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OFFICE OF CIVILIAN RADIOACTIVE WASTE MANAGEMENT
CALCULATION COVER SHEET

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Waste Package Outer Barrier Stress Due to Thermal Expansion with Various Barrier Gap Sizes

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FIGURES

TABLES

1. PURPOSE

The objective of this activity is to determine the tangential stresses of the outer shell, due to uneven thermal expansion of the inner and outer shells of the current waste package (WP) designs. Based on the results of the calculation *Waste Package Barrier Stresses Due to Thermal Expansion*, CAL-EBS-ME-000008 (Ref. 10), only tangential stresses are considered for this calculation. The tangential stresses are significantly larger than the radial stresses associated with thermal expansion, and at the WP outer surface the radial stresses are equal to zero. The scope of this activity is limited to determining the tangential stresses the waste package outer shell is subject to due to the interference fit, produced by having two different shell coefficients of thermal expansions. The inner shell has a greater coefficient of thermal expansion than the outer shell, producing a pressure between the two shells. This calculation is associated with Waste Package Project.

The calculations are performed for the 21-PWR (pressurized water reactor), 44-BWR (boiling water reactor), 24-BWR, 12-PWR Long, 5 DHLW/DOE SNF - Short (defense high-level waste/Department of Energy spent nuclear fuel), 2-MCO/2-DHLW (multi-canister overpack), and Naval SNF Long WP designs. The information provided by the sketches attached to this calculation is that of the potential design for the types of WPs considered in this calculation.

This calculation is performed in accordance with the *Technical Work Plan for: Waste Package Design Description for SR* (Ref. 7). The calculation is documented, reviewed, and approved in accordance with AP-3.12Q, *Calculations* (Ref. 1).

2. METHOD

The method and the analytical approach for this calculation are performed through the use of basic equations of solid mechanics. With regard to the development of this calculation, the control of electronic management of data was evaluated in accordance with AP-SV.1Q, *Control of the Electronic Management of Information* (Ref. 3). The electronic management of data is controlled in accordance with Ref. 7, Section 10.

3. ASSUMPTIONS

In the course of developing this document, assumptions were made regarding the thermal expansion calculations. These are identified below.

3.1 One temperature range (20°C – 239°C) (Ref. 9, Table 6-7) is used throughout this calculation for all the waste packages in this study. Although this temperature range pertains to the 21-PWR, it is the largest range among all the waste packages. The rationale for this assumption is that the interference created from thermal expansion with this temperature range will be larger, compared to the interference created from the smaller temperature ranges associated with the other waste packages. This assumption provides bounding results in terms of tangential stresses in the outer shell due to thermal expansion. This assumption is used in Section 5.1.4.

3.2 The 21-PWR WP overall heat transfer rates are used throughout this calculation for all the waste packages in this study. Although these overall heat transfer rates pertain to the 21-PWR WP, they are the greatest among all the waste packages (Ref. 9, Table 6-7). The rationale for this assumption is that larger overall heat transfer rates produce a larger difference in temperature between the inner and outer shells. The inner shell results in having a higher temperature than the outer shell, causing the thermal expansion to be greater for the former. This leads to a greater interference between the shells, yielding higher tangential stresses in the outer shell. This assumption provides bounding results in terms of tangential stresses in the outer shell due to thermal expansion. This assumption is used in Section 5.1.5.

3.3 Room temperature elastic moduli are used for calculating the pressure due to the interference. The rationale for this assumption is that the pressure calculation yields greater pressures when the elastic moduli are larger. At the maximum temperature, the elastic moduli are less than those at room temperature, resulting in a smaller pressure. Therefore, using the larger elastic moduli will provide a higher pressure along with higher stresses in the outer shell. This assumption provides bounding results in terms of tangential stresses in the outer shell due to thermal expansion. This assumption is used in Sections 5.1.1 and 5.1.2.

3.4 The initial temperature of the waste packages is room temperature, at 20°C (68°F and 293 K). The rationale for this assumption is that the waste packages are manufactured at room temperature or warmer. Room temperature will provide a low initial temperature for the waste package shells, yielding a greater change in temperature than that at a higher temperature. This assumption provides bounding results in terms of tangential stresses in the outer shell due to thermal expansion. This assumption is used in Section 5.1.4.

3.5 When calculating the inner shell inner surface temperature, the inner shell outer surface and the outer shell inner surface have the same radius and are in complete contact with each other. The rationale for this assumption is that there is no stress associated with thermal expansion if the two surfaces are not in contact with each other. Calculating the temperature of the inner shell when the shells are not in contact is not of concern for this calculation. This assumption is used in Attachments II through VIII.

4. USE OF COMPUTER SOFTWARE AND MODELS

4.1 SOFTWARE

Section 8 contains computations using the standard functions of a commercial-off-the-shelf software program, Mathcad 2000 Professional. The results of the computation can be reproduced and checked by hand; therefore, the software use is considered exempt from the requirements of AP-SI.1Q, *Software Management* (Ref. 2) (see Ref. 1, Attachment 2, Section 4.A). These computations are performed using Mathcad 2000 Professional on a personal computer. The filenames for each computation are documented in the header for the attachments. Formulas, algorithms, listings of inputs and outputs, and numerical solution techniques are described in comments in the calculation itself, as applicable. The form of the computation files is such that the routines lend themselves to easy verification by visual inspection.

4.2 MODELS

None used.

5. CALCULATION

5.1 CALCULATION DATA

The material properties of the inner and outer shell are given below.

5.1.1 Inner Shell Properties

- 316 SS NG (Stainless Steel Nuclear Grade), SA-240, inner shell material (Attachment I) 316 NG SS, which is 316 SS [SA-240 S31600] with tightened control on carbon and nitrogen content and has the same material properties as 316 SS (Ref. 5, page 931 and Ref. 6, Section II, SA-240 Table 1)
- Modulus of elasticity, $E_i = 195.1\, GPa$ at $20°C$ ($28.3 \cdot 10^6\ psi$) (Ref. 6, Table TM-1, Material Group G) (Assumption 3.3)
- Poisson's ratio, $\nu_i = 0.298$ at 20^oC (Ref. 4, page 755, Fig. 15)
- Mean coefficient of thermal expansion, $\alpha_{ss} = 17 \cdot 10^{-6}\ m/m \cdot K$ at 260^oC ($9.7 \cdot 10^{-6}\ in/in \cdot °F$) (Ref. 6, Table TE-1, 16CR-12Ni-2Mo at 500^oF, Coefficient B)
- Thermal conductivity, $K_i = 17.3\ W/m \cdot K$ at 232^oC ($10.0\ BTU/hr \cdot ft \cdot °F$) (Ref. 6, Table TCD, 16CR-12Ni-2Mo at 450^oF).

5.1.2 Outer Shell Properties

- Alloy 22, SB-575 N06022, outer shell material (Attachment I)
- Modulus of elasticity, $E_o = 206\ GPa$ at $20°C$ ($29.9 \cdot 10^6\ psi$) (Ref. 15, page 14, Average Dynamic Modulus of Elasticity) (Assumption 3.3)
- Poisson's ratio, $\nu_o = 0.278$ at 21^oC (Ref. 4, page 143, Mechanical Properties)
- Mean coefficient of thermal expansion, $\alpha_{alloy22} = 12.6 \cdot 10^{-6}\ m/m \cdot K$ from 24^o to 316^oC ($7.0 \cdot 10^{-6}\ in/in \cdot °F$) (Ref. 15, page 13, Average Physical Properties, Mean Coefficient of Thermal Expansion)
- Thermal conductivity, $K_o = 13.4\ W/m \cdot K$ at 200^oC ($7.75\ BTU/hr \cdot ft \cdot °F$) (Ref. 15, page 13, Average Physical Properties, Thermal Conductivity)

- Yield strength $\sigma_y = 222\,MPa$ at $260°C\,(32.2 \cdot 10^3\ psi)$ (Ref. 6, Table Y-1, 55Ni-21Cr-13.5Mo at $500^{o}F$).

5.1.3 Shell Dimensions

The dimensions of the inner and outer shells for various waste packages (Attachment I) are given in Table 1.

Table 1. Dimensions of the Inner and Outer Shell for Various Waste Packages

Waste Package Type	Inner Shell Inner Radius		Inner Shell Thickness		Outer Shell Thickness	
	(m)	(in.)	(m)	(in.)	(m)	(in.)
21-PWR	0.712	28.0	0.050	1.97	0.020	0.79
44-BWR	0.727	28.6	0.050	1.97	0.020	0.79
24-BWR	0.549	21.6	0.050	1.97	0.020	0.79
12-PWR Long	0.555	21.9	0.050	1.97	0.020	0.79
5 DHLW/DOE SNF - Short	0.940	37.0	0.050	1.97	0.025	0.98
2-MCO/2-DHLW	0.792	31.2	0.050	1.97	0.025	0.98
Naval SNF Long	0.8595	33.8	0.050	1.97	0.025	0.98

Table 2 provides the inner cavity length of the inner shell for various waste packages (Attachment I).

Table 2. Inner Cavity Length of the Inner Shell for Various Waste Packages

Waste Package Type	Inner Cavity Length	
	(m)	(in.)
21-PWR	4.585	180.5
44-BWR	4.585	180.5
24-BWR	4.585	180.5
12-PWR Long	5.121	201.6
5 DHLW/DOE SNF - Short	3.590	141.3
2-MCO/2-DHLW	4.617	181.8
Naval SNF Long	5.415	213.2

5.1.4 Temperature Range

The upper boundary of the temperature range for the 21-PWR WP is 239°C (462 degrees F and 512 K) at the outer shell outer surface, occurring 35 years after emplacement (Ref. 9, Table 6-7). This waste package outer surface upper boundary temperature is the maximum among all the waste packages and will be used for all the thermal expansion calculations (Assumption

3.1). The lower boundary temperature is room temperature at 20°C (68°F and 293 K) (Assumption 3.4) representing the shells before the spent nuclear fuel is inserted.

5.1.5 Overall Heat Transfer Rate

The overall heat transfer rates for the 21-PWR WP are presented in Table 3 along with the corresponding outer shell outer surface temperatures (Ref. 9). These values are used throughout this calculation for all the waste packages in this study (Assumption 3.2).

Table 3. Overall Heat Transfer Rates

Heat, q_r (W)	Outer Shell Outer Surface Temperature, T_{os} (K)	(°C)
0.0	293	20
11799.9	330	57
11762.5	357	84
10846.7	381	108
7192.8	411	138
7191.7	426	153
7182.4	443	170
7102.3	468	195
6856.1	493	220
6540.6	502	229
6158.3	512	239

5.2 TECHNICAL APPROACH

Seven different potential WP designs are evaluated in this document: 21-PWR, 44-BWR, 24-BWR, 12-PWR Long, 5 DHLW/DOE SNF - Short, 2-MCO/2-DHLW, and Naval SNF Long. For each one of these potential WP designs, a parametric study is performed by calculating the interference produced by the thermal expansion of the inner and outer shells. The interference between the two shells causes a pressure at the interface of the two shell surfaces. This pressure is used to calculate the outer shell tangential stresses at the inner and outer surfaces.

5.3 THERMAL EXPANSION CALCULATIONS

Thermal expansion occurs with a change in temperature and is represented by the following equation: $\delta = \alpha R \Delta T$; where δ is the change in radial length; α is the coefficient of thermal expansion; R is the radial length; and, ΔT is the change in temperature (see Attachments II through VIII). Attachment IX verifies this equation for thermal expansion. This calculation is a parametric study that determines the resulting interference between the waste package shells due to thermal expansion based on various gap sizes at room temperature. The calculations for the tangential stresses of the outer shell at the outer and inner surfaces are presented in Attachments II through VIII. The results are presented in Section 6.

6. RESULTS

This document may be affected by technical product input information that requires confirmation. Any changes to the document that may occur as a result of completing the confirmation activities will be reflected in subsequent revisions. The status of the technical product input information quality may be confirmed by review of the DIRS database.

6.1 MAXIMUM OUTER SHELL TANGENTIAL STRESS

At the maximum temperature, the waste package shells are subject to the greatest pressure created by the interference fit caused by uneven thermal expansion of the waste package inner and outer shells. This interference fit subjects the shells to a tangential stress. The locations of these outer shell stresses are depicted in Figure 1.

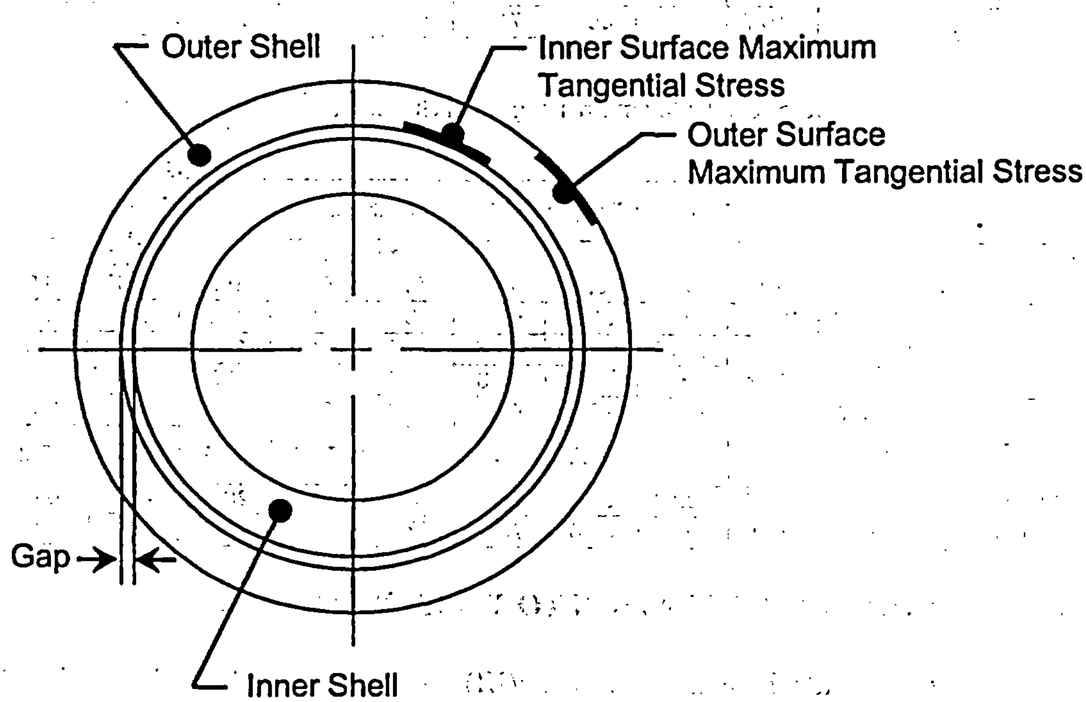

Figure 1. The Locations of the Outer Shell Inner Surface and Outer Surface Maximum Tangential Stresses

The outer shell maximum tangential stresses at the outer and inner surfaces for a corresponding gap size (Attachments II through VIII) are shown in Table 4 and Table 5.

Table 4. Outer Shell Maximum Tangential Stress at the Outer Surface

	Maximum Tangential Stress at the Outer Surface, σ_{os} (MPa)										
Waste Package Type	Gap Size (mm) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
21-PWR	140.9	122.1	103.2	84.4	65.6	46.8	27.9	9.1	0.0	0.0	0.0
44-BWR	140.9	122.4	103.9	85.5	67.0	48.5	30.1	11.6	0.0	0.0	0.0
24-BWR	141.3	117.4	93.5	69.6	45.8	21.9	0.0	0.0	0.0	0.0	0.0
12-PWR Long	140.8	117.2	93.6	69.9	46.3	22.7	0.0	0.0	0.0	0.0	0.0
5 DHLW/DOE SNF - Short	131.4	117.9	104.4	90.9	77.4	63.9	50.4	36.9	23.4	9.9	0.0
2-MCO/2-DHLW	130.9	115.0	99.2	83.4	67.5	51.7	35.8	20.0	4.2	0.0	0.0
Naval SNF Long	130.4	115.7	101.1	86.4	71.7	57.0	42.4	27.7	13.0	0.0	0.0

Table 5. Outer Shell Maximum Tangential Stress at the Inner Surface

	Maximum Tangential Stress at the Outer Surface, σ_{is} (MPa)										
Waste Package Type	Gap Size (mm) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
21-PWR	144.6	125.3	106.0	86.6	67.3	48.0	28.7	9.4	0.0	0.0	0.0
44-BWR	144.5	125.6	106.6	87.7	68.7	49.8	30.8	11.9	0.0	0.0	0.0
24-BWR	146.1	121.4	96.7	72.0	47.3	22.7	0.0	0.0	0.0	0.0	0.0
12-PWR Long	145.6	121.1	96.7	72.3	47.8	23.4	0.0	0.0	0.0	0.0	0.0
5 DHLW/DOE SNF - Short	134.8	120.9	107.1	93.2	79.4	65.5	51.7	37.9	24.0	10.2	0.0
2-MCO/2-DHLW	134.8	118.5	102.2	85.9	69.5	53.2	36.9	20.6	4.3	0.0	0.0
Naval SNF Long	134.1	119.0	103.9	88.8	73.7	58.6	43.5	28.5	13.4	0.0	0.0

6.2 TANGENTIAL STRESS RELATION TO TEMPERATURE

The calculation results (Attachments II through VIII) are reported in the following sections for each WP. The waste package outer shell tangential stresses at the inner and outer surfaces due to thermal expansion are reported using plots, illustrating the tangential stress (MPa) with respect to temperature (°C). The plots depict the stress/temperature curves for a range of shell gap sizes.

6.2.1 21-PWR WP

Figure 2 illustrates the 21-PWR WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa (see Section 5.1.2), respectively, for informational purposes.

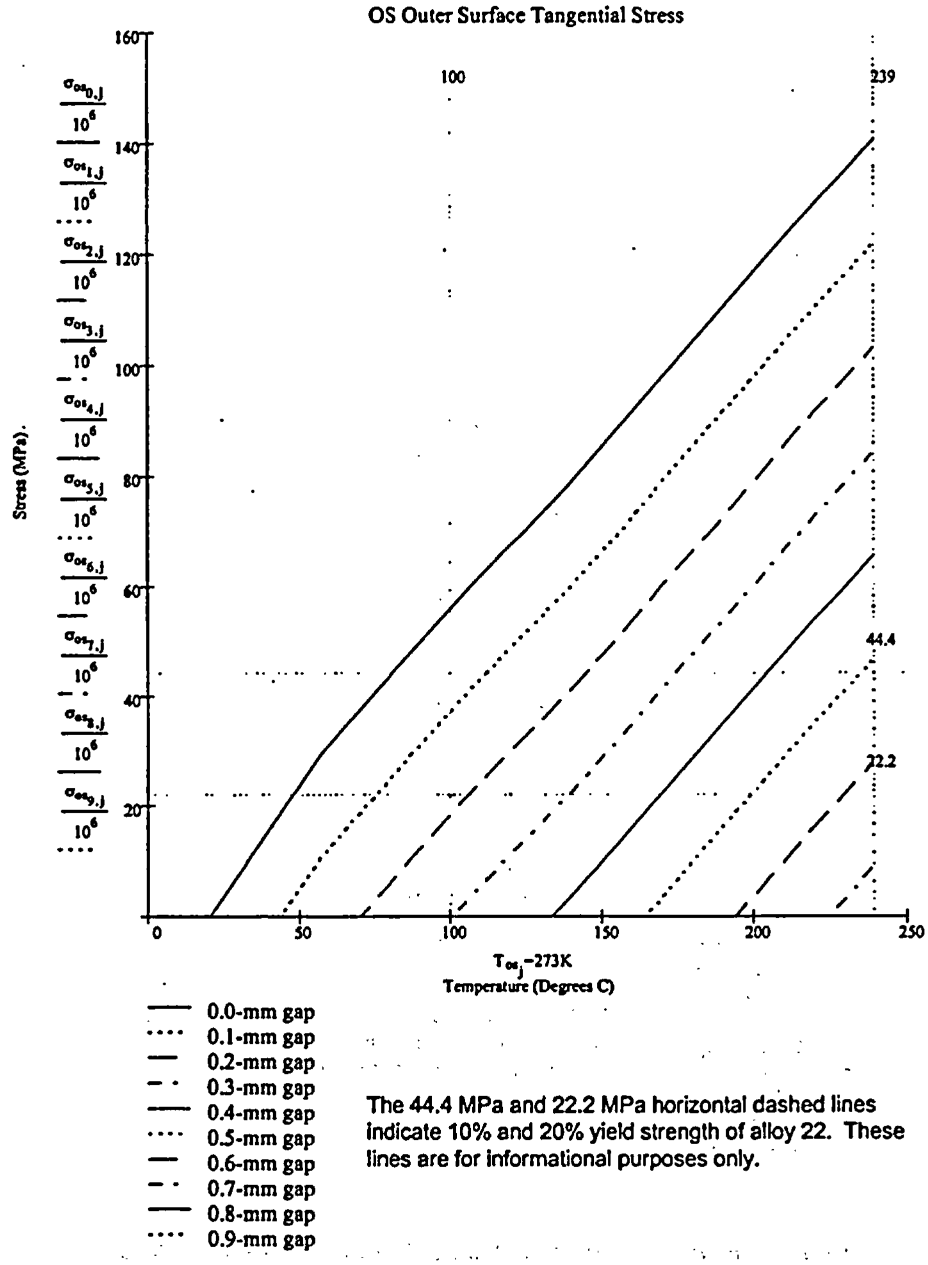

Figure 2. 21-PWR WP Outer Shell Outer Surface Tangential Stress

Figure 3 illustrates the 21-PWR WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

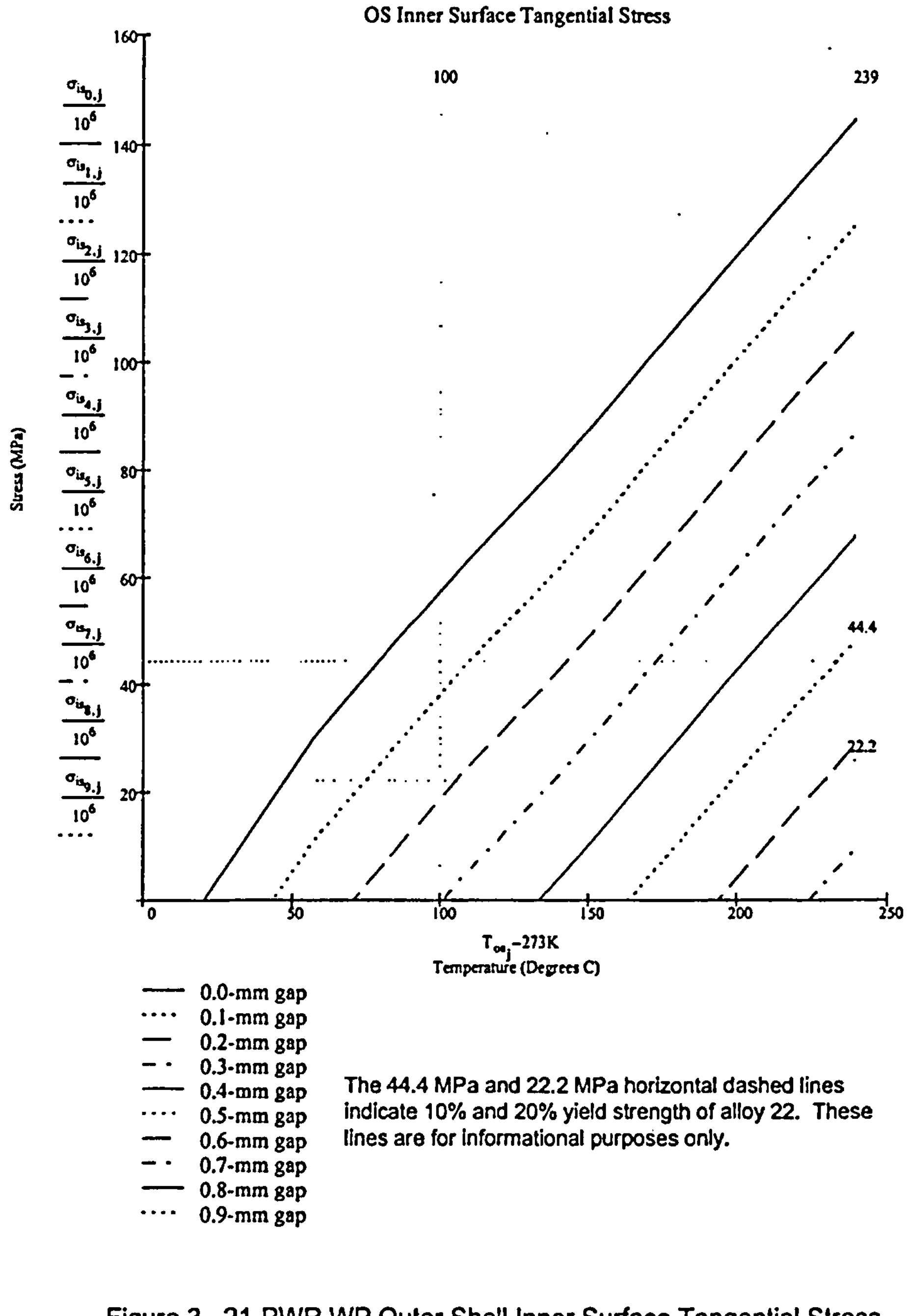

Figure 3. 21-PWR WP Outer Shell Inner Surface Tangential Stress

6.2.2 44-BWR WP

Figure 4 illustrates the 44-BWR WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

