading is cylindrical in the calculation. Two-group parameters are computed for both the test and reference region. These consist of the slow diffusion coefficient D_S , the fast diffusion coefficient D_F , the Fermi age τ , the diffusion area L^2 , and the resonance escape probability p . The material buckling of the reference region is determined by the extrapolated radius, the vertical extrapolation distance, and the measured critical moderator height in the one-region reference loading. The specification of the critical vertical buckling with the test lattice in place (from the critical moderator height) then completely determines the problem according to two-group theory, and the test region material buckling can be computed. Even a two-region, two-group calculation is not suitable for hand calculation, however, and an appropriate code for the IBM 704 is used to perform the two-group calculation. The mathematical basis of the code is discussed to some extent in a previous report⁽³⁾.

Additional accuracy can be achieved by calibrating the test region with lattices of known bucklings, and two-group parameters quite similar to those of the reference region. Such a calibration is used to adjust one of the constants used in the calculation. At SRL, the constant traditionally adjusted by this means is the test region radius, which thus would differ slightly from the cylindricalized radius. This choice of constant to be adjusted is not unique. The test region calibration procedure has been described elsewhere^(2,4). It will not be further discussed here for two reasons:

- Suitable calibration lattices are rarely available, thus restricting the applicability of the technique.
- In the present experiments, for which suitable calibration assemblies were available^(2,4), the difference between the test region radius resulting from the calibration and the cylindricalized test region radius was negligible.

Corrections to the bucklings obtained by the two-group method will be discussed in a later section.

THREE-REGION, TWO-GROUP METHOD

This method was an attempt to combine some of the features of the previous two methods. It has in common with the Persson method the necessity for measurements using two different sized test regions, and the assumption of an intermediate region of unknown buckling. However, it uses two-group theory rather than one-group perturbation theory to analyze the data. In

one sense this is more restrictive on the experiment, in that it requires all test regions to be concentric with the reference lattice. This need not be true if perturbation theory is used, although it is desirable.

The measurements consist of critical moderator height measurements for the one-region reference lattice, the lattice with one test assembly in the center of the pile, and the lattice with more than one test assembly in the center of the pile. The boundaries of test, intermediate, and reference regions are determined as in the perturbation method (Figures 1 and 2), and then cylindricalized. Two-group parameters for reference and test regions are calculated, as in the two-region, two-group method. The two-group parameters of the intermediate region are then determined by the equations given below. Subscript 1 refers to reference region, 2 to intermediate region, and 3 to test region.

$$D_{s_2} = (D_{s_1} + D_{s_3})/2 \quad (13)$$

$$D_{f_2} = (D_{f_1} + D_{f_3})/2 \quad (14)$$

$$L_2^2 = \frac{D_{s_1} + D_{s_3}}{\frac{D_{s_1}}{L_1^2} + \frac{D_{s_3}}{L_3^2}} \quad (15)$$

$$\tau_2 = \frac{D_{f_1} + D_{f_3}}{\frac{D_{f_1}}{\tau_1} + \frac{D_{f_3}}{\tau_3}} \quad (16)$$

$$p_2 = \frac{(1 + L_2^2 B_{m_2}^2)(1 + \tau_2 B_{m_2}^2)}{\frac{1}{2}[(\eta \epsilon f)_1 + (\eta \epsilon f)_3]} \quad (17)$$

The symbols have their conventional meanings. The quantity $B_{m_2}^2$ is not known initially, so that an initial guess must be made for p_2 .

In the case where only one test assembly is used, there are only two types of regions present: reference and intermediate. The buckling of the intermediate region can therefore be calculated by two-group theory just as in the two-region, two-group method. The only difference is that an iterative process must be used in which the buckling solution is substituted into equation (17) to obtain a new value of p_2 , the two-group calculation repeated with the new value of p_2 , etc. until the process has converged.

Once the intermediate region buckling has been obtained, it can be used in a two-group, three-region calculation of the measurement in which more than one test fuel assembly is present. All parameters in this calculation except the test region buckling would be known at this point, and therefore the test region buckling could be computed.

Corrections to the bucklings obtained by this method will be discussed in a later section.

EXPERIMENTAL PROCEDURE

Four types of fuel assemblies were used in these experiments: a cluster of 31 UO_2 rods in a 5-inch-OD aluminum housing, a cluster of 19 UO_2 rods in a 5-inch-OD aluminum housing, a cluster of 19 UO_2 rods in a 4-inch-OD aluminum housing, and a single 1-inch-diameter rod of natural uranium metal. All of the fuel assemblies had been used in one-region pile buckling measurements^(5,6). Fuel assembly details are given in Table I, and cross sections of the fuel assemblies are shown in Figure 4.

TABLE I

Fuel Assembly Details

UO_2 density = 10.4 gm/cm³ (effective over rod length)
OD of UO_2 = 0.500 inch
OD of 6063 aluminum cladding = 0.547 inch
 UO_2 cladding thickness = 0.020 inch
Center-to-center spacing of UO_2 rods = 0.650 inch
OD of small aluminum tubes in 31-rod clusters = 0.250 inch
Wall thickness of small aluminum tubes in 31-rod clusters = 0.035 inch
OD of large 6063 aluminum housing tube = 5.00 inches
Wall thickness of large 6063 aluminum housing tube = 0.056 inch
OD of small 6063 aluminum housing tube = 4.00 inches
Wall thickness of small 6063 aluminum housing tube = 0.054 inch
OD of uranium metal rods = 0.998 inch
OD of 1100 aluminum cladding = 1.090 inches
Wall thickness of 1100 aluminum cladding = 0.032 inch

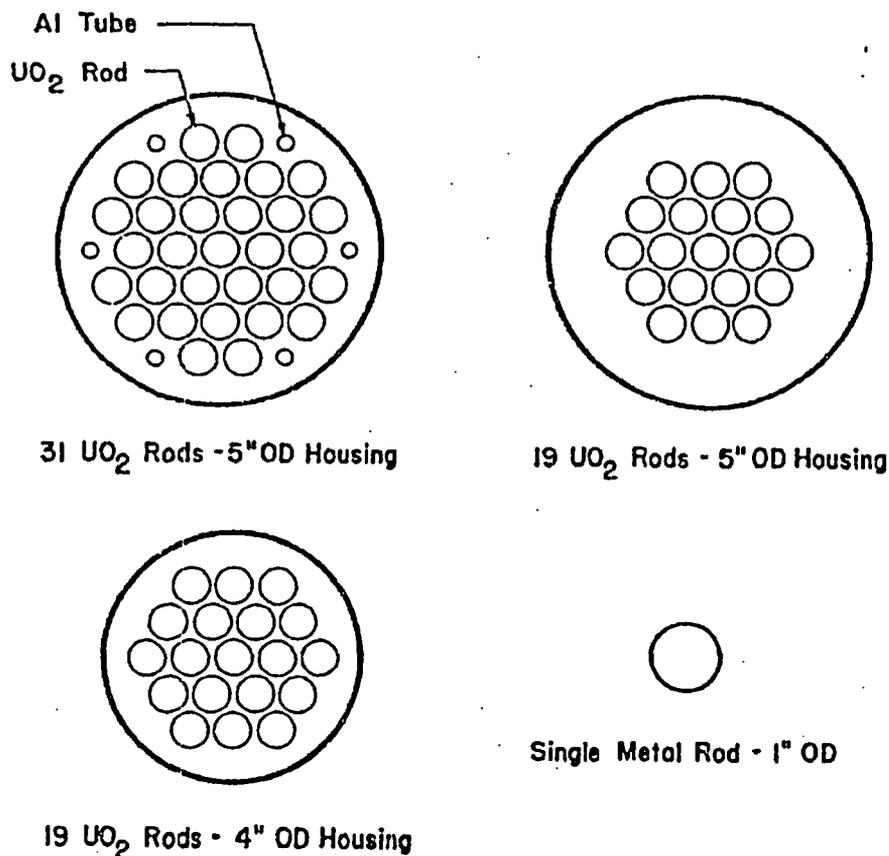


FIG. 4 CROSS SECTIONS OF FUEL ASSEMBLIES

All experiments were performed in the Process Development Pile (PDP)⁽³⁾ on a 9.33-inch triangular pitch in the lattice geometry shown in Figure 1. Two different reference fuel assemblies were used: the 31-rod UO₂ cluster in a D₂O-filled housing, and the 31-rod UO₂ cluster in an air-filled housing. The extrapolated pile radii and vertical extrapolation distances were measured by the flux profile method^(5,6). The moderator temperature during the experiments was 22.5 ± 1.0°C, and the moderator isotopic purity was 99.58 mol % D₂O with an absolute uncertainty of ±0.03 mol % D₂O.

The experiments were of two types: the measurement of $\delta D/D$ values by the Persson method on voiding the coolant channels of test assemblies, and the substitution measurements proper. The $\delta D_2/D$ measurements were made by placing a single test fuel assembly in the central fuel position. The bottom of the test assembly housing was open to the moderator. The

pile was brought to critical and held there for at least 30 minutes, after which the moderator height was measured. Helium pressure was then applied at the top of the test fuel assembly to partially expel the D₂O coolant. The expulsion was performed in several steps, and the moderator height was changed at each step to maintain criticality. At each step the critical moderator height and the helium pressure were measured, the latter with a light water manometer. The height of the D₂O coolant in the test assembly was inferred from the helium pressure. After all of the coolant had been expelled, the pressure was reduced to zero and the critical moderator height was remeasured. The final critical moderator height was usually within about 0.03 cm of the initial one, and the two were averaged to obtain the reference vertical buckling α^{*2} (see equation (11)).

The measurement of $\delta D_r/D$ was made using three test fuel assemblies at symmetrical 120° positions instead of a single assembly. The purpose of this was to increase the precision of the measurement by obtaining larger changes in vertical buckling. The smallest distance of test assemblies from the center of the pile was two lattices pitches, so that the perturbations would not be expected to interact. The modification of the analysis to account for three test assemblies rather than one was therefore trivial.

With the three test assemblies at a single radius in the pile, the critical moderator height was measured with D₂O in the test assembly housing. The D₂O was then completely expelled by means of helium pressure, criticality being maintained by changing the moderator height. After the new critical moderator height had been measured, the helium pressure was reduced to zero and the initial condition remeasured. The pile was then shut down and the test assemblies were moved to a different radius. Another critical run was then made, etc.

The substitution measurements proper, unlike the $\delta D/D$ measurements, depended on comparing critical moderator heights of separate reactor runs. Five types of test fuel configurations were used: a single test fuel assembly in the central fuel position; three test fuel assemblies in an equilateral triangle one lattice pitch on the side, with one of the test fuel assemblies in the central fuel position; three fuel assemblies at 120° to each other and one lattice pitch from the central fuel position; seven fuel assemblies in the central seven positions; and nineteen fuel assemblies in the central nineteen positions. Whenever the same configuration was to be used to measure test lattices both with D₂O and air in the housings, the helium pressure system was used to obtain both

measurements in the same critical run. The critical moderator height with the one-region reference lattice was measured frequently. Each measurement included a determination of the (relative) critical moderator height to within about 0.005 cm and a determination of the (relative) moderator temperature to about 0.03°C. All vertical bucklings were then corrected to a common temperature by means of calculated temperature coefficients. When this had been done, it was found that the critical vertical buckling of the one-region reference lattice decreased smoothly with run number. This decrease in buckling (in μB^*) is shown over a limited range of run numbers in Figure 5. The effect is presumably due to moderator degradation. Plots such as Figure 5 were used to correct vertical bucklings with test fuel assemblies in place to the run number of the nearest reference lattice measurement.

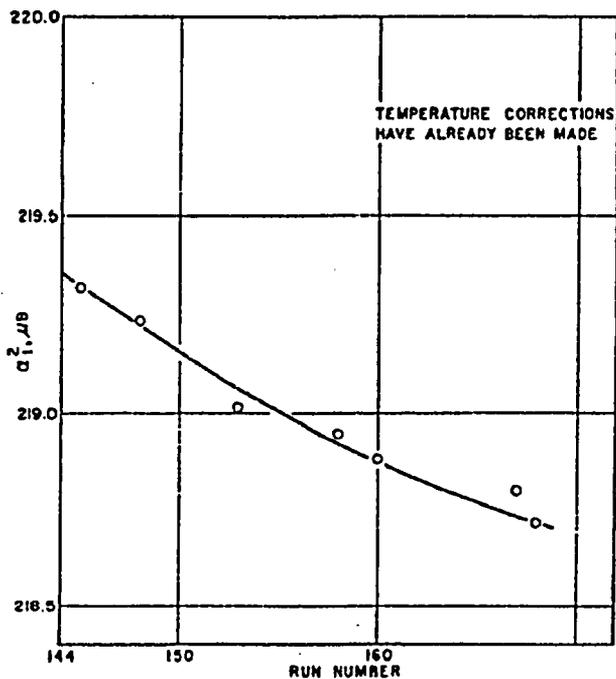


FIG. 5 CRITICAL VERTICAL BUCKLING OF
REFERENCE LOADING vs RUN NO.

*1 $\mu B = 10^{-6} \text{ cm}^{-2}$

RESULTS

$\delta D/D$ MEASUREMENTS

It was assumed that the one-group diffusion coefficients of all lattices containing no void were the same. The majority of the $\delta D/D$ measurements were made in a reference lattice of 31-rod UO_2 clusters with D_2O in the housings. In two measurements the reference lattice contained the same fuel assemblies with air-filled housings. In order to facilitate comparison, all results are presented in the form of the ratio of δD to the one-group diffusion coefficient of the 31-rod cluster lattice with D_2O in the housings. Sample plots of a $\delta D_z/D$ measurement and a $\delta D_r/D$ measurement are shown in Figures 5 and 7, respectively. Complete numerical results are given in Table II.

TABLE II

Changes in Neutron Diffusion Coefficient on D_2O Removal
from UO_2 Rod Cluster Fuel Assemblies

Rods per cluster		19	19	31
OD of housing, inches		4	5	5
$\delta D_z/D_{z,ref}$	Exp. 1	-	0.533 \pm 0.011	0.287 \pm 0.005
	Exp. 2	0.203 \pm 0.005	0.524 \pm 0.010	0.277 \pm 0.004
	Average	0.203 \pm 0.010	0.529 \pm 0.015	0.282 \pm 0.010
	Calc.	0.187	0.598	0.345
$\delta D_r/D_{z,ref}$	Exp. 2	0.134 \pm 0.017	0.271 \pm 0.020	0.199 \pm 0.011
	Calc.	0.128	0.328	0.232
D_z/D_r (anisotropy)	Exp. 2	1.061 \pm 0.018	1.203 \pm 0.022	1.069 \pm 0.013
	Calc.	1.052	1.203	1.092

(a) Experiment 1 was performed in a reference lattice of 31-rod clusters in air-filled housing tubes. Experiment 2 was performed in a reference lattice of 31-rod clusters in D_2O -filled housing tubes. In order to obtain the value listed under Experiment 1 for the 19-rod cluster in a 5-inch housing, a value of $D_{31,air}/D_{31,D_2O} = 1.282$ was used.

(b) The errors on the average values of Experiment 1 and Experiment 2 were increased slightly over the estimated errors of individual measurements, to take into account small systematic differences between the results of Experiment 1 and Experiment 2.

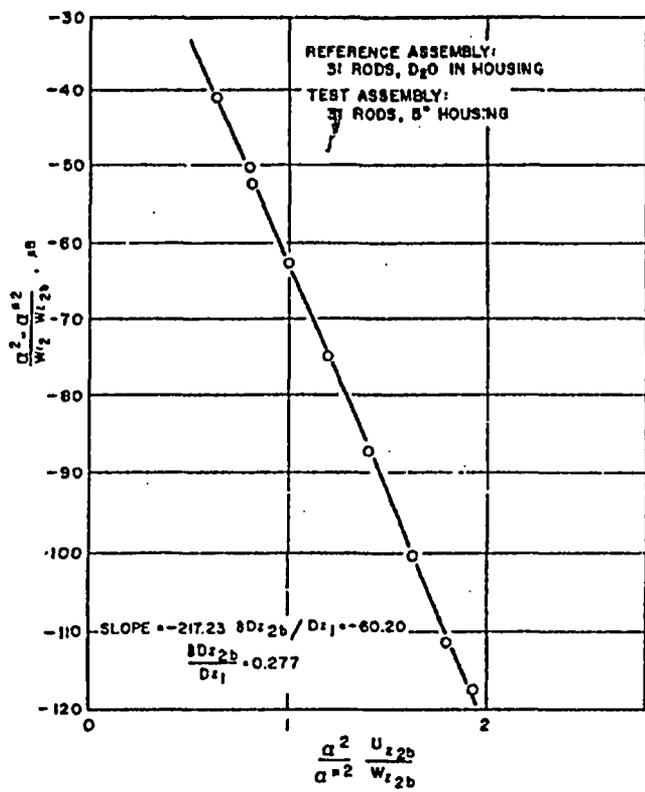


FIG. 6 EXAMPLE OF MEASUREMENT OF $\delta D_z / D_{z1}$

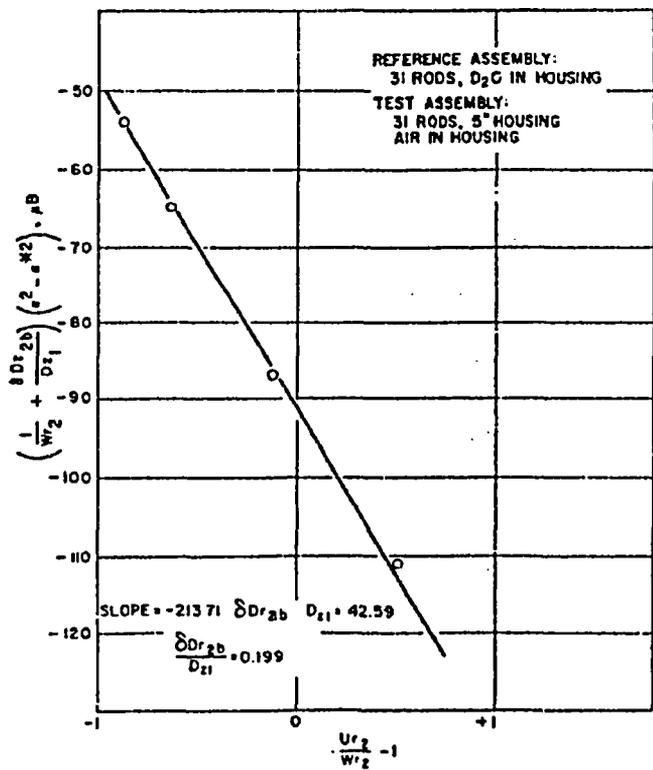


FIG. 7 EXAMPLE OF MEASUREMENT OF $\delta D_r / D_{z1}$

The theory of Benoist⁽⁷⁾ was used to obtain theoretical values of $\delta D/D$ to compare with experiment. A recent modification⁽⁸⁾ in the calculation of the radial component was included. The transport mean free paths used in the computation are given in Reference 5. The results of the calculation are in two-group form, i.e., fast and slow diffusion coefficients for the lattices. These were combined to obtain one-group values by the following equation:

$$\frac{\delta D}{D_{\text{ref}}} = \left(\frac{L^2}{L^2 + \tau} \right)_{\text{test}, D_2O} \left(\frac{D_{s,\text{air}} - D_{s,D_2O}}{D_{s,\text{ref}}} \right) + \left(\frac{\tau}{L^2 + \tau} \right)_{\text{test}, D_2O} \left(\frac{D_{f,\text{air}} - D_{f,D_2O}}{D_{f,\text{ref}}} \right) \quad (18)$$

The calculated results are also given in Table II. The agreement between experiment and calculation is excellent in the case with the smallest void fraction (19-rod cluster in 4-inch housing). In the other two lattices, in which the void fraction is larger, the agreement is poorer. The calculated anisotropy is still in good accord with experiment however.

