From: Mahendra Shah
To: Bernard White; Daniel Huang; Robert Shewmaker; Ron Parkhill
Date: 10/23/02 12:09PM
Subject: Fwd: Cask exit velocity

Fowarded herewith please find Doug's simple calculations for an estimate of the cask velocty, based on the momentum transfer. Thanks.

Mahendra

CC: Earl Easton; Jack Guttmann

Portions of attachment - Ex 2 E/52

From: "Ammerman, Douglas J" <djammer@sandia.gov>
To: "'Shah, Mahendra'" <MJS3@nrc.gov>
Date: 10/23/02 10:50AM
Subject: Cask exit velocity

Mahendra,

The attached document has a discussion of the method I used to derive an exit velocity for a cask being impacted by an airplane.
The resulting exit velocity of agrees fairly closely to the results from the CTH analysis. I think that this method could also be used to estimate the velocity at partial crush distances. Ex 2

Let me know if you have any questions or comments.

Doug

<<cons_momentum .doc>>

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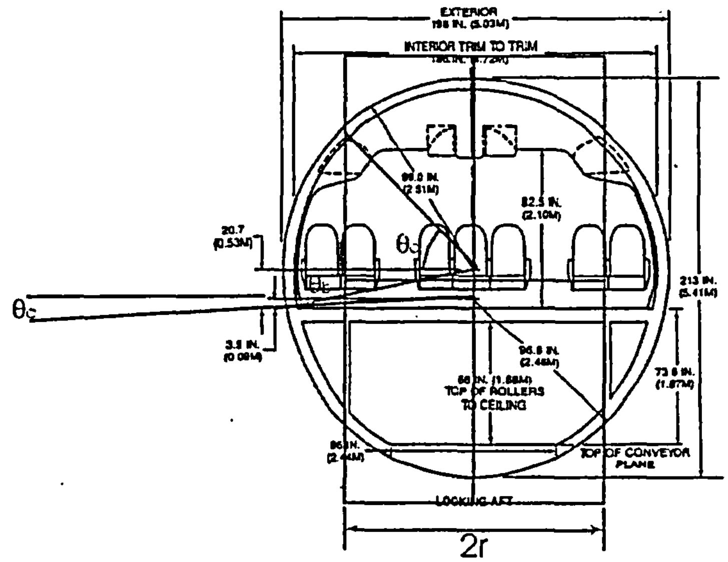
Portion Ex 2



The red rectangle in the above figure represents the outline of a Hi-Storm storage cask.

To calculate the amount of the fuselage that is impacting onto the cask the portion of the fuselage that is inside the red rectangle is calculated as follows:

$\theta_a = \cos^{-1}(r/96) = 45.8^\circ = 0.799$ *radians*
$\theta_b = \sin^{-1}(20.7/99) = 12.1^\circ = 0.211$ *radians*
$\theta_c = \sin^{-1}(3.5/96.8) = 2.07^\circ = 0.0362$ *radians*

Circumference of fuselage = $99.0 * [\pi + (2) * (0.211)] + 96.8 * [\pi - (2) * (0.0362)] = 649.8$ in
Length of floor = (2)*(90.8) = 181.6 in
Total length of fuselage structure = 649.8 + 181.6 = 831.4 in

Length of fuselage impacting cask = (2)*(99)*[π-(2)*(0.799)] = 305.5 in
Length of floor impacting cask = 2r = 133.875 in
Total length of fuselage structure impacting cask = 305.5 + 133.9 = 439.4 in

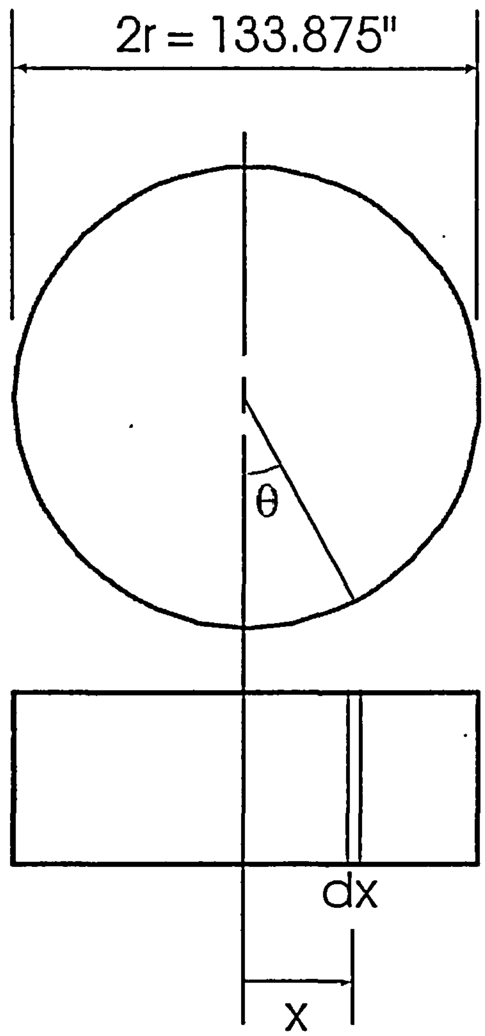
Ratio of fuselage mass that impacts cask = 439.4/831.4 = 0.529

Total mass of airplane impacting cask

Component	Portion Impacting	Total Mass (kg)
Center fuselage	0.529	
Front wheel	1	
Center fuel tank	1	
Total Impacting Mass		

Ex 2

Because the cask is not flat, not all of the impacting mass is totally effective at imparting its momentum to the cask (the mass near the edge of the cask strikes a glancing blow).



$\sin\theta = x/r$, $\cos\theta = \sqrt{1 - x^2/r^2}$

relative effectiveness of mass = $\cos\theta$

Effective mass ratio = $\displaystyle\int_0^r \frac{\cos\theta}{r}dx = \frac{1}{r}\int_0^r \sqrt{1 - x^2/r^2}\,dx$

$$= \frac{1}{r}\left[\frac{x}{2}\sqrt{1 - x^2/r^2} + \frac{r}{2}\sin^{-1}\left(\frac{x}{r}\right)\right]_0^r$$

$= \pi/4 = \mathbf{0.785}$

Therefore, the total effective mass impacting the cask is 0.785*( ) Ex 2

Portion Ex 2

For a perfectly plastic collision, the conservation of momentum equation gives us:

$m_a v_o = (m_a + m_c) v_f$

For the analysis in consideration,

m_a = the effective mass of the airplane striking the cask = ( ) Ex 2
m_c = the mass of the Hi-Storm = 150,000 kg
v_o = the initial velocity of the airplane = ( ) Ex 2
v_f = the exit velocity of the cask

Solving for v_f:

$$v_f = \frac{m_a}{m_a + m_c} v_o = \left(\frac{\quad}{\quad + 150000} * \quad \right)$$ Ex 2

Equation 1

Portions Ex 2