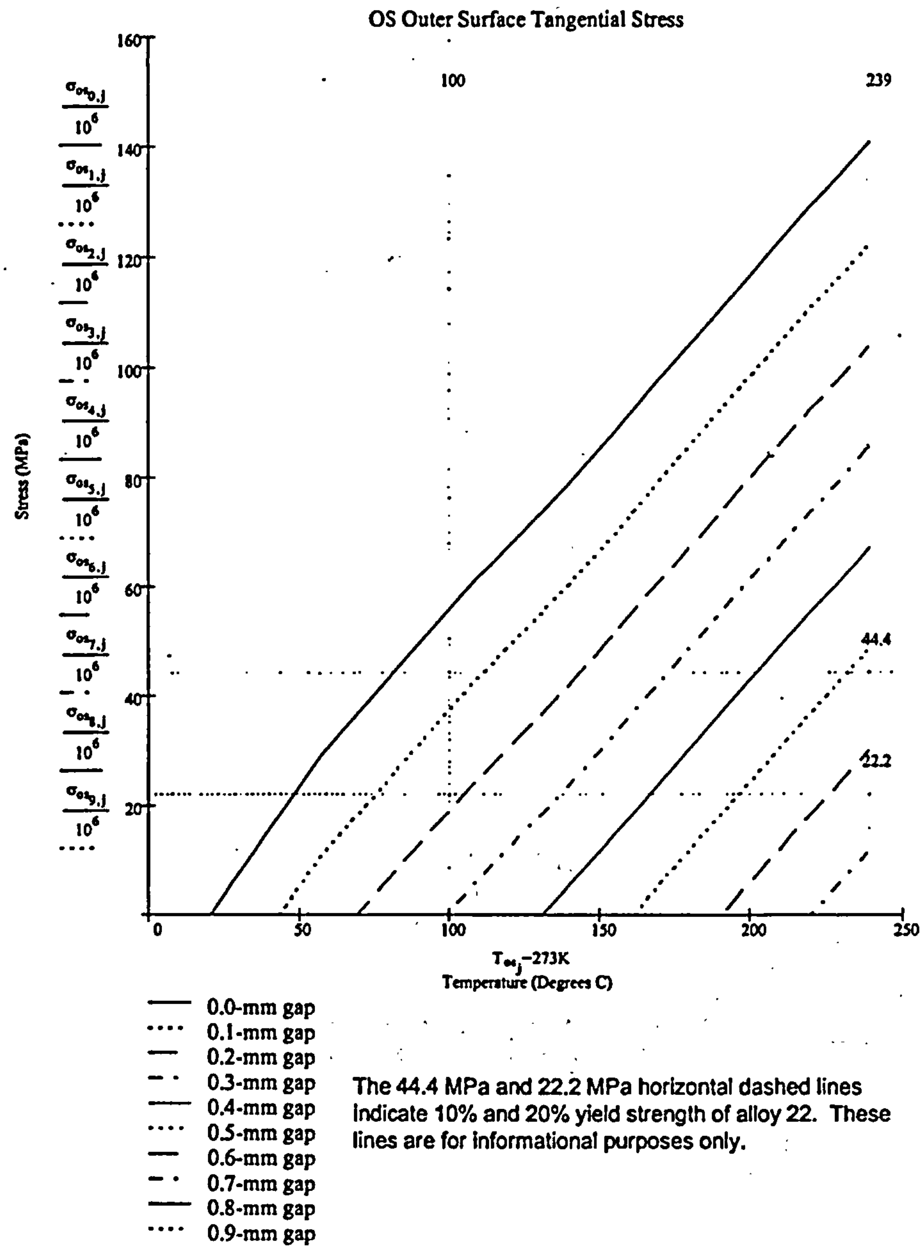

Figure 4. 44-BWR WP Outer Shell Outer Surface Tangential Stress

Figure 5 illustrates the 44-BWR WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

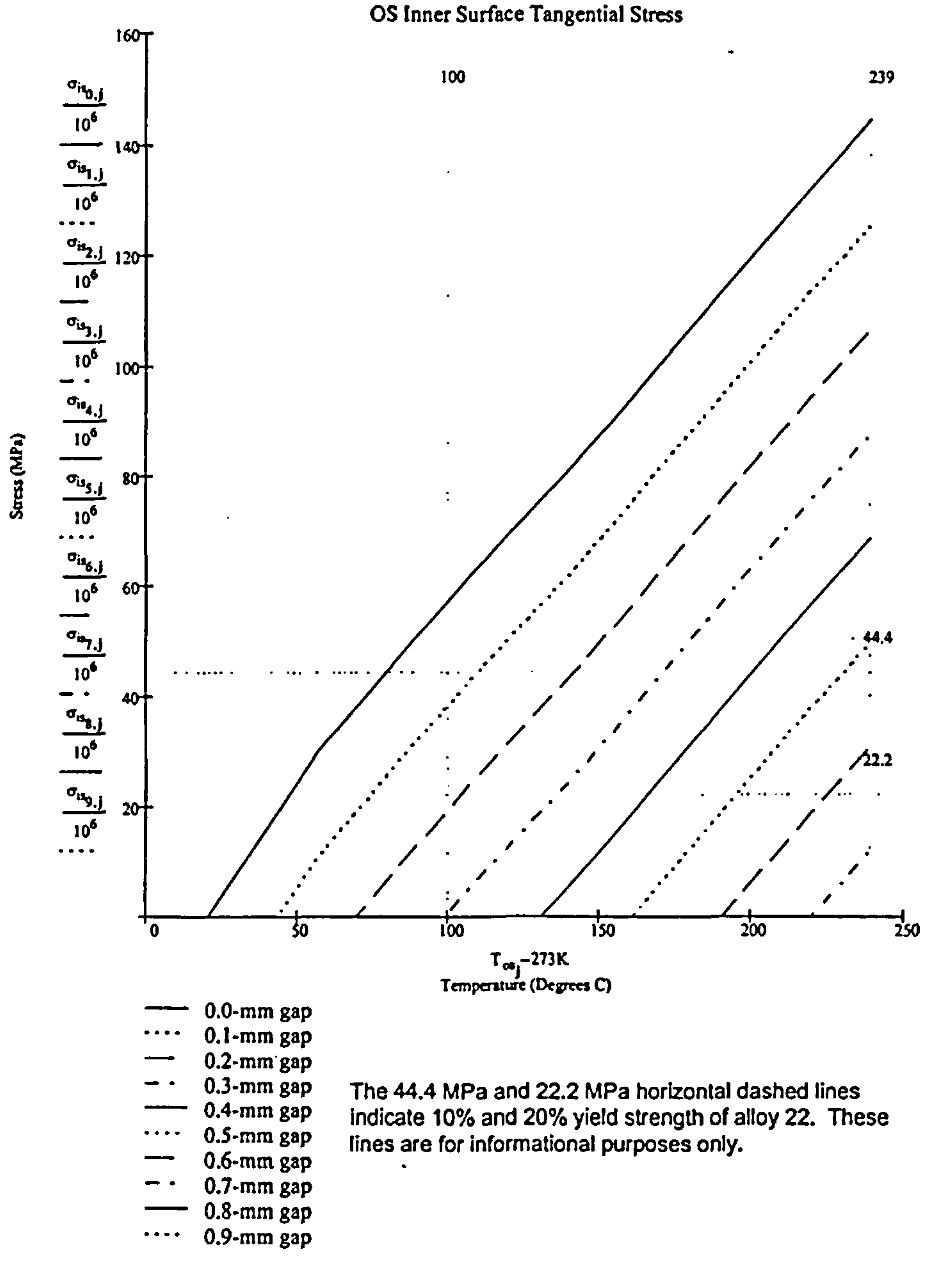

Figure 5. 44-BWR WP Outer Shell Inner Surface Tangential Stress

6.2.3 24-BWR WP

Figure 6 illustrates the 24-BWR WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

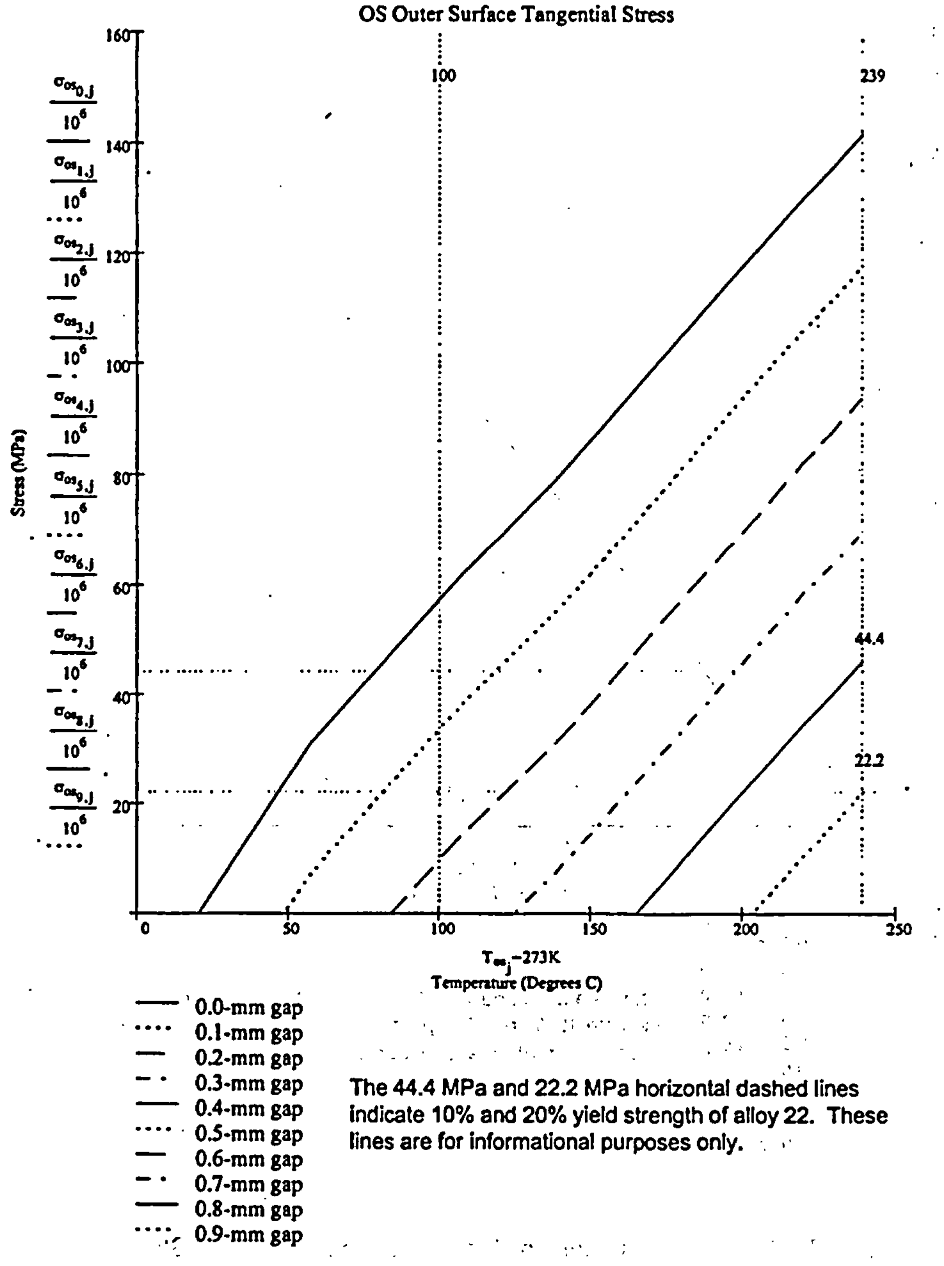

Figure 6. 24-BWR WP Outer Shell Outer Surface Tangential Stress

Figure 7 illustrates the 24-BWR WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

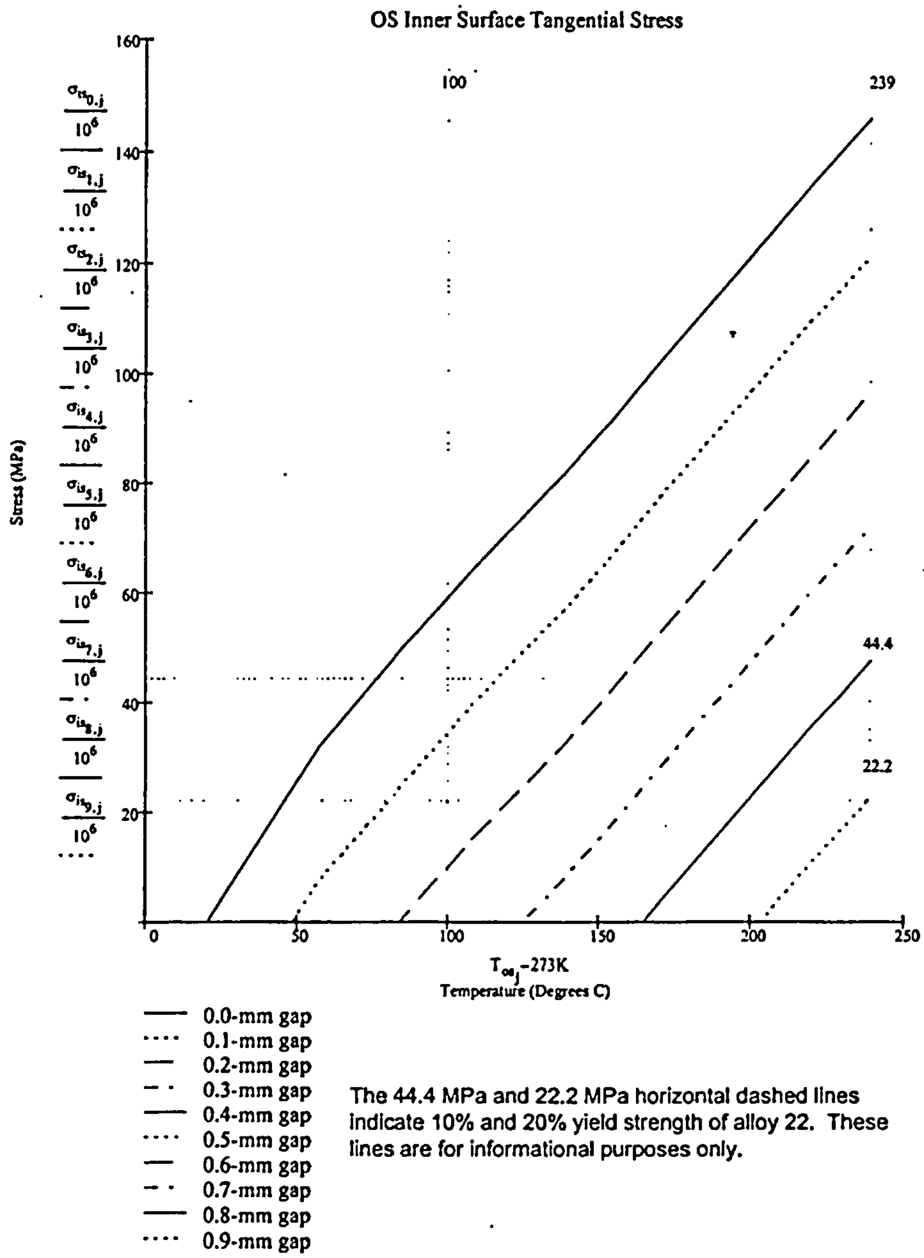

Figure 7. 24-BWR WP Outer Shell Inner Surface Tangential Stress

6.2.4 12-PWR LONG WP

Figure 8 illustrates the 12-PWR Long WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

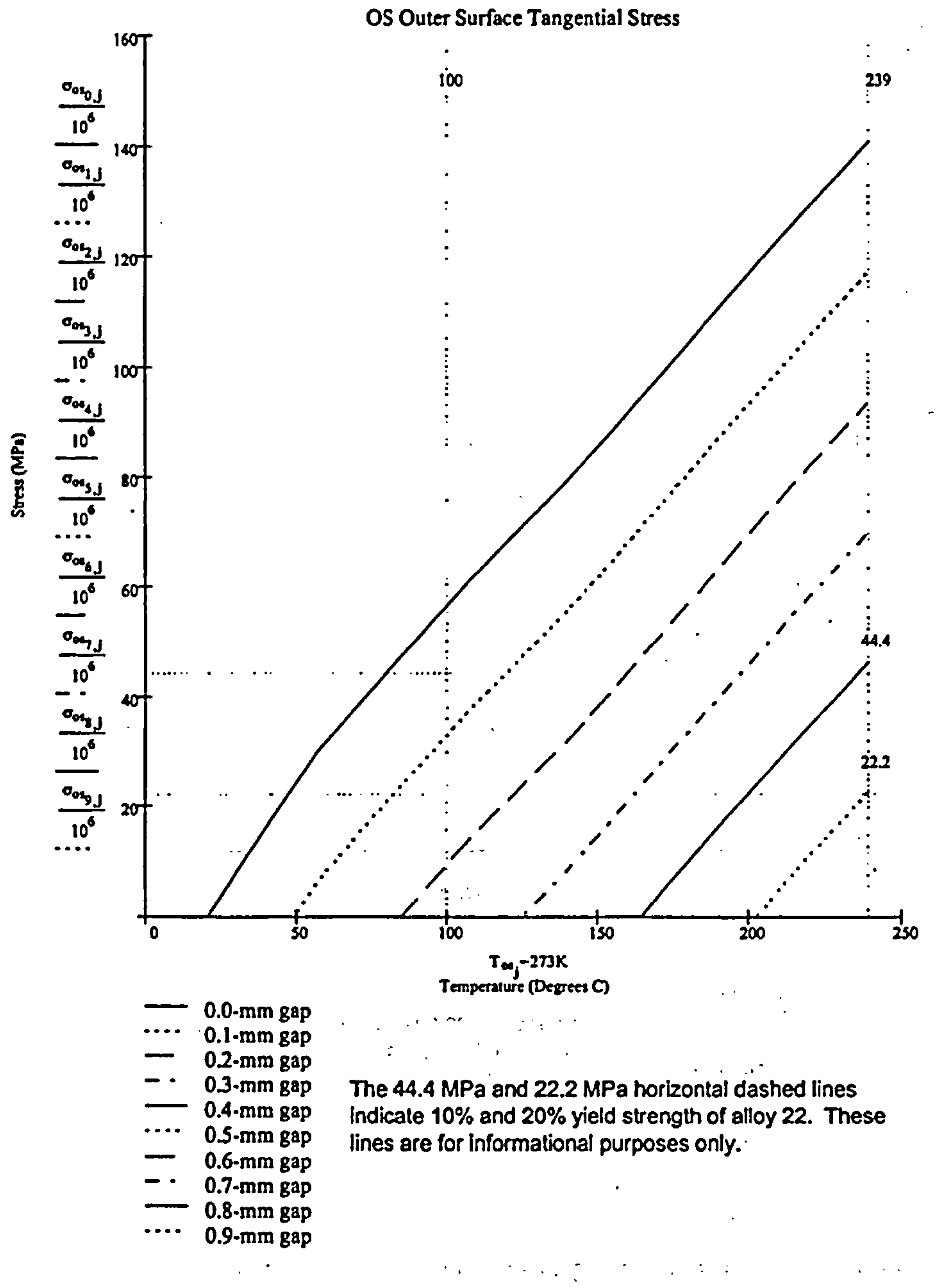

Figure 8. 12-PWR Long WP Outer Shell Outer Surface Tangential Stress

Figure 9 illustrates the 12-PWR Long WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

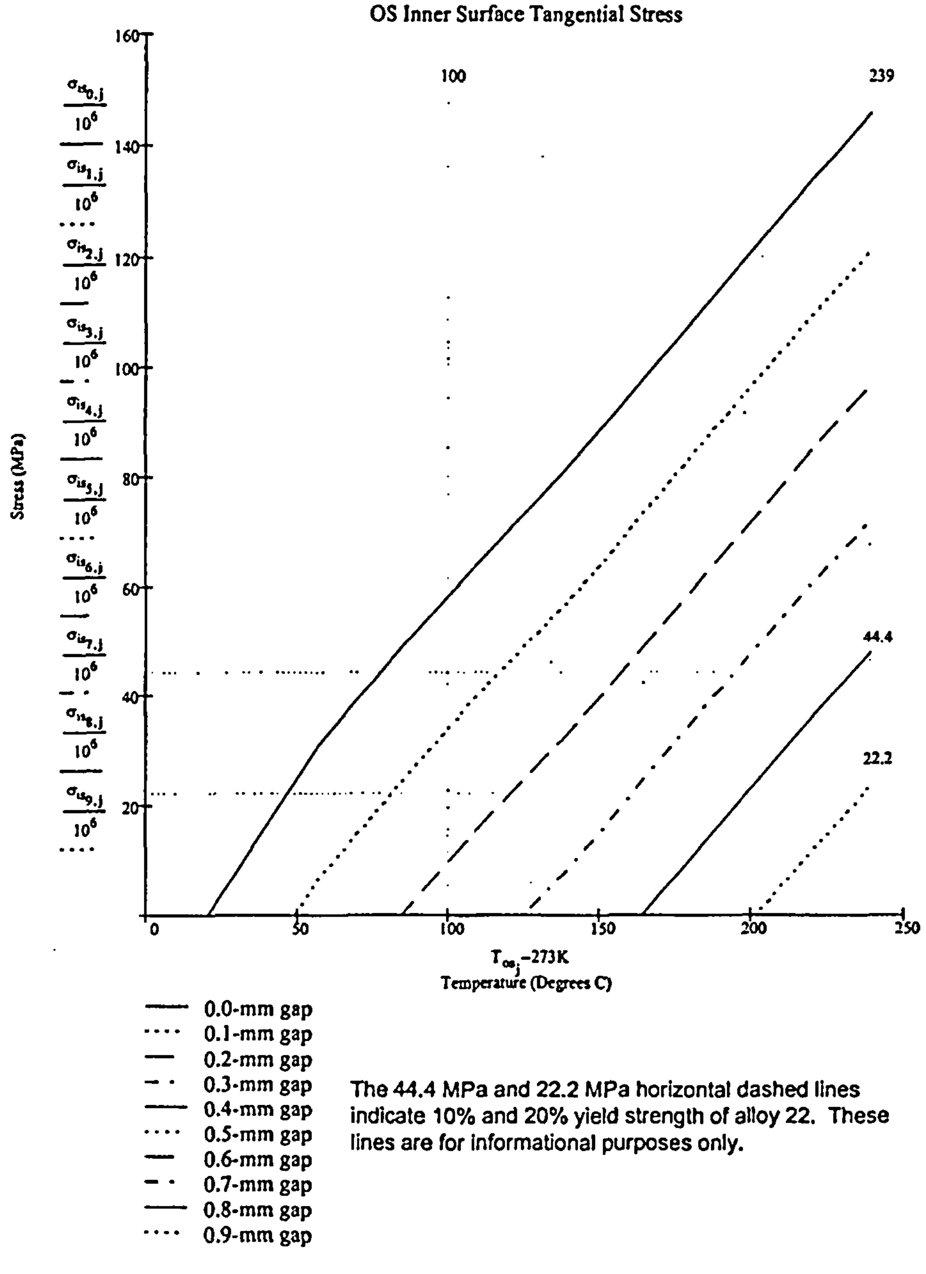

Figure 9. 12-PWR Long WP Outer Shell Inner Surface Tangential Stress

6.2.5 5 DHLW/DOE SNF - Short WP

Figure 10 illustrates the 5 DHLW/DOE SNF - Short WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

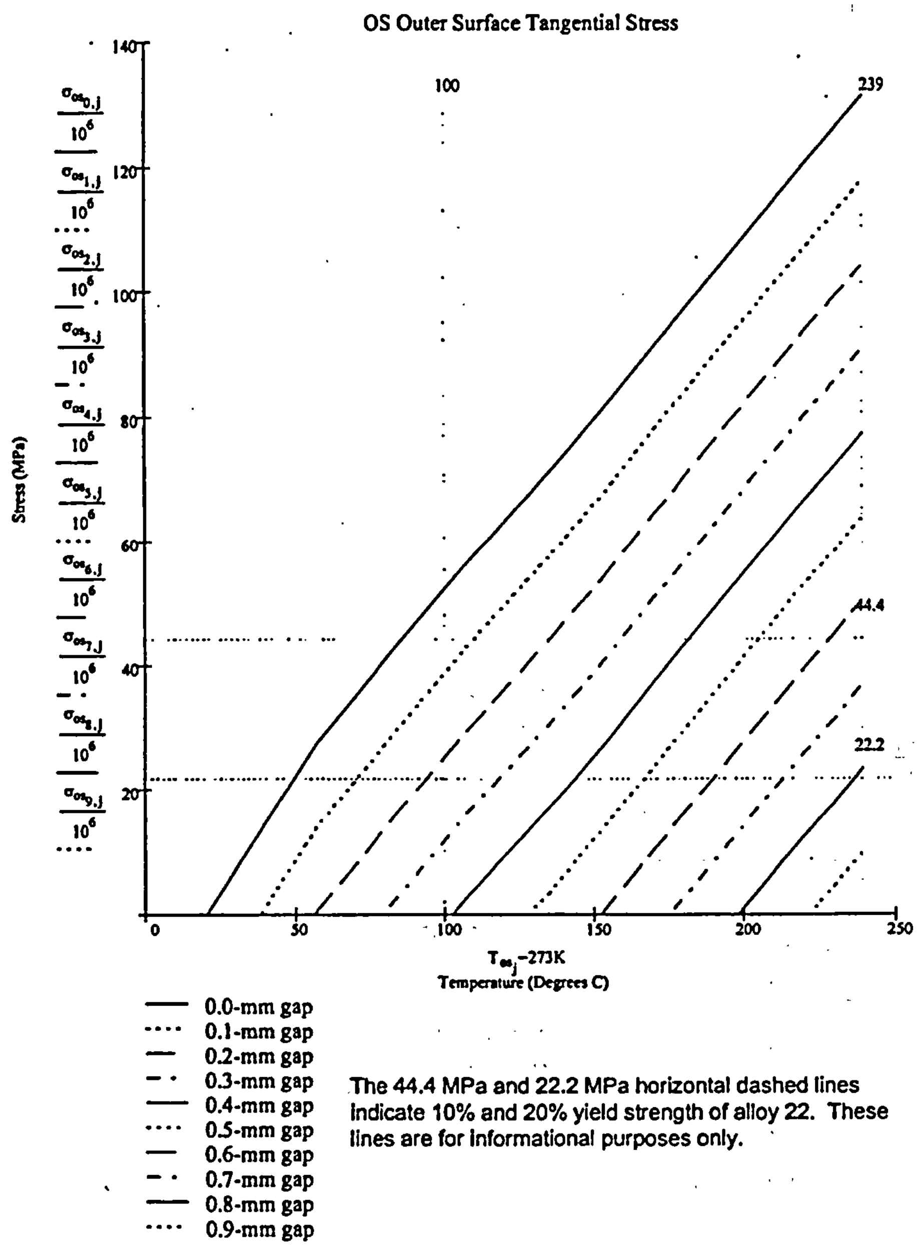

Figure 10. 5 DHLW/DOE SNF - Short WP Outer Shell Outer Surface Tangential Stress

Figure 11 illustrates the 5 DHLW/DOE SNF - Short WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