SUBSTITUTION MEASUREMENTS

The basic experimental data from the substitution measurements are given in Table III. The radial bucklings of the two reference lattices, obtained from flux traverses, were 202.15 μB for the lattice with air-filled housing and 213.71 μB for the lattice with D_2O -filled housings. Neither the vertical bucklings in Table III nor the above radial bucklings are actually known to the given number of significant figures, of course, but all significant figures were carried in the calculations for consistency. The changes in vertical buckling listed in the last column of Table III have been corrected for temperature and moderator degradation.

Two-group parameters used in the calculations are listed in Table IV. In anisotropic lattices, the parameters listed are for the radial direction. The Benoist theory^(7,8) was used to obtain D_s and D_f . Values of L^2 were obtained from D_s values and values of $\Sigma_{a,\text{eff}}$ computed by the P-3 approximation to transport theory. Values of τ are normalized to $\tau = 128.4 \text{ cm}^2$ for 99.6 mol % D_2O .

TABLE III
Basic Data from Substitution Measurements

Reference Lattice	Test Lattice			Number of Test Assemblies	Vertical Buckling of Reference Lattice $\alpha_1^2, \mu B$	Change in Vertical Buckling $\alpha^2 - \alpha_1^2, \mu B$	
	Rods per Cluster	OD of Housing, inches	D ₂ O or Air in Housing				
31 rods, 5" OD housing, air-filled	31	5	D ₂ O	1	127.317	+0.981	
				3(a)	127.193	+3.009	
				7	127.193	+6.346	
				19	127.317	+16.217	
	19	5	D ₂ O	1	126.901	+3.174	
				3(a)	126.901	+8.949	
				7	126.901	+14.988	
	19	5	Air	1	126.901	+0.813	
				3(a)	126.901	+2.228	
				7	126.901	+3.187	
	31 rods, 5" OD housing, D ₂ O-filled	31	5	Air	1	217.621	-1.029
					3(a)	217.621	-3.007
3(b)					217.621	-3.123	
7					217.621	-7.632	
19		5	D ₂ O	1	217.457	+1.706	
				3(a)	217.409	+3.942	
				3(b)	217.409	+3.272	
				7	217.457	+6.070	
19		5	Air	1	217.457	-1.154	
				3(a)	217.409	-3.659	
				3(b)	217.409	-4.046	
				7	217.457	-8.994	
19		4	D ₂ O	3(a)	217.469	+5.851	
				3(b)	217.400	+5.437	
				7	217.469	+10.579	
19		4	Air	3(a)	217.469	+3.524	
				3(b)	217.400	+3.568	
				7	217.469	+7.158	
1(metal) None		None	-	1	219.220	+3.447	
				3(b)	219.220	+5.835	
				7	219.220	+5.078	

(a) Test fuel at 120°, one pitch from center of pile.

(b) Test fuel in equilateral triangle one pitch on a side; one fuel assembly in center of pile.

TABLE IV
Two-Group Parameters

Rods per Cluster	Fuel Assembly			$D_B, \text{ cm}$	$D_F, \text{ cm}$	$L^2, \text{ cm}^2$	$\tau, \text{ cm}^2$	ρ
	OD of Housing, inches	D ₂ O or Air in Housing						
31	5	D ₂ O		0.8430	1.2061	115.9	142.6	0.854
31	5	Air		1.0417	1.4827	130.2	212.1	0.848
19	5	D ₂ O		0.8399	1.2137	173.6	137.9	0.900
19	5	Air		1.1198	1.6027	202.9	229.3	0.887
19	4	D ₂ O		0.8381	1.2112	174.5	137.3	0.901
19	4	Air		0.9467	1.3651	181.9	171.6	0.899
1(metal)	None	-		0.8257	1.2130	456.0	129.3	0.972

The one-group perturbation analysis was performed using the data of Tables II and III. The plots of the Persson type are shown in Figure 8. The numbers identifying the data points refer to the number of test assemblies. Straight lines were fitted to the data points by a least squares analysis, with functions weighted for the individual points proportional to $(\alpha^2 - \alpha_1^2)^2$. Results are given in Table V. Results of the other two methods of analysis are given in Tables VI and VII. The one-region results^(5,6) to which they are to be compared are given in Table VIII. A number of corrections are listed in Tables V-VIII. These are described below.

The first correction, called $\Delta(1)$, accounts for the presence in the lattice of aluminum guide tubes for control rods. (The rods themselves were never in the lattice at the time of the measurements.) The test lattice bucklings obtained from all these methods of analysis corresponded to the presence of these tubes in a one-region loading of test lattice. The values of $\Delta(1)$ used to correct to the condition of no tubes were based on experiments with one-region loadings⁽⁵⁾.

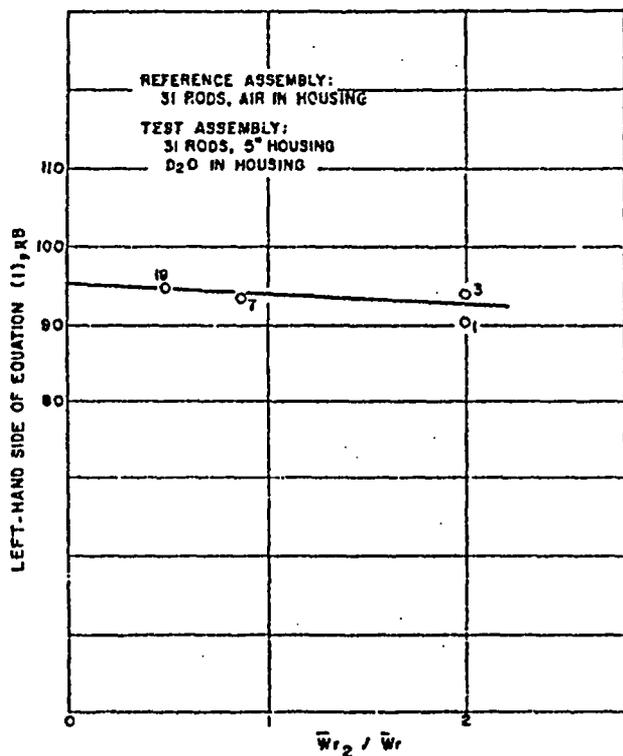


FIG. 8 ONE-GROUP PERTURBATION ANALYSIS PLOTS

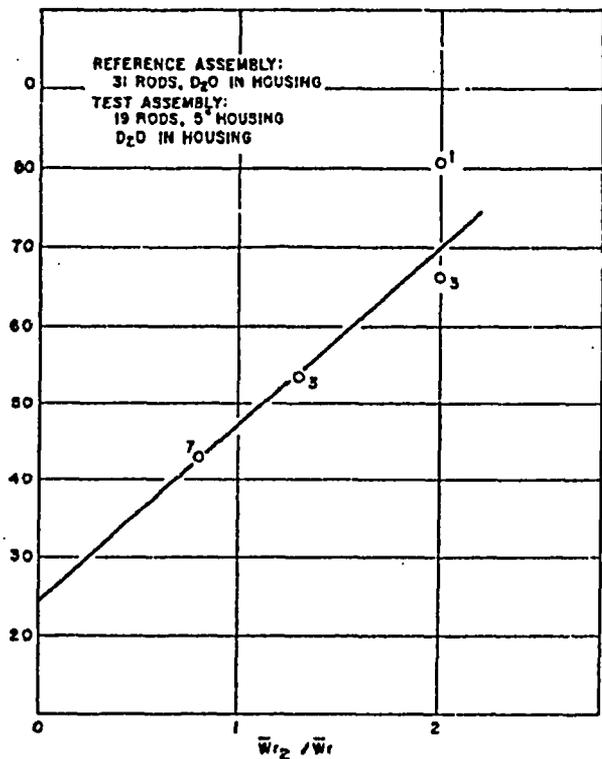
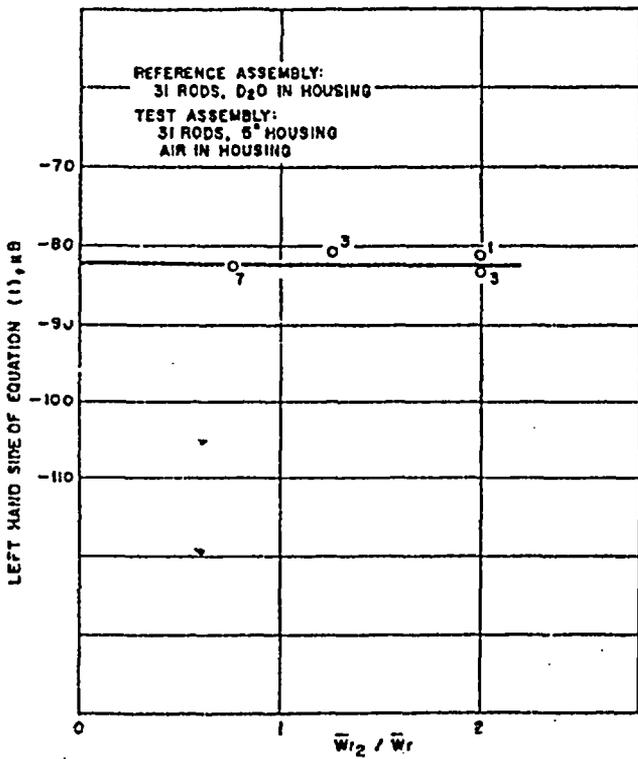
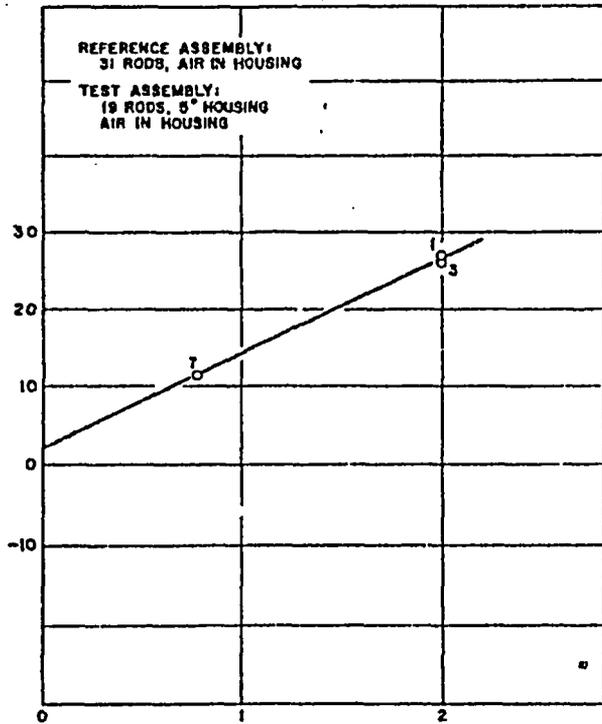
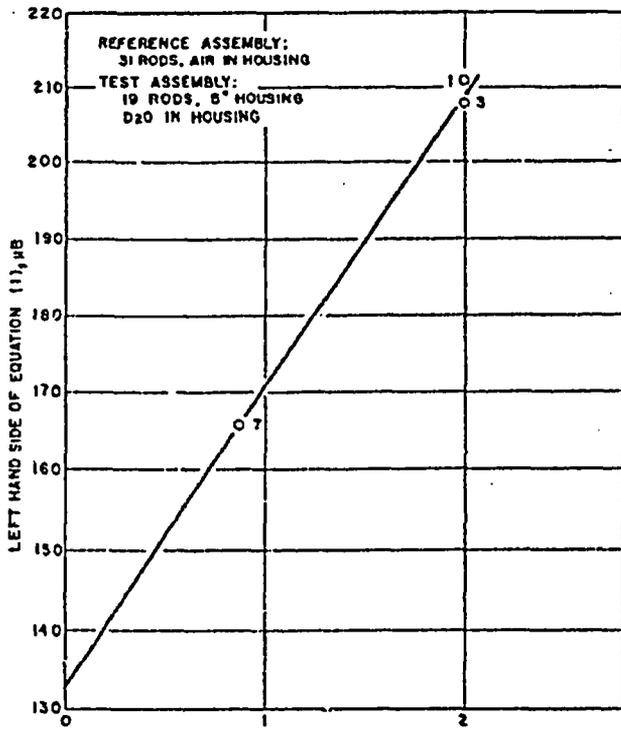


FIG. 8 CONTINUED

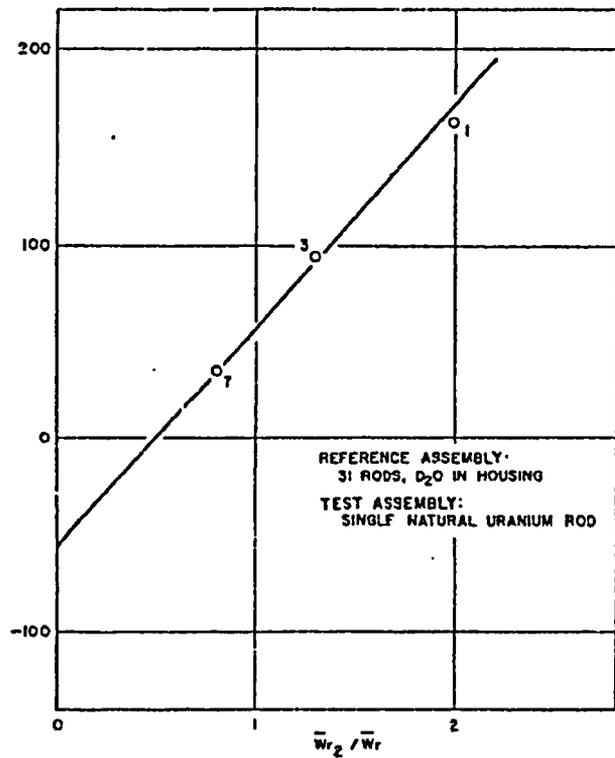
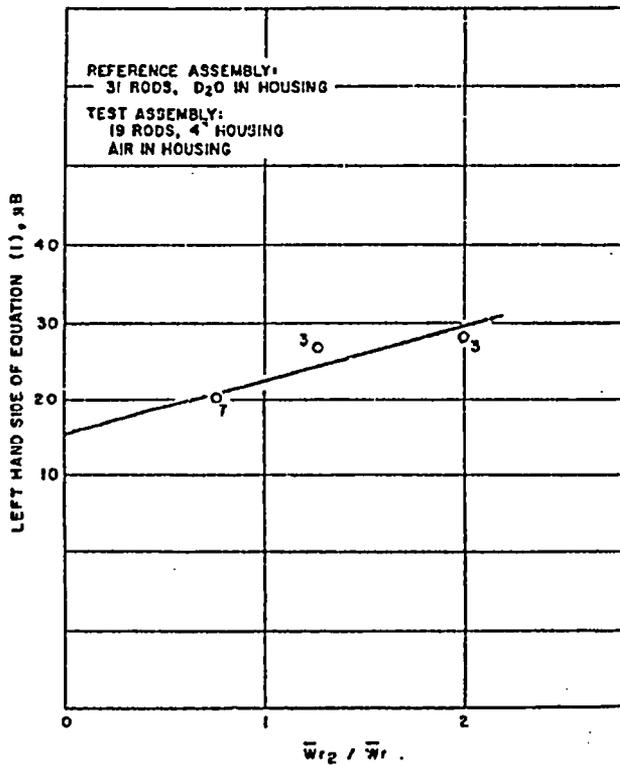
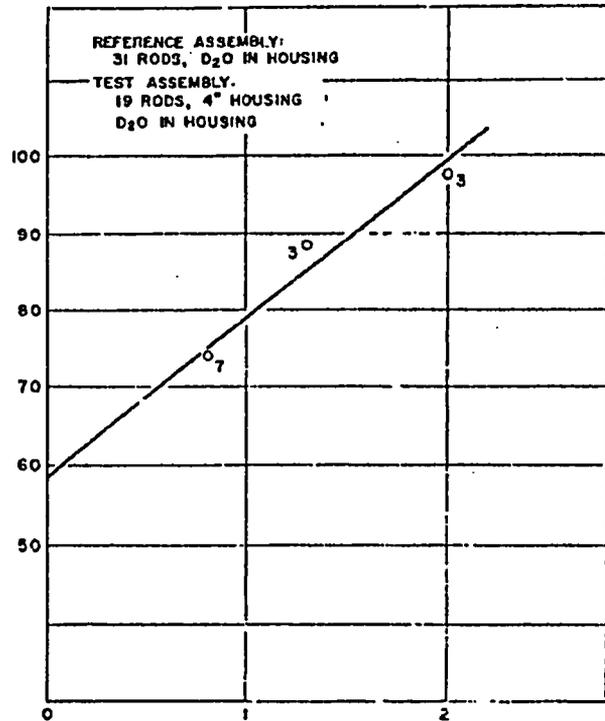
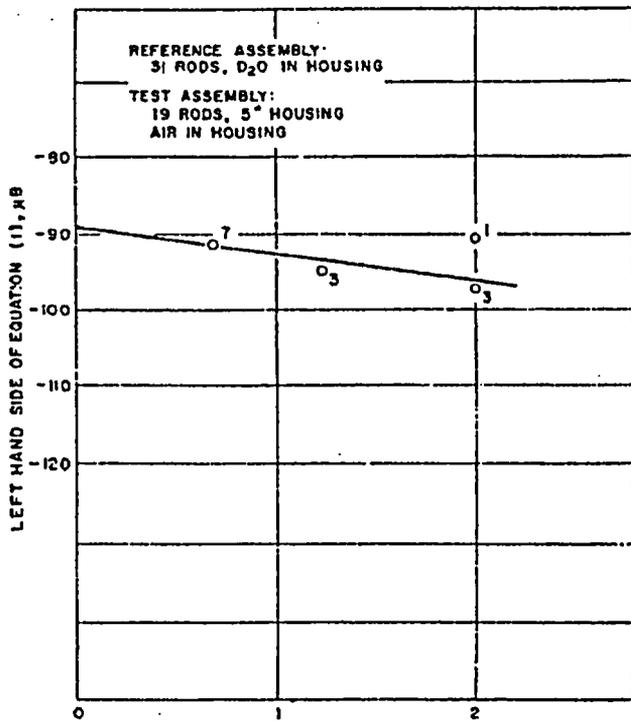