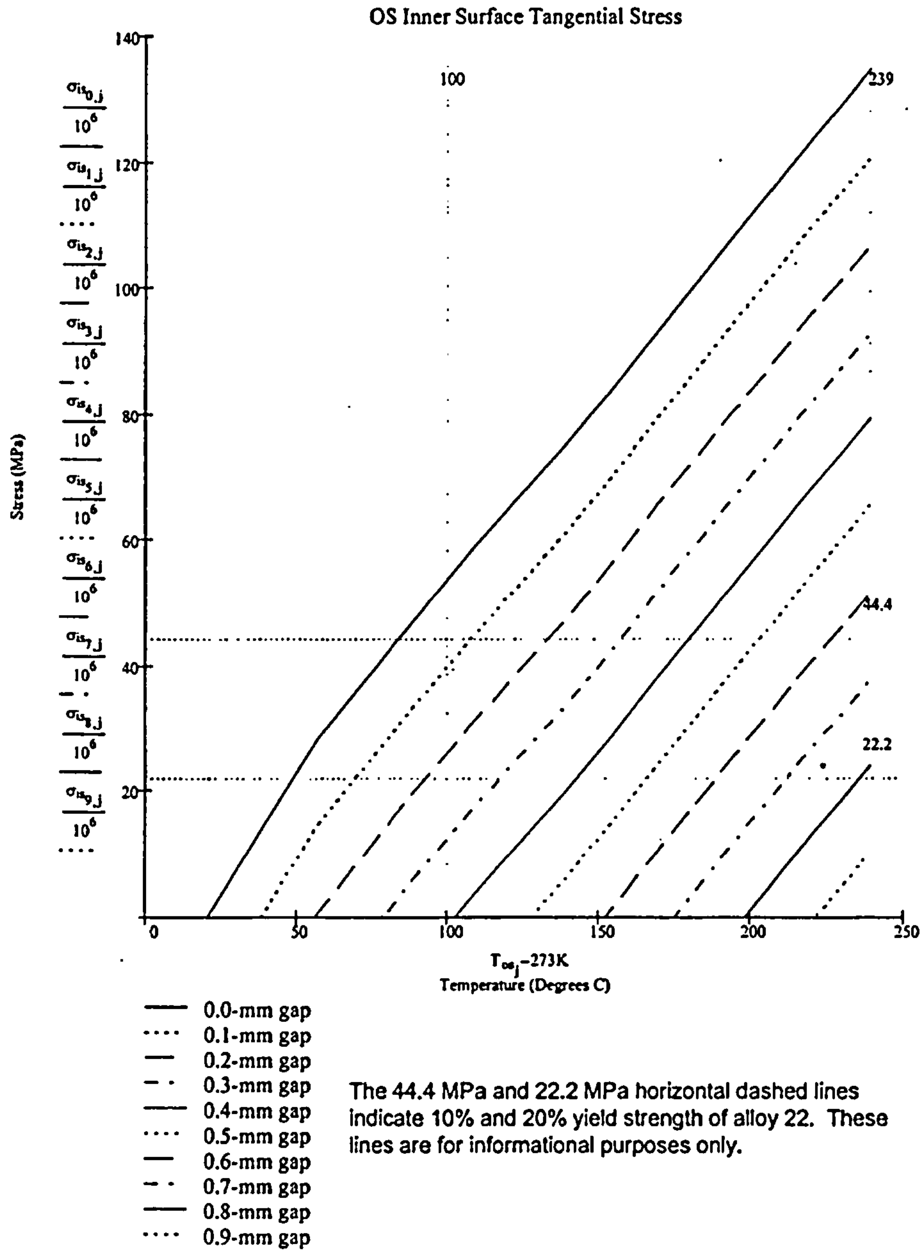

Figure 11. 5 DHLW/DOE SNF - Short WP Outer Shell Inner Surface Tangential Stress

6.2.6 2-MCO/2-DHLW WP

Figure 12 illustrates the 2-MCO/2-DHLW WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

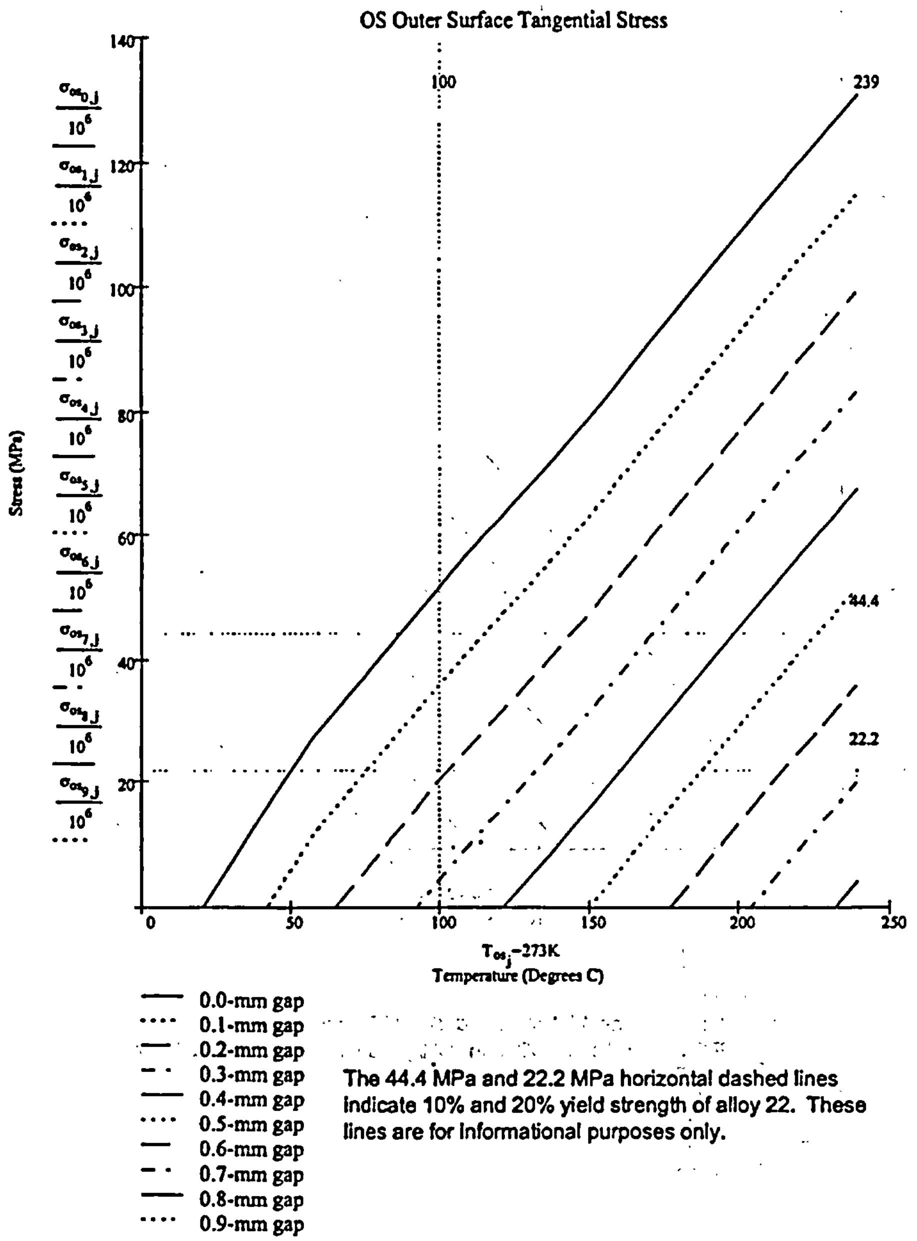

Figure 12. 2-MCO/2-DHLW WP Outer Shell Outer Surface Tangential Stress

Figure 13 illustrates the 2-MCO/2-DHLW WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

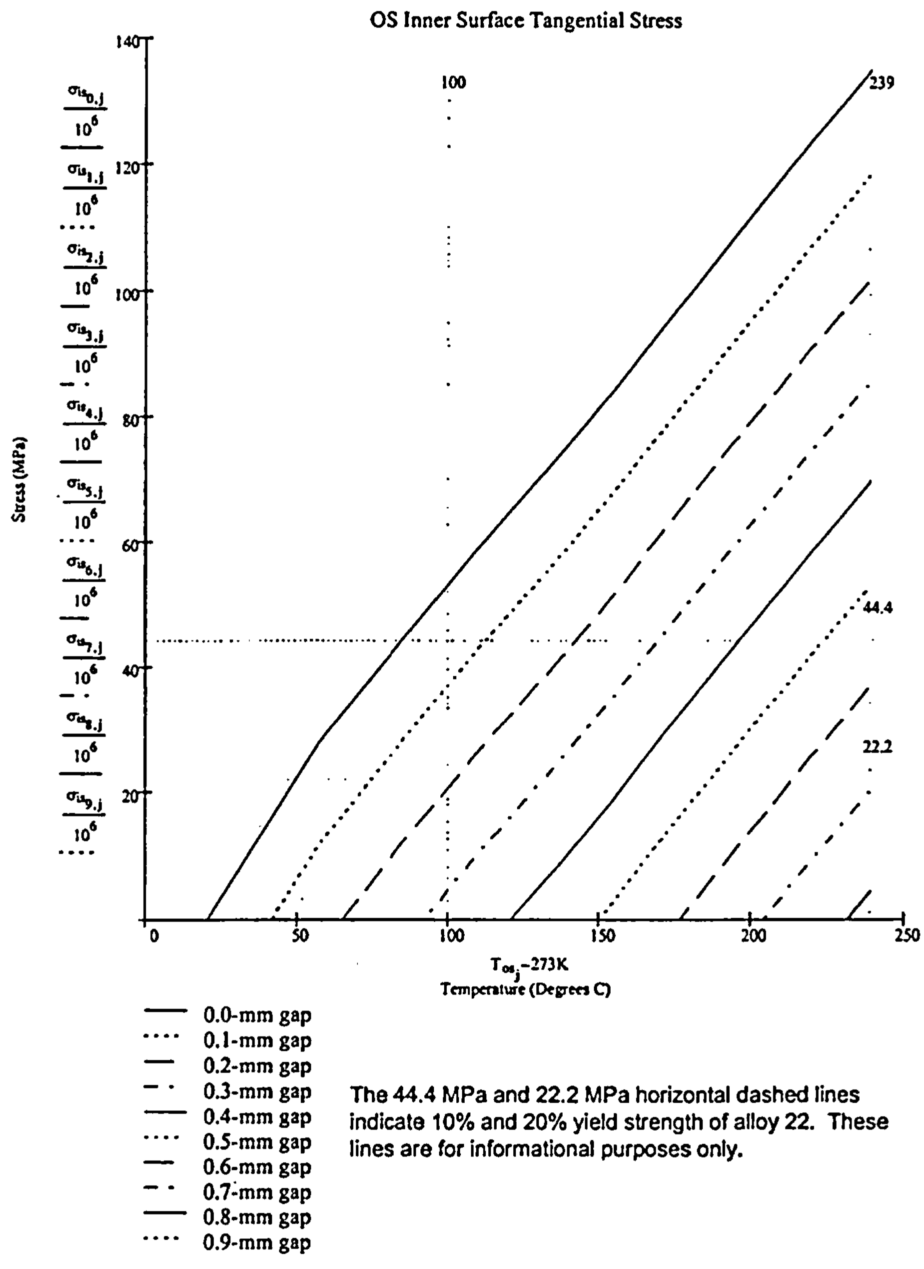

Figure 13. 2-MCO/2-DHLW WP Outer Shell Inner Surface Tangential Stress

6.2.7 NAVAL SNF-Long WP

Figure 14 illustrates the Naval SNF-Long WP outer shell outer surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

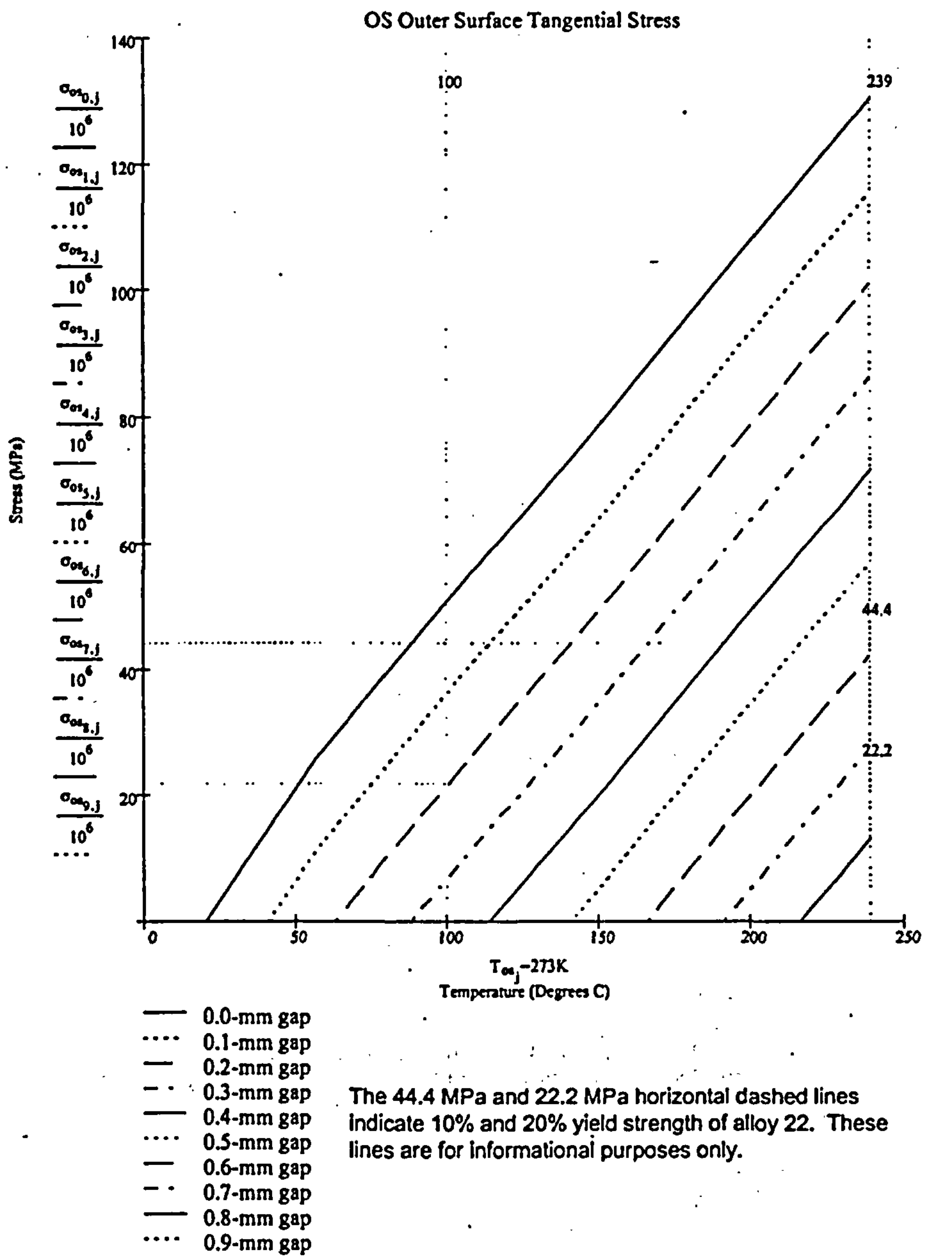

Figure 14. Naval SNF-Long WP Outer Shell Outer Surface Tangential Stress

Figure 15 illustrates the Naval SNF-Long WP outer shell inner surface tangential stress (MPa) with respect to temperature (°C) for a range of gap sizes. The Alloy 22 yield strength 10% and 20% levels are indicated on the plots at 22.2 and 44.4 MPa, respectively, for informational purposes.

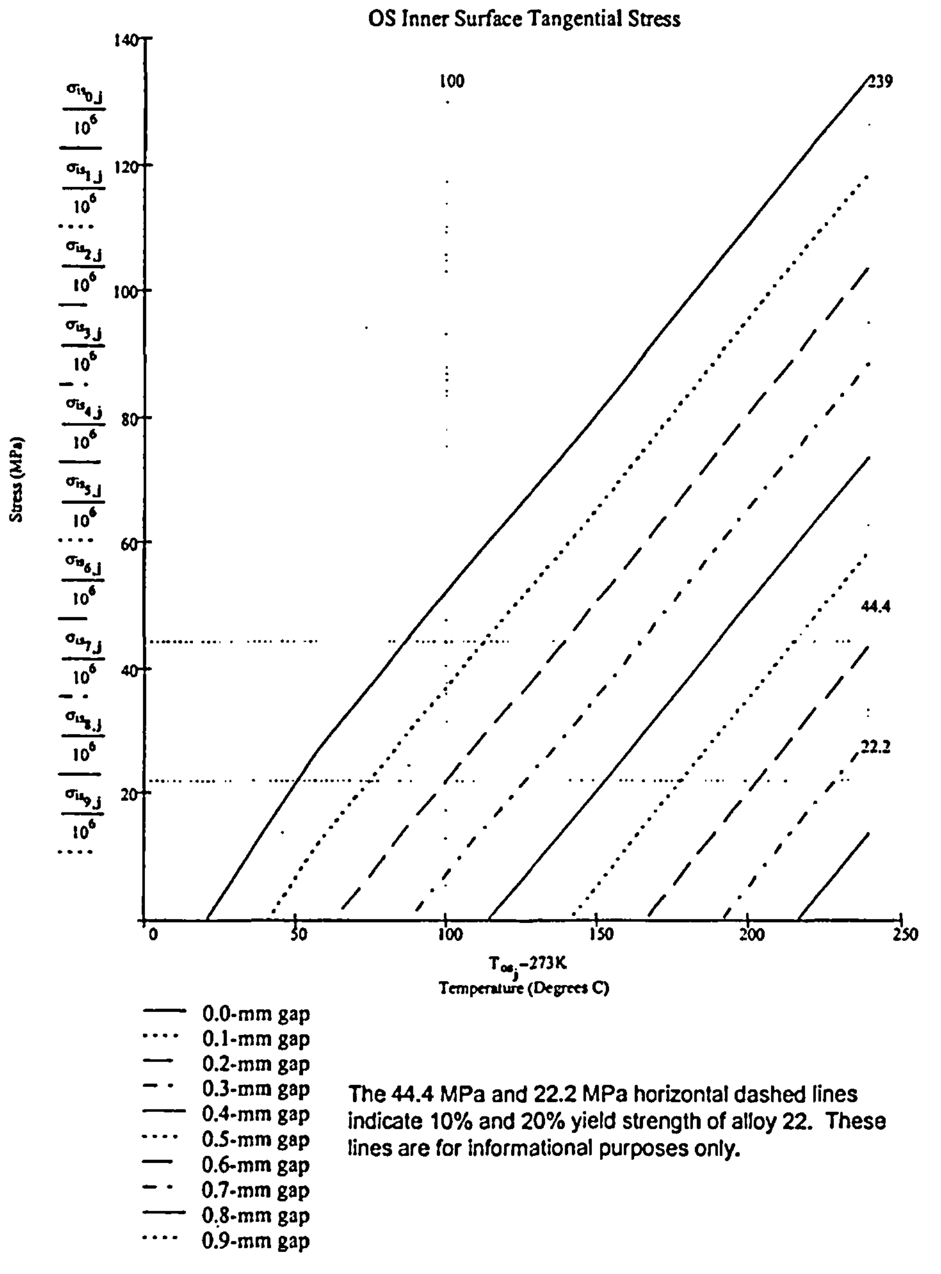

Figure 15. Naval SNF-Long WP Outer Shell Inner Surface Tangential Stress

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8. ATTACHMENTS

Attachment I (22 pages): Design sketches. Table 4 lists the potential design sketches used in this calculation.

Table 4. Potential Design Sketches Used

Design Sketch Title	Sketch Number	Rev.	Pages
21-PWR Waste Package Configurations for Site Recommendation	SK-0175	02	I-1 to I-2
21-PWR Waste Package Weld Configuration	SK-0191	00	I-3
44-BWR Waste Package Configuration for Site Recommendation	SK-0192	00	I-4 to I-5
44-BWR Waste Package Assembly Weld Configuration	SK-0193	00	I-6
24-BWR Waste Package Configuration for Site Recommendation	SK-0184	00	I-7 to I-8
24-BWR Waste Package Assembly Weld Configuration	SK-0202	00	I-9
12-PWR Long Waste Package Configuration for Site Recommendation	SK-0183	01	I-10 to I-11
12-PWR Long Waste Package Weld Configuration	SK-0205	00	I-12
5 DHLW/DOE SNF - Short WP Assembly Configuration for Site Recommendation	SK-0196	03	I-13 to I-14
5 DHLW/DOE SNF - Short Weld Configuration	SK-0197	00	I-15
2-MCO/2-DHLW Waste Package Configuration for Site Recommendation	SK-0198	04	I-16 to I-18
2-MCO/2-DHLW Waste Package Weld Configuration	SK-0199	01	I-19
Naval SNF Long Waste Package Configuration for Site Recommendation	SK-0194	01	I-20 to I-21
Naval SNF Long Waste Package Weld Configuration	SK-0195	00	I-22

Attachment II (10 pages): Mathcad thermal expansion calculations for the 21-PWR WP

Attachment III (10 pages): Mathcad thermal expansion calculations for the 44-BWR WP

Attachment IV (10 pages): Mathcad thermal expansion calculations for the 24-BWR WP

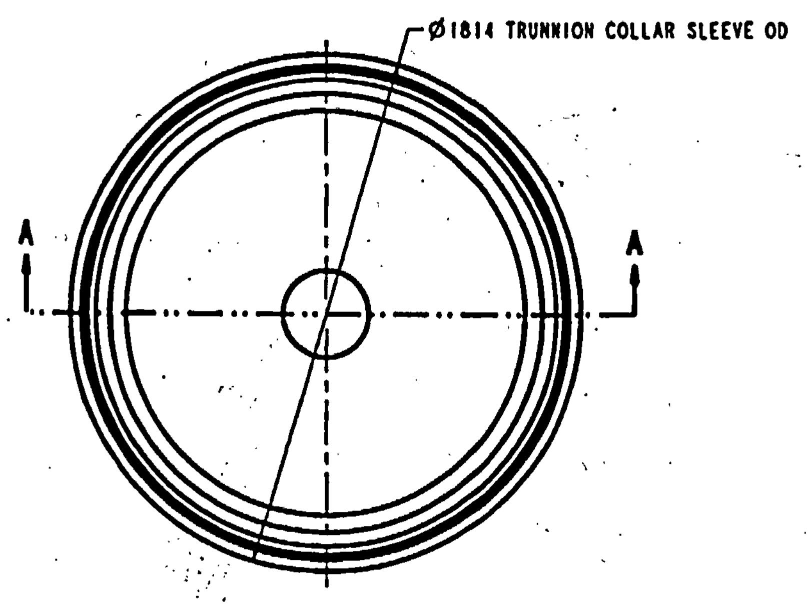
Attachment V (10 pages): Mathcad thermal expansion calculations for the 12-PWR Long WP

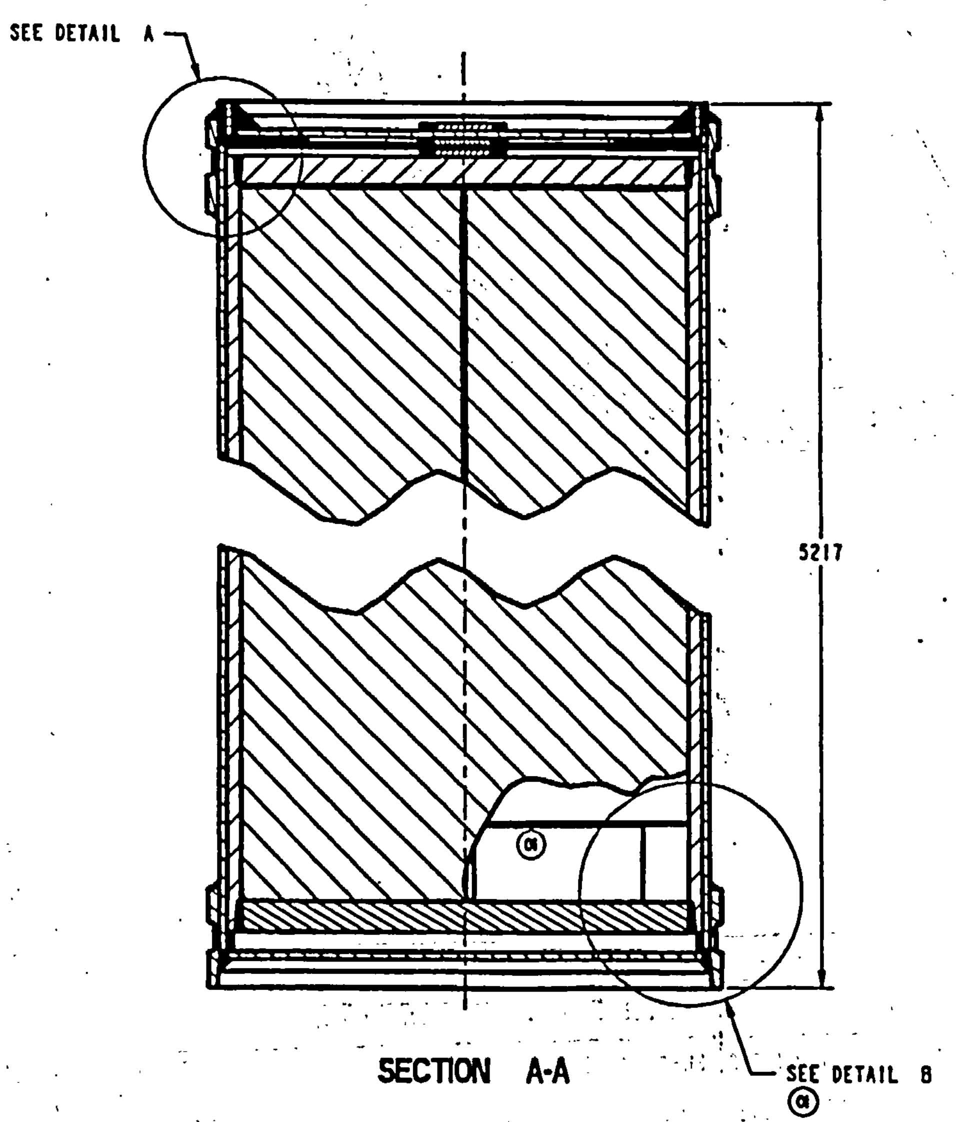
Attachment VI (10 pages): Mathcad thermal expansion calculations for the 5 DHLW/DOE SNF - Short WP

Attachment VII (10 pages): Mathcad thermal expansion calculations for the 2-MCO/2-DHLW WP

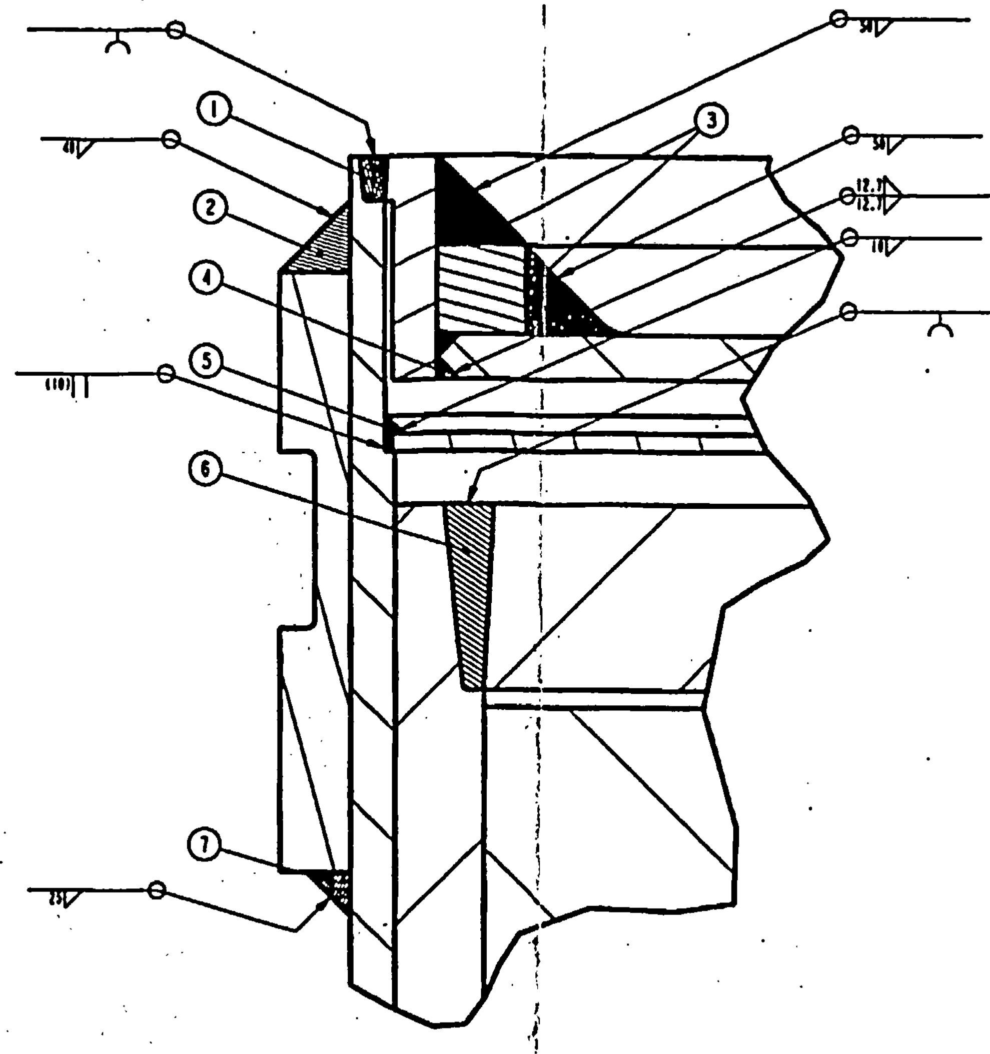
Attachment VIII (10 pages): Mathcad thermal expansion calculations for the Naval SNF Long WP

Attachment IX (5 pages): Mathcad verification for the equation of thermal expansion through a radius, using the theory of elasticity.

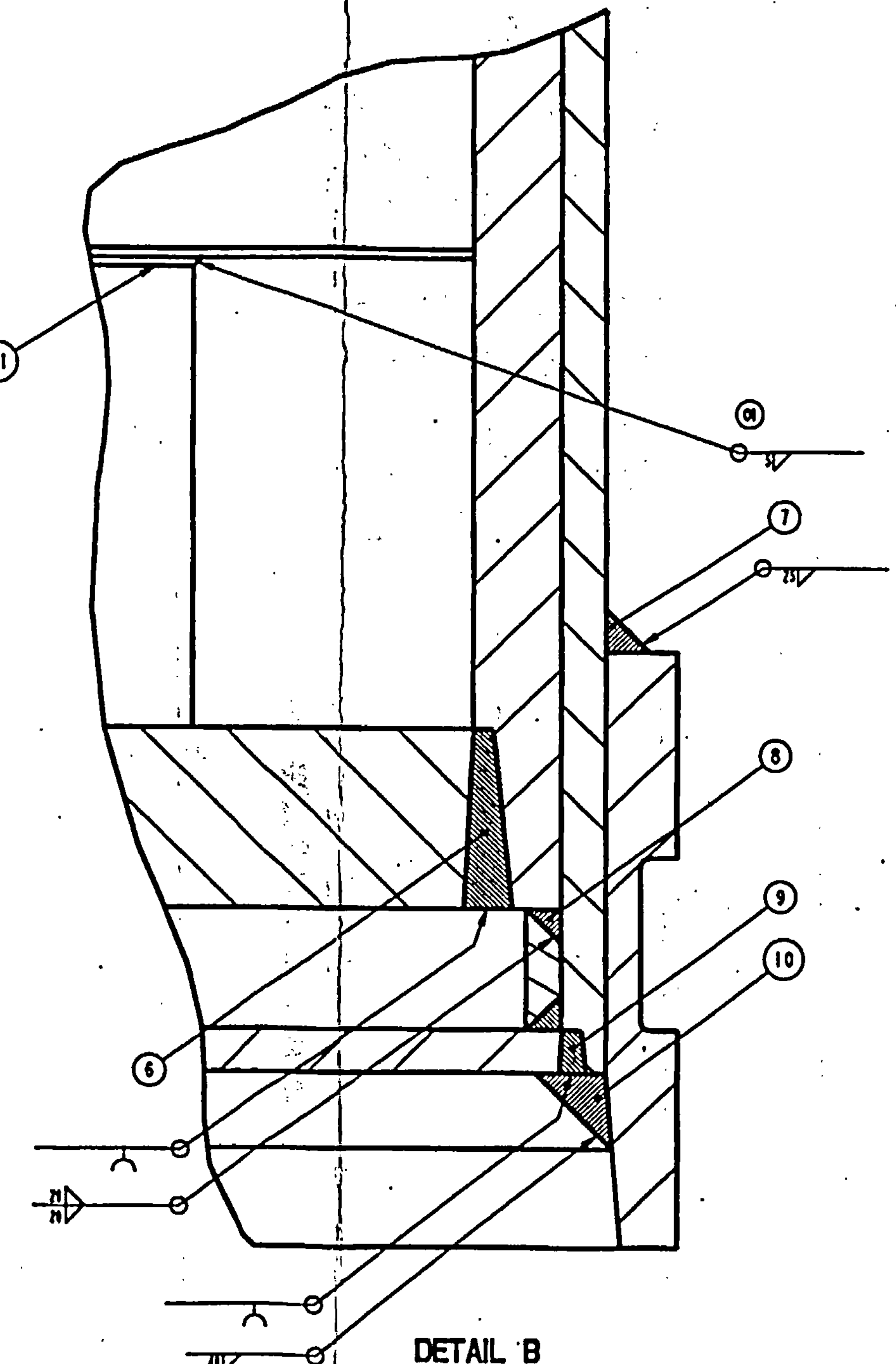




SECTION A-A



DETAIL A



DETAIL B

REVISION TABLE			
REV	DESCRIPTION	DWN BY	DATE
00	ISSUED APPROVED	BH	03/06/00
01	FUEL SUPPORT ASSEMBLY WELD ADDED TO WP ASSEMBLY.	BH	05/09/00
01	WELD SYMBOL FOR FUEL SUPPORT ASSEMBLY WELD ADDED TO DETAIL B	BH	05/09/00
01	FUEL SUPPORT ASSEMBLY WELD ADDED TO WELD TABLE AS WELD 11	BH	05/09/00
01	TOTAL CARBON STEEL WELDS ADDED TO WELD TABLE	BH	05/09/00
01	LOCATION OF DETAIL B FROM SECTION A-A WAS MODIFIED	BH	05/09/00

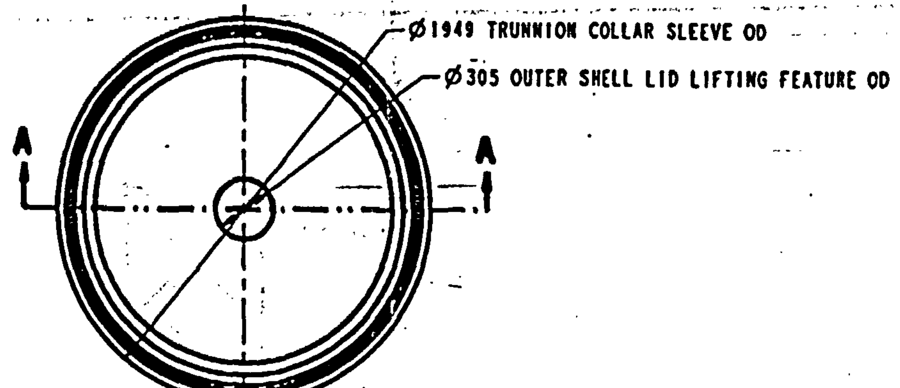
WELD	MATERIAL	MASS (KG)	QTY ROD
1	SFA-5.14 N06022	15	1
2	SFA-5.14 N06022	38	1
3	SFA-5.14 N06022	107	1
4	SFA-5.14 N06022	3.5	2
5	SFA-5.14 N06022	4.2	1
6	SFA-5.9 S31680	82	2
7	SFA-5.14 N06022	15	2
8	SFA-5.14 N06022	9.1	2
9	SFA-5.14 N06022	15	1
10	SFA-5.14 N06022	41	1
(01) 11	SFA-5.18 K10726	0.19	2
TOTAL ALLOY 22 WELDS	SFA-5.14 N06022	276	-
TOTAL 316 WELDS	SFA-5.9 S31680	164	-
(01) TOTAL CARBON STEEL WELDS	SFA-5.18 K10726	0.38	-

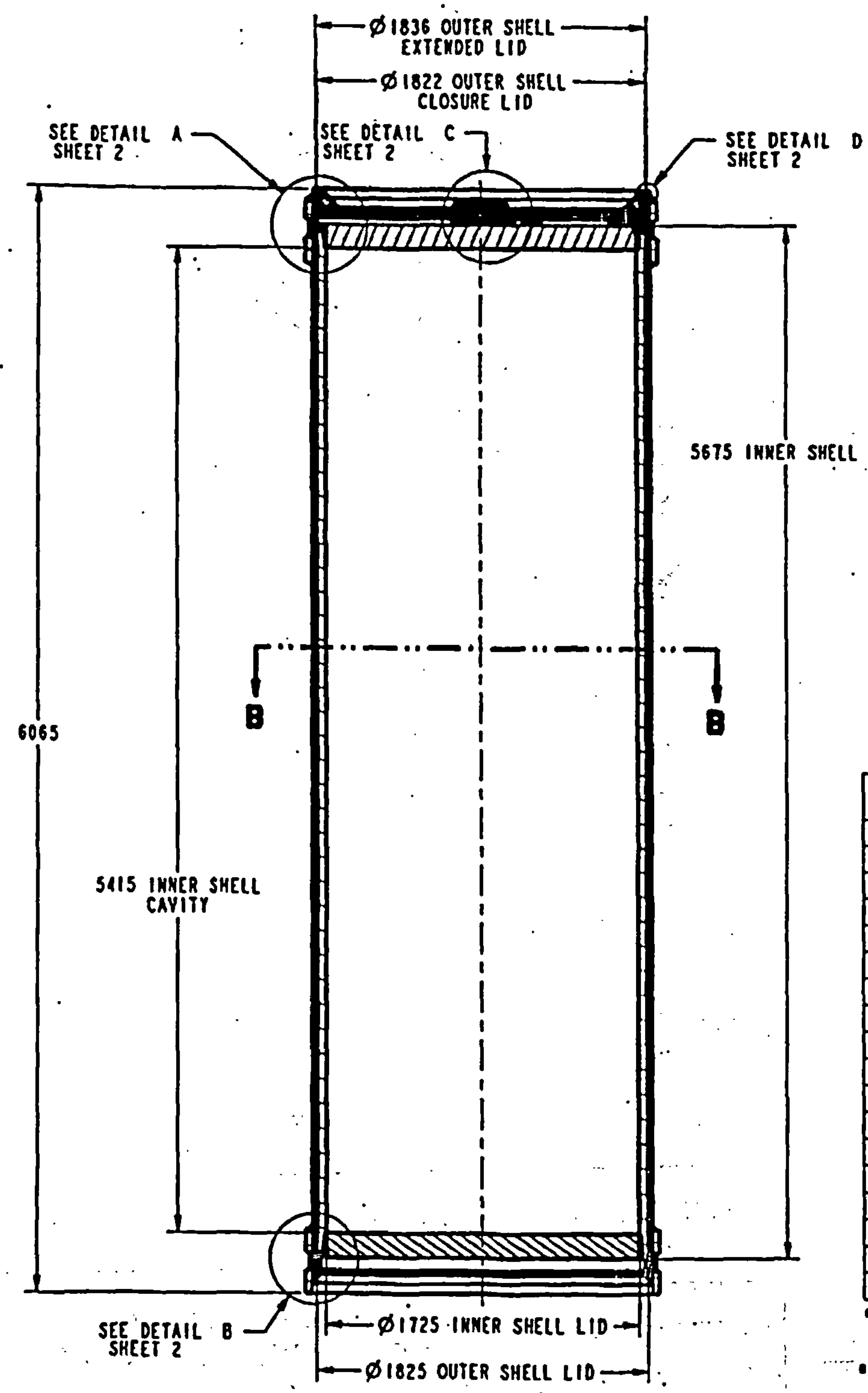
UNITS: mm

DO NOT SCALE FROM SKETCH

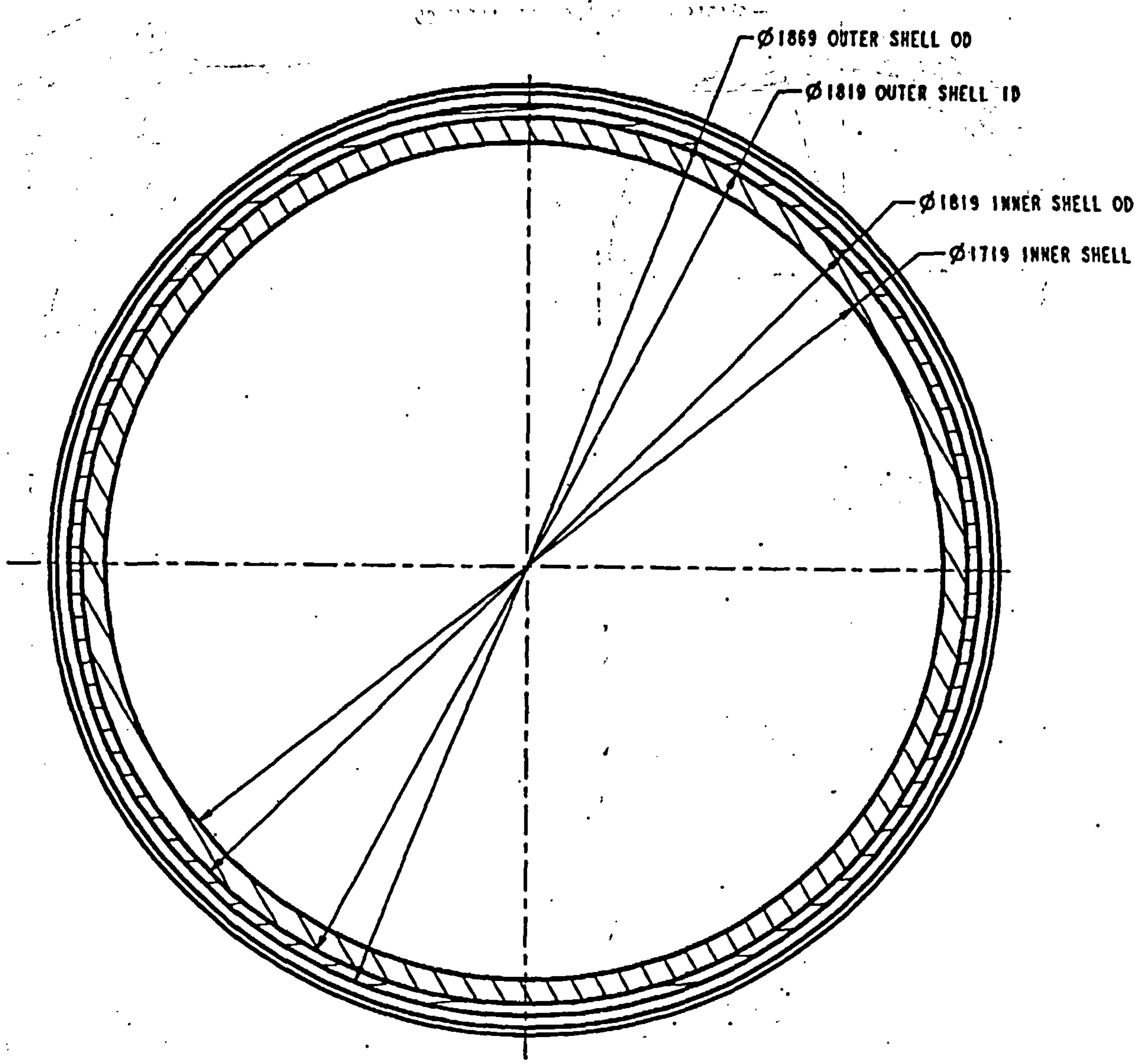
"FOR INFORMATION ONLY"

2-MCO / 2-DHLW WASTE PACKAGE WELD CONFIGURATION	
SKETCH NUMBER:	SK-0199 REV 01
SKETCHED BY:	BRYAN HARKINS BH 24 May 00 SMB 05/24/00
DATE:	05/09/00
FILE:	/home/pro_library/checkout/sketches/2mco_2dhlw/sk-0199rev01.dwg





SECTION A-A



SECTION B-B

COMPONENT NAME	MATERIAL	THICKNESS	MASS (KG)	QTY ROD
INNER SHELL	SA-240 S31600	50	12372	1
INNER SHELL LID	SA-240 S31600	130	2390	2
INNER LID LIFTING FEATURE	SA-240 S31600	27	12	1
OUTER SHELL	SB-575 N06022	25	7430	1
EXTENDED OUTER SHELL LID	SB-575 N06022	25	158	1
EXTENDED OUTER SHELL LID BASE	SB-575 N06022	25	528	1
EXTENDED LID REINFORCEMENT RING	SB-575 N06022	50	118	1
OUTER LID LIFTING FEATURE	SB-575 N06022	27	13	2
OUTER SHELL FLAT CLOSURE LID	SB-575 N06022	10	227	1
OUTER SHELL FLAT BOTTOM LID	SB-575 N06022	25	564	1
UPPER TRUNNION COLLAR SLEEVE	SB-575 N06022	40	604	1
LOWER TRUNNION COLLAR SLEEVE	SB-575 N06022	40	592	1
INNER SHELL SUPPORT RING	SB-575 N06022	20	49	1
TOTAL ALLOY 22 WELDS	SFA-5.14 N06022	-	298	**
TOTAL 316 WELDS	SFA-5.9 S31680	-	243	**
WASTE PACKAGE ASSEMBLY	-	-	28005	1
NAVAL SNF	-	-	44452*	1
WASTE PACKAGE WITH SNF	-	-	72457	1

*"MAXIMUM EXPECTED PARAMETERS FOR NAVAL REACTORS CANISTERS" 10/29/97 FROM: RICHARD GUIDA TO: RUSSELL DYER, MOL.19980121.0011

**REFER TO SK-0195 REV 00 "NAVAL SNF LONG WASTE PACKAGE WELD CONFIGURATION"

UNITS: mm

DO NOT SCALE FROM SKETCH

"FOR INFORMATION ONLY"

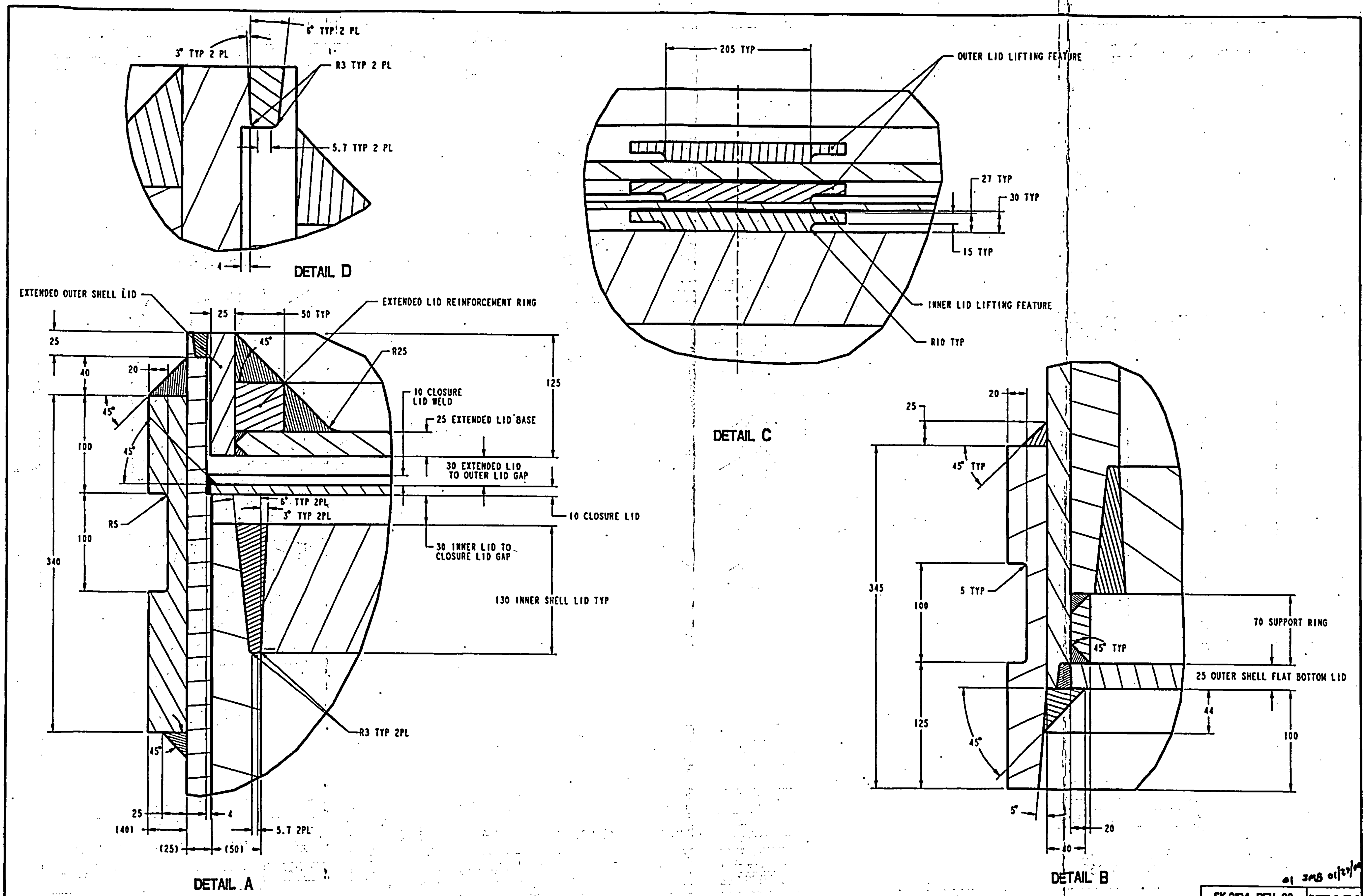
NAVAL SNF LONG WASTE PACKAGE CONFIGURATION FOR SITE RECOMMENDATION

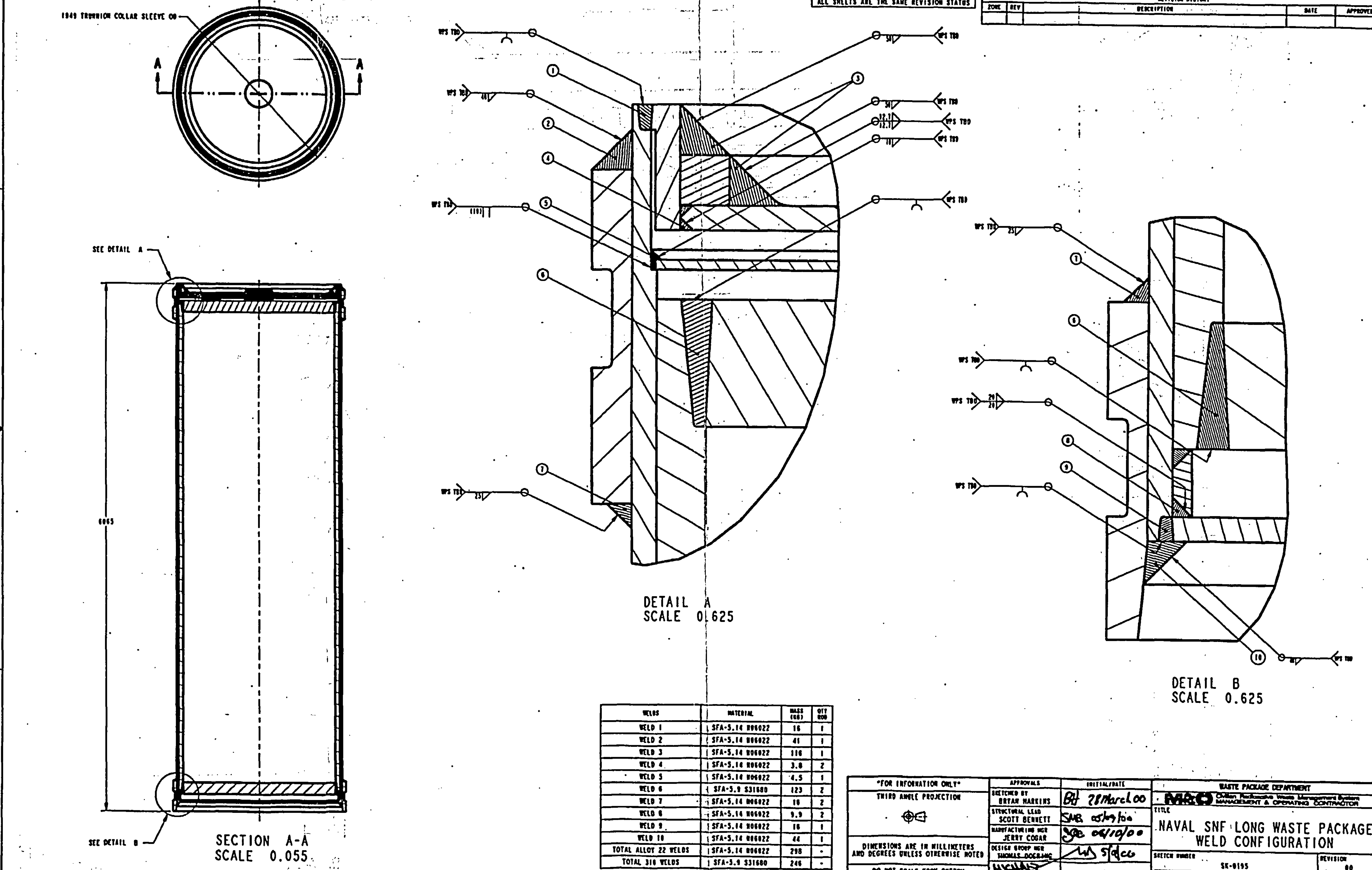
SKETCH NUMBER: SK-0194 REV.01 — SHEET 1 OF 2