FIG. 8 CONTINUED

TABLE V

Results of One-Group Perturbation Analysis

Reference Lattice	31 Rods, 5" OD Housing, Air-Filled			31 Rods, 5" OD Housing, D ₂ O-Filled						
	Rods per cluster	31	19	19	31	19	19	19	19	1 (metal)
Test Lattice	OD of housing, inches	5	5	5	5	5	5	4	4	None
	D ₂ O or air in housing	D ₂ O	D ₂ O	Air	Air	D ₂ O	Air	D ₂ O	Air	-
ρ^2 , μB	202.2	202.2	202.2	213.7	213.7	213.7	213.7	213.7	213.7	213.7
α_1^2 , μB	127.2	126.9	126.9	217.6	217.4	217.4	217.4	217.4	217.4	219.2
$\alpha_3^2 - \alpha_1^2$, μB	95.4	132.6	2.0	-82.2	24.4	-89.0	58.4	15.3	-57.3	
α_5^2 , μB	222.6	259.5	128.9	135.4	241.8	128.4	275.8	232.7	161.9	
$\Delta(1)$, μB	5.9	7.7	5.2	4.9	7.7	5.2	7.7	3.4	7.8	
$\Delta(2)$, μB	+5.1	+6.2	-3.1	-5.1	0	-8.2	0	-3.0	0	
α_3^2 , μB (corrected)	233.6	273.4	131.0	135.2	249.5	125.4	283.5	233.1	169.7	
$\beta^2 + \alpha_3^2$ (corrected)	435.8	475.6	333.2	348.9	463.2	339.1	497.2	446.8	383.4	
ΔB_m^2 , μB	0	0	+3.8	+1.5	0	+2.4	0	+4.6	0	
B_m^2 , μB (corrected)	435.8	475.6	337.0	350.4	463.2	341.5	497.2	451.4	383.4	

TABLE VI

Results of Two-Region, Two-Group Analysis

Reference Lattice	31 Rods, 5" OD Housing, Air-Filled			31 rods, 5" OD Housing, D ₂ O-Filled						
	Rods per cluster	31	19	19	31	19	19	19	19	1 (metal)
Test Lattice	OD of housing, inches	5	5	5	5	5	5	4	4	None
	D ₂ O or air in housing	D ₂ O	D ₂ O	Air	Air	D ₂ O	Air	D ₂ O	Air	-
B_m^2 , μB (reference lattice)	329.34	329.05	329.05	431.33	431.17	431.17	431.18	431.18	432.93	
B_m^2 , μB (calculated test)	427.3	457.9	325.6	341.3	453.5	324.7	484.7	444.0	-	
α^2 , μB	133.5	141.7	130.1	210.0	223.5	208.5	228.0	224.6	224.3	
$\beta^2 = B_m^2 - \alpha^2$, μB (test region)	293.8	316.0	195.5	131.3	230.0	116.2	256.7	219.4	81.7	
$\Delta(1)$, μB	5.9	7.7	5.2	4.9	7.7	5.2	7.7	3.4	7.8	
$\Delta(2)$, μB	+5.1	+6.2	-3.1	-5.1	0	-8.2	0	-3.0	0	
$\Delta(3)$, μB	+3.3	+7.5	+1.7	0	0	0	0	0	0	
α^2 , μB (corrected test region)	138.6	148.1	127.0	204.9	223.5	200.3	228.0	221.6	224.3	
β^2 , μB (corrected test region)	303.0	331.2	202.4	136.2	237.7	121.4	264.4	222.8	89.5	
ΔB_m^2 , μB	0	0	+3.3	+4.1	0	+17.7	0	+4.0	0	
B_m^2 , μB (corrected)	441.6	479.3	332.7	348.2	461.2	339.4	492.4	448.4	313.8	

TABLE VII

Results of Three-Region, Two-Group Analysis

Reference Lattice	31 Rods, 5" OD Housing, Air-Filled			31 Rods, 5" OD Housing, D ₂ O-Filled			
Test Lattice	Rods per cluster	31	19	19	31	19	19
	OD of housing, inches	5	5	5	5	5	5
	D ₂ O or air in housing	D ₂ O	D ₂ O	Air	Air	D ₂ O	Air
B_m^2 , μB (calculated test)	427.3	454.1	322.5	340.5	442.0	321.5	
α^2 , μB	133.5	141.9	130.1	210.0	223.5	208.5	
$\beta^2 = B_m^2 - \alpha^2$, μB (test region)	293.8	312.2	192.4	130.5	218.5	113.0	
$\Delta(1)$, μB	5.9	7.7	5.2	4.9	7.7	5.2	
$\Delta(2)$, μB	+5.1	+6.2	-3.1	-5.1	0	-8.2	
$\Delta(3)$, μB	+3.3	+7.5	+1.7	0	0	0	
α^2 , μB (corrected test region)	138.5	148.1	127.0	204.9	223.5	200.3	
β^2 , μB (corrected test region)	303.0	327.4	199.3	135.4	226.2	118.2	
ΔB_m^2 , μB	0	0	+3.5	+7.1	0	+17.9	
B_m^2 , μB (corrected)	441.6	475.5	329.8	347.4	449.7	336.4	

TABLE VIII

Results of One-Region Measurements^(a)

Test Lattice	Rods per cluster	31	31	19	19	19	19	1(metal)
	OD of housing, inches	5	5	5	5	4	4	None
	D ₂ O or air in housing	D ₂ O	Air	D ₂ O	Air	D ₂ O	Air	-
β^2 , μB	210.9	201.9	213.6	176.3	212.7	213.1	-	
α^2 , μB	228.2	131.7	254.8	145.7	288.8	234.5	-	
$\beta^2 + \alpha^2$, μB	439.1	333.6	468.4	322.0	501.5	447.6	398.9 ^(a)	
ΔB_m^2 , μB	0	+1.6	0	+7.3	0	+4.7	0	
B_m^2 , μB (corrected)	439.1	335.2	468.4	329.3	501.5	452.3	398.9 ^(a)	
Correction to 99.58 mol % D ₂ O, μB	-	-	-	-	-	-	-21.7 ^(b)	
B_m^2 , μB (final corrected)	-	-	-	-	-	-	377.2	

(a) At 99.70 mol % D₂O⁽⁶⁾
 (b) See Reference 6

vert:
latt:
ment:
air-:

two-
latt:
cal
was
this
regio
To a
regio

where
usini

latt:
inde;
must
was
trop
corr

The
the :

with
on t
dime
ment
wall
impo
blie
mati
used
a re
asse
This

The second correction, called $\Delta(2)$, accounts for different vertical extrapolations in the reference lattice and test lattice. Values of $\Delta(2)$ were obtained from one-region experiments⁽⁵⁾ and were taken as zero when neither lattice contained air-filled housings.

The third correction, called $\Delta(3)$, applied only to the two-group methods, and only to the cases in which the reference lattice was anisotropic. When a change was made in the vertical buckling, the relation $\Delta\beta^2 = -\Delta\alpha^2$ in the reference region was used for the two-group codes. For an anisotropic lattice this leads to an error in the radial buckling of the reference region of $(\gamma - 1)\Delta\alpha^2$, where γ is the anisotropy (Table II). To a first approximation, this leads to an error in the test region buckling given by

$$\Delta(3) = \frac{1 - W_r}{W_r} (\gamma - 1)(\alpha^2 - \alpha_1^2) \quad (19)$$

where $W_r \equiv W_3 + \frac{1}{2}W_2$. The correction given by equation (19), using $\gamma = 1.08$, was applied to the data in Tables VI and VII.

The final correction, ΔB_m^2 , was applied when the test lattice was anisotropic. A material buckling does not exist independent of geometry in this case, and a standard geometry must be chosen in order to compare data. The geometry chosen was that which gives a minimum critical volume in the isotropic cylindrical pile; i.e., $\beta^2 = 2\alpha^2$. The necessary correction to $\alpha^2 + \beta^2$ to obtain $\alpha^2 + \beta^2$ for this geometry is

$$\Delta B_m^2 = (2\alpha^2 - \beta^2) \frac{(\gamma - 1)}{3 + (\gamma - 1)} \quad (20)$$

The one-region results of Table VIII were also corrected to the minimum critical volume geometry by equation (20).

Table IX compares the corrected substitution results with the corrected one-region bucklings. The quality control on the UO_2 fuel assemblies was not particularly good, and the dimensions given in Table I represent the average of measurements made on a large number of assemblies. Variation of the wall thickness of the aluminum housing tubes is the most important irregularity. Thus the small samples of fuel assemblies used in these experiments could have differed systematically from the average of the large number of assemblies used in the measurements of Reference 5. In terms of buckling, a reasonable expectation of the deviation of a single fuel assembly from the average is felt to be at least $\pm 5 \mu B$. This results in two types of errors:

TABLE IX

Comparison of Corrected Results

Reference Lattice	Test Lattice	31 Rods, 5" OD Housing, Air-Filled			31 Rods, 5" OD Housing, D ₂ O-Filled					1 (metal)	
		Rods per cluster	OD of housing, inches	D ₂ O or air in housing	Rods per cluster	OD of housing, inches	D ₂ O or air in housing	Rods per cluster	OD of housing, inches		D ₂ O or air in housing
		31	19	19	31	19	19	19	19	19	
		5	5	5	5	5	5	4	4	4	None
		D ₂ O	D ₂ O	Air	Air	D ₂ O	Air	D ₂ O	Air	Air	-
	$B_{m_1}^2, \mu B$	435.8	475.6	337.0	350.4	463.2	341.5	497.2	451.4	451.4	383.4
	$B_{m_2}^2, \mu B$	441.6	479.3	332.7	348.2	461.2	339.4	492.4	448.4	448.4	313.8
	$B_{m_3}^2, \mu B$	441.6	475.5	329.8	347.4	449.7	336.4	-	-	-	-
	$B_{m_4}^2, \mu B$	439.1	468.4	329.3	335.2	468.4	329.3	501.5	452.3	452.3	377.2
	$B_{m_1}^2 - B_{m_4}^2, \mu B$	-3.3	+7.2	+7.7	+15.2	-5.2	+12.2	-4.3	-0.9	-0.9	+6.2
	$B_{m_2}^2 - B_{m_4}^2, \mu B$	+2.5	+10.9	+3.4	+13.0	-7.2	+10.1	-9.1	-3.9	-3.9	-63.4
	$B_{m_3}^2 - B_{m_4}^2, \mu B$	+2.5	+7.1	+0.5	+12.2	-18.7	+7.1	-	-	-	-

Note: $B_{m_1}^2$ = One-group perturbation result from Table V
 $B_{m_2}^2$ = Two-region, two-group result from Table VI
 $B_{m_3}^2$ = Three-region, two-group result from Table VII
 $B_{m_4}^2$ = One-region result from Table VIII

- The seven fuel assemblies of a single type may differ appreciably from the average of a larger number. This would give rise to systematic discrepancies between the one-region results and the substitution results even in the two-group, two-region method of analysis, which uses only the seven-assembly data.
- If fuel assemblies within the group of seven differ appreciably from each other, another type of error would give rise to incorrect slopes of the lines in the one-group perturbation method and incorrect intermediate region bucklings in the three-region, two-group method. The latter method is particularly vulnerable on this score.

It is the author's opinion that the scatter of data points from the straight lines in Figure 8 can be ascribed almost entirely to these errors. With this point in mind, one can draw the following conclusions:

- The one-group perturbation method gave satisfactory results in all cases.
- The two-region, two-group method gave satisfactory results when diffusion coefficient mismatches were the major problem (as in all of the UO_2 experiments). In the single experiment in which there was no diffusion coefficient mismatch but there was a large mismatch in resonance escape probability, this method failed rather badly. There is other evidence from this laboratory indicating the failure of the two-region, two-group method for this particular situation⁽⁸⁾.
- The three-region, two-group method does not appear to offer any particular advantage to offset the increased complexity of the analysis.

ACKNOWLEDGMENT

The author is indebted to F. D. Benton for running the three-region, two-group problems.

W. E. Graves

W. E. Graves

Approved by

J. L. Crandall

J. L. Crandall, Research Manager
Experimental Physics Division

Originalien

Critical and exponential experiments on 19-rod clusters (R3 fuel) in heavy water

By ROLF PERSSON, CARL-ERIK WIKDAHL, and ZENON ZADWÓRSKI

(AB Atomenergi, Studsvik, Tystberga, Sweden)

With 12 Figures

(Received February 14, 1962)

Summary. Buckling measurements on clusters of 19 UO_2 rods in heavy water have been performed in an exponential assembly and by means of substitution measurements in a critical facility. The material buckling was determined as a function of lattice pitch (range of V_{mod}/V_{fuel} : 7–22), internal spacing, void, and temperature ($20 < T < 90^\circ C$). The change of diffusion coefficients (about 6–8 per cent) caused by voids was studied with single test fuel assemblies. The progressive substitution measurements have been analysed by means of a modified one-group perturbation theory in combination with an unconventional cell definition. The buckling differences between test and reference lattices are of the order of -1.0 to $-3.5 m^{-2}$. The results of the exponential and the critical experiments are compared with similar measurements on the same kind of fuel at the Savannah River Laboratory. This comparison shows that the results of the various experiments agree quite well, whereas theoretical predictions fall in the extreme ranges.

1. Introduction

Fuel designed for the Swedish power reactor R3 [1], which is under construction at a southern suburb of Stockholm and planned to go critical in 1963, has been used for substitution measurements in the critical facility R0 [2] and for exponential experiments in a rebuilt arrangement of the subcritical facility ZEBRA [3], [4] at Studsvik, Sweden.

The fuel investigated consisted of natural uranium oxide (diameter 1.70 cm, density $10.5 \pm 0.1 g/cm^3$) canned in zircaloy-2 (ID 1.72 cm, OD 1.87 cm) and arranged in the form of clusters. Each cluster was composed of 19 rods placed in a hexagonal pattern with a c-c distance of either 2.11 or 2.21 cm, except six of the outer rods, which were farther from the centre in order to make the outer contour more circular (compare Fig. 1). The rods were 77.1 cm long and four such rods had to be screwed together in order to have an assembly length of about 3 m, which was necessary for the experiments. The actual length of the clusters to be used in the R3 reactor will also be 3 m. At every joint there was a section (about 4.5 cm long) containing zircaloy and He-filled space instead of uranium oxide.

The measurements reported here were restricted to determinations of the material buckling as a function of lattice pitch, internal spacing, temperature and void.

The same fuel with the smaller of the two internal spacings was also investigated at the Savannah River Laboratory, USA [14]. Buckling measurements and intra-cell flux studies were undertaken up to $215^\circ C$.

2. Critical substitution experiments

2.1 Experimental procedure

The critical assembly R0 [2] is without a reflector, heavy-water moderated and contained in an aluminium tank (ID 2.25 m, height 3.2 m, wall thickness 0.6 cm). The water level in the reactor tank is measured with an accuracy of 0.01 cm by a level meter in a communicating tube. Temperature variations are determined within $\pm 0.01^\circ C$ by means of resistance thermometers

placed in various positions. The heavy water can be heated by circulation through an electric heater.

The fuel suspension system and the safety mechanisms are placed in a square top box, with inspection windows on all four sides. The fuel assemblies are

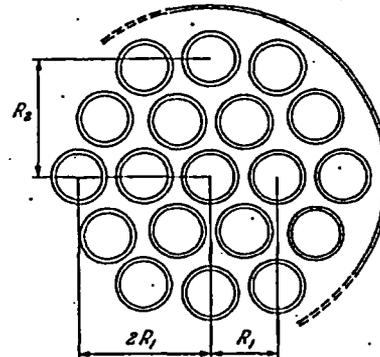


Fig. 1. Cross-section of R3 fuel assembly (test version). $R_1 = 2.11$ cm or 2.21 cm, $R_2 = 3.90$ cm or 4.08 cm. Fuel: natural UO_2 , density $10.50 \pm 0.10 g/cm^3$, diameter 1.70 cm. Cladding: Zircaloy-2, ID 1.72 cm, OD 1.87 cm. Shroud: aluminium (Z3), ID 11.27 cm, OD 11.50 cm

suspended from rollers resting on beams, which are provided with wheels at both ends. The position of the rollers and the beams can easily be adjusted from outside without opening the system. The fuel is put in or taken out through suitable openings which can be uncovered along a diameter in a large revolving lid.

Uranium metal rods, diameter 3.05 cm, density $18.76 \pm 0.14 g/cm^3$, in the form of lumps placed in aluminium tubes, ID 3.16 cm, OD 3.45 cm, are used as reference fuel. The buckling of the reference lattice is measured with the aid of BF_3 counters ($0.02 atm B^{10}F_3$, length 40 cm, diameter 1.0 cm) placed in travelling probes. The technique is the same as is used in the exponential assembly (see 3-1 below). However, in R0 we need a monitor to normalize the intensity. The monitor, a larger and more sensitive BF_3 counter, is placed outside the tank. The counting rate from

Nukleonik
 im in der Schiffsreaktor-
 Anhaltswerte für Abmes-
 sungen der schiffbau-
 Anlagen
 urform- und Anrege-
 ktiv a-
 prochen werden.
 eninteressen durchgeführt
 ichtung dienten ausgeführte
 unnah-Entwicklung.
 ng der einzelnen Systeme
 igerungen auf Grund der
 ichtig zu bewerten. Erst
 s Abbrandwerte vorliegen,
 reysteme sich mit der glei-
 nd die Anlagen mit ihren
 der Technik entsprechen.
 en. Die aufgeführten Ver-
 elativwerte, die zum Teil
 in Anlagenteilen ermittelt
 ng der nächsten 15 Jahre
 ngesehen werden, daß nur
 ichten in der Anwendung
 ah" bringt die bisher ge-
 lieses ersten Kernenergie-
 n zeigen neue Wege der
 ne Gewichtsersparnis von
 tufe des „Savannah“-Pro-
 n der organisch moderierte
 schiffsreaktor, der Sied-
 kühlte Schiffsreaktor be-
 nder Vortrag von Herrn
 und Aufgaben des For-
 leitwort der Herron Bun-
 r. Ing. Sæverom und die
 lichkeiten aus dem nord-
 m die wirtschaftliche und
 1 Kernreaktor-Schiffsent-
 schiffahrt betrifft, sondern
 is Binneland hineinträgt
 kann t jeder Vortrag
 Prof. E. Baggø und
 m ist für die Herausgabe
 bildlicher Weise hat der
 Schriftsatz und in den
 ausgabe des Buches, fast
 g, die Aktualität verloren
 gliche Buch sicher einen
 reaktorbau interessierte
 CHAFSTALL (Mannheim)

Uro. 852 Erlangen, Günther
 Jöttingen • Hoidelberg.

o about fifteen minutes

r level is measured with
tem ture variations
it \pm ° C, if the errors
ng o. Jay of measure
we have found that an
develops. Even though
s important when high

efficient measurements
as cooled down to about
temperature dependence
dings of the level meter
regard to the difference
he two communicating
temperature there was
vel tube.

e heavy water may also
For the reference lattices
critical height which is
ase of the D₂O content
on a special reference
we have a more or less
heavy water correspond
onth [8].

above mentioned syste-
ferential measurements
be performed within as

was always measured at
(0.1 W). We are mainly
ter levels and the source
neutrons was then found
ng a the experimental
e SAL ny.

analysis

periments we define the
conventional way, such
l at the corners of the
s will be more kinds of
fuel elements. Thus we
efined transition region
æ and the test regions
cept has been combined
theory, but we have
perturbed radial buck-
as also been taken into

regions have the same
he well-known result of
y [9].

$$\beta W_i \quad (1)$$

ic buckling,

1 i,
proportional to $\int \Phi \Phi'$
e unperturbed flux of the
uniform core and Φ' is
c: By definition we have

the perturbation theory is
urbations.

In substitution measurements we have, if Φ' is put
equal to Φ and $\Phi = A J_0(\beta r) \sin \alpha z$,

$$W_i = C \int J_0^2(\beta r) dS,$$

where $1/C = \pi R^2 J_1^2(\beta R)$ and $\beta = 2.405/R$.