SKETCHED BY: GENE CONNELL EGC 01/26/00 SMB 01/26/00 JAC 1/26/00 TAB 1,26,00

DATE: 01/11/00

FILE: /home/pro_library/checkout/sketches/naval/naval snf long/SK-0194.dwg


6° TYP 2 PL
3° TYP 2 PL
R3 TYP 2 PL
5.7 TYP 2 PL
4
DETAIL D
205 TYP
OUTER LID LIFTING FEATURE
27 TYP
30 TYP
15 TYP
INNER LID LIFTING FEATURE
R10 TYP
DETAIL C
EXTENDED OUTER SHELL LID
25
50 TYP
EXTENDED LID REINFORCEMENT RING
45°
R25
125
40
20
10 CLOSURE LID WELD
25 EXTENDED LID BASE
100
30 EXTENDED LID TO OUTER LID GAP
6° TYP 2PL
3° TYP 2PL
10 CLOSURE LID
R5
340
30 INNER LID TO CLOSURE LID GAP
130 INNER SHELL LID TYP
R3 TYP 2PL
(40)
(25)
(50)
5.7 2PL
DETAIL A
45° TYP
345
5 TYP
70 SUPPORT RING
25 OUTER SHELL FLAT BOTTOM LID
44
5°
DETAIL B
SK-0194 REV 00
SHEET 2 OF 2



WELDS	MATERIAL	MASS (KG)	QTY ROD
WELD 1	SFA-5.14 N06022	16	1
WELD 2	SFA-5.14 N06022	41	1
WELD 3	SFA-5.14 N06022	116	1
WELD 4	SFA-5.14 N06022	3.8	2
WELD 5	SFA-5.14 N06022	4.5	1
WELD 6	SFA-5.9 S31680	123	2
WELD 7	SFA-5.14 N06022	18	2
WELD 8	SFA-5.14 N06022	9.9	2
WELD 9	SFA-5.14 N06022	16	1
WELD 10	SFA-5.14 N06022	44	1
TOTAL ALLOY 22 WELDS	SFA-5.14 N06022	298	-
TOTAL 316 WELDS	SFA-5.9 S31680	246	-

ALL SHEETS ARE THE SAME REVISION STATUS

ZONE	REV	DESCRIPTION	DATE	APPROVED

REVISION HISTORY
SKETCH NUMBER SK-0195 SHEET 1 OF 1 REVISION 00

"FOR INFORMATION ONLY"
THIRD ANGLE PROJECTION
DIMENSIONS ARE IN MILLIMETERS AND DEGREES UNLESS OTHERWISE NOTED
DO NOT SCALE FROM SKETCH

APPROVALS	INITIAL/DATE
SKETCHED BY BRYAN MARKINS	BH 28March00
STRUCTURAL LEAD SCOTT BENNETT	SMB 05/09/00
MANUFACTURING MGR JERRY COGAR	JC 05/10/00
DESIGN GROUP MGR THOMAS DOERING	MA 5/10/00
MICHAEL ANDERSON	

WASTE PACKAGE DEPARTMENT
M&O Civilian Radioactive Waste Management System MANAGEMENT & OPERATING CONTRACTOR
TITLE
NAVAL SNF LONG WASTE PACKAGE
WELD CONFIGURATION
SKETCH NUMBER SK-0195
REVISION 00
SCALE .055
SHEET 1 OF 1

Various Shell Gap Sizes for the 21-PWR WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

mm

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):

$r_i := 0.712 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

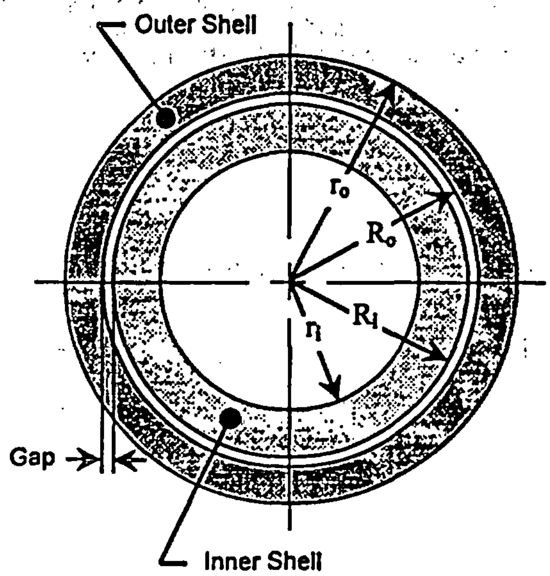
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.020 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 4.585m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\ 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\ 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2\,ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

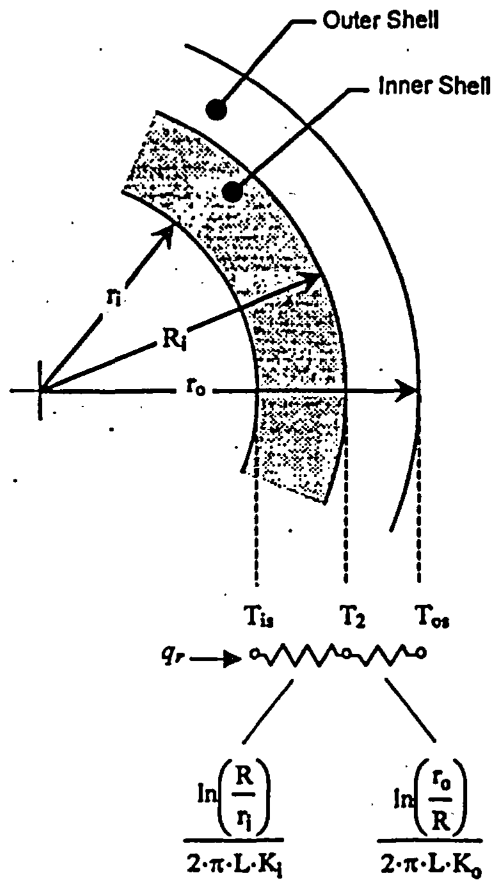
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



T_{is} = (K)

	0
0	293.0
1	332.4
2	359.4
3	383.2
4	412.5
5	427.5
6	444.5
7	469.4
8	494.4
9	503.3
10	513.3

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot \Delta T_{os}^{T}$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^{T} \cdot \delta_i^{T} - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o}\cdot\left(\frac{r_o^2 + R_o^2}{r_o^2 - R_o^2} + \nu_o\right) + \frac{R_i}{E_i}\cdot\left(\frac{R_i^2 + r_i^2}{R_i^2 - r_i^2} - \nu_i\right)\right]}\right]}$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{r_o^{\,2}}\right)\right]}$$

outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{R_o^{\,2}}\right)\right]}$$

outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\%\cdot\sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

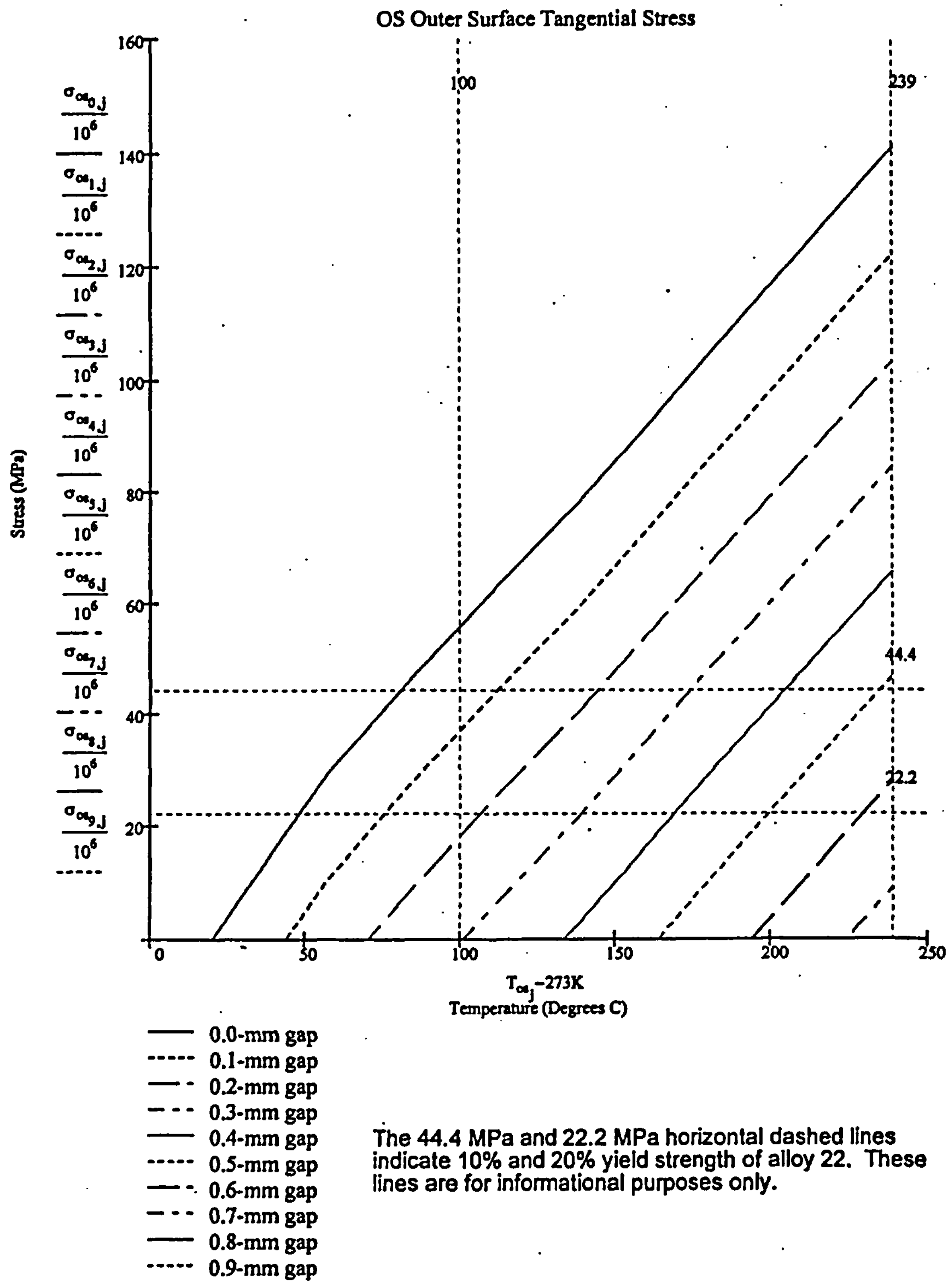
$\sigma_{20\%} := 20\%\cdot\sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	140.9
0.1	122.1
0.2	103.2
0.3	84.4
0.4	65.6
0.5	46.8
0.6	27.9
0.7	9.1
0.8	-9.7
0.9	-28.5
1.0	-47.3

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

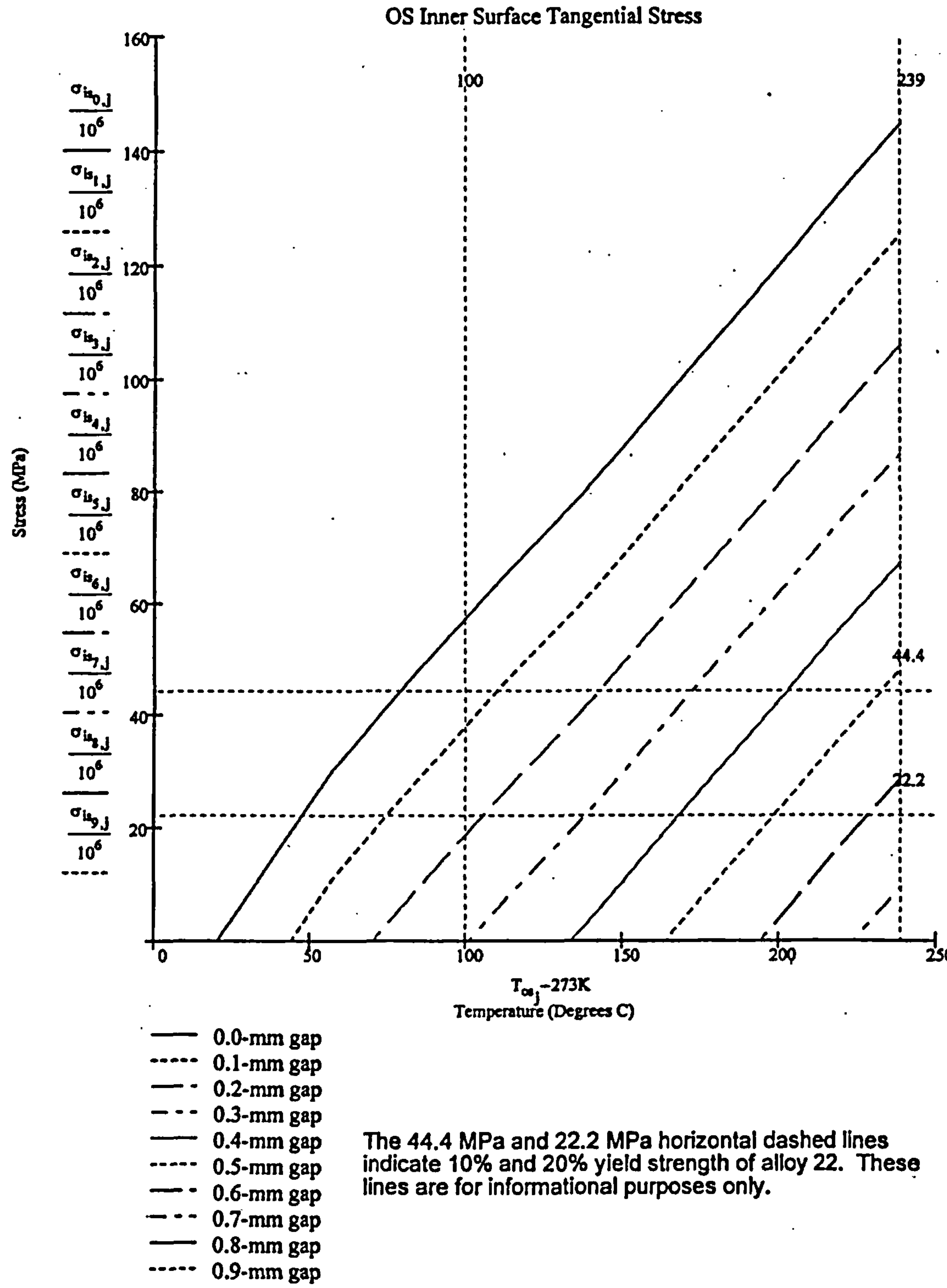


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{is_{j,10}}$ = (MPa)
0.0	144.6
0.1	125.3
0.2	106.0
0.3	86.6
0.4	67.3
0.5	48.0
0.6	28.7
0.7	9.4
0.8	-10.0
0.9	-29.3
1.0	-48.6

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the 44-BWR WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):

$r_i := 0.727 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

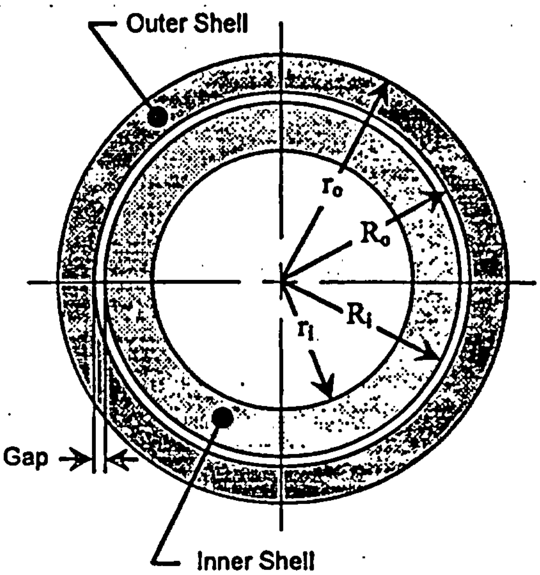
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.020 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 4.585m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\,10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\,10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2\,ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

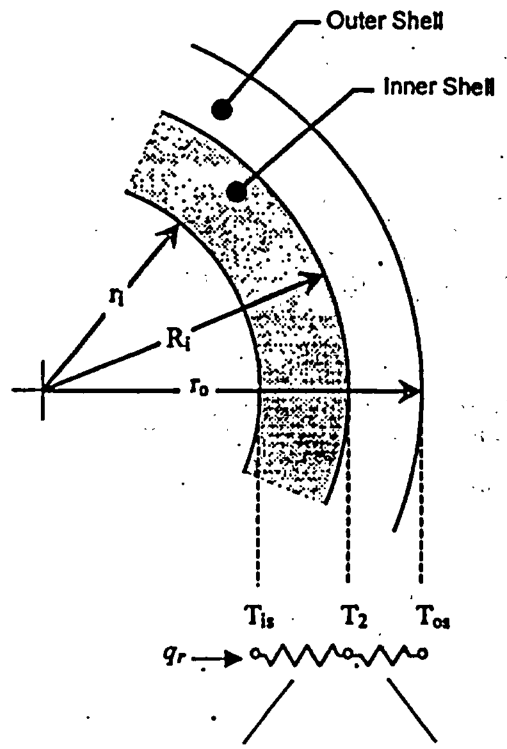
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



$T_{is} =$

	0
0	293.0
1	332.4
2	359.3
3	383.2
4	412.4
5	427.4
6	444.4
7	469.4
8	494.4
9	503.3
10	513.2

K

$$\frac{\ln\left(\frac{R}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} \qquad \frac{\ln\left(\frac{r_o}{R}\right)}{2\cdot\pi\cdot L\cdot K_o}$$

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot \Delta T_{os}^{\,T}$ change in size of the outer shell inner radius

$A := (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot \delta_i^{\,T} - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o}\cdot\left(\frac{r_o^2 + R_o^2}{r_o^2 - R_o^2} + \nu_o\right) + \frac{R_i}{E_i}\cdot\left(\frac{R_i^2 + r_i^2}{R_i^2 - r_i^2} - \nu_i\right)\right]}\right]}$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2}-R_o^{\,2}}\cdot\left(1+\frac{r_o^{\,2}}{r_o^{\,2}}\right)\right]}$$ outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2}-R_o^{\,2}}\cdot\left(1+\frac{r_o^{\,2}}{R_o^{\,2}}\right)\right]}$$ outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\%\cdot\sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

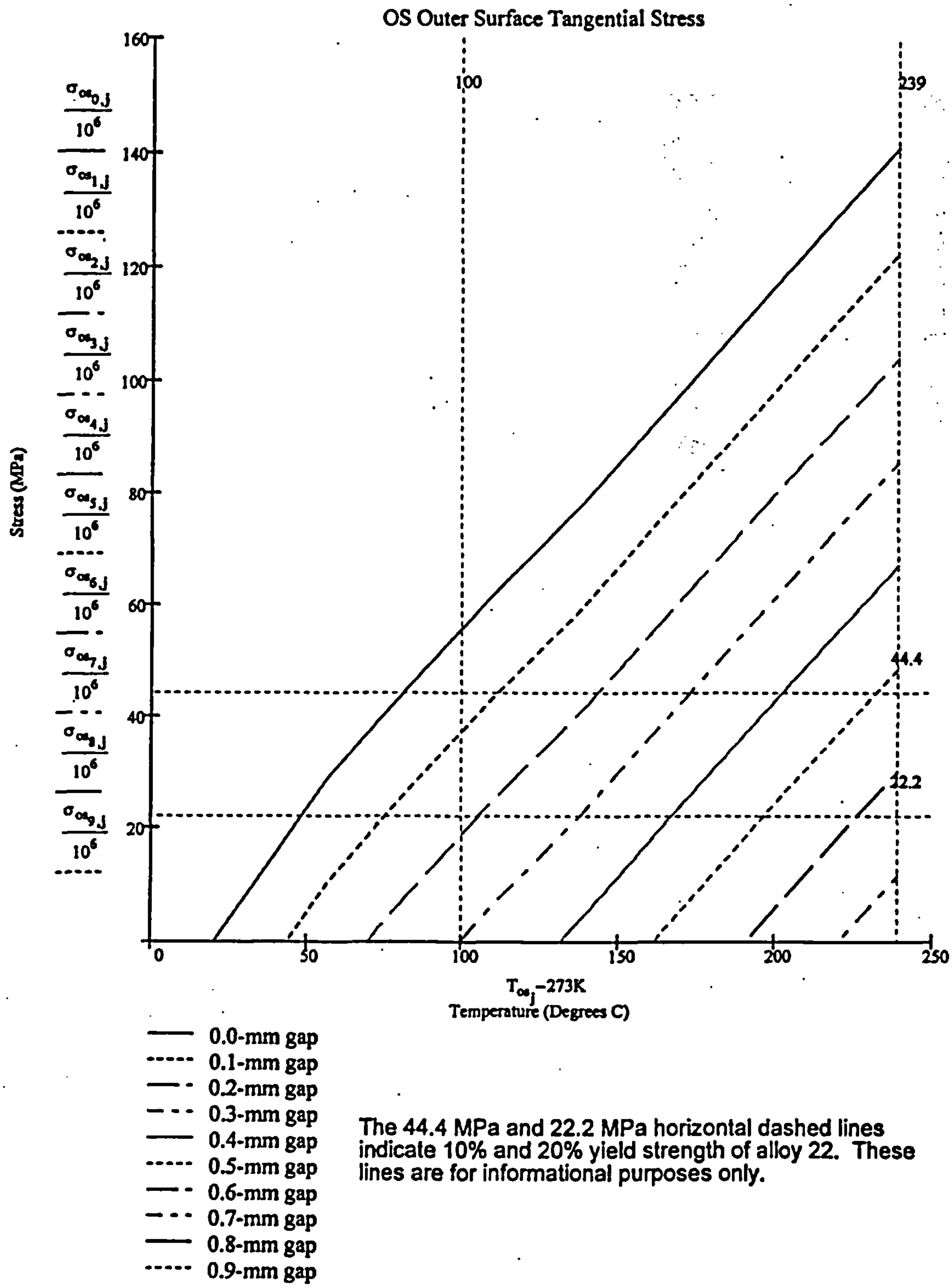
$\sigma_{20\%} := 20\%\cdot\sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	140.9
0.1	122.4
0.2	103.9
0.3	85.5
0.4	67.0
0.5	48.5
0.6	30.1
0.7	11.6
0.8	-6.9
0.9	-25.3
1.0	-43.8

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

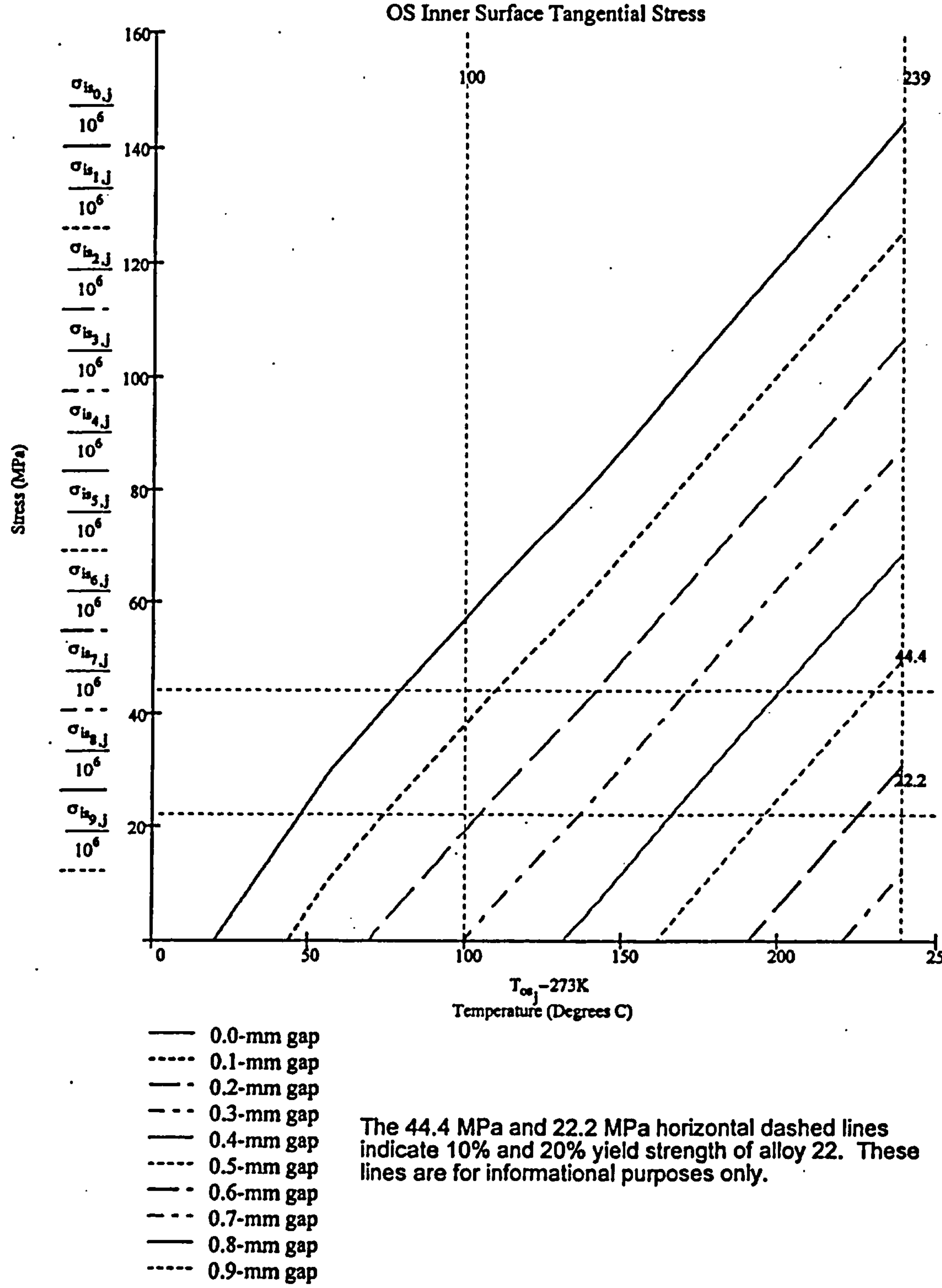


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

$gap_j =$ (mm)	$\sigma_{is_{j,10}} =$ (MPa)
0.0	144.5
0.1	125.6
0.2	106.6
0.3	87.7
0.4	68.7
0.5	49.8
0.6	30.8
0.7	11.9
0.8	-7.0
0.9	-26.0
1.0	-44.9

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the 24-BWR WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):

$r_i := 0.549 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

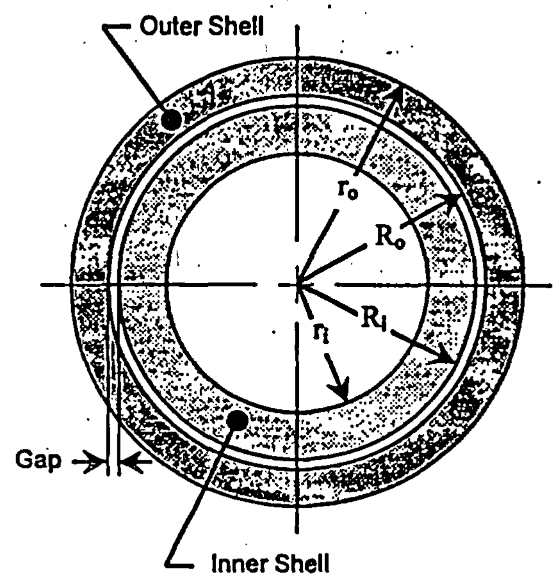
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.020 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 4.585m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\ 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\ 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2\,ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

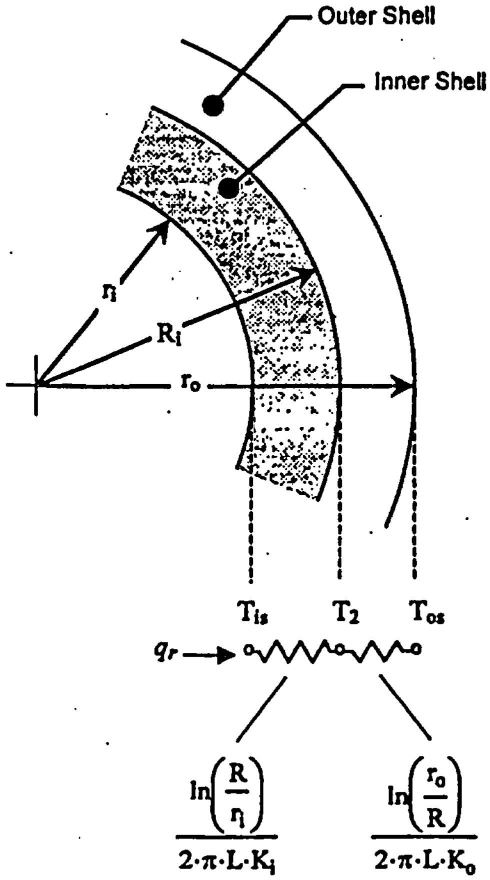
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



$T_{is} =$ (K)

	0
0	293.0
1	333.1
2	360.1
3	383.8
4	412.9
5	427.9
6	444.9
7	469.8
8	494.8
9	503.7
10	513.6

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot {\Delta T_{os}}^T$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot {\delta_i}^T - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o}\cdot\left(\frac{{r_o}^2 + {R_o}^2}{{r_o}^2 - {R_o}^2} + \nu_o\right) + \frac{R_i}{E_i}\cdot\left(\frac{{R_i}^2 + {r_i}^2}{{R_i}^2 - {r_i}^2} - \nu_i\right)\right]}\right]}$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{r_o^2}\right)\right]}$$

outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{R_o^2}\right)\right]}$$

outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\% \cdot \sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

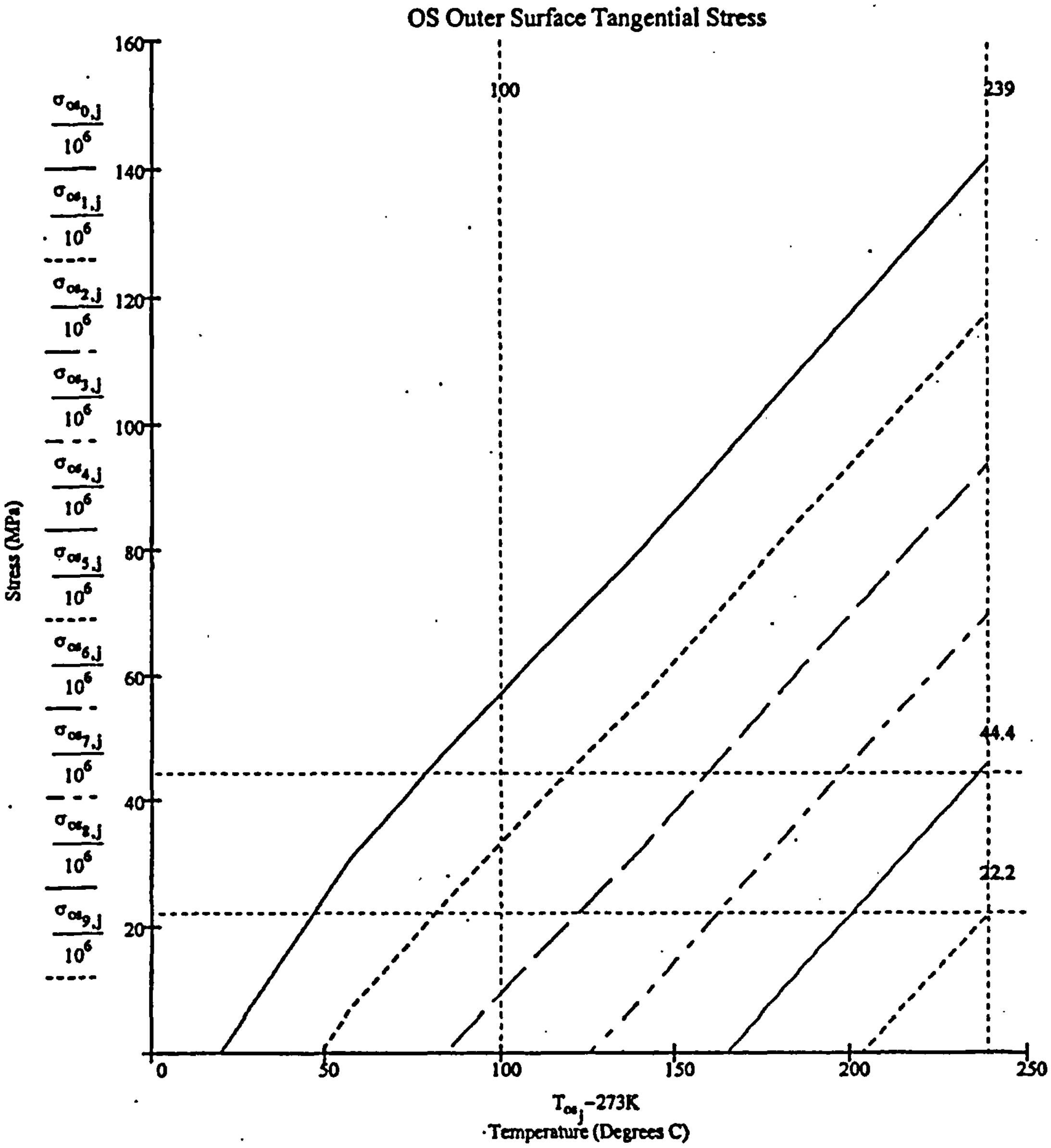
$\sigma_{20\%} := 20\% \cdot \sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	141.3
0.1	117.4
0.2	93.5
0.3	69.6
0.4	45.8
0.5	21.9
0.6	-1.9
0.7	-25.8
0.8	-49.6
0.9	-73.5
1.0	-97.3

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.



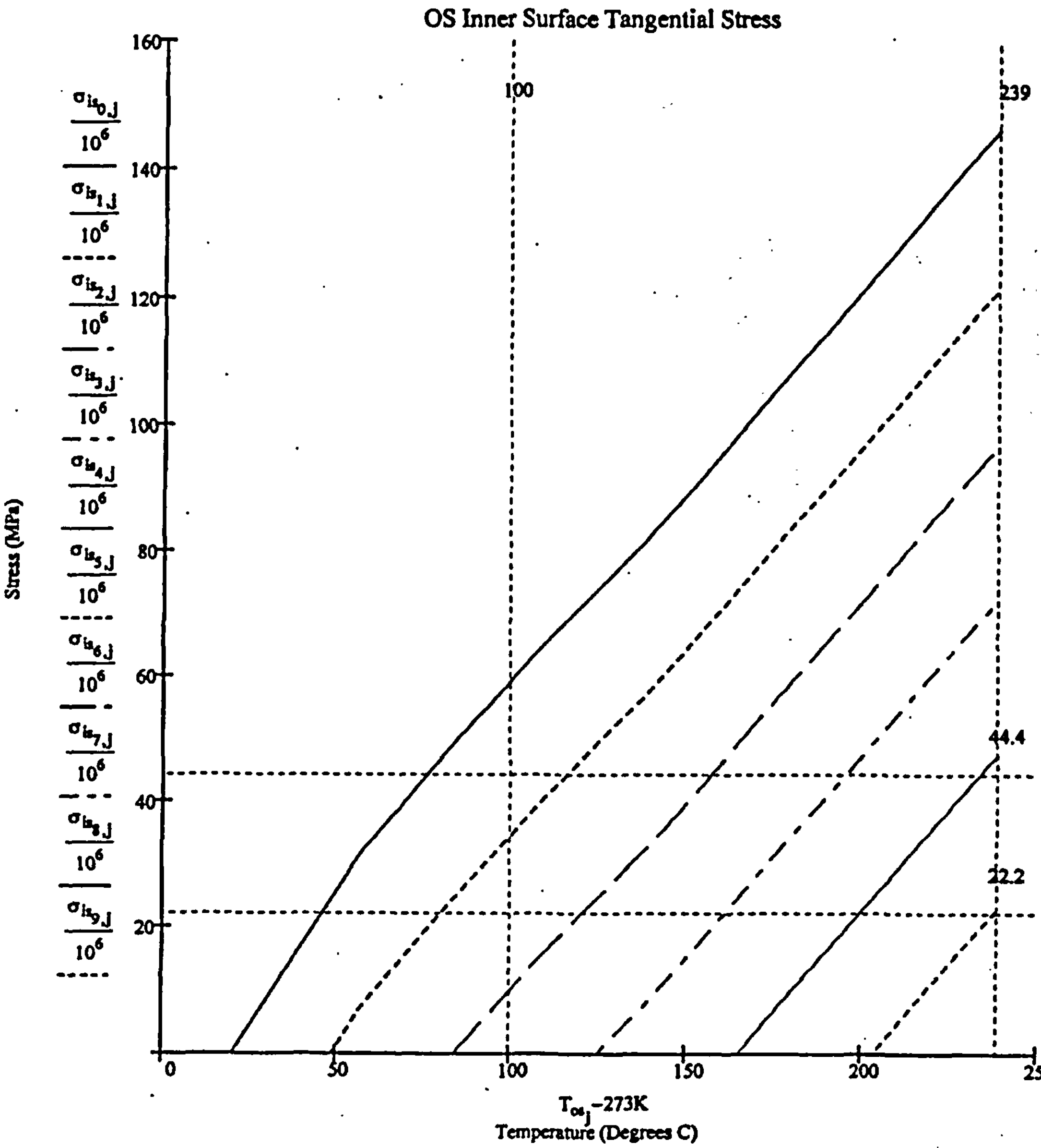
— 0.0-mm gap
----- 0.1-mm gap
—· 0.2-mm gap
- - - 0.3-mm gap
— 0.4-mm gap
----- 0.5-mm gap
—· 0.6-mm gap
- - - 0.7-mm gap
— 0.8-mm gap
----- 0.9-mm gap

The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{is_{j,10}}$ = (MPa)
0.0	146.1
0.1	121.4
0.2	96.7
0.3	72.0
0.4	47.3
0.5	22.7
0.6	-2.0
0.7	-26.7
0.8	-51.3
0.9	-76.0
1.0	-100.6

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



- 0.0-mm gap
- 0.1-mm gap
- 0.2-mm gap
- 0.3-mm gap
- 0.4-mm gap
- 0.5-mm gap
- 0.6-mm gap
- 0.7-mm gap
- 0.8-mm gap
- 0.9-mm gap

The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the 12-PWR Long WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):

$r_i := 0.555 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

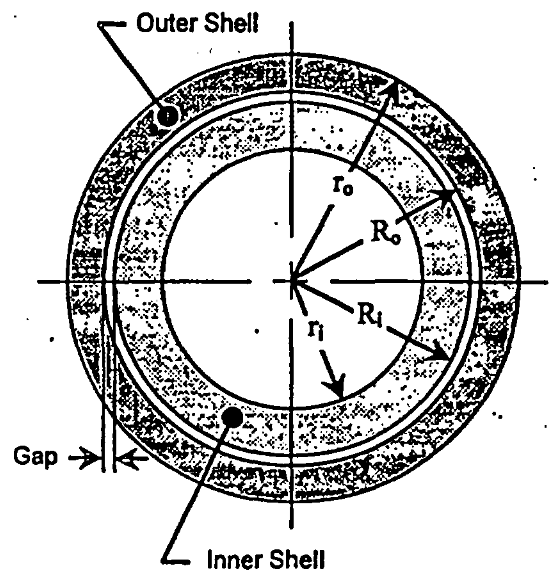
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.020 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 5.121m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot degF}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\ 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\ 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2 ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

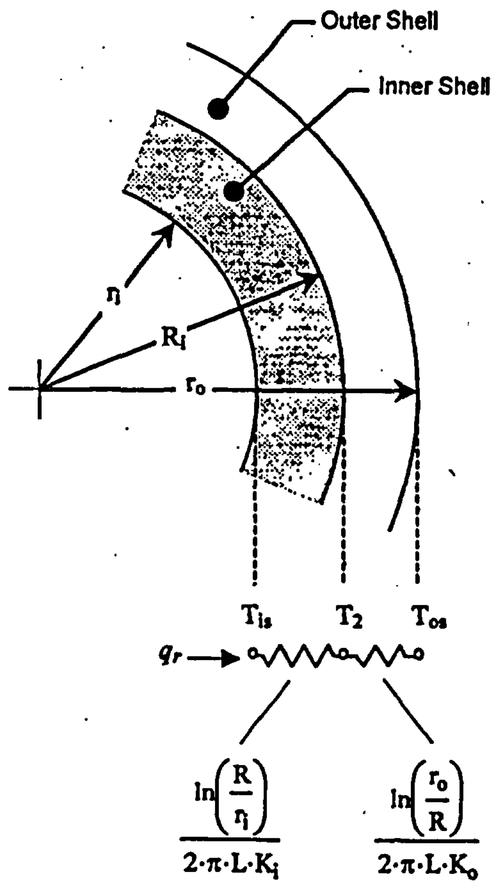
Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_{o_0}}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$

$T_{is} =$

	0
0	293.0
1	332.7
2	359.7
3	383.5
4	412.7
5	427.7
6	444.7
7	469.6
8	494.6
9	503.5
10	513.4

K



The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot {\Delta T_{os}}^T$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot {\delta_i}^T - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o} \cdot \left(\frac{{r_o}^2 + {R_o}^2}{{r_o}^2 - {R_o}^2} + \nu_o\right) + \frac{R_i}{E_i} \cdot \left(\frac{{R_i}^2 + {r_i}^2}{{R_i}^2 - {r_i}^2} - \nu_i\right)\right]}\right]}$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{r_o^{\,2}}\right)\right]}$$

outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{R_o^{\,2}}\right)\right]}$$

outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\%\cdot\sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

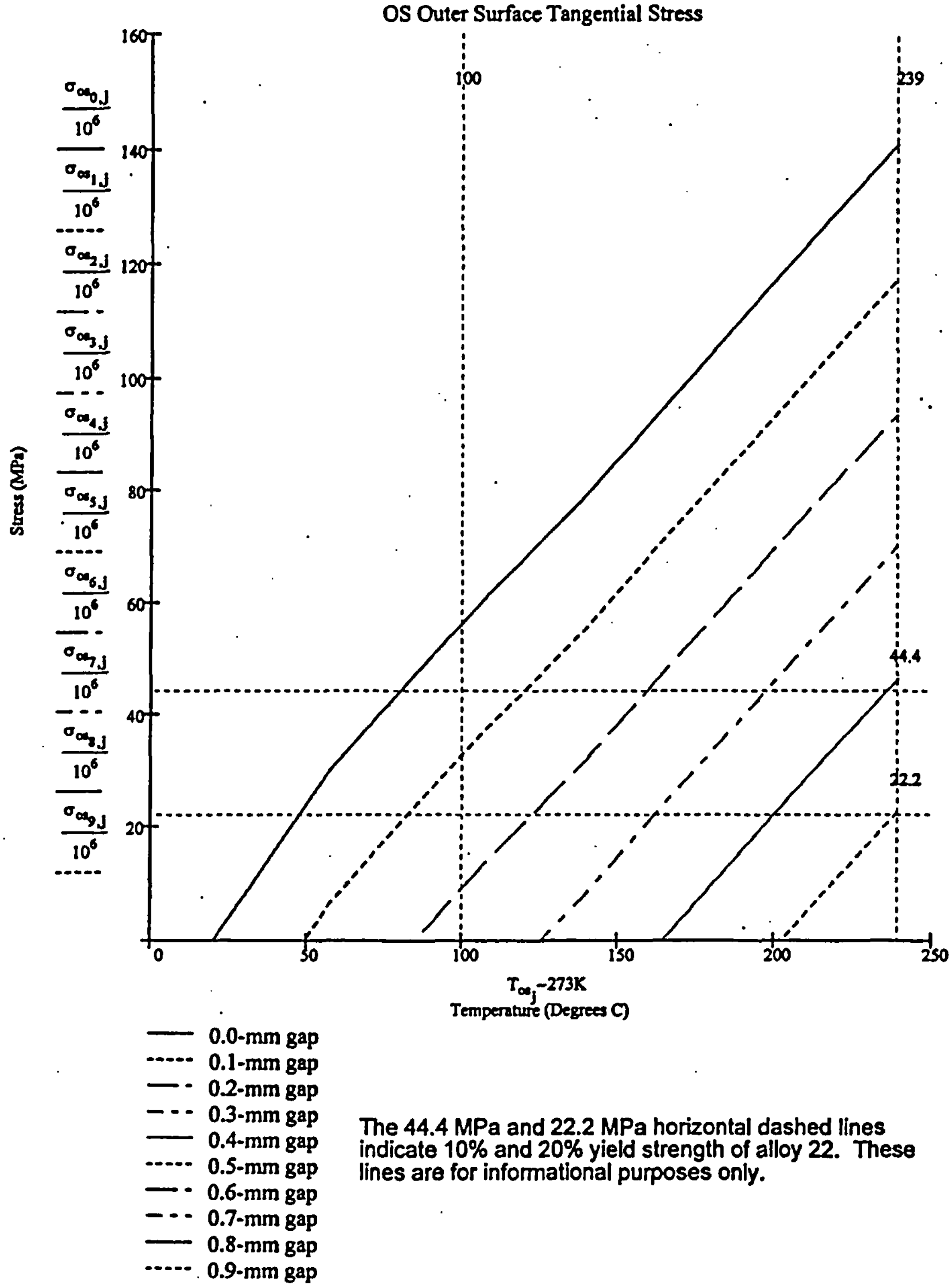
$\sigma_{20\%} := 20\%\cdot\sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	140.8
0.1	117.2
0.2	93.6
0.3	69.9
0.4	46.3
0.5	22.7
0.6	-1.0
0.7	-24.6
0.8	-48.2
0.9	-71.8
1.0	-95.4

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

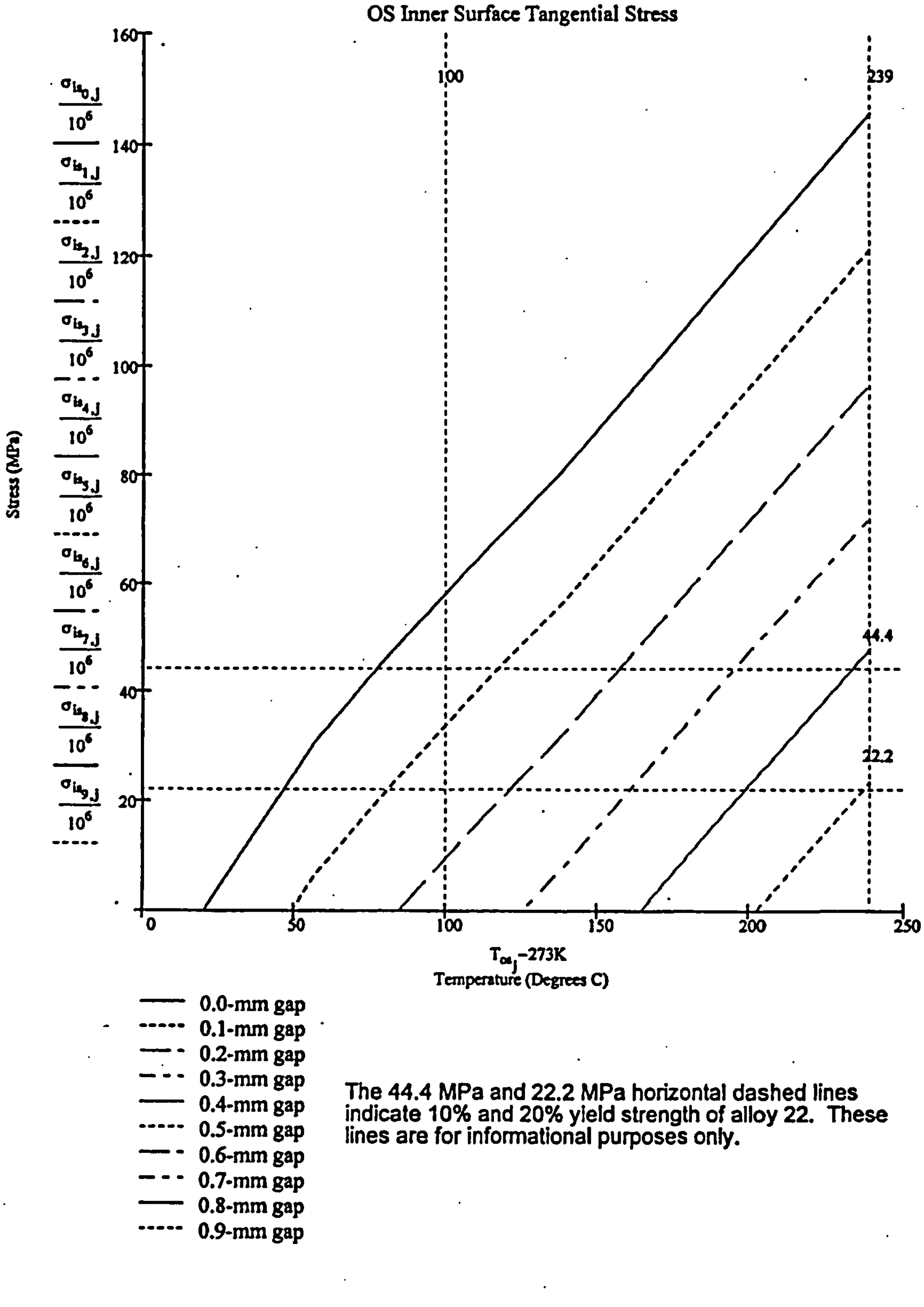


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{is_{j,10}}$ = (MPa)
0.0	145.6
0.1	121.1
0.2	96.7
0.3	72.3
0.4	47.8
0.5	23.4
0.6	-1.0
0.7	-25.4
0.8	-49.8
0.9	-74.2
1.0	-98.6

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the 5-DHLW/DOE SNF-LONG WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

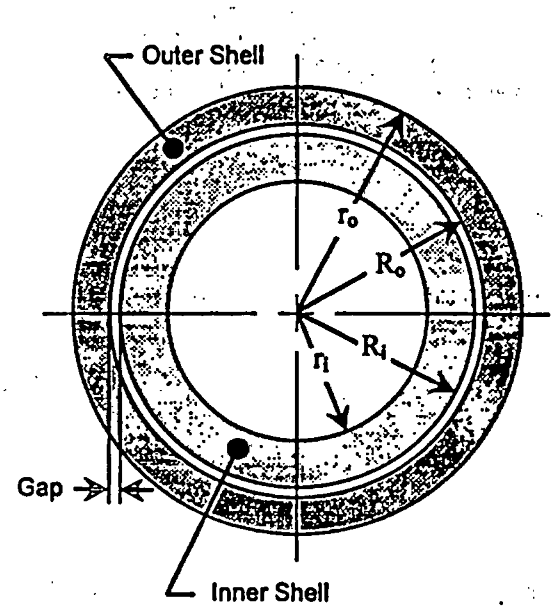
$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):