If there are only three different regions, eq. (1) may
be written as

$$\frac{B^2 - B_1^2}{W_2} = (B_2^2 - B_1^2) \frac{W_2}{W_2} + (B_3^2 - B_1^2) \quad (2)$$

where the indices 1, 2, and 3 mean the reference, the
transition, and the test regions respectively (compare
Fig. 2). From eq. (2) it is clear that $(B^2 - B_1^2)/W_2$ is
a linear function of W_2/W_2 , such that the intercept
gives $B_3^2 - B_1^2$ and the slope is $B_2^2 - B_1^2$. Eq. (2) may
be transformed to a form, which is more suitable
when applying corrections due to the perturbed flux
and more appropriate for graphical representation.
If we write

$$B_2^2 - B_1^2 = \frac{1}{2} (B_2^2 - B_1^2) + \delta B_2^2 \quad (3)$$

we get the expression

$$\frac{B^2 - B_1^2}{W} = \delta B_2^2 \frac{W_2}{W} + (B_3^2 - B_1^2), \quad (4)$$

where

$$\bar{W} = W_2 + \frac{1}{2} W_2. \quad (4a)$$

The relation (4) with adequate corrections [6] is used
in the analysis, when differences between diffusion
coefficients are negligible.

If we use two-group perturbation theory we are
able to derive a formula approximately identical with
eq. (4) except as regards a small two-group correction
term. Instead of eq. (4) we get [6]

$$\frac{B^2 - B_1^2}{W} \{1 + (1 - \bar{W}) \Gamma\} = \delta B_2^2 \frac{W_2}{W} + (B_3^2 - B_1^2) \quad (4b)$$

where Γ is approximately independent on the size of
the test region but mainly a function of the coupling
coefficients. We may use the following expression

$$\Gamma \cong \frac{D_1/D_2 + \bar{T}}{D_1/D_2 + S_{test}} \cdot \frac{S_{test} - S_{ref}}{S_{ref} - \bar{T}} \quad (4c)$$

where S and T are the coupling coefficients S_1 and S_2
respectively of ref. 9 and $\bar{T} = (T_{ref} + T_{test})/2$. We
notice that $T < 0$.

In the case of substitution measurements with
voided fuel assemblies we are usually forced to take
the diffusion coefficient into account. We may derive [6] the following expression

$$\left. \begin{aligned} \sum_i (\alpha^2 - \alpha_i^2) (W_{ri} D_{st} - U_{ri} \delta D_{ri}/2) \\ = \beta^2 \sum_i (W_{ri} - U_{ri}) \delta D_{ri} \end{aligned} \right\} \quad (5)$$

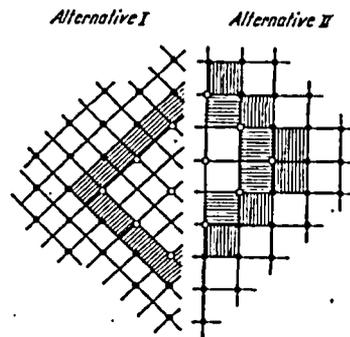
where

- α^2 = axial buckling of the whole core in the perturbed state,
- α_i^2 = axial buckling of region i , when it occupies the whole tank,
- β^2 = radial buckling = $(2.405/R)^2$,
- W_r = radial part of the Φ^2 -dependent statistical weight
 $\cong \int J_0^2(\beta r) dS_r / \pi R^2 J_1^2(\beta R)$,

U_r = radial part of the $(\nabla \Phi)^2$ -dependent weight function

- $\cong \int J_1^2(\beta r) dS_r / \pi R^2 J_1^2(\beta R)$,
- D_i = diffusion coefficient of region i ,
- $\delta D_i = D_i - D_1$, where index 1 is related to the reference region.

The change of the diffusion coefficients according to the one-group model may be estimated by theoretical calculations, but it can also be measured in separate experiments with a single test fuel assembly. Measurements on a single test fuel assembly in a reference lattice mean that we investigate a transition cell as defined in Fig. 2. Having studied the axial void effect of the central fuel assembly in a test



- Reference fuel: lattice pitch l
- Test fuel (R3 cluster): lattice pitch $\sqrt{2}l$
- Reference cell \diamond (index 1) \square (index 1)
- Transition cell(s) \diamond (index 2) \blacksquare (index 2a) \square (index 2b)
- Test cell \diamond (index 3) \square (index 3)

Fig. 2. Examples of cell definitions used in substitution experiments

region of 3×3 test fuel assemblies we assume that we
are able to take the structure of the cell into account
fairly well.

In order to have agreement between one-group and
two-group expressions we find that

$$\frac{\delta D}{D_{ref}} = \frac{\delta D_1 + S_{ref} \delta D_2}{D_{1ref} + S_{ref} D_{2ref}} \quad (5a)$$

where $\delta D = D_{test} - D_{ref}$ and the indices 1 and 2 refer
to the fast and thermal groups respectively. S is the
coupling coefficient between fundamental modes
(= S_1 in ref. 9).

The following relation has been used to analyse
the single-rod experiments [6].

$$\left. \begin{aligned} \sum_{i=1,2} [D_{ri} \beta^2 (W_{ri} - U_{ri}) W_{st} + \\ + D_{st} (\alpha_i^2 W_{ri} - \alpha^2 U_{st}) W_{ri}] = 0 \end{aligned} \right\} \quad (6)$$

where index 1 represents the reference region and
index 2 the region investigated, which is just equal to
the transition region mentioned before. The functions
 W_r and U_r are defined as

- $W_r = \int \sin^2 \alpha z dz / (H/2) =$ axial part of the Φ^2 -dependent statistical weight,
- $U_r = \int \cos^2 \alpha z dz / (H/2) =$ axial part of the $(\nabla \Phi)^2$ -dependent weight function.

When the perturbation penetrates the whole core we have a purely radial dependence, i.e. $W_{r2} = U_{r2} = 1$. We now introduce the extra indices e and f representing the cases with void (empty shroud) and without void (filled shroud) respectively. Using eq. (6) and assuming $W_{r2} \ll 1$ and $\alpha^2 - \alpha_1^2 \ll \alpha_1^2$ we subtract the equations for the two cases from each other and after rearrangement we get

$$\frac{\alpha^2 - \alpha_1^2}{U_{r2}} = -\beta^2 \frac{D_{r2e} - D_{r2f}}{D_{r1}} + F(z) \cdot \frac{W_{r2}}{U_{r2}} \quad (7)$$

from which relation the change of the radial diffusion coefficient is determined. Using more common symbols we see that $[(\pi/H_e)^2 - (\pi/H_f)^2]/J_0^2(\beta r)$ is a linear

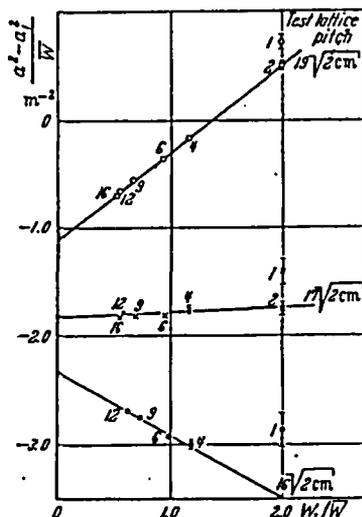


Fig. 3. Substitution analysis in graphical form with the cell definition of the alternative I in Fig. 2. Test lattice pitches: 22.63, 24.04 and 26.87 cm. The figures next to the points represent the number of clusters in the test region. The error limits indicated correspond to ± 0.1 cm in height differences

function of $J_0^2(\beta r)/J_1^2(\beta r)$ and $(D_{r2e} - D_{r2f})/D_{r1}$ is found from the intercept.

When the change of the axial diffusion coefficient (δD_{z2}) is wanted, we just place the void perturbation close to the center at a fixed radial position and investigate its axial dependence by varying its depth. From eq. (6) we get

$$\frac{\alpha^2 - \alpha_1^2}{\alpha^2} \cdot \frac{1}{U_{z2}} = G(r) \frac{1}{\alpha^2} \cdot \frac{W_{z2}}{U_{z2}} - \frac{\delta D_{z2}}{D_{z1}} \cdot W_{r2} \quad (8)$$

2.3. Results of critical substitution experiments

2.3.1. Measurements on fuel without shroud at room temperature. Most of the measurements were carried out with clusters having an internal c-c spacing of 2.21 cm. The various test regions investigated comprised 1, 2, 4, 6, 9, 12, and 16 clusters. Datas of the lattices used are collected in Table 1a. The two-group parameters necessary for evaluation of Γ and $\delta D/D$ were obtained from ref. 16. Experiments to determine $\delta D/D$ with R3 fuel assemblies without voids did not give sufficient accuracy. From the values of $\delta D/D$ given in Table 1a we see that the effect of different diffusion coefficients in the various regions cannot be neglected.

Table 1a. Lattices used in critical substitution measurements on R3 clusters (internal c-c spacing 2.21 cm) without shrouds at room temperature

Square lattice pitch in cm		Max. number of test fuel assemblies	Number of ref. fuel rods in unperturbed core	Extrapolated radius in cm	Γ acc. to eq. (4c)*	$\delta D/D$ acc. to eq. (5a)*
Test	Ref.					
19.0	19.0	12	112	115.2	-0.071	-0.040
20.0	20.0	12	96	116.2	-0.065	-0.035
22.63	16.0	12	132	114.8	-0.029	-0.021
24.04	17.0	16	120	115.0	-0.025	-0.018
26.87	19.0	16	120	115.2	-0.019	-0.012
32.0	16.0	12	132	114.8	+0.017	-0.003

* Input datas from ref. 16.

The experimental points have been analysed by combining eqs. (4b) and (5) with corrections applied for perturbed radial bucklings.

Both alternatives of cell definitions (I and II) shown in Fig. 2 were used in the analysis. In the case of 19.0 and 20.0 cm test lattices there were discrepancies of the order of 0.10 to 0.15 m^{-2} between results obtained with the two cell definitions. Alternative II is thought to give the most accurate results in these cases. The analysis of the test lattices with the pitches $16.0\sqrt{2}$, $17.0\sqrt{2}$, and $19.0\sqrt{2}$ cm gave good agreement between the two alternatives, if the points corresponding to single test elements were rejected (cf. Fig. 3). The 32.0 cm test lattice could only be analysed with cells according to alternative II.

The results of all these substitution measurements are given in Table 1b.

Table 1b. Results of critical substitution measurements on R3 clusters (internal c-c spacing 2.21 cm) without shrouds at room temperature

Square lattice pitch in cm		D_{ref}^2	$D_{test}^2 - D_{ref}^2$	D_{test}^2	Temp.	D ₂ O	Γ_{test} corrected*
Test	Ref.	m^{-2}	m^{-2}	m^{-2}	$^{\circ}C$	mol-%	m^{-2}
19.0	19.0	6.26	-2.76 \pm 0.04	3.50	19.4	99.05	3.46 \pm 0.06
20.0	20.0	5.79	-1.40 \pm 0.08	4.30	19.9	99.65	4.29 \pm 0.09
22.63	16.0	7.63	-2.22 \pm 0.06	5.31	19.9	99.65	5.34 \pm 0.08
24.04	17.0	8.17	-1.74 \pm 0.04	6.43	21.8	99.72	5.46 \pm 0.06
26.87	19.0	6.33	-1.05 \pm 0.03	5.28	21.5	99.72	5.32 \pm 0.05
32.0	16.0	7.63	-3.42 \pm 0.03	4.11	19.2	99.65	4.24 \pm 0.05

± 0.04

* Values normalized to 20 $^{\circ}$ C and 99.75 mol-% D₂O.

2.3.2. Effect of internal spacing. Special experiments were carried out in order to get the effect of the internal spacing (c-c: 2.11 or 2.21 cm) on the material buckling. Three different square lattice pitches were investigated, namely 19.80, 22.63, and 26.87 cm respectively. With a test region of nine (3x3) R3 clusters, c-c: 2.21 cm, the central test fuel assembly was exchanged with another having an internal spacing of 2.11 cm. In order to minimize temperature drift the moderator was not lowered during the fuel exchange. Because the fuel assemblies are equal except as regards the internal spacing we are convinced that the perturbation analysis of the experiments with a single fuel assembly is reliable. However, at the lattice pitch 26.87 cm progressive substitutions with a maximum number of 9 clusters of each kind were also

investigated (Fig. 4). T formed during two consecutive water degradation a collected in Table 2 at close-packed alternative region investigated.

Table 2. Difference in μ 2.11 cm) and R3 (c-c:

Square lattice pitch cm	Max. number of assemblies
19.80	0.21
22.63	0.11
26.87	0.01

2.3.3. Void experiment. The fuel, metal rods (2.21 cm), were furnished

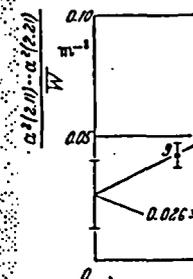


Fig. 4. Progressive substitution results for different lattice pitches with 2.11 cm and 2.21 cm. The figures next to the points represent the number of clusters in the test region. The error limits indicated correspond to ± 0.1 cm in height differences

(reference shrouds: I. Shrouds: 1D 11.27 cm, pitch of the reference lattice: the test region 26.87 cm)

The radial statistical cluster was measured at different radial positions in the shroud and water in the critical level as a function of the same day, so that the same good accuracy was carried out in Fig. 6b and the radial diffusion coefficient was found to be δD .

The axial statistical cluster was measured near the reactor center assembly was in this reference rods, square shows the critical height. The analysis according to the form in Fig. 6b and the value refers to a

* In Fig. 6b the points represent the void depths. Larger void depths means that the slopes of the

substitution measurements on 2.21 cm) without shrouds at room temperature

Ex. no. in cm.	β acc. to eq. (4c)*	$\delta D_{r3}/D$ acc. to eq. (5a)*
115-2	-0.071	-0.040
115-2	-0.065	-0.035
114-8	-0.029	-0.021
115-0	-0.025	-0.018
115-2	-0.019	-0.012
114-8	+0.017	-0.003

have been analysed by with corrections applied

definitions (I and II) the analysis. In the case lattices there were discre- 0.15 m⁻² between results finitions. Alternative II accurate results in these lattices with the pitches m gave good agreement as, if the points corre- spondence were rejected (cf. alternative II.

substitution measurements

substitution measurements on R3 (cm) and shrouds at room temperature

Ex. no.	Temp. °C	D ₂ O mol-%	B ₁₀₀₀ corrected m ⁻²
50	19.4	99.65	3.46 ± 0.06
30	19.9	99.65	4.29 ± 0.09
31	19.9	99.65	5.34 ± 0.08
43	21.8	99.72	5.46 ± 0.06
28	21.6	99.72	5.32 ± 0.05
11	19.2	99.65	4.24 ± 0.06

and 99.75 mol-% D₂O.

spacing. Special experi- ment to get the effect of the 2.21 cm) on the material are lattice pitches were 22.63, and 26.87 cm region of nine (3 × 3) R3 central test fuel assembly having an internal spacing minimize temperature drift during the fuel ex- periments are equal except we are convinced that the experiments with a. However, at the lattice substitutions with a of each kind were also

investigated (Fig. 4). These measurements were per- formed during two consecutive days in order to keep the water degradation at a minimum. The results are collected in Table 2 and they show that the most close-packed alternative has the larger buckling in the region investigated.

Table 2. Difference in material buckling between R3 (c-c: 2.11 cm) and R3 (c-c: 2.21 cm) at room temperature

Square lattice pitch cm	B ⁰ (2.11) - B ⁰ (2.21)	
	Meas. with single fuel assembly in test region m ⁻²	Progressive subst. meas. (see Fig. 4) m ⁻²
19.80	0.212 ± 0.013	-
22.63	0.117 ± 0.010	-
26.87	0.027 ± 0.004	0.026 ± 0.014

2.3.3. Void experiments. In the void experiments all the fuel, metal rods as well as R3 clusters (c-c: 2.21 cm), were furnished with aluminium shrouds

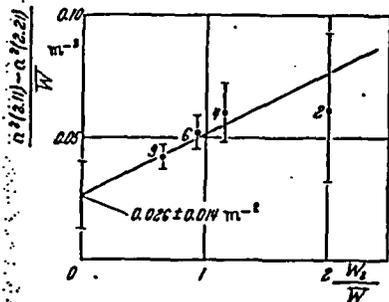


Fig. 4. Progressive substitution measurement of buckling difference between clusters with 2.11 cm and 2.21 cm internal spacing. Test lattice pitch: 26.87 cm. The figures next to the points represent the number of clusters tested. The error limits indicated correspond to ±0.1 cm in the water height

(reference shrouds: ID 6.30 cm, OD 6.50 cm; test shrouds: ID 11.27 cm, OD 11.50 cm). The square pitch of the reference lattice was 19.0 cm and that of the test region 26.87 (= 19.0 · √2) cm.

The radial statistical weight of one single R3 cluster was measured in two steps at each of six different radial positions: 1. air in the whole R3 shroud and water in the reference shrouds, and 2. water in all the shrouds. In Fig. 5a we have plotted the critical level as a function of the radius for the two cases. It was not possible to measure all the points on the same day, so that these curves do not show the same good accuracy as found by the differential analysis carried out in Fig. 5b according to eq. (7). The change of the radial diffusion coefficient of a transition cell was found to be $\delta D_{r3}/D = 0.077 \pm 0.002$.

The axial statistical weight of void in a single R3 cluster was measured with the test fuel in a position near the reactor centre (r=13.43 cm). The test fuel assembly was in this case first surrounded only by reference rods, square lattice pitch 19.0 cm. Fig. 6a shows the critical height versus the void depth (z). The analysis according to eq. (8) is given in graphical form in Fig. 6b and the result is $\delta D_{r3}/D = 0.103 \pm 0.007$ which value refers to a transition cell. In a second part

* In Fig. 6b the points given correspond only to small void depths. Larger void depths were also taken into account which means that the slopes of the lines in Fig. 6b are well defined.

of the axial void experiment nine (3 × 3) R3 clusters formed a test region with a square lattice pitch of 19.0 · √2 cm. The void effect of the central cluster was investigated and we found (cf. Fig. 6b) that $\delta D_{r3}/D = 0.085 \pm 0.007$. This value should be valid

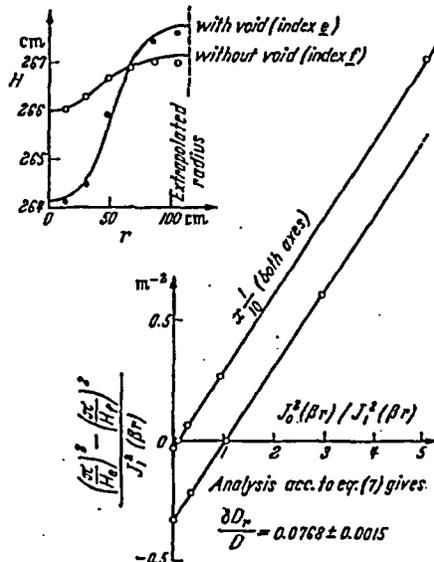


Fig. 5a and b. Determination of $\delta D_{r3}/D$ from single-cluster experiments. Reference lattice pitch: 19.0 cm

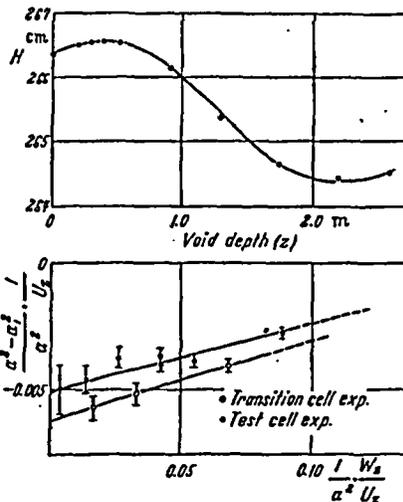


Fig. 6a and b. Determination of $\delta D_{r3}/D$ from single-cluster experiments. Reference lattice pitch: 19.0 cm. Test lattice pitch: 26.87 cm

for a test cell. We now assume that we are able to correct the radial value $\delta D_{r3}/D$ found in Fig. 5b and get $\delta D_{r3}/D$ as follows.

$$\delta D_{r3}/D = \frac{0.085 \pm 0.007}{0.103 \pm 0.007} \cdot (0.077 \pm 0.002) = 0.063 \pm 0.007.$$

The substitution measurements on R3 fuel assemblies with shrouds were carried out in three steps at every substitution: 1. air in all the shrouds, 2. air in the R3 shrouds and water in the reference shrouds and, finally, 3. water in all the shrouds. The cases 2. and 3.

were here analysed by means of eq. (5) with corrections made for perturbed fluxes. We found that a complete void in the R3 fuel caused a positive reactivity change.