$r_i := 0.940 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.025 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 3.590m$ inner cavity length

Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\, 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\, 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2\,ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

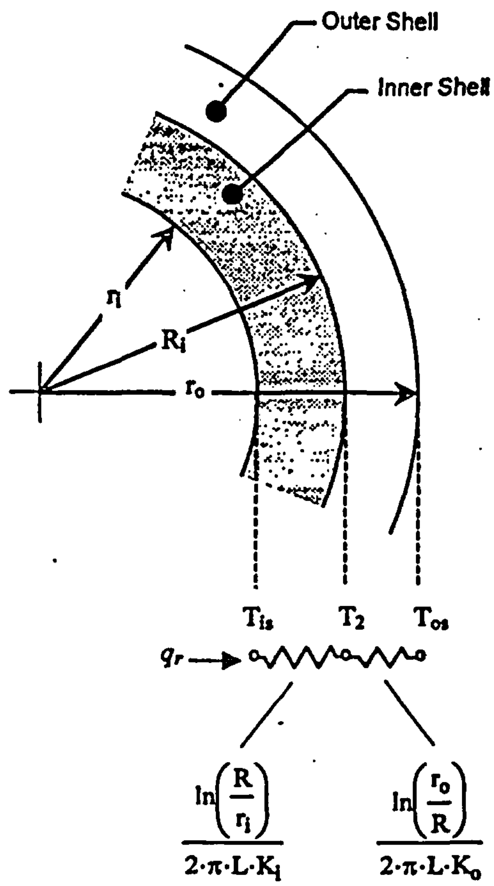
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



$T_{is} =$ (K)

	0
0	293.0
1	332.5
2	359.5
3	383.3
4	412.5
5	427.5
6	444.5
7	469.5
8	494.5
9	503.4
10	513.3

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot \Delta T_{os}^{T}$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot \delta_i^{T} - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o} \cdot \left(\frac{r_o^2 + R_o^2}{r_o^2 - R_o^2} + \nu_o\right) + \frac{R_i}{E_i} \cdot \left(\frac{R_i^2 + r_i^2}{R_i^2 - r_i^2} - \nu_i\right)\right]}\right]}$$

· The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{r_o^{\,2}}\right)\right]}$$

outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p\cdot(R_o)^2}{r_o^{\,2} - R_o^{\,2}}\cdot\left(1 + \frac{r_o^{\,2}}{R_o^{\,2}}\right)\right]}$$

outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\%\cdot\sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

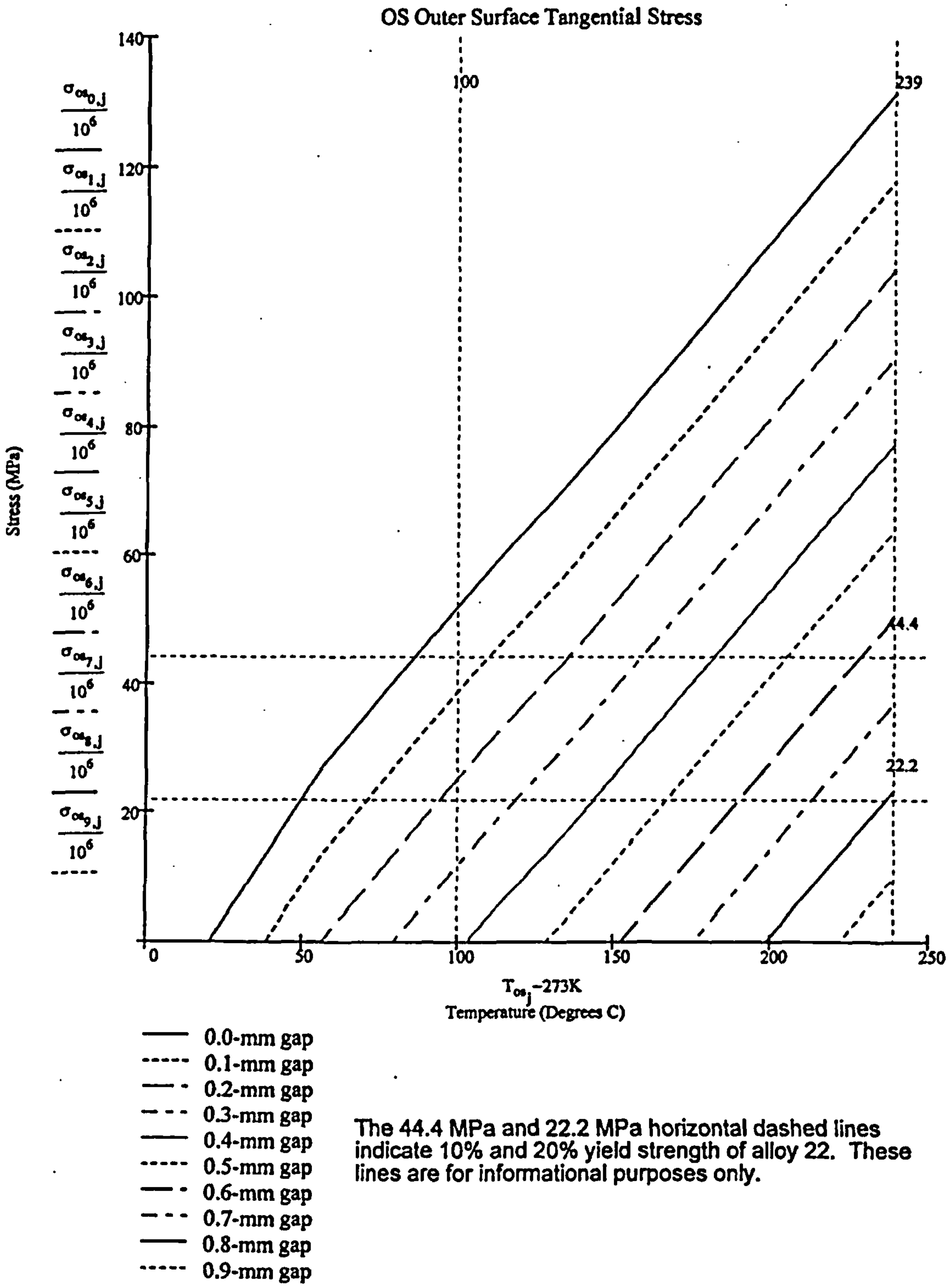
$\sigma_{20\%} := 20\%\cdot\sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	131.4
0.1	117.9
0.2	104.4
0.3	90.9
0.4	77.4
0.5	63.9
0.6	50.4
0.7	36.9
0.8	23.4
0.9	9.9
1.0	-3.6

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

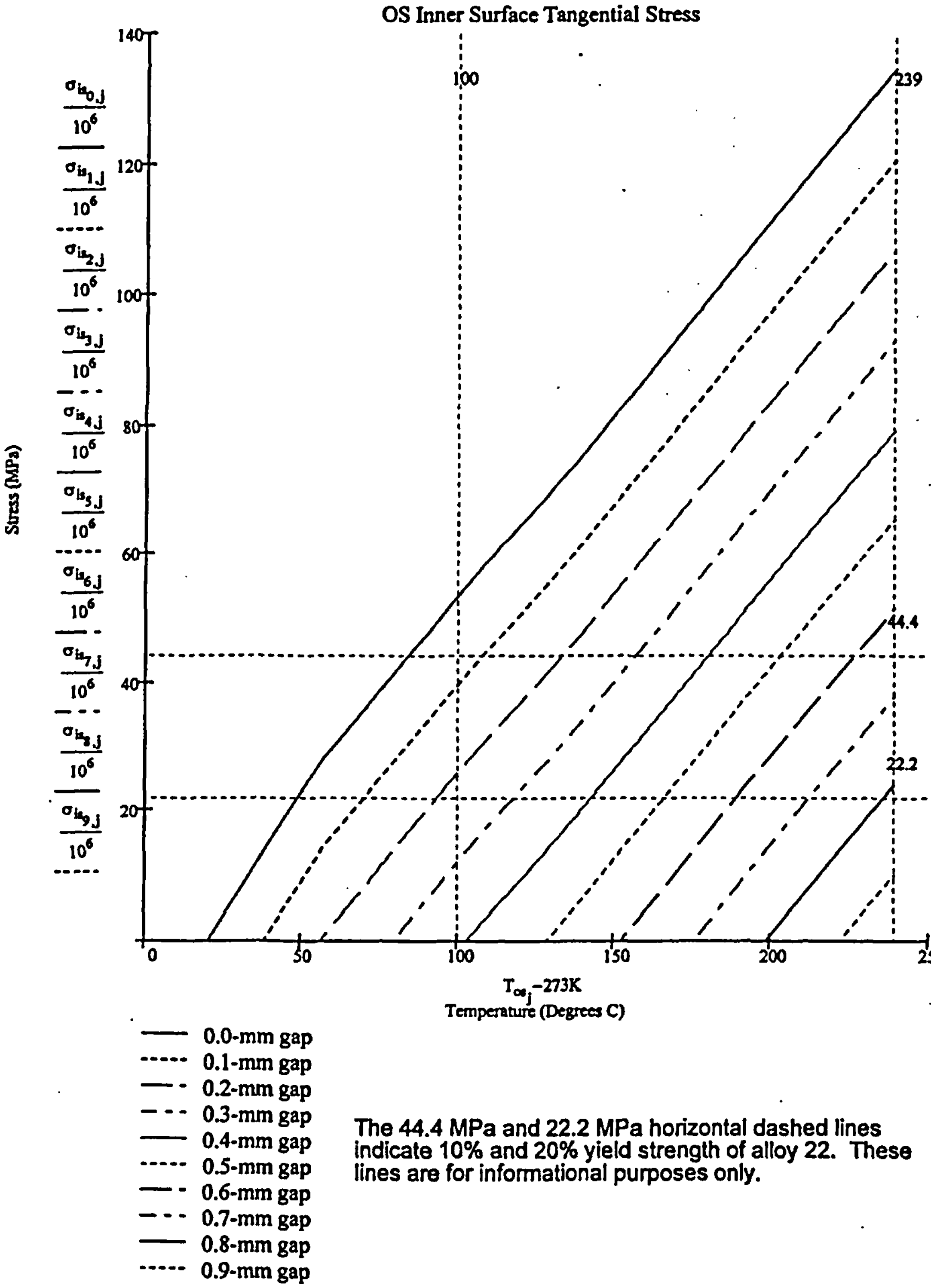


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{ls_{j,10}}$ = (MPa)
0.0	134.8
0.1	120.9
0.2	107.1
0.3	93.2
0.4	79.4
0.5	65.5
0.6	51.7
0.7	37.9
0.8	24.0
0.9	10.2
1.0	-3.7

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the 2-MCO/2-DHLW WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section and the inner cavity length (Section 5.1.3):

$r_i := 0.792 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

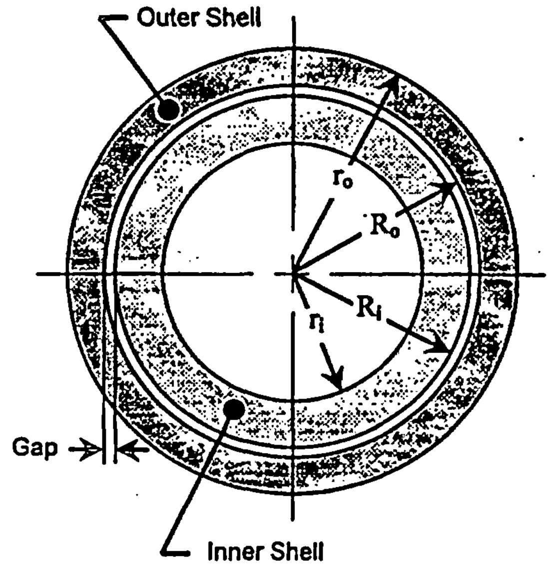
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.025 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 4.617m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\ 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\ 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$v_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$v_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2 ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

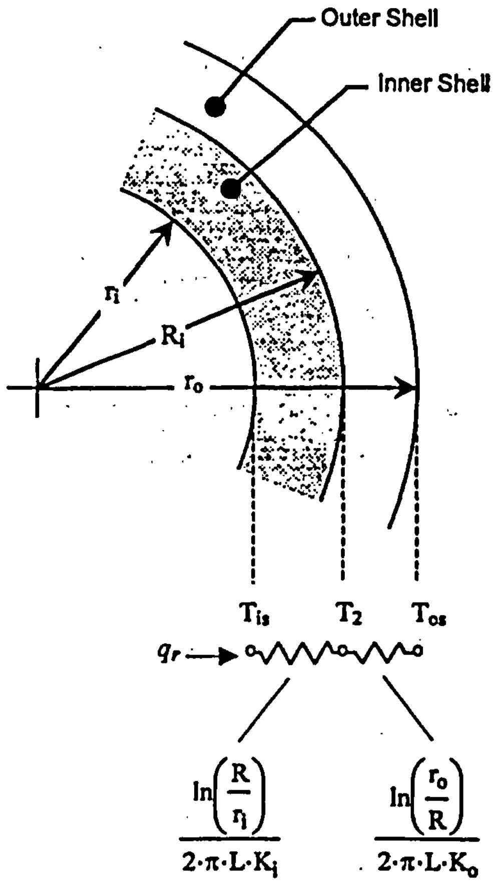
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



$T_{is} =$

	0
0	293.0
1	332.3
2	359.3
3	383.1
4	412.4
5	427.4
6	444.4
7	469.4
8	494.4
9	503.3
10	513.2

K

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\, R\, \Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot {\Delta T_{os}}^T$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot {\delta_i}^T - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \left[\overrightarrow{\frac{\delta}{\left[\frac{R_o}{E_o} \cdot \left(\frac{{r_o}^2 + {R_o}^2}{{r_o}^2 - {R_o}^2} + \nu_o \right) + \frac{R_i}{E_i} \cdot \left(\frac{{R_i}^2 + {r_i}^2}{{R_i}^2 - {r_i}^2} - \nu_i \right) \right]}} \right]$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{r_o^2}\right)\right]}$$ outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{R_o^2}\right)\right]}$$ outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\% \cdot \sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,MPa$

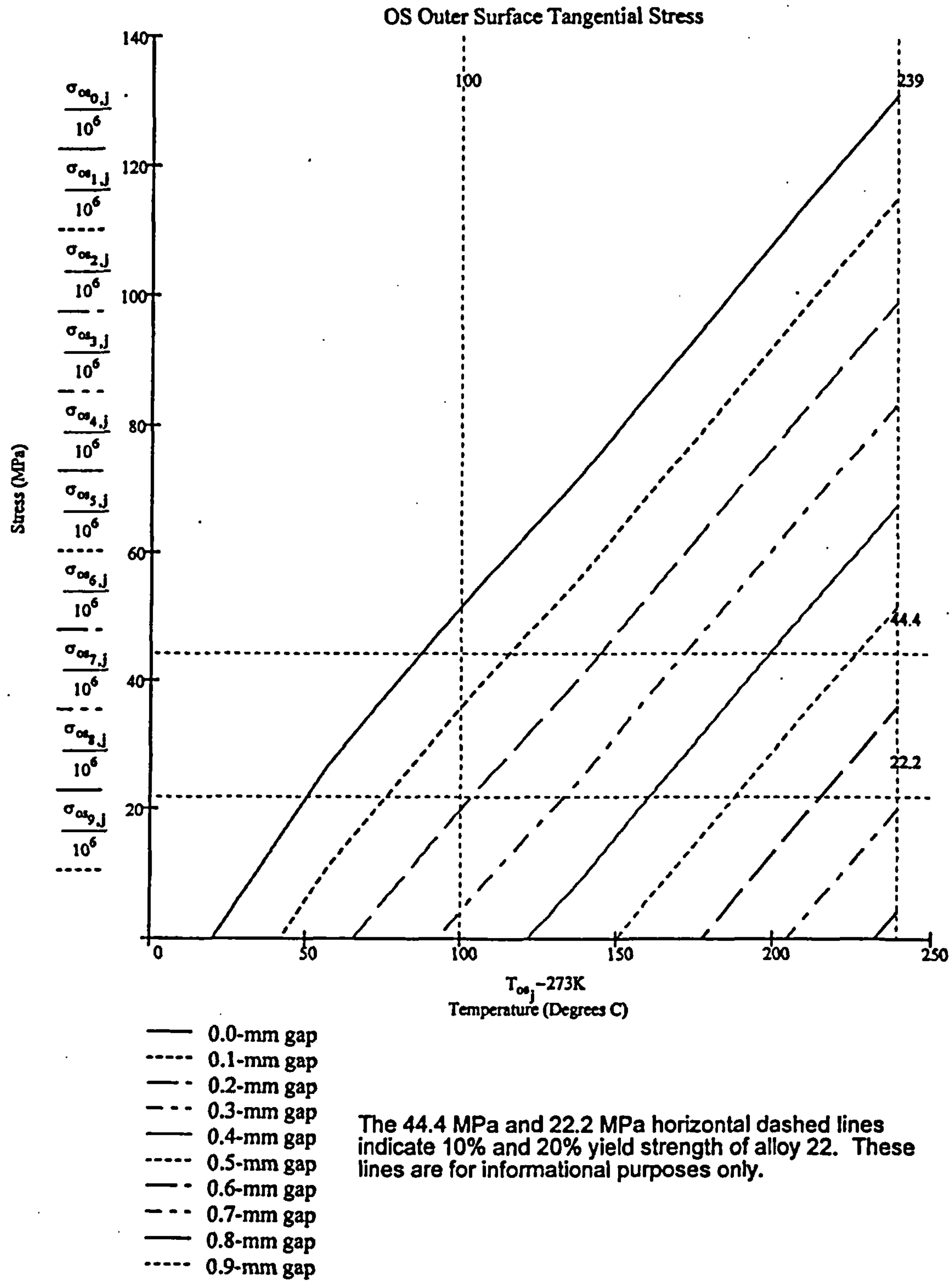
$\sigma_{20\%} := 20\% \cdot \sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,MPa$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	130.9
0.1	115.0
0.2	99.2
0.3	83.4
0.4	67.5
0.5	51.7
0.6	35.8
0.7	20.0
0.8	4.2
0.9	-11.6
1.0	-27.5

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

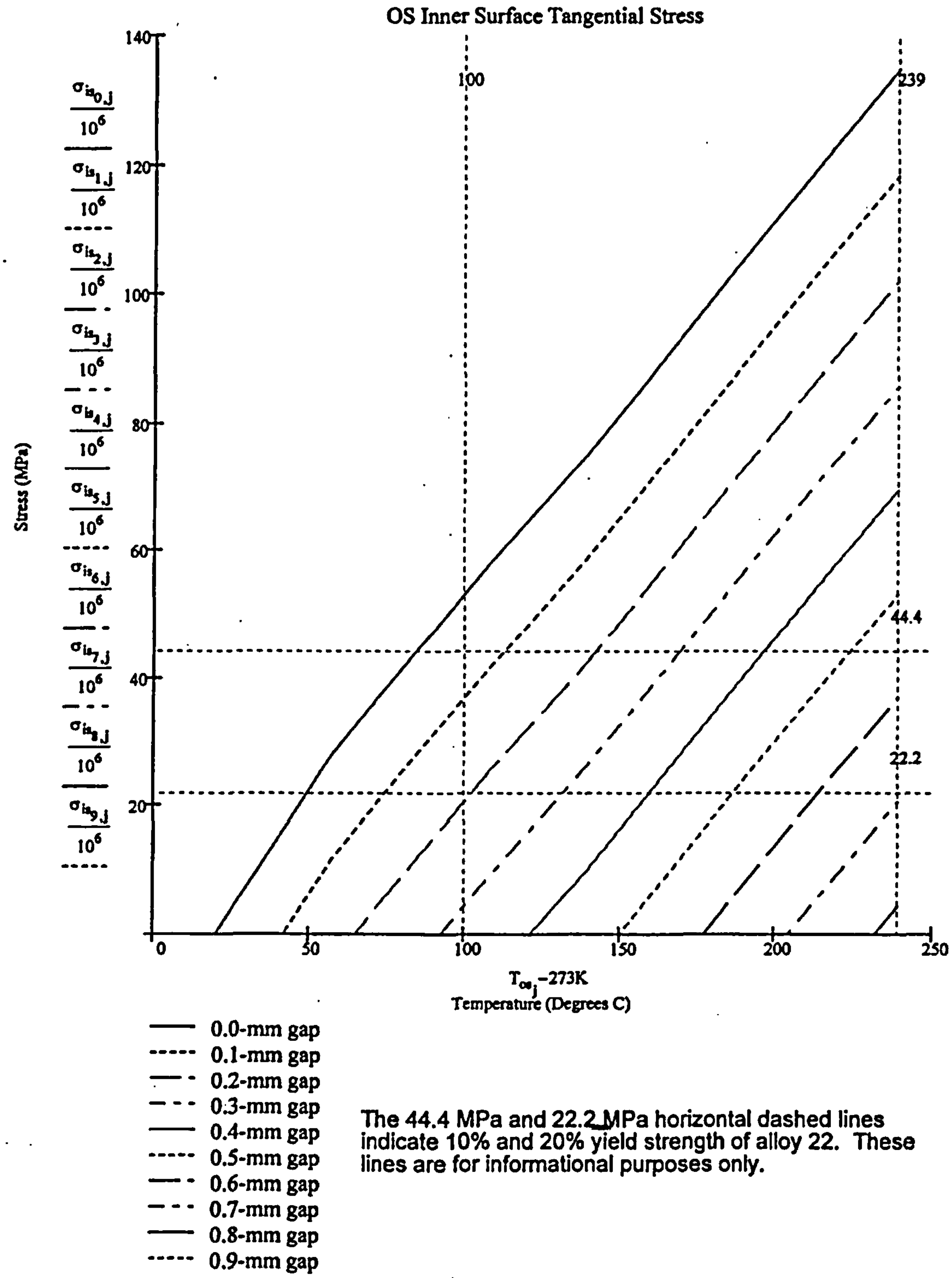


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{is_{j,10}}$ = (MPa)
0.0	134.8
0.1	118.5
0.2	102.2
0.3	85.9
0.4	69.5
0.5	53.2
0.6	36.9
0.7	20.6
0.8	4.3
0.9	-12.0
1.0	-28.3

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Various Shell Gap Sizes for the Naval SNF Long WP

This calculation determines the outer shell stresses due to uneven thermal expansion of the inner and outer shells. The inner shell is constructed of 316 Stainless Steel Nuclear Grade (Section 5.1.1), and the outer shell is constructed of Alloy 22 (Section 5.1.2). Various shell gap sizes are used to calculate the resulting outer shell stresses.

Parameter j provides a range from 0 to 10 with an interval of 1.

$j := 0..10$ range from 0 to 10 with an interval of 1

$gap_j := j \cdot 0.0001 \cdot m$ range of shell gap sizes between the shells from which the outer shell stresses are to be calculated

$gap_j =$ (mm)

	0
0	0.0
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	1.0

Dimensions of the waste package cross section
and the inner cavity length (Section 5.1.3):

$r_i := 0.8595 \cdot m$ inner shell inner radius

$th_i := 0.050 \cdot m$ inner shell thickness

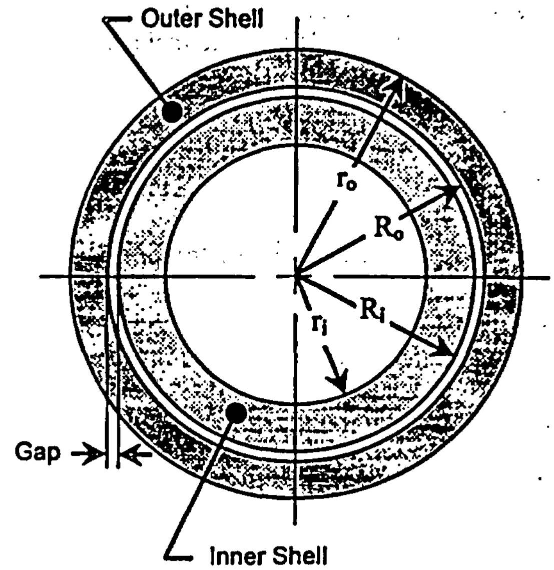
$R_i := r_i + th_i$ inner shell outer radius

$R_o := R_i + gap$ outer shell inner radius

$th_o := 0.025 \cdot m$ outer shell thickness

$r_o := R_o + th_o$ outer shell outer radius

$L := 5.415m$ inner cavity length



Material Properties.

$\alpha_{ss} := 17 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for 316NG SS $\left(9.7 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.1)

$\alpha_{alloy22} := 12.6 \cdot 10^{-6} \frac{m}{m \cdot K}$ mean coefficient of thermal expansion for Alloy 22 $\left(7.0 \cdot 10^{-6} \frac{in}{in \cdot deg F}\right)$ (Section 5.1.2)

$GPa := 10^9 \cdot Pa$ $MPa := 10^6 \cdot Pa$ $ksi := 10^3 \cdot psi$

$E_o := 206 \cdot GPa$ $E_o = 29.9\ 10^6 \cdot psi$ outer shell elastic modulus (Section 5.1.2)

$E_i := 195.1 \cdot GPa$ $E_i = 28.3\ 10^6 \cdot psi$ inner shell elastic modulus (Section 5.1.1)

$\nu_o := 0.278$ outer shell Poisson's ratio (Section 5.1.2)

$\nu_i := 0.298$ inner shell Poisson's ratio (Section 5.1.1)

$K_i := 17.3 \cdot \frac{W}{m \cdot K}$ inner shell thermal conductivity $\left(10.0 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.1)

$K_o := 13.4 \frac{W}{m \cdot K}$ outer shell thermal conductivity $\left(7.75 \frac{BTU}{hr \cdot ft \cdot degF}\right)$ (Section 5.1.2)

$\sigma_{y.alloy22} := 222MPa$ $\sigma_{y.alloy22} = 32.2\,ksi$ outer shell yield strength at 260 degrees C (Section 5.1.2)

The upper boundary of the temperature range that the outer shell of the 21-PWR waste package is subject to is 239 degrees C (462 degrees F and 512 K) (Section 5.1.4), occurring 35 years after emplacement. This upper boundary temperature range is used for all waste packages (Section 5.1.4). Room temperature at 20 degrees C (68 degrees F and 293 K) is the initial temperature (Section 5.1.4) of the shells before the spent nuclear fuel is inserted.

T_{os} represents the temperature range values (Kelvin) of the calculation. q_r represents the corresponding overall heat transfer rates (Watts) for each temperature (Section 5.1.5). At room temperature the overall heat transfer rate is equal to zero, representing the shells before insertion of the spent nuclear fuel.

$$T_{os} := \begin{pmatrix} 293 \\ 330 \\ 357 \\ 381 \\ 411 \\ 426 \\ 443 \\ 468 \\ 493 \\ 502 \\ 512 \end{pmatrix} K$$

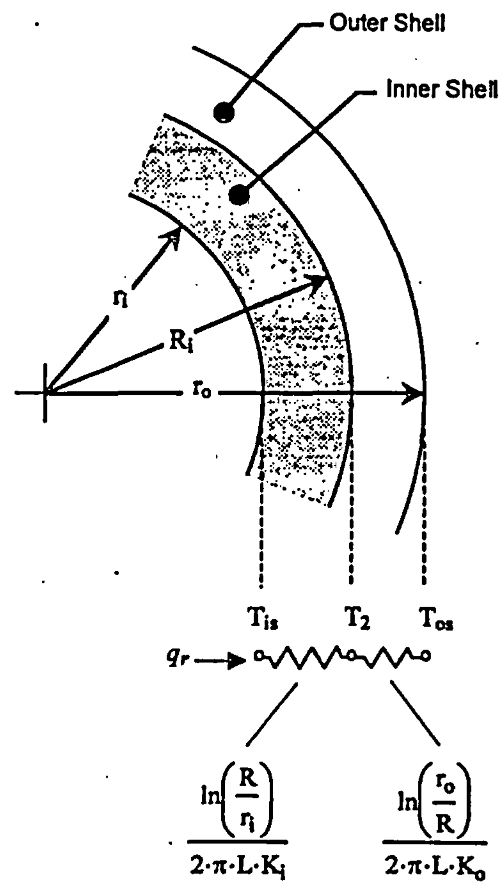
outer shell outer surface temperature

$$q_r := \begin{pmatrix} 0.0 \\ 11799.9 \\ 11762.5 \\ 10846.7 \\ 7192.8 \\ 7191.7 \\ 7182.4 \\ 7102.3 \\ 6856.1 \\ 6540.6 \\ 6158.3 \end{pmatrix} W$$

overall heat transfer rates (Section 5.1.5)

Using heat transfer methods for a composite cylindrical wall, illustrated here, the inner shell inner surface temperature range, T_{is}, is found by the following equation (Ref. 16, page 92, eq. (3-29)). These temperature values correspond with T_{os} values. For this part of the calculation R_i and R_o are equal to each other (Assumption 3.5).

$$T_{is} := \overrightarrow{\left[\left(\frac{\ln\left(\frac{R_i}{r_i}\right)}{2\cdot\pi\cdot L\cdot K_i} + \frac{\ln\left(\frac{r_o}{R_i}\right)}{2\cdot\pi\cdot L\cdot K_o}\right)\cdot q_r\right]} + T_{os}$$



T_{is} = (K)

	0
0	293.0
1	331.8
2	358.8
3	382.7
4	412.1
5	427.1
6	444.1
7	469.1
8	494.1
9	503.0
10	513.0

The temperature change is found for both the inner shell at the inner surface and outer shell at the outer surface:

$\Delta T_{is} := T_{is} - 293K$ inner shell inner surface temperature change

$\Delta T_{os} := T_{os} - 293K$ outer shell outer surface temperature change

Interference between the two shells along the radius is determined using the basic definition of thermal expansion:

$\varepsilon = \alpha\,(\Delta T)$ where ε is the strain (change in length per length), α is the coefficient of thermal expansion, and ΔT is the change in temperature (Ref. 17, page 63, eq. (2-61)).

Since $\varepsilon = \delta / L$ (change in length per length), solving for the change in length, δ, yields the equation for thermal expansion along a radius:

$\delta = \alpha\,R\,\Delta T$ where δ is the change in radial length, α is the coefficient of thermal expansion, R is the radial length, and ΔT is the change in temperature.

$\delta_i := \alpha_{ss} \cdot R_i \cdot \Delta T_{is}$ change in size of the inner shell outer radius

$\delta_o := \alpha_{alloy22} \cdot R_o \cdot {\Delta T_{os}}^{T}$ change in size of the outer shell inner radius

$A := (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$ This 1x11 row vector is used to expand the 11x1 column vectors into matrices compatible with the δ_o 11x11 matrix.

$\delta := A^T \cdot {\delta_i}^T - \delta_o - gap \cdot A$ interference between shells

$R_o := R_o \cdot A$ outer shell inner surface radii 11x1 column vector, expanded to an 11x11 matrix

$r_o := r_o \cdot A$ outer shell outer surface radii 11x1 column vector, expanded to an 11x11 matrix

Pressure, *p*, due to an interference fit is found by the following equation (Ref. 17, pages 62 to 63, eq. (2-59)):

$$p := \overrightarrow{\left[\frac{\delta}{\left[\frac{R_o}{E_o} \cdot \left(\frac{{r_o}^2 + {R_o}^2}{{r_o}^2 - {R_o}^2} + \nu_o\right) + \frac{R_i}{E_i} \cdot \left(\frac{{R_i}^2 + {r_i}^2}{{R_i}^2 - {r_i}^2} - \nu_i\right)\right]}\right]}$$

The tangential stresses at the inner and outer surfaces of the outer shell are found in this section.

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the outer surface:

$$\sigma_{os} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{r_o^2}\right)\right]}$$ outer shell outer surface tangential stress (MPa)

Outer shell tangential stress (Ref. 17, page 59, eq. (2-50), first equation) at the inner surface:

$$\sigma_{is} := \overrightarrow{\left[\frac{p \cdot (R_o)^2}{r_o^2 - R_o^2} \cdot \left(1 + \frac{r_o^2}{R_o^2}\right)\right]}$$ outer shell inner surface tangential stress (MPa)

The following calculations determine the outer shell 10% and 20% yield strength values. These values are marked on the resulting plots and are for informational purposes only.

$\sigma_{10\%} := 10\% \cdot \sigma_{y.alloy22}$ 10% yield strength

$\sigma_{10\%} = 22.2\,\text{MPa}$

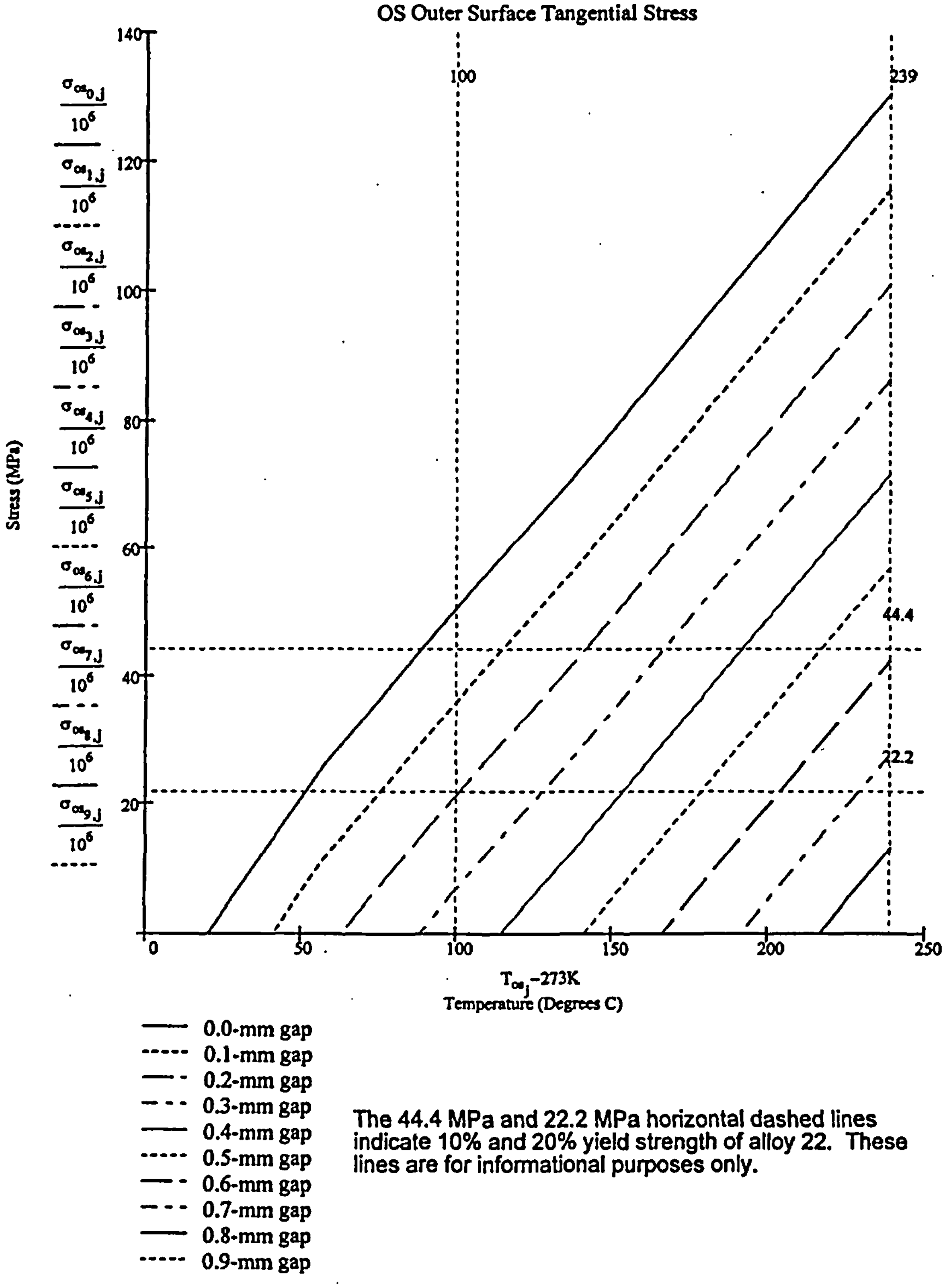
$\sigma_{20\%} := 20\% \cdot \sigma_{y.alloy22}$ 20% yield strength

$\sigma_{20\%} = 44.4\,\text{MPa}$

Maximum stress at 239 degrees C at the outer surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

gap_j = (mm)	$\sigma_{os_{j,10}}$ = (MPa)
0.0	130.4
0.1	115.7
0.2	101.1
0.3	86.4
0.4	71.7
0.5	57.0
0.6	42.4
0.7	27.7
0.8	13.0
0.9	-1.7
1.0	-16.3

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the outer surface of the outer shell for various gap sizes.

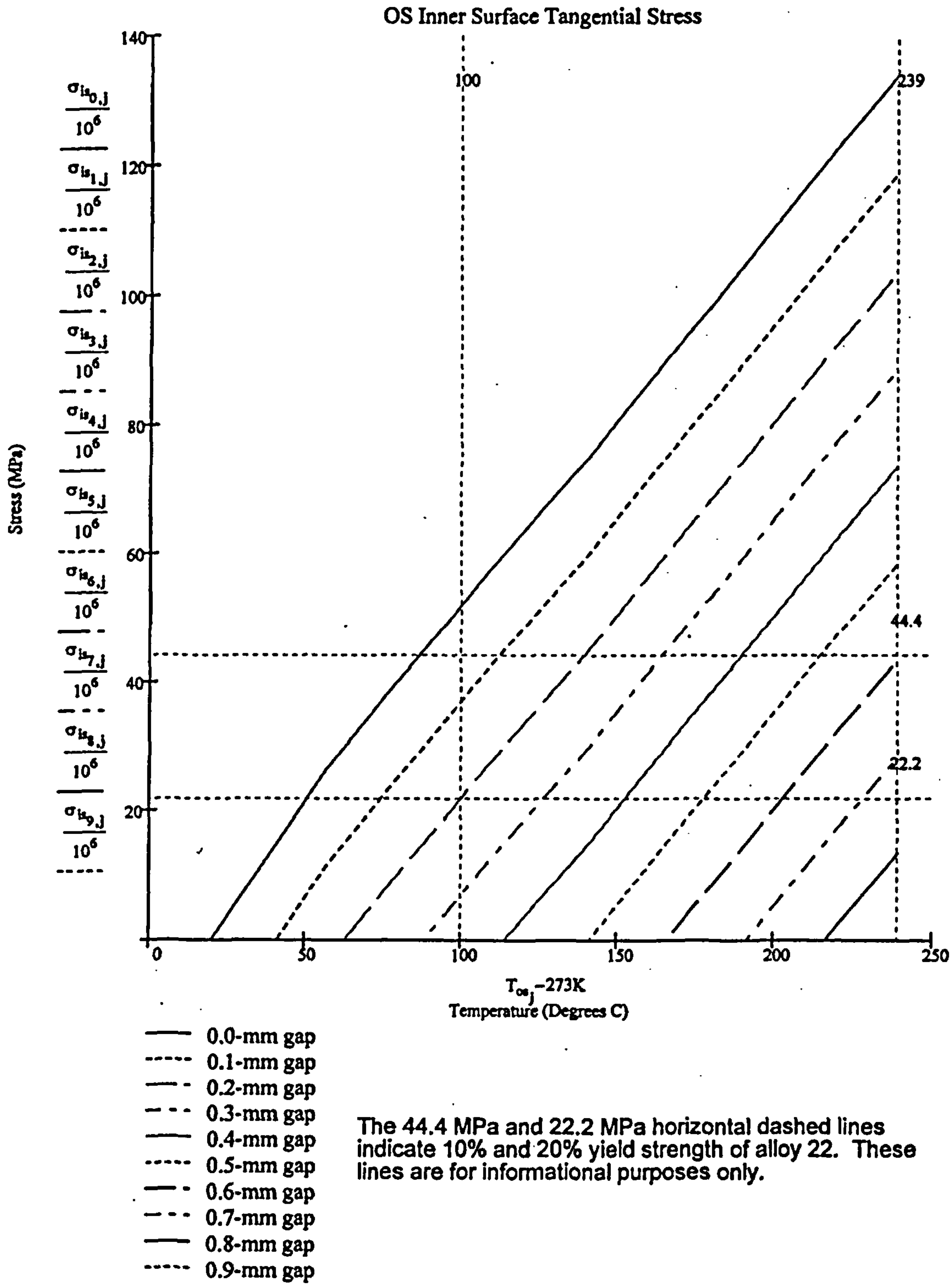


The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Maximum stress at 239 degrees C at the inner surface for a corresponding gap size. Negative stresses signify that there is no contact between the shells for the corresponding gap size.

$gap_j =$ (mm)	$\sigma_{is_{j,10}} =$ (MPa)
0.0	134.1
0.1	119.0
0.2	103.9
0.3	88.8
0.4	73.7
0.5	58.6
0.6	43.5
0.7	28.5
0.8	13.4
0.9	-1.7
1.0	-16.8

This plot illustrates the stress (MPa) with respect to temperature (degrees C) of the inner surface of the outer shell for various gap sizes.



The 44.4 MPa and 22.2 MPa horizontal dashed lines indicate 10% and 20% yield strength of alloy 22. These lines are for informational purposes only.

Thermal Expansion for a Long Circular Cylinder

This attachment will verify the basic equation for thermal expansion through the radius of a cylinder. The equation is as follows:

$$\delta = \alpha \cdot R \cdot \Delta T$$

where
δ is the change in radial length,
α is the coefficient of thermal expansion,
R is the radial length,
and ΔT is the change in temperature.

The following equations (Ref. 19, page 444, eq. (c) through (f)) are used to determine the stress in the radial, angular, and axial directions, represented by σ_r, σ_θ, and σ_z, respectively. The displacement due to thermal expansion is given by u. Since the temperature gradient through the barrier thickness is negligibly small, ΔT is independent of the radius, r.

$$u = \frac{1+\nu}{1-\nu} \cdot \alpha \cdot \frac{1}{r} \cdot \int_a^r \Delta T \cdot r\,dr + C_1 \cdot r + \frac{C_2}{r} \qquad (1)$$

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \int_a^r \Delta T \cdot r\,dr + \frac{E}{1+\nu} \cdot \left(\frac{C_1}{1-2\nu} - \frac{C_2}{r^2}\right) \qquad (2)$$

$$\sigma_\theta = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \cdot \int_a^r \Delta T \cdot r\,dr - \frac{\alpha \cdot E \cdot \Delta T}{1-\nu} + \frac{E}{1+\nu}\left(\frac{C_1}{1-2\nu} + \frac{C_2}{r^2}\right) \qquad (3)$$

$$\sigma_z = \frac{\alpha \cdot E \cdot \Delta T}{1-\nu} + \frac{2 \cdot \nu \cdot E \cdot C_1}{(1+\nu)(1-2\nu)} \qquad (4)$$

where

ν is Poisson's ratio,
α is the coefficient of thermal expansion,
E is the elastic modulus,
r is the radial length,
a is the inner radius,
and ΔT is the change in temperature.

Integrating and simplifying equation (2) gives

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \int_a^r \Delta T \cdot r\,dr + \frac{E}{1+\nu} \cdot \left(\frac{C_1}{1-2\nu} - \frac{C_2}{r^2} \right)$$

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \left(\frac{1}{2} \cdot r^2 \cdot \Delta T - \frac{1}{2} \cdot a^2 \cdot \Delta T \right) + \frac{E}{1+\nu} \cdot \left(\frac{C_1}{1-2\nu} - \frac{C_2}{r^2} \right)$$

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{\Delta T}{2} \cdot \left(1 - \frac{a^2}{r^2} \right) + \frac{E}{1+\nu} \cdot \left(\frac{C_1}{1-2\nu} - \frac{C_2}{r^2} \right) \qquad (5)$$

Using eq. (5), C_2 is found in terms of C_1 by using the following boundary condition:

For $r = a$, $\sigma_r = 0$.

$$C_2 = \frac{1}{1-2\nu} \cdot a^2 \cdot C_1 \qquad (6)$$

C_2 is substituted into eq. (5).

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{\Delta T}{2} \cdot \left(1 - \frac{a^2}{r^2} \right) + \frac{E}{1+\nu} \left[\frac{C_1}{1-2\nu} - \left(\frac{1}{1-2\nu} \cdot a^2 \cdot C_1 \right) \frac{1}{r^2} \right]$$

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{\Delta T}{2} \cdot \left(1 - \frac{a^2}{r^2} \right) + \frac{C_1 \cdot E}{(1+\nu) \cdot (1 - 2 \cdot \nu)} \cdot \left(1 - \frac{a^2}{r^2} \right)$$

$$\sigma_r = \left[\frac{\alpha \cdot E}{1-\nu} \cdot \frac{\Delta T}{2} + \frac{C_1 \cdot E}{(1+\nu) \cdot (1 - 2 \cdot \nu)} \right] \cdot \left(1 - \frac{a^2}{r^2} \right) \qquad (7)$$

Using eq. (7), C_1 is found by using the following boundary condition:

For $r = b$, $\sigma_r = 0$, where b is the outer radius.

$$C_1 = \frac{(1+\nu) \cdot (1 - 2 \cdot \nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T \qquad (8)$$

Substituting C_1 into eq. (6) produces C_2.

$$C_2 = \frac{1}{1-2\nu} \cdot a^2 \cdot \frac{(1+\nu)\cdot(1-2\cdot\nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T$$

$$C_2 = \frac{(1+\nu)}{2(1-\nu)} \cdot a^2 \cdot \alpha \cdot \Delta T \qquad (9)$$

C_1 and C_2 are inserted into eq. (5) to determine the radial stress, σ_r.

$$\sigma_r = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{\Delta T}{2} \cdot \left(1 - \frac{a^2}{r^2}\right) + \frac{E}{1+\nu} \left[\frac{1}{1-2\nu} \cdot \frac{(1+\nu)\cdot(1-2\cdot\nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T - \frac{1}{r^2} \cdot \frac{(1+\nu)}{2(1-\nu)} \cdot a^2 \cdot \alpha \cdot \Delta T \right]$$

Reducing the equation yields

$$\sigma_r = \frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 - \frac{a^2}{r^2}\right) + \frac{E}{1+\nu} \cdot \left[\frac{(1+\nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T - \frac{(1+\nu)}{2(1-\nu)} \cdot \frac{a^2}{r^2} \cdot \alpha \cdot \Delta T \right]$$

$$\sigma_r = \frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 - \frac{a^2}{r^2}\right) + \frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_r = 0 \qquad (10)$$

C_1 and C_2 are inserted into eq. (3) to determine the angular stress, σ_θ.

$$\sigma_\theta = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \cdot \int_a^r \Delta T \cdot r \, dr - \frac{\alpha \cdot E \cdot \Delta T}{1-\nu} + \frac{E}{1+\nu} \left[\frac{1}{1-2\nu} \cdot \frac{(1+\nu)\cdot(1-2\cdot\nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T + \frac{(1+\nu)}{2(1-\nu)} \cdot \frac{a^2}{r^2} \cdot \alpha \cdot \Delta T \right]$$

Reducing the equation yields

$$\sigma_\theta = \frac{\alpha \cdot E}{1-\nu} \cdot \frac{1}{r^2} \cdot \left[\frac{1}{2} \cdot \Delta T \cdot \left(r^2 - a^2\right)\right] - \frac{\alpha \cdot E \cdot \Delta T}{1-\nu} + \frac{E}{1+\nu} \cdot \left[\frac{(1+\nu)}{2 \cdot (1-\nu)} \cdot \alpha \cdot \Delta T + \frac{(1+\nu)}{2 \cdot (1-\nu)} \cdot \frac{a^2}{r^2} \cdot \alpha \cdot \Delta T\right]$$

$$\sigma_\theta = \frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 - \frac{a^2}{r^2}\right) - \frac{E}{1-\nu} \cdot \alpha \cdot \Delta T + \frac{E}{2 \cdot (1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 + \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = \frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 - \frac{a^2}{r^2} - 2\right) + \frac{E}{2 \cdot (1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 + \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = -\frac{E}{2(1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 + \frac{a^2}{r^2}\right) + \frac{E}{2 \cdot (1-\nu)} \cdot \alpha \cdot \Delta T \cdot \left(1 + \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = 0 \qquad (11)$$

A uniform axial stress $\sigma_z = C_3$ is superposed onto eq. (4), choosing C_3 so that the resultant force on the ends is zero (Ref. 19, page 444).

$$\sigma_z = -\frac{\alpha \cdot E \cdot \Delta T}{1-\nu} + \frac{2 \cdot \nu \cdot E \cdot C_1}{(1+\nu)(1-2\nu)} + C_3 = 0 \qquad (12)$$

Using eq. (12), C_3 is found by substituting C_1 into the equation.

$$C_3 = \frac{\alpha \cdot E \cdot \Delta T}{1-\nu} - \frac{2 \cdot \nu \cdot E}{(1+\nu) \cdot (1-2 \cdot \nu)} \cdot \frac{(1+\nu) \cdot (1-2 \cdot \nu)}{2(1-\nu)} \cdot \alpha \cdot \Delta T$$

Reducing the equation yields

$$C_3 = \frac{E}{1-\nu} \cdot \alpha \cdot \Delta T - \frac{\nu \cdot E}{1-\nu} \cdot \alpha \cdot \Delta T$$

$$C_3 = \frac{E}{1-\nu} \cdot \alpha \cdot \Delta T (1-\nu)$$

$$C_3 = E \cdot \alpha \cdot \Delta T \qquad (13)$$

The displacement u, is affected by the axial stress C_3. A term $-\nu C_3 r/E$ must be added on the right of eq. (1) (Ref. 19, page 445).

$$u = \frac{1+\nu}{1-\nu}\cdot\alpha\cdot\frac{1}{r}\cdot\int_a^r \Delta T\cdot r\,dr + C_1\cdot r + \frac{C_2}{r} + \left(\frac{\nu\cdot C_3\cdot r}{E}\right) \quad (14)$$

C_1, C_2, and C_3 are inserted into eq. (14) to determine the radial displacement.

$$u = \frac{1+\nu}{1-\nu}\cdot\alpha\cdot\frac{1}{r}\cdot\int_a^r \Delta T\cdot r\,dr + \frac{(1+\nu)\cdot(1-2\cdot\nu)}{2(1-\nu)}\cdot\alpha\cdot\Delta T\cdot r + \frac{1}{r}\cdot\frac{(1+\nu)}{2(1-\nu)}\cdot a^2\cdot\alpha\cdot\Delta T + \left(\frac{\nu\cdot E\cdot\alpha\cdot\Delta T\cdot r}{E}\right)$$

Reducing the equation yields

$$u = \frac{1+\nu}{2(1-\nu)}\cdot\frac{1}{r}\cdot\alpha\cdot\Delta T\cdot\left(r^2 - a^2\right) + \frac{(1+\nu)\cdot(1-2\cdot\nu)}{2\cdot(1-\nu)}\cdot\alpha\cdot\Delta T\cdot r + \frac{(1+\nu)}{2\cdot(1-\nu)}\cdot\frac{a^2}{r}\cdot\alpha\cdot\Delta T - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = \frac{1+\nu}{2\cdot(1-\nu)}\cdot\alpha\cdot\Delta T\cdot\left[\left(r - \frac{a^2}{r}\right) + (1-2\cdot\nu)\cdot r + \frac{a^2}{r}\right] - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = \frac{1+\nu}{2\cdot(1-\nu)}\cdot\alpha\cdot\Delta T\cdot\left(r - \frac{a^2}{r} + r - 2\cdot\nu\cdot r + \frac{a^2}{r}\right) - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = \frac{1+\nu}{2\cdot(1-\nu)}\cdot\alpha\cdot\Delta T\cdot(2r - 2\cdot\nu\cdot r) - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = \frac{1+\nu}{(1-\nu)}\cdot\alpha\cdot r\cdot\Delta T\cdot(1-\nu) - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = (1+\nu)\cdot\alpha\cdot r\cdot\Delta T - \nu\cdot\alpha\cdot r\cdot\Delta T$$

$$u = \alpha\cdot r\cdot\Delta T \quad (15)$$