All the results of void experiments on R3 fuel are given in Table 3. The notation δB_{void}^2 means the change of buckling caused by void, but since $M_2^2/M_1^2 \neq 1$ the value $\delta B_{void}^2 = 0.19 \text{ m}^{-2}$ is related only to a system with a radial buckling of 4.37 m^{-2} . In the case of anisotropy ($M_2^2/M_1^2 \neq 1$) the buckling of a critical assembly is dependent on the geometry of the system, i.e. the ratio between height and radius. If the radial buckling is e.g. 3.00 m^{-2} we get $\delta B_{void}^2 = 0.16 \text{ m}^{-2}$ instead of 0.19 m^{-2} .

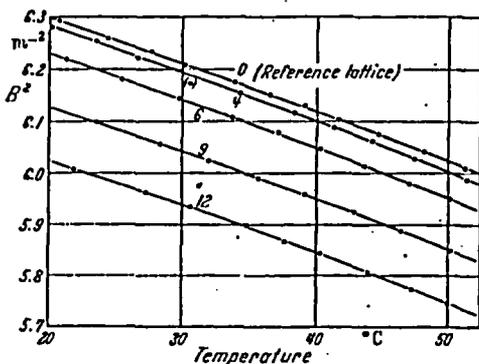


Fig. 7. Critical substitution measurements of buckling vs temperature. Reference lattice pitch: 19.0 cm. Test (R3) lattice pitch: $19.0 \cdot \sqrt{2}$ cm. Figures by the curves represent numbers of R3 clusters used in the test regions. Concentration of D_2O : 99.67 mol-%.

Comparison with the result without shrouds in 2.3.1 yielded the negative effect caused by shrouds (without void). The plastic tubes, which fed the pneumatic bottom valves with compressed air, were estimated to decrease the buckling by 0.03 m^{-2} .

Table 3. Results from the measurements on R3 clusters (c-c: 2.21 cm) with shroud at 22°C and 99.70 mol-% D_2O . Square lattice pitch: 26.87 cm. The meaning of δB_{void}^2 is given in the text

$\frac{\delta T_c}{D}$	$\frac{\delta D_c}{D}$	$\frac{M_2^2}{M_1^2}$	δB_{void}^2 m^{-2}	δB_{shroud}^2 m^{-2}
0.085 ± 0.007	0.063 ± 0.007	1.021 ± 0.010	+0.19 ± 0.03	-0.70 ± 0.07

2.3.4. Temperature coefficient. The critical height was measured periodically during the warming-up period. In some cases the temperature was stabilized by adjusting the electric power just to counterbalance the heat losses. A comparison between the two ways of measuring showed only a small systematic difference.

The temperature variation of the radial buckling has been calculated according to ref. 4. The main contribution is due to the thermal expansion of the reactor tank, whereas the change of the effective extrapolation length is quite small.

The investigation was first carried out with 6, 12, and 16 R3 clusters up to $80-90^\circ\text{C}$. However, the results of these measurements indicated that there was something wrong. It was therefore decided to repeat the experiments. Because of mechanical difficulties (undesirable pressure phenomena at scram) it was not

¹ We have: $0.19 - (4.37 - 3.00) (1 - M_2^2/M_1^2) = 0.16 \text{ m}^{-2}$.

permissible to raise the temperature above 50°C . The new test regions consisted of 4, 6, 9, and 12 R3 clusters in order to get more points for the analysis. From a comparison between the two investigations we conclude that the first run of the reference lattice had some systematic error.

The buckling points of each lattice arrangement are plotted versus temperature in Fig. 7. The following simple function of the temperature was then fitted to each set of experimental points.

$$B^2 = a + b \Delta T + c(\Delta T)^2 \quad (9)$$

The analysis according to eq. (4) may be applied to each of the coefficients a , b and c separately. The temperature coefficients b and c are given in Table 4.

Table 4. Temperature coefficients b and c according to eq. (9) within the temperature range $20^\circ\text{C} \leq T \leq 50^\circ\text{C}$ and with $\Delta T = T - 35^\circ\text{C}$. Concentration of D_2O : 99.67 mol-%.

Fuel	Square lattice pitch cm	$b \cdot 10^4$ $\text{m}^{-2}/^\circ\text{C}$	$c \cdot 10^4$ $\text{m}^{-2}/(^\circ\text{C})^2$
Reference	19.0	-9.23 ± 0.03	-0.023 ± 0.003
R 3	26.87	-7.93 ± 0.24	-0.02 ± 0.02

3. Exponential measurements

3.1. Experimental procedure

The exponential assembly ZEBRA has been described previously [3], [4]. However, since then it has been moved from Stockholm and rebuilt at the Studsvik site. The diameter is still only 1.00 m, but the height has been increased in order to make it possible to use the same kind of fuel as in R0. The Ra- α -Be sources of about one curie have been exchanged for Sb-Be sources giving about the same neutron intensity. The sources are now placed in a graphite pedestal below the tank.

The water can be heated up by circulation through an external electric heater. The minimum heating-up time from 20 to 90°C is about 10 hours.

The axial flux distribution in the exponential assembly is measured by a travelling probe containing two sensitive BF_3 counters (filled with 0.8 atm B^{10}F_3 , length 40 cm, diameter 1.0 cm). The counters are placed horizontally in the U-shaped probe which is driven up and down by a synchronous motor. The counting rate is integrated over predetermined height intervals. The pulse-counting channels have a dead-time of about $1.0 \mu\text{sec}$ (determined by the main amplifier) and the registered maximum rates are such that corrections due to the dead-time have to be applied. Pulses from the two separate counters are fed to a mixer via their pre-amplifiers. The mixed rate as well as the individual counting rates are registered simultaneously by means of a printer. Such an arrangement makes it possible to get a practically instantaneous measure of the dead-time and the constancy of the electronic system.

Two different lattices with 21.0 cm and 27.0 cm square pitch were investigated. The number of fuel assemblies used to fill up the ZEBRA tank was 16 and 12 respectively. The cross-section of the tank was not exactly filled by the cells, but special experiments indicate that the uncertainty caused by this mismatch is probably less than 0.1 m^{-2} . In the first case the central axis of the tank was situated between two clusters

($r=10.5$ cm) and in the second case between two clusters ($r=19.09$ cm). The probe coincides with the fuel in the first case and cannot place any fuel in the second case.

3.2. Method

The radial buckling a is calculated according to eqs. 3 and 4. However, effective extrapolation length L is here changed from 1.5 ± 0.3 cm (new probe) to 1.5 ± 0.3 cm (old probe) to give a better fit to the data between the exponential and the reference lattice. The main reason for this is that the tank wall is 0.3 to 0.4 cm thick, and the wall with regard to effective extrapolation length is not known. Since we have quite few elements we cannot calculate the radial buckling with a high accuracy.

The axial neutron flux distribution is suitable for measurement by

$$N(z; T) = A(T) \varphi(z; T)$$

where $\varphi(z; T)$ is the axial flux distribution which can be found theoretically. The axial flux distribution will be discussed below.

The analysis of the exponential measurements is made by the method of

$$\frac{N(z; T)}{N(z; T + \delta T)} = e^{-\lambda \delta T} \frac{\varphi(z; T)}{\varphi(z; T + \delta T)}$$

where z_i is varied, δz is a constant (half the distance between the left-hand side of the probe and the right-hand side of the probe), λ is a function of $1/N(z; T)$ and δz is the background intensity of z or a pure cosine distribution but something between.

In order to be able to measure the axial flux distribution caused by the joints we need to be just equal to the background intensity. This means that there must be a constant background intensity available if such a probe is used. After the determination of the microstructure of the axial flux distribution by means of the travelling probe, we can use the method of

$$\frac{N(z; T + \delta T)}{N(z; T)} = \frac{A(T + \delta T)}{A(T)} \frac{1 + \varphi(z; T)}{1 + \varphi(z; T + \delta T)}$$

As the changes generalise we can allow to write

$$\frac{N(z; T + \delta T)}{N(z; T)} \left(1 - \frac{1}{A} \frac{dA}{dT} \right) \times \left[1 - \frac{d}{dT} \left(\frac{\varphi}{N - \varphi} \right) \right]$$

turo above 50° C. The
, 9, and 12 R3 clusters
the analysis. From a
investigations we con-
reference lattice had

lattice arrangement
Fig. 7. The following
was then fitted to

$$(\Delta T)^2 \quad (9)$$

(4) may be applied to
d c separately. The
are given in Table 4.

nd c according to eq. (9)
T ≤ 50° C and with
D₂O: 99.67 mol-%

	$\sigma \cdot 10^3$ m ⁻¹ (°C)
0.03	-0.023 ± 0.003
0.24	-0.02 ± 0.02

iments

cedure

EBRA has been des-
ever, since then it
n and rebuilt at the
still only 1.00 m, but
n order to make it
fuel as in R0. The
urio e been ex-
ng s t the same
re n placed in s

7 circulation through
minimum heating-up
hours.

in the exponential
ing probe containing
with 0.8 atm B¹⁰F₃,
e counters are placed
e which is driven up
r. The counting rate
eight intervals. The
dead-time of about
1 amplifier) and the
that corrections due
ied. Pulses from the
-mixer via their pre-
ell as the individual
laneously by means
nt makes it possible
measure of the dead-
ectronic system.

1.0 cm and 27.0 cm
RA tank was 16 and
of the tank was not
cial experiments in-
by this mismatch is
first case the central
tween two clusters

(r=10.5 cm) and in the other case between four
clusters (r=19.09 cm). Because the horizontal part of
the probe coincides with a diameter of the tank we
cannot place any fuel in the center.

3.2. Method of analysis

The radial buckling and its temperature dependence
is calculated according to the procedures given in
refs. 3 and 4. However, the differences between the
effective extrapolation radii of the fast and the slow
groups is here changed from 1.05 ± 0.30 cm (old value)
to 1.5 ± 0.3 cm (new preliminary value) in order to
give a better fit to intercomparison measurements
between the exponential assembly and the critical
facility. The main reason for such an adjustment is
that the tank wall is covered with borated plastic,
0.3 to 0.4 cm thick, and the effective thickness of the
wall with regard to epithermal neutrons is not well
known. Since we have a heterogeneous system with
quite few elements we are not able to measure the
radial buckling with any high degree of accuracy.

The axial neutron distribution $N(z; T)$ in the
region suitable for measurements is assumed to be
given by

$$N(z; T) = A(T) \cdot e^{-\gamma(T) \cdot (z-z_0)} + \varphi(z; T), \quad (10)$$

where $\varphi(z; T)$ is the background. The value of z_0 can
be found theoretically [3], [4]. (The microstructure
caused by the joints is here neglected but this problem
will be discussed below.)

The analysis of measurements at any constant
temperature is made by taking the following ratio

$$\frac{N(z_i)}{N(z_i + n \cdot \delta z)} = e^{\gamma \cdot n \delta z} = \frac{e^{\gamma \cdot n \delta z} \varphi(z_i + n \delta z) - \varphi(z_i)}{N(z_i + n \delta z)}, \quad (11)$$

where z_i is varied, δz is the length of one interval and
 n is a constant (half the total number of intervals).
The left-hand side of eq. (11) is obviously a linear
function of $1/N(z_i + n \delta z)$, if φ is constant. However,
the background intensity $\varphi(z)$ is neither independent
of z nor a pure cosine distribution (fundamental mode)
but something between.

In order to be able to neglect the flux perturbation
caused by the joints we have to choose $n \cdot \delta z$ in eq. (11)
to be just equal to the length of one fuel section. It
means that there must be at least three fuel sections
available if such a procedure should be successful.
After the determination of γ and φ by means of eq. (11)
the microstructure of the flux is easily found.

If measurements of $N(z; T)$ are carried out by
means of the travelling probe during a continuously
rising temperature, we may utilize the following ratio
for an analysis.

$$\frac{N(z; T + \delta T)}{N(z; T)} = \frac{A(T + \delta T)}{A(T)} \cdot e^{-[\gamma(T + \delta T) - \gamma(T)](z-z_0)} \times \left. \begin{aligned} & \times \frac{1 + \varphi(z; T + \delta T) \cdot e^{\gamma(T + \delta T) \cdot (z-z_0)/A}}{1 + \varphi(z; T) \cdot e^{\gamma(T) \cdot (z-z_0)/A}} \end{aligned} \right\} \quad (12)$$

As the changes generally are rather small, we are
allowed to write

$$\frac{N(z; T + \delta T)}{N(z; T)} \left(1 - \frac{1}{A} \frac{dA}{dT} \delta T \right) \times \left. \begin{aligned} & \times \left[1 - \frac{d}{dT} \left(\frac{\varphi}{N - \varphi} \right) \delta T \right] = e^{-\frac{d\gamma}{dT} \cdot \delta T (z-z_0)} \end{aligned} \right\} \quad (13)$$

The quantities dA/dT and $d(\varphi/N)/dT$ can be
evaluated once for all from measurements at various
constant temperatures.

3.3. Results of exponential measurements

3.3.1. Measurements at room temperature. We chose
a total number of 20 intervals, 7.71 cm each, corre-
sponding to a total range of 154.2 cm, which is just

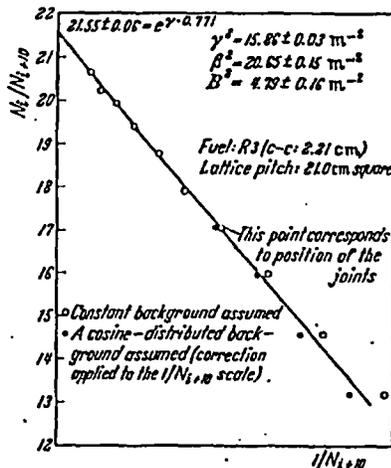


Fig. 8. Evaluation of axial buckling measurements at 22° C in ZEBRA. Lattice pitch: 21.0 cm

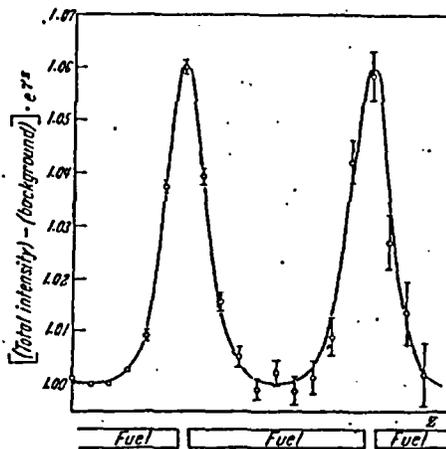


Fig. 9. The microstructure in the axial flux distribution measured with BF₃ counters in ZEBRA. The counters were placed horizontally in the moderator midway between fuel assemblies. Square lattice pitch: 21.0 cm

equal to the length of two fuel sections. The ratio
 $N(z_i)/N(z_i + n \delta z)$ according to eq. (11) was taken
between points in equivalent axial positions, i.e.
 $n=10$ and $n \delta z=77.1$ cm.

An example of the analysis by means of eq. (11)
is shown in Fig. 8, where $N_i=N(z_i)$ and $N_{i+10}=N(z_i+10 \delta z)$. The effect of joints is practically
eliminated. The buckling values corrected to 20° C and
99.75 mol-% D₂O [8] are given in Table 5.

The microstructure in the exponential distribution
is shown in Fig. 9. The flux peak caused by a joint is
as large as 6% in the moderator between the clusters,
where the detectors were placed.

Table 6. Results from exponential measurements on R3 fuel (c-c: 2.21 cm) at 20° C and corrected to 99.75 mol-% D₂O

Square lattice pitch cm	(2.4048)/M ² m ⁻²	γ ² m ⁻²	B ² m ⁻²
21.0	20.65 ± 0.15	15.84 ± 0.03	4.81 ± 0.16
27.0	20.83 ± 0.15	15.47 ± 0.03	5.36 ± 0.16

3.3.2. Temperature coefficient. These experiments were performed in two ways:

- a) two runs at constant temperatures, 22 and 89° C respectively, and analysis acc. to eq. (11);
- b) measurements during the temperature rise up to 89° C and analysis acc. to eq. (13).

The results are presented in Table 6 and in Fig. 10, which shows the graphical analysis acc. to eq. (13).

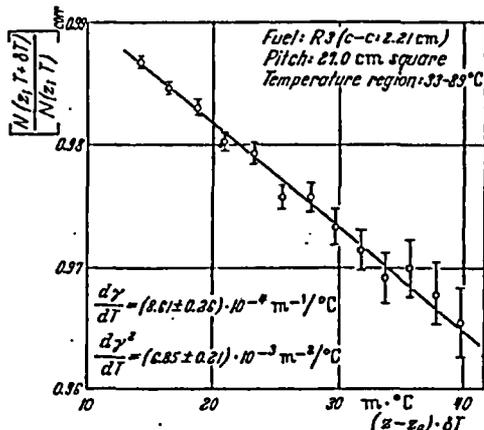


Fig. 10. Evaluation of temperature coefficient measurements in ZEBRA with continuously rising temperature. Temperature region: 33–89° C. Pitch: 27.0 cm

Table 6. Temperature coefficient measurements of R3 fuel (c-c: 2.21 cm) in the exponential assembly. Square lattice pitch: 27.0 cm. Concentration of D₂O: 99.67 mol-%

Temp. °C	γ ² m ⁻²	dγ/dT × 10 ⁶ m ⁻² /°C	dγ ² /dT × 10 ³ m ⁻² /°C	Mean temp. °C
22	15.88 ± 0.02	-1.5 ± 0.3	-7.9 ± 0.8	55
89	16.01 ± 0.04	-1.5 ± 0.3	-8.4 ± 0.4	61
33–89	(See Fig. 10)	-1.5 ± 0.3	-8.4 ± 0.4	61
	Mean value	-1.5 ± 0.3	-8.3 ± 0.4	60

The points analysed in Fig. 10 correspond to four consecutive runs (upwards-downwards-upwards-downwards) over a length of about 1 m. The two runs upwards are combined acc. to eq. (13) as are also the two runs downwards. The points of Fig. 10 represent the mean of the two combinations. In this way the uncertainty of the correction due to dA/dT is reduced to a minimum. The axial temperature gradient during the heating-up period is less than 1° C/m and assumed to be negligible.

4. Discussion of results

A bare critical facility like R0 is found to be very suitable for high-precision buckling measurements by means of the substitution technique. The minimum geometric buckling (~5.4 m⁻²) is comparatively high for fuel assemblies proposed for power reactors, but the core geometry is very clean and corrections due to neutron reflection are negligible.

The modified one-group perturbation theory combined with a new definition of cells is of course not the only way to analyse the substitution measurements, but we have found the method to be at least as good as any other theoretical treatment applied [7]; [10], [11] and the computational work is comparatively simple. It is also worth while to point out that with the usual definition of cells it is difficult to get a uniquely determined boundary between the test and the reference regions when the pitches are unequal. Since we take the real form of the regions into account we have found that the shape of the test region may vary considerably with a constant number of rods and the results are still interpretable [6].

The method of analysis has been checked in two cases using only reference fuel where complete critical substitutions were obtainable. The reference lattice pitches were in these cases 21.0 and 15.0 cm, while the pitches of the test regions were 21.0/√2 cm and 15.0/√2 cm respectively. The agreement was good within 0.05 m⁻², when the total difference of the buckling was about 2.5 m⁻² and the largest test region used in the extrapolation procedure was less than 15% of the core [6]. With a test lattice pitch of 19.0/√2 cm the maximum number of 16 R3 clusters corresponds to roughly 20% of the core.

The information we get about the transition region may also be of some value. The slope of any of the lines in Figs. 3 and 4 tells us how much the buckling B₀² of the corresponding transition region deviates from the mean value (B₁² + B₂²)/2 as may be seen from eqs. (3) and (4). The buckling of the transition region is approximately given as the mean buckling of the test and the reference regions taken at a moderator-to-fuel ratio which is equal to that of the transition region.

The measurements of the 19.0 · √2 cm lattice give a positive slope in Fig. 3, but at the 17.0 · √2 cm lattice we get an approximately horizontal line. This fact is possible to explain by realizing that the moderator-to-fuel ratio of the transition cell is smaller than that of any of the other two cell configurations. If we decrease the test pitch to 16.0/√2 cm (reference pitch 16.0 cm) we get, as we expect, a negative slope.

From the positive slope of the line in Fig. 4 we conclude that the buckling difference between the two fuel alternatives increases with decreasing lattice pitch (moderator-to-fuel ratio) as is also found experimentally (see Table 2).

Our interpretation of void experiments with single clusters in a reference lattice involves systematic errors. The method has been checked on a 19.0 cm lattice containing only reference fuel surrounded by aluminium shrouds, ID 6.3 cm and OD 6.5 cm [6]. The changes of the one-group diffusion coefficients when a void was introduced were found to be δD₀/D₀ = 0.120 ± 0.018 and δD₁/D₁ = 0.083 ± 0.004. These values were then used to analyse substitution measurements with voids. The buckling difference measured directly was 0.274 ± 0.004 m⁻² and the substitution experiments gave results, which agreed within about 0.01 m⁻² when the test region occupied up to 10% of the core. However, in this case the test fuel and the reference fuel were identical except regarding the void. By making also axial void measurements on the cen-

tral cluster in a test region we hope to be able to get a

The effect of the substitution compared with theoretical pile-oscillator measurement of the absorption of the aluminium is higher than expected discrepancy.



Fig. 11. Buckling vs lattice pitch R3 clusters at room temperature taken from ref. 14 and a 99.75 mol-% D₂O, a

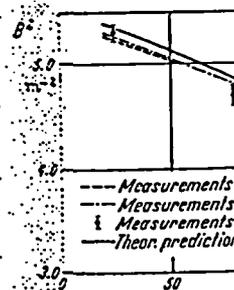


Fig. 12. Buckling vs temperature measurements on R3 clusters to be distan-

The fuel gaps in the axial regions with large correction should be positive. We have tried to measure by means of spectrometry but were not able to during the time available.

Buckling values at (c-c: 2.21 cm) without axial voids. The results obtained at [14] are also given for different internal spac-

turbation theory com-
cells is of course not the
tution measurements.
to 1 least as good
men plied [7], [10],
work — comparatively
point out that with the
ficult to get a uniquely
m the test and the
hes are unequal. Since
egions into account we
f the test region may
nt number of rods and
e [6].

s been checked in two
fuel where complete
ainable. The reference
ases 21.0 and 16.0 cm
gions were $21.0/\sqrt{2}$ cm
re agreement was good
stal difference of the
the largest test region
cedure was less than
test lattice pitch of
ber of 16 R3 clusters
the core.

it the transition region
he slope of any of the
ow much the buckling
sition region deviates
2 as may be seen from
f the transition region
mean buckling of the
ken r' moderator-to-
f the isition region.
1.0 · $\sqrt{2}$ cm lattice
ntal line. This fact is
that the moderator-to-
is smaller than that of
ratios. If we decrease
ference pitch 16.0 cm
s slope.

the line in Fig. 4 we
fference between the
with decreasing lattice
s is also found experi-

periments with single
involves systematic
checked on a 19.0 cm
fuel surrounded by
and OD 6.5 cm [6].
diffusion coefficients
are found to be $\delta D/D$
 0.083 ± 0.004 . These
substitution measure-
g difference measured
and the substitution
agreed within about
occupied up to 10% of
the test fuel and the
pt regarding the void-
measurements on the cen-

tral cluster in a test region of 3×3 similar clusters we
hope to be able to get an appropriate correction factor.

The effect of the shroud ($-0.70 m^{-2}$) is rather large
compared with theoretical calculations [12]. However,
pilo-oscillator measurements showed [13] that the
absorption of the aluminium shroud was about 30 per
cent higher than expected, which partly accounts for
the discrepancy.

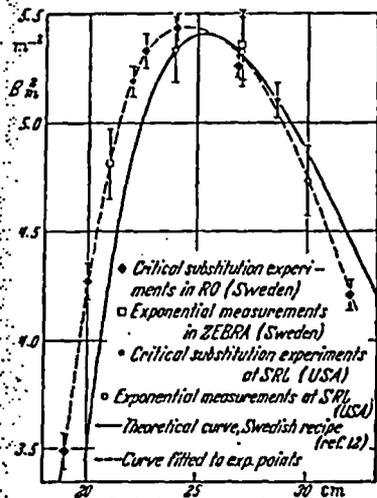


Fig. 11. Buckling vs lattice pitch. Comparison between measurements on 2.2 clusters at room temperature in different facilities. The BRL values were taken from ref. 14 and all the results have been adjusted to 20° C, 99.75 mol-% D₂O, and internal c-c spacing 2.21 cm.

Fig. 11. Buckling vs lattice pitch. Comparison between measurements on 2.2 clusters at room temperature in different facilities. The BRL values were taken from ref. 14 and all the results have been adjusted to 20° C, 99.75 mol-% D₂O, and internal c-c spacing 2.21 cm.

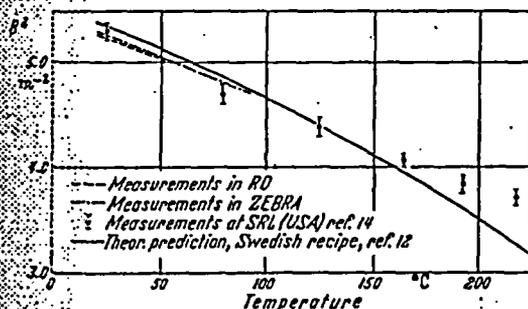


Fig. 12. Buckling vs temperature. Intercomparison between different measurements on R3 clusters. The curves are shifted somewhat vertically to be distinguished from each other.

The fuel gaps in the joints between fuel sections
introduce perturbations, which can be interpreted as
axial regions with larger moderator-to-fuel ratio. We
have not made any correction for these gaps. Such
a correction should be negative for close-packed
lattices and positive for larger lattice pitches [14].
We have tried to measure the gap effect on the buck-
ling by means of special perturbation measurements
but were not able to do so with sufficient accuracy
during the time available for the experiments.

Buckling values at room temperature for clusters
(c-c: 2.21 cm) without shrouds are plotted in Fig. 11.
The results obtained at the Savannah River Labora-
tory [14] are also given, but they are corrected for the
different internal spacing (c-c: 2.11 cm) by means of

the experimental values in Table 2. The theoretical
curve in Fig. 11 was calculated according to the
Swedish recipe [12].

However, the gap effect was treated in a simplified
way, the change of resonance absorption being neglect-
ed [16]. Part of the discrepancy between theoretical
and experimental curves is certainly due to this fact.

The temperature coefficient of the buckling has
been studied at three different facilities, R0 (20 to
50° C), ZEBRA (20 to 90° C) and the pressurized,
high-temperature subcritical arrangement PSE at the
SRL (20 to 215° C) [14]. An intercomparison is made
in Fig. 12 where the experimental values are plotted
versus temperature together with a theoretical curve
[12]. The curves are shifted somewhat vertically to be
distinguished from each other. The slopes, i.e. dB^2/dT ,
agree fairly well below 50° C but the discrepancy
between experiments and theory increases with tem-
perature.

For comparison with our temperature coefficient
measurements we may also mention experiments done
in ZEEP [15] on 19-element UO₂ clusters (UO₂ diam.
1.32 cm, hexagonal lattice pitch 21.59 cm), which
gave the result $dB^2/dT = -0.0113 \pm 0.0009 m^{-2}/°C$
(20° < T < 70° C). The measurements were made with
55 clusters in the central part of ZEEP, and the
buckling was determined by flux mapping.

5. Acknowledgement

We are much indebted to Dr. I. NILSSON, Messrs.
E. BLOMSJÖ, O. OLOFSSON, S. SJÖDQVIST, L.-G. STRÖM-
BERG, and the operating crew for their valuable con-
tributions to the experiments.

Reference: [1] MARGEN, P. H., et al.: R3 - A Natural Uranium, Heavy Water Reactor for Combined Electrical Production and District Heating. Proc. 2nd Intern. Conf. Peaceful Uses Atomic Energy 8, 220-237 (1958); — Directory of Nuclear Reactors, Vol. 1, p. 103. Vienna, Austria, IAEA, 1959. — [2] LANDERGÅRD, O., K. CAVALLIN and G. JONSSON: The Swedish Zero Power Reactor R0. AE-55 (1961). — [3] PERSSON, R., E. BLOMSJÖ, M. BUSTRAAN and R. MEIER: J. Nucl. Energy I 8, 188-206 (1956). — [4] PERSSON, R., et al.: Exponential Experiments on Heavy Water Natural Uranium Metal and Oxide Lattices. Proc. 2nd Intern. Conf. Peaceful Uses Atomic Energy 12, 364-373 (1958). — [5] COHEN, E. R.: Exponential Experiments on D₂O-Uranium Lattices. Proc. 1st Intern. Conf. Peaceful Uses Atomic Energy 5, 268-278 (1955). — [6] PERSSON, R.: Paper on perturbation methods (in preparation). To be published. — [7] BACHER, P., et R. NAUDET: J. Nucl. Energy, Part A 13, 112-127 (1961). — [8] WIKDAHL, C.-E.: The Variation of the Buckling with the H₂O-Concentration of Moderator in R0. RFX-68 (1961). — [9] GLASSTONE, S., and M. C. EDLUND: The Elements of Nuclear Reactor Theory (pp. 372-383). New York: P. van Nostrand Company 1952. — [10] JONSSON, A.: Analysis of Substitution Measurements in R0 by Means of Two-Group Diffusion Theory. RFR-84 (1960). Report in Swedish. — [11] JONSSON, A., and G. NILSUND: Heterogeneous Two-Group Diffusion Theory for a Finite Cylindrical Reactor. AE-57 (1961). — [12] APÉLQVIST, G., and P.-E. ÅHSTRÖM: Comparison Studies between Calculated and Measured Buckling Values in Heavy-Water Moderated Lattices. RFR-183 (1962). Report in Swedish. — [13] ERLANDSSON, I.: Neutron Absorption in Aluminium Measured by a Pilo-Oscillator. Memo (in preparation). — [14] BAUMANN, N. P., W. E. GRAVES, E. J. HENNELLY and G. F. O'NEILL: High Temperature Exponential Measurements and Room Temperature Critical Substitution Measurements on Swedish R3/Adam Fuel Assemblies in Heavy Water. This issue of Nukleonik. — [15] GREEN, R. E., D. F. ALLEN, D. W. HONE and A. OKAZAKI: Temperature Coefficient for a Lattice of 19-Element UO₂ Clusters. CRRP-978, AECL No. 1121 (1960). — [16] APÉLQVIST, G., AB Atomenergi: Private communication.

High Temperature Exponential Measurements and Room Temperature Critical Substitution Measurements on Swedish R3/Adam Fuel Assemblies in Heavy Water

By NORMANN P. BAUMANN, WILLIAM E. GRAVES, EDWARD J. HENNELLY and GEORGE F. O'NEILL

(Savannah River Laboratory, Aiken, South Carolina, U.S.A.)*

With 11 Figures in the Text

(Received March 5, 1962)

Summary. The Savannah River Laboratory has performed physics studies on D_2O lattices of fuel assemblies intended for the R3/Adam reactor being built by A. B. Atomenergi. Buckling temperature coefficients up to $220^\circ C$ were measured at square lattice pitches of 24.0, 27.0 and 30.0 cm center-to-center, between fuel assemblies, in the Pressurized Subcritical Experiment (PSE). Central cell foil activation traverses were also made at room temperature and at $217^\circ C$. Two buckling measurements at room temperature were made in the Process Development Pile (PDP) at triangular pitches of 23.7 and 30.8 cm using a substitution technique in which R3/Adam fuel replaces similar assemblies composed of smaller diameter oxide rods. The experimental results compared well with calculations.

Introduction

As part of a Swedish-U.S. exchange program on D_2O moderated power reactors, the Savannah River Laboratory (SRL) was requested to measure the bucklings of D_2O moderated lattices utilizing natural

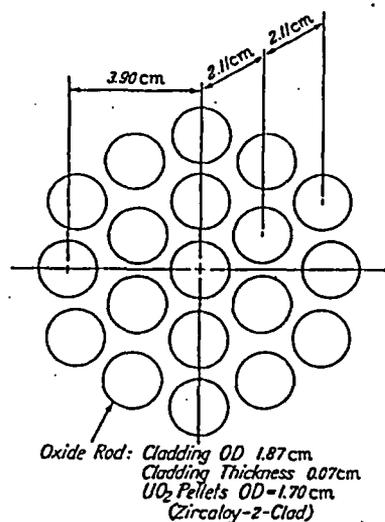


Fig. 1. Swedish UO_2 fuel assembly for R3/Adam

uranium oxide fuel assemblies of Swedish design. These fuel assemblies, which are intended for later use in the R3/Adam reactor [1] being built south of Stockholm, were furnished to SRL by A. B. Atomenergi. The primary purpose of the experiments was to obtain measurements of the temperature coefficient of buckling in SRL's unique pressurized exponential, the PSE [2]. Secondary purposes included the measurement of lattice cell flux distributions at room temperature and at $217^\circ C$ and the determination of room temperature bucklings in SRL's critical facility, the PDP [3]. In a separate paper [4] from A. B. Atomenergi the buckling measurements are compared with experiments that were done later in the Swedish reactor, RO, and in the Swedish Exponential, ZEBRA.

* The information contained in this article was developed during the course of work under contract AT(07-2)-1 with the U.S. Atomic Energy Commission.

Discussion

Description of R3/Adam Fuel Assemblies

The Swedish UO_2 fuel was in the form of sintered pellets 1.70 cm in diameter contained in Zircaloy-2 tubes having an outer diameter of 1.87 cm and a wall thickness of 0.075 cm. The density of the oxide in the individual pellets was 10.4 to 10.6 g/cm³. Each tube was about 77 cm in length and was welded closed at both ends. Helium gas filled the unused space inside the tube. Fuel assemblies were clusters of 19 such rods arranged as shown in Figure 1. No shroud tubes were used on the clusters. In the exponential experiments two lengths of rods were connected together to make fuel assemblies approximately 164 cm in length. In the PDP experiments, three lengths were connected together to make assemblies that were about 231 cm in length. At the junctions between fuel rods there were vertical gaps 4.5 cm long in which no fuel was present.

Experimental Facilities

Exponential buckling measurements were made in the Subcritical Experiment (SE) [5] and the Pressurized Subcritical Experiment (PSE) [2]. The SE consists of a cylindrical aluminum tank, with an inside diameter of 152.4 cm; it is sheathed with 0.076 cm of cadmium and placed vertically above the Standard Pile (SP), a graphite, enriched uranium reactor [6], [7]. The PSE consists of self-pressurizing carbon steel tank approximately 168 cm in diameter that can be heated up to $220^\circ C$ by steam coils. The tank is lined with stainless steel. Neutrons are supplied by 5 retractable Sb sources positioned inside fixed Be tubes.

The substitution critical measurements were made in the Process Development Pile (PDP) [3]. The PDP consists of a bare stainless steel tank 494.7 cm in diameter. Its effective height is controlled by D_2O level and was limited by the length of the fuel assemblies and their vertical position in the tank.

Lattice Arrangements

For the exponential experiments, the 19-rod clusters of R3/Adam fuel were arranged in square lattice arrays. At the 30 cm and 27 cm pitches, 21 assemblies were arranged in a five-by-five, fuel-centered lattice with an assembly removed from each corner. At the

24 cm pitch, 24 assemblies were arranged in a six-by-six, fuel-centered lattice with an assembly removed from each corner. The center of a fuel assembly was at the center of a square, formed by the center of the four adjacent assemblies.

Measurements in the PSE were made using a substitution technique in which R3/Adam fuel assemblies replaced similar assemblies composed of smaller diameter oxide rods. The experimental results compared well with calculations. The substitution technique of UO_2 rod clusters in a region containing the fuel was matched to the experimental conditions as far as possible for comparison of thermal and fast and slow neutron spectra. A desired match is shown in Figure 3. The copper poison position was under the PDP.

In the PDP, the triangular lattice pitch was 30.8 cm. The lattice arrangement is shown in Figure 3. The outermost region consisted of lithium-aluminum rods. These were used to get maximum sensitivity. The next region consisted of rods with a pitch as the test fuel. The test fuel. At each test region were assemblies and the other 12 were essentially identical.

PSE Buc.

Ver.

The exponential buckling measurements were made primarily on measurements. In the PSE by activating indium foils 0.051 cm thick in an aluminum tube was approximately equidistant. The inner diameter of the wall thickness was support provided for which were an alloy of foils were counted on an automatic counter the corresponding position. An IBM 650 routine was used to fit a least squares fit of a to the experimental data. The amplitude, κ is the neutron multiplication factor, t is the chosen from earlier neutron measurements. A and κ are variables that differ by less than 1%. The background and dead time were corrected. A set of foils were treated in a similar manner combined to give average values. Assumptions constant foil

al Sr⁹⁰ substitution after

F. O'NEILL

semblies intended for the re measured at square subcritical Experiment buckling measurements at 8 cm using a substitution. The experimental

el Assemblies

he form of sintered lined in Zircaloy-2 1.87 cm and a wall of the oxide in the 3 g/cm³. Each tube as welded closed at unused space inside sters of 19 such rods shroud tubes were ential experiments d together to make 4 cm in length. In ths were connected wert out 231 cm een rods there which, no fuel was

ities nents were made in 5] and the Pressur) [2]. The SE con nk, with an inside ed with 0.076 cm of ove the Standard um reactor [6], [7] ng carbon steel tank : that can be heated e tank is lined with ied by 5 retractable l Be tubes.

rements were made PDP) [3]. The PDP nk 494.7 cm in dia olled by D₂O level the fuel assemblies tank.

nts, the 19-rod clus ed in square lattice itches, 21 assemblies fuel-centered lattice ach corner. At the

24 cm pitch, 24 assemblies were arranged in a six-by-six moderator-centered lattice with three assemblies removed from each corner. For a fuel-centered lattice, the center of a fuel assembly coincided with the axis of the tank; for a moderator-centered lattice the center of a square, formed by four assemblies, was placed at the center of the tank.

Measurements in the PDP were performed by substitution techniques. An outer "driver" lattice of UO₂ rod clusters was used surrounding a central region containing the test lattice. The driver lattice fuel was matched to the R3/Adam fuel as closely as possible for comparable bucklings, resonance, escape probabilities, thermal migration areas, neutron ages, and fast and slow diffusion coefficients. The 31 rod cluster of 1.27 cm oxide rods chosen to achieve the desired match is shown in Figure 2. The use of the copper poison positions shown in the Figure is discussed under the PDP calibrations.

In the PDP, the R3/Adam fuel was studied at triangular lattice pitches of 23.7 cm and 30.8 cm. The lattice arrangement at the 30.8 cm pitch is shown in Figure 3. The outermost region of the pile was loaded with lithium-aluminum poison rods 2.54 cm in diameter. These were used to size the loading for approximately equal radial and vertical bucklings so as to get maximum sensitivity in the buckling measurements. The next region consisted of driver fuel at the same pitch as the test fuel. The innermost region contained the test fuel. At each lattice pitch two different sized test regions were used, one containing 7 fuel assemblies and the other 12. Loadings for the 23.7 cm pitch were essentially identical except for the change in pitch.

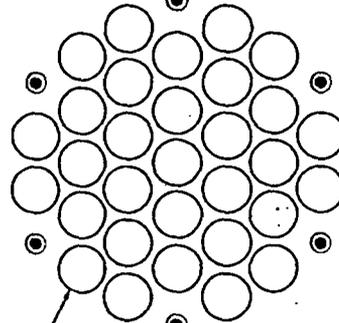
PSE Buckling Measurements

Vertical Bucklings

The exponential buckling measurements depended primarily on measurements of the vertical flux distributions. In the PSE these measurements were made by activating indium foils 1.27 cm in diameter and 0.051 cm thick in an air-filled aluminum tube. The aluminum tube was mounted slightly off-center approximately equidistant from four fuel assemblies. The inner diameter of the tube was 2.093 cm and the wall thickness was 0.287 cm. An inner aluminum support provided for accurate positioning of the foils, which were an alloy of 20 wt % indium in lead. The foils were counted on NaI(Tl) scintillation crystals in an automatic counter system. The activity data with the corresponding position data were processed using an IBM 650 routine which obtained a two parameter, least squares fit of a theoretical $A \sinh \kappa(t-z)$ curve to the experimental data. In this expression, A is an amplitude, κ is the reciprocal of the vertical relaxation length, t is the extrapolated upper boundary chosen from earlier measurements to be equal to the water height plus 5.4 cm, and z is the position coordinate. A and κ are varied by the code until successive iterations differ by less than 1×10^{-6} in κ . The code also corrects for time decay of the foils and for counter background and deadtime. Multiple counts of the same set of foils were treated as separate problems and were combined to give average values for κ . Since the code assumes constant foil positions, a small correction was

needed for the lengthening of the aluminum foil support with temperature. The correction was calculated assuming the vertical flux follows an exponential function.

0.366-cm Copper Wire in 0.635-cm OD x 0.089-cm Wall Thickness Al Tube (6061)



Oxide Rod: Cladding OD 1.389 cm, Cladding Thickness 0.051 cm, UO₂ Pellets OD=1.27 cm (Al Cladding 6063T6)

Fig. 2. 31-Rod cluster of UO₂ rods with copper poison

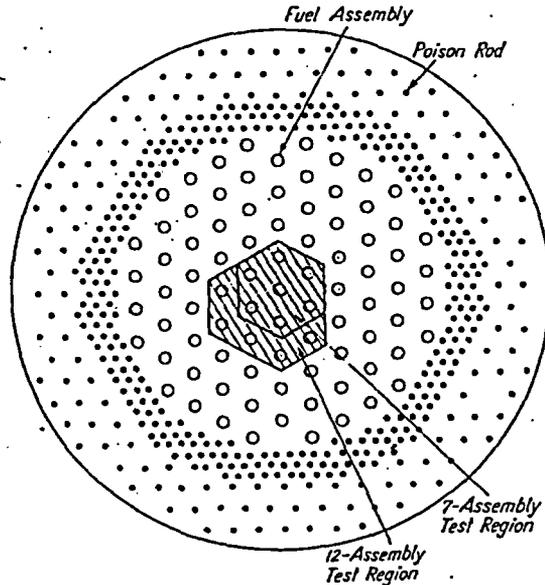


Fig. 3. PDP test regions used in the 30.8 cm pitch substitution measurements

Radial Bucklings

Material bucklings for the exponential lattices were obtained from the usual expression $B_m^2 = B_r^2 - \kappa^2$, where B_r^2 is the radial buckling of the exponential. In the PSE the value of the B_r^2 was determined by intercomparisons with the PDP and SE by substituting in known values of B_m^2 for several lattices with measured κ^2 values, to obtain the average radial buckling. The interior of the PSE is lined with coiled stainless steel piping that is used to contain high pressure steam during the heating cycle and cold water during the cooling cycle of operation. The coils present a vary

irregular nuclear boundary and consequently it is difficult to obtain the radial buckling by conventional flux measurements. A room temperature value of $8.45 \pm 0.15 \text{ m}^{-2}$ was chosen from the intercomparison.

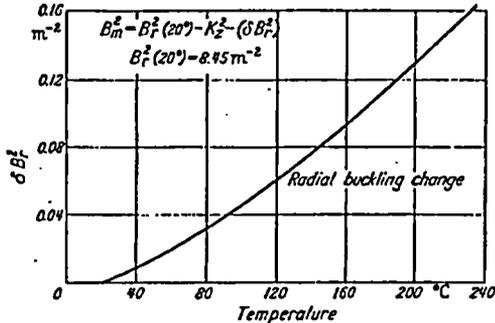


Fig. 4. Corrections to PSE buckling measurements from change of B_r^2 with temperature

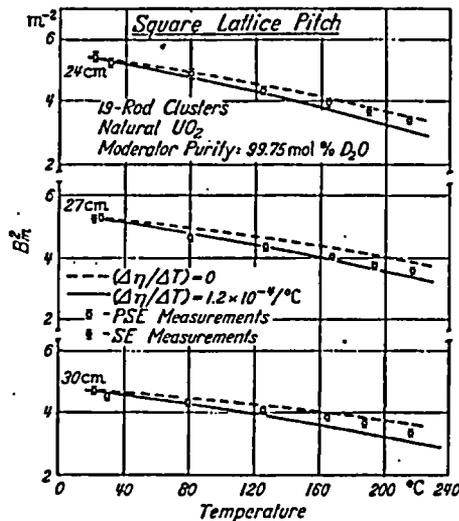


Fig. 5. Temperature variation of buckling for R3/Adam lattices

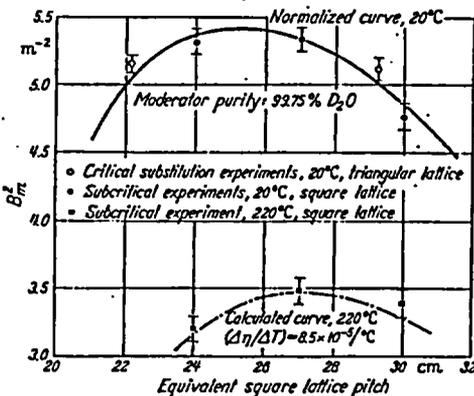


Fig. 6. Bucklings of R3/Adam lattices

The intercomparison of the measurements reported in this paper provides additional confirmation of the value of B_r^2 that was chosen.

The change in radial buckling resulting from heating the PSE was calculated according to the procedure

given by PERSSON [8]. This procedure takes into account both the physical expansion of the tank and coils and the changes in the diffusion lengths for fast and slow neutrons. The calculated change was found to give good agreement with foil activation measurements in D_2O moderator with no fuel in the PSE. The calculated curve is given in Figure 4.

Experimental Difficulties

Because it was necessary to adapt the experimental measurements to fuel elements that were designed for eventual use in the R3/Adam reactor, two difficulties were encountered in performing the experiments in the PSE. First the elements were about 154 cm in length rather than the 180 cm for which the PSE was designed. This necessitated having water levels above the top of the fuel at the highest temperature. Some adjustment for this effect could have been made except that the gap in the fuel column between the two fuel sections caused an exaggerated flux peak which perturbed the vertical exponential traverse. This peak required that several activating foils be placed above and below the gap in order to obtain sufficient unperturbed foil measurements to make a vertical traverse. (There were not enough fuel elements available to make longer assemblies for the PSE measurements.) Therefore a compromise was arranged in which the water level was selected to coincide with the top of the fuel at about 140 degrees C, which is midway in the expansion of the moderator over the heating cycle. Subsequent measurements at room temperature showed that the errors due to the effect of the reflector were both random and small, i.e., $\pm 0.06 \text{ m}^{-2}$, if the reflector savings were made equal to the reflector thicknesses. In addition theoretical calculations, using two-group theory and taking into account the neutron absorption of the stainless-steel fuel-element supports that extended through the reflector, showed that for the maximum reflector thickness of 15 cm, the correction to the buckling was less than 0.02 m^{-2} . No such correction was applied to the data because of the uncertainty in the approximations that had to be used to estimate the neutron absorptions of the support rods.

The effect of the gap was further minimized at the 24 and 30 cm pitches by using foils located at 35, 40, 45, 95, 100, 105, 110 and 115 cm above the bottom of the fuel. Measurements were made to show that the cadmium ratio of indium was constant over this region. At the 27 cm pitch the full set of foils was used, and the room temperature measurements were compared with measurements in the same lattice in the SE. As described later the results tended to substantiate the PSE procedure.

Results

The results of the PSE buckling measurements are given in Table I. and are summarized in Figures 5 and 6. Each data point is an average of several successive counts on the same set of foils¹.

¹ The calculated curve at 20 degrees C was normalized to the experimental results at the 27 cm pitch by adjustments to the value of the resonance integral. In addition, all the experimental points in Figure 6 were corrected for the effect of the gaps in the fuel column. The calculated curve at 220 degrees C was computed from the theoretical evaluation of the temperature coefficient normalized to the values of the buckling at 20 degrees C.

Table I. PSE buckling 99

24 cm Pitch			
Temp. °C	B_r^2, m^{-2}	B_r^2, m^{-2}	Temp. °C
20	—	(5.42)	20
22	2.98	5.47	25
28	3.13	5.32	80
60	3.63	4.82	125
125	4.07	4.38	165
165	4.46	3.99	193
190	4.71	3.74	218
217	4.99	3.46	220
220	—	(3.30)	—

Buckling values in j

SE Buc

SE buckling mea to verify the PSE tri in the fuel. They wer

Vertical buckling made with a traveling compensated, neutron s moved vertically in : 2.54 cm in outer dia 0.076 cm. The tube approximately equidi The neutron flux w vertical position as through the tank. F traverse were made i background traverse exponential was shu shutter, 0.076 cm thi The background tra photon neutrons result are initiated by gamm any residual gamma s well as for leakage of the SE. The backgro is separated from th 42.5 cm graphite ped

Measurements wi inherently more acc Consequently, the ments was to verify t points on either side results as the travel Such was indeed the direct comparison of the 27 cm room temp verify the radial buc buckling was adjust 0.08 m^-2 to correct fc was made in 99.65 m

PDP Buc

Ref

The PDP subst: R3/Adam lattice bucl got's substitution mea These reference latt driver fuel assemblies containing 6, 4, 2 or 0

procedure takes into account the tank and diffusion lengths for alcu change was with all activation or with no fuel in the given in Figure 4.

liculties

adapt the experimental that were designed for actor, two difficulties g the experiments in vere about 154 cm in or which the PSE was ng water levels above at temperature. Some uld have been made column between the aggerated flux peak exponential traverse l activating foils be p in order to obtain urement to make a not enough fuel e-sssemblies for the PSE promise was arranged to coincide with °C, which is midway ator over the heating at room temperature effect of the reflector e., ±0.06 m⁻², if the qual to the reflector cal c lations, using acc : the neutron fuel-e-ment supports ctor, showed that for of 15 cm, the correc n 0.02 m⁻². No such data because of the ions that had to be sorptions of the sup-

ther minimized at the g foils located at 35 cm above the bottom made to show that s constant over this l set of foils was used, urement were com ame lattice in the SE nded to substantiate

ng measurements are arized in Figures 5 i average of several set of foils².

was normalized to the itch by adjustments to addition, all the experi ted for the effect of the led curve at 220° C was tion of the temperature if the buckling at 20° C.

Table 1. PSE buckling measurements of R3/Adam lattices for 99.75 mol-% D₂O

25 cm Pitch			27 cm Pitch			30 cm Pitch		
Temp. °C	k_{eff} , m ⁻²	B_{eff} , m ⁻¹	Temp. °C	k_{eff} , m ⁻²	B_{eff} , m ⁻¹	Temp. °C	k_{eff} , m ⁻²	B_{eff} , m ⁻¹
20	—	(5.42)	20	—	(5.35)	20	—	(4.71)
25	2.98	5.47	25	3.16	5.29	22	3.76	4.69
30	3.13	5.32	80	3.74	4.71	31	3.97	4.48
36	3.63	4.82	125	4.08	4.39	80	4.12	4.37
125	4.07	4.38	165	4.37	4.08	125	4.30	4.15
165	4.46	3.99	193	4.61	3.84	164	4.57	3.88
190	4.71	3.74	218	4.74	3.71	180	4.72	3.73
217	4.99	3.46	220	—	(3.60)	217	5.01	3.46
220	—	(3.30)	—	—	—	220	—	(3.36)

Buckling values in parentheses are extrapolated values.

SE Buckling Measurements

SE buckling measurements were intended only to verify the PSE treatment of the effect of the gap in the fuel. They were restricted to the 27 cm lattice.

Vertical buckling measurements in the SE were made with a traveling monitor [5], [9], a gamma compensated, neutron sensitive ion chamber which is moved vertically in an air-filled aluminum thimble, 2.54 cm in outer diameter with a wall thickness of 0.076 cm. The tube was mounted slightly off center approximately equidistant from four fuel assemblies. The neutron flux was recorded as a function of vertical position as the ion chamber was moved through the tank. Four traverses and a background traverse were made for the present studies. For the background traverse the thermal neutron feed to the exponential was shut off by inserting a cadmium shutter, 0.076 cm thick, between the SE and the SP. The background traverse corrects principally for photoneutrons resulting from D(γ, n)H reactions that are initiated by gamma rays from the SP and also for any residual gamma sensitivity of the ion chamber as well as for leakage of fast neutrons from the SP into the SE. The background is important because the SE is separated from the surface of the SP by only a 42.5 cm graphite pedestal.

Measurements with the traveling monitor are inherently more accurate than those made with foils. Consequently, the main purpose of the SE measurements was to verify that the restricted number of foil points on either side of the fuel gap gave the same results as the traveling monitor flux distributions. Such was indeed the case. Also as shown in Figure 5, direct comparison of the SE and PSE bucklings for the 27 cm room temperature lattice further served to verify the radial buckling intercalibration. The SE buckling was adjusted upwards by a calculated 0.08 m⁻² to correct for the fact the SE measurement was made in 99.65 mol-% D₂O.

PDP Buckling Measurements

Reference Lattices

The PDP substitution measurements of the R3/Adam lattice bucklings were calibrated to analogous substitution measurements on reference lattices. These reference lattices consisted of the standard driver fuel assemblies of 31 rod UO₂ fuel clusters each containing 6, 4, 2 or 0 copper poison wires. The wires

were inserted symmetrically in the thin-walled aluminum tubes shown in Figure 2. Use of the copper wires made it possible to cover a wide range of bucklings with a minimum disturbance of the lattice parameters other than f and L^2 .

Critical moderator height measurements were made on one-region loadings of each reference lattice. In order to obtain material bucklings from these data, it was necessary to know the vertical extrapolation distances as well as the radial bucklings of the lattices. These quantities were measured by irradiating gold pins and thus obtaining the radial and vertical flux distributions. The irradiations were made at both pitches in the lattices containing 0 and 6 copper wires per assembly. The radial bucklings and vertical extrapolation distances were independent of the amount of copper poison present, and were therefore applied directly to the critical moderator height measurements made on the full set of reference lattices. Buckling results are shown in Table 2.

Table 2. Buckling results for reference pile loadings of PDP

Triangular Lattice Pitch, cm	Number of Copper Rods Per Fuel Assembly	Measured Bucklings, m ⁻²	ΔB^2 for Aluminum Tubes, m ⁻²	Corrected B_{eff} , m ⁻²
30.8	6	3.732	0.055	3.787
30.8	4	4.101	0.058	4.157
30.8	2	4.456	0.058	4.514
30.8	0	4.818	0.059	4.877
23.7	6	3.679	0.025	3.704
23.7	4	4.199	0.025	4.224
23.7	2	4.707	0.026	4.733
23.7	0	5.209	0.026	5.235

The aluminum tube correction accounts for the safety and control rod guides used in the PDP. The moderator temperature was 22.5 ± 1.0° C and the purity was 99.68 ± 0.02 mol-% D₂O at the time of these measurements. An analysis of experimental uncertainties leads to an error of ±0.05 m⁻² on the reference lattice bucklings.

Test Region Calibration

Test region calibrations consisted of measurements of the PDP critical water heights with the four reference lattices loaded in turn into each of the test regions. These calibration data were analyzed by requiring them to fit two-region, two-group diffusion theory. The measured water heights were increased by the measured extrapolation distances and substituted into an IBM 650 code using calculated input lattice parameters. Results of the two-group calculations were extremely insensitive to the methods used in calculating the lattice parameters. The radius of the boundary between regions and the buckling of the "driver" region were varied until the buckling solutions of the two-group calculations gave the best fit to the previously measured bucklings of the four lattices used in the test regions. Results are given in Table 3. They reveal that the calculated "driver" region bucklings were slightly lower than the bucklings measured for the one-region loadings, uncorrected for the presence of aluminum thimbles. The differences are reasonable because the thimbles are concentrated in the "driver" region.

Table 3. Input parameters for two-group calculation of bucklings of test regions

Triangular Lattice Pitch, cm	Number of Copper Rods For Assembly	Extrapolated Pile Height, cm	
		7-Assembly Test Region	12-Assembly Test Region
23.7	6	183.41	187.13
23.7	4	181.47	184.03
23.7	2	179.51	180.78
23.7	0	177.52	177.63
30.8	6	202.82	208.64
30.8	4	199.72	203.43
30.8	2	196.63	198.42
30.8	0	193.43	193.55
23.7	R3/Adam	177.42	177.67
30.8	R3/Adam	192.32	191.85

Triangular Lattice Pitch, cm	Number of Assemblies in Test Region	R ₁ , cm	R ₂ , cm	B _m ² , Outer Region, m ⁻²
30.8	7	43.0	163.4	4.7826
30.8	12	64.8	163.4	4.7588
23.7	7	33.1	166.9	5.2034
23.7	12	42.5	166.9	5.1956

R3/Adam Lattice Bucklings

Following the calibration experiments, the R3/Adam lattices were inserted in the test regions and the critical water heights measured. The extrapolated pile

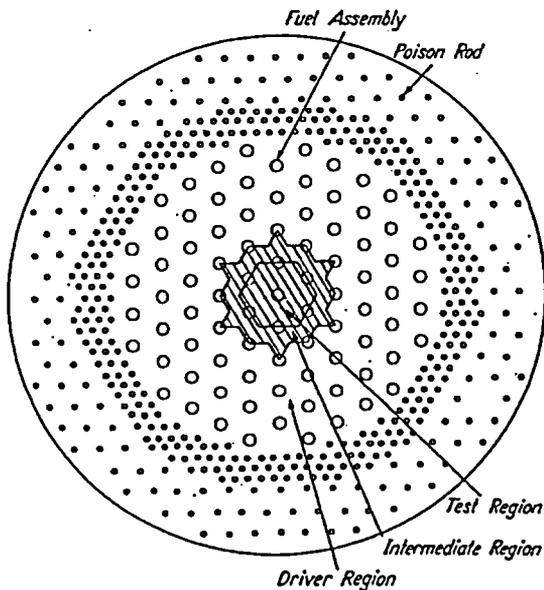


Fig. 7. Seven-assembly PDP test region. 30.8 cm pitch substitution measurements

heights obtained are given in Table 3. Bucklings were obtained in two ways. In the two-group method they were calculated from the same IBM 650 code used for the calibration calculations. The one-group results were obtained by using the same geometrical boundaries and outer region bucklings as in the two-group calculations. The slow neutron diffusion coefficient was assumed to be the same in both regions.

Results are given in Table 4. The small size of the differences between the one- and two-group results is an indication of the good "match" between the R3/Adam lattices and the calibration lattices.

Table 4. R3/Adam lattices — PDP buckling results for 99.68 mol-% D₂O and 22.5° C

Triangular Lattice Pitch, cm	Number of Energy Groups Used in Calculations	B _m ² , m ⁻²		B _m ² , m ⁻² (Average)
		7-Assembly Test Region	12-Assembly Test Region	
23.7	2	5.275	5.258	5.267
23.7	1	5.262	5.244	5.253
30.8	2	5.014	5.010	5.015
30.8	1	5.003	5.006	5.005

It is estimated that the precision of the substitution measurements, assuming the bucklings of the calibration lattices are exact, is ±0.03 m⁻². When combined with the uncertainty of ±0.05 m⁻² in the calibration buckling, an absolute uncertainty of about ±0.06 m⁻² is obtained for the bucklings in Table 4.

Correcting these data to standard conditions of 99.75 mol-% D₂O and 20° C gives values of buckling of 5.32 ± 0.07 m⁻² for the 23.7 cm pitch and 5.11 ± 0.08 m⁻² for the 30.8 cm pitch. The water purity corrections were calculated and the temperature corrections were obtained from PSE data.

Perturbation Theory Analysis of PDP Data

An alternate method of analyzing the PDP substitution data is provided by a perturbation theory approach developed by PERSSON [10]. For this method the substitution loadings were broken into test, intermediate and driver regions as shown by the example in Figure 7. Unit cells were drawn as rhomboids with the two vertices at the acute (60°) angles terminating on fuel pieces. Rhomboids with both vertices terminating on test fuel were assigned to the test region, those with one vertex on a test fuel assembly and the other on a driver fuel assembly were assigned to the intermediate region, and those with both vertices on driver fuel to the driver region. In obtaining statistical weights for use in the perturbation calculations each rhomboid was broken into two equilateral triangles. The statistical weights were then calculated for the flux at the centroid of each triangle. Since the test regions closely approximated the driver region in material buckling, a uniform radial flux distribution was assumed corresponding to the flux distribution J₀(B, r). The statistical weights, w, were then obtained as

$$w = \frac{\sum_i A_i J_0^2(B, r_i)}{\int_0^R 2\pi r J_0^2(B, r) dr} = \left(\frac{A}{\pi R^2 J_0^2(2.4048)} \right) \sum_i J_0^2(B, r_i)$$

where A is the area of the unit triangle, R is the extrapolated radius of the pile taken from Table 3, B is the square root of the radial buckling corresponding to that radius, and r_i is the distance from the center of the reactor to the centroid of the ith unit triangle. The summation is taken over all unit triangles in the region of interest.

Numerical results are given in Table 5. The vertical bucklings α³ and α₁² are those corresponding to the measured critical water heights for the substitution and full driver loadings respectively. The quoted errors on the (α² - α₁²) values are standard deviations corresponding to an uncertainty of one millimeter in the difference between the two moderator heights.

Table 5. Re

	30.8 cm Pitch
W ₀ (test)	0
W ₀ (intermediate)	0
α ² , m ⁻²	2
α ₁ ² , m ⁻²	2
α ² - α ₁ ² , m ⁻²	±0
W ₀ ÷ ½ W ₀	0
W ₀ ÷ ¼ W ₀	0
α ² - α ₁ ²	0
α ² - α ₁ ² / (W ₀ ÷ ¼ W ₀)	±0

In the method of function $\frac{\alpha^2 - \alpha_1^2}{W_0 + \frac{1}{4}W_0}$ i $\frac{W_0}{W_0 + \frac{1}{4}W_0}$ for the dif substitution lattice. are the statistical we mediate regions resp value of $\frac{W_0}{W_0 + \frac{1}{4}W_0}$ d a full test loading difference also corre rial bucklings betwe polations are graphe Figure the results s error of those obtain analyses.

As an alternative through the two dat of the lines can be these slopes can be data points. Such a at the 30.8 cm pitch value of (α² - α₁²)/(W₀ ÷ ¼ W₀) 0.00 m⁻². This furt between the different r

Bucklings f

The bucklings as exponentials are for blies which have gap the experiments and sis, corrections were for fuel assemblies w

The calculation effect of the fuel gap val 20 cm on either that the effect was The difference in th interval from that c lated by the SRL activations in the vi and L². For the ex flux profiles were theory for the three calculated flux profi lattice. The correcti ches were -0.09, 0.1 20° C and -0.10, - A measurement of t the SE confirmed, wi

PDP buckling results for $T = 22.5^\circ\text{C}$

λ, m^{-2}	Assembly	B_1, m^{-2} (Average)
5.258		5.257
5.244		5.253
5.016		5.015
5.006		5.005

tion of the substitution
cklings of the calibra-
 m^{-2} . When combined
 m^{-2} in the calibration
y of about $\pm 0.06 \text{ m}^{-2}$
1 Table 4.

andard conditions of
es values of buckling
cm pitch and $5.11 \pm$
1. The water purity
nd the temperature
PSE data.

sis of PDP Data

lyzing the PDP sub-
turbation theory
SON [10]. For this
gs were broken into
ions as shown by the
were drawn as rhom-
the acute (60°) angles
homboids with both
ael were assigned to
vert on a test fuel
er fu assembly were
gion, and those with
the driver region. In
se in the perturbation
as broken into two
ical weights were then
roid of each triangle
proximated the driver
uniform radial flux
sponding to the flux
tical weights, w , were

$$\frac{1}{2(4048)} \sum_j J_0^2(B_r, r_i)$$

it triangle, R is the
taken from Table 3.
l buckling correspond-
he distance from the
ntroid of the i^{th} unit
over all unit triangles

in Table 5. The ver-
ose corresponding to
ghts for the substitu-
ectively. The quoted
o standard deviations
of one millimeter in
moderator heights.

Table 5. Results of perturbation method

	30.8 cm Pitch 7 Assemblies	30.8 cm Pitch 12 Assemblies	23.7 cm Pitch 7 Assemblies	23.7 cm Pitch 12 Assemblies
W_2 (test)	0.1376	0.2559	0.0800	0.1532
W_2 (intermediate)	0.1855	0.2152	0.1130	0.1383
α^2, m^{-2}	2.6684	2.6815	3.1354	3.1301
$\alpha_1^2, \text{m}^{-2}$	2.6379	2.6346	3.1319	3.1280
$\alpha^2 - \alpha_1^2, \text{m}^{-2}$	0.0305	0.0469	0.0035	0.0021
	± 0.0028	± 0.0028	± 0.0035	± 0.0035
$W_2 + \frac{1}{2} W_3$	0.2304	0.3635	0.1365	0.2224
$\frac{\alpha^2 - \alpha_1^2}{W_2 + \frac{1}{2} W_3}$	0.805	0.592	0.828	0.622
$\alpha^2 - \alpha_1^2$	0.132	0.129	0.026	0.009
$\frac{\alpha^2 - \alpha_1^2}{W_2 + \frac{1}{2} W_3}, \text{m}^{-2}$	± 0.012	± 0.008	± 0.026	± 0.016

In the method developed by PERSSON [10] the function $\frac{\alpha^2 - \alpha_1^2}{W_2 + \frac{1}{2} W_3}$ is plotted against the function $\frac{W_2}{W_2 + \frac{1}{2} W_3}$ for the different sized test loading of each substitution lattice. In these functions, W_2 and W_3 are the statistical weights of the test region and intermediate regions respectively. Extrapolation to a zero value of $\frac{W_2}{W_2 + \frac{1}{2} W_3}$ then gives the difference between a full test loading and a full driver loading. This difference also corresponds to the difference in material bucklings between the two lattices. The extrapolations are graphed in Figure 8. As seen from the figure the results are well within the experimental error of those obtained from the one- and two-group analyses.

As an alternative to the straight line extrapolation through the two data points at each pitch, the slopes of the lines can be computed and straight lines with these slopes can be drawn to give a best fit to the data points. Such an approach gives the same result at the 30.8 cm pitch, but changes the extrapolated value of $(\alpha^2 - \alpha_1^2)/(W_2 + \frac{1}{2} W_3)$ for the 23.7 cm pitch to 0.00 m^{-2} . This further improves the agreement between the different methods of measuring the buckling.

Bucklings for Uniform Fuel Columns

The bucklings as measured in the PDP and the exponentials are for the actual R3/Adam fuel assemblies which have gaps in the fuel columns. To put all the experiments and the calculations on the same basis, corrections were calculated to give the bucklings for fuel assemblies without gaps.

The calculation was made by assuming that the effect of the fuel gap extended uniformly over an interval 20 cm on either side of the center of the gap and that the effect was negligible beyond this interval. The difference in the average buckling of the 40 cm interval from that of the uniform lattice was calculated by the SRL recipe using the measured foil activations in the vicinity of the gap for calculating f and L^2 . For the exponential measurements, vertical flux profiles were calculated by one-energy group theory for the three region loading and compared to calculated flux profiles for a loading with a uniform lattice. The corrections for the 24, 27 and 30 cm pitches were $-0.09, 0.00$ and $+0.05 \text{ m}^{-2}$ respectively at 20°C and $-0.10, -0.01$, and $+0.03 \text{ m}^{-2}$ at 220°C . A measurement of the effect of the gap at 27 cm in the SE confirmed, within $\pm 0.05 \text{ m}^{-2}$, that the vertical

traverse was unaffected by the presence of the gap at that pitch.

The corrections for the critical substitution measurements in the PDP were also made by assuming that the effect of the flux perturbation due to the gaps in the fuel columns extended 20 cm each side of the gap. By statistically weighting these various regions according to the over-all flux distribution in the PDP, the corrections to the 23.7 and 30.8 cm pitches were determined to be -0.16 and $+0.02 \text{ m}^{-2}$ respectively.

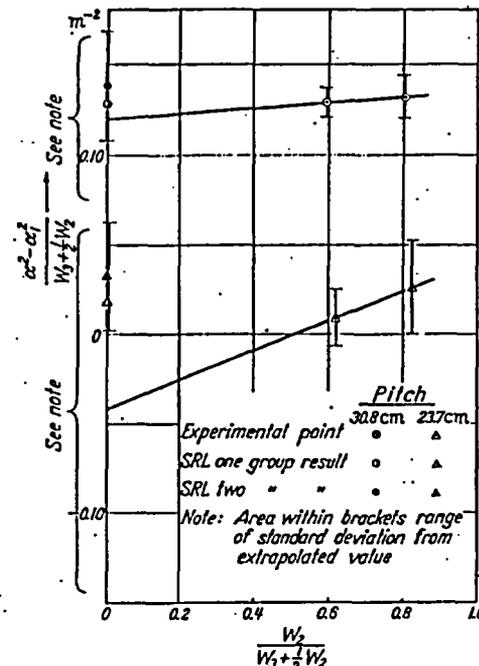


Fig. 8. Analysis of R3/Adam substitution measurements by perturbation method

These corrections were used in obtaining the experimental points that are compared with the calculated bucklings in Figure 6.

Intracell Activation Measurements

The thermal neutron flux profiles in the central cell of the 27 cm lattice in the PSE were measured by activations of manganese in "P-metal"¹ pins 1.27 cm long and 0.162 cm in diameter. A schematic of the pin loading is shown in Figure 9. Pins inside the cluster were supported by a circular plate of Zircaloy-2, 0.318 cm thick, a quarter of which is shown in Figure 9. Pin holders, or ladders, were cantilevered from the circular plate into the moderator and perpendicular to the fuel column. The ladders were Zircaloy rods, 0.475 cm in diameter, containing holes for the pins. Ladders 1 and 4 were pointed to the nearest adjacent fuel assemblies. A lesser number of pins covered with 0.076 cm of cadmium were irradiated in a similar fashion at a level 8 inches below the bare pins. Radial plots of the subcadmium and epicadmium activations

¹P-metal is a commercially available alloy containing 72% Mn, 18% Cu, and 10% Ni. Under the condition of the experiment essentially only Mn activations are detected in the alloy.

ma.
ica
+1

are shown for the cold and hot lattices in Figures 10 and 11. These plots do not contain corrections for the over-all radial flux shape in the PSE. Accordingly the comparison flux distributions obtained from P-3 calculations, to be discussed later, were multiplied by $J_0(B, r_i)$.

Actual locations of the pins within the clusters were in the coolant between fuel rods. The P-3 fluxes,

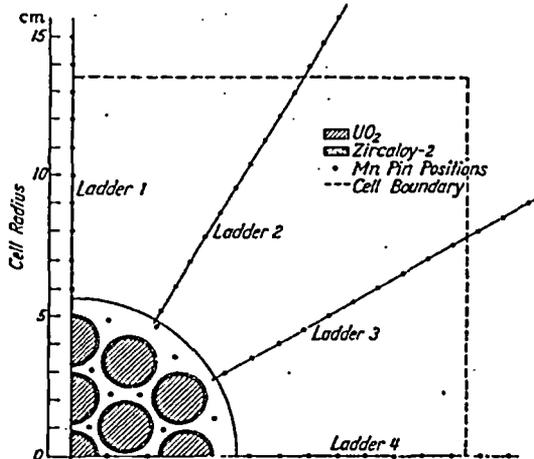


Fig. 9. Quarter of unit cell for R3/Adam lattice. Showing pin positions used in activation measurements

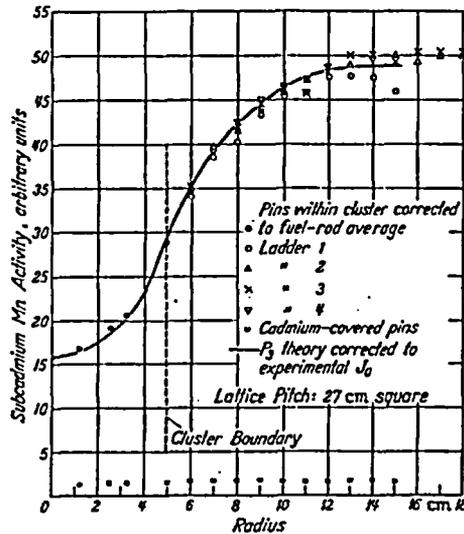


Fig. 10. Thermal flux profile for R3/Adam lattice at 20° C

however, correspond very nearly to average values within the uranium oxide of the individual rods. The experimental points were therefore corrected to average uranium oxide values. Diffusion theory was used to obtain the detailed flux within a small cell consisting of a single rod, its cladding, and a varying boundary radius equal to the distance of each pin from its associated fuel rod. These flux profiles were then used to reduce the pin activations at the small cell boundary to the average in the fuel. The same profiles were also used to obtain average fluxes in the Zircaloy-2 cladding from the pin activations.

For the purpose of calculating disadvantage factors for determining L^2 , the over-all $J_0(B, r)$ flux correction was applied to the experimental data. These data were then transferred to a two-dimensional plot and isoflux lines plotted over the cell. Numerical integration was used to obtain the average flux in the various regions. The results are shown in the tables of disadvantage factors, Tables 6 and 7. In these tables, the cell average diffusion coefficient, D_{eff} , and macroscopic absorption cross section, $(\Sigma_a)_{eff}$, are defined by

$$(\Sigma_a)_{eff} = \sum_i d_i \frac{V_i}{V_{cell}} \Sigma_{a_i} \quad \text{and} \quad 1/D_{eff} = \sum_i d_i \frac{V_i}{V_{cell}} \frac{1}{D_i}$$

where the terms d_i and V_i/V_{cell} are defined in the tables.

The P-3 calculation of L^2 is described in the next section. The value of L^2 obtained from the experimental flux profiles agrees precisely with the P-3 value for

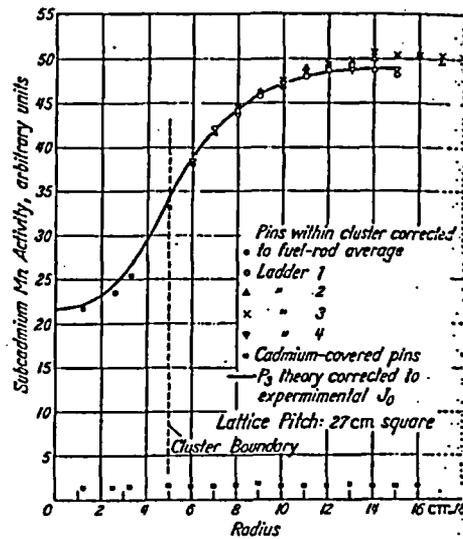


Fig. 11. Thermal flux profile for R3/Adam lattice at 217° C

the room temperature case. For the high temperature case, the experimental value of L^2 is about 3% higher than the P-3 value. This discrepancy is within the estimated combined errors of such variables as counting statistics, ladder placement, and the calculated corrections to average fuel fluxes. An inspection of Figures 10 and 11 also shows that the P-3 method is very good at predicting the flux shapes within the oxide rod clusters.

Calculation of Bucklings and Temperature Coefficients

The calculation of room temperature lattice bucklings was based on the recipe currently used at SRL [11]. Temperature coefficients of buckling were based on the same recipe with the following extensions and modifications.

Neutron Age, τ

The neutron age, τ , was increased at the higher temperatures by the square of the ratio of D₂O density at 20° C to that at the higher temperatures. The effect of light water was obtained from other measurements [12].

Table 6.

Material	D_{eff}	$\Sigma_{a,eff}$
D ₂ O (bulk moderator)	1.062	0.004
D ₂ O (fuel coolant)		
CO ₂		
Zircaloy-2		
Cell		

Table 7. C

Material	D_{eff}	$\Sigma_{a,eff}$
D ₂ O (bulk moderator)	0.828	0.004
D ₂ O (fuel coolant)		
CO ₂		
Zircaloy-2		
Cell		

Therm and Ther

L^2 and J were based on the one-to-transport theory of fuel rods was the difference in the cladding was obtained theory calculation of single rod within the

Standard values of sections were used for cross sections for manufacturer's correlation average value and $\Sigma_s = 0.3418$ cm⁻¹ was taken to have increasing temperatures for natural uranium were derived from

The moderator temperature P-3 procedure followed macroscopic scatterer's first moment, Σ_1^* as differential not, however, permit these two quantities parameters for the to choose average v combine to give the combined $D = 1/[3(\Sigma_s - \Sigma_a)]$ over Maxwellian en