SOFTWARE RELEASE NOTICE

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CNWRA Form TOP-6 (06/95)

SOFTWARE SUMMARY FORM

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CNWRA Form TOP-4-1

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MULTIPHASE-MULTICOMPONENT CHEMICAL TRANSPORT MODEL

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dcm demonstration with YM parameters Aug 27, 1999

*GRID---> Co-ordinate Geometry : DCMXYZ

Number of Elements in I-direction NX = Number of Elements in J-direction NY = Number of Elements in K-direction...... NZ = 1 1 80 $\ddot{}$

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Developed for the U.S. NRC

Version 1.5.1

June 2002

MULTIPHASE-MULTICOMPONENT CHEMICAL TRANSPORT MODEL

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--> Initializatize METRA --> open input file VAldgvtl.dat ------------------------------------ [METRA I--------------------------------- --> Initializatize GEM \leftarrow read input file: icode = 3 Notice: For ICODE > 3, IPOR must be negative, have reset IPOR = -1 --> master species for time-step control: ALL --> finish reading input data Total Work Space Required for [aa] in MULTIFLO = 26638 Assigned = 10000000

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Wednesday, June 12, 2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-2002 04-20
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c Contract Number: NRC 02-97-009 Scott Painter (210-522-3348) c Center for Nuclear Waste Regulatory Analyses c San Antonio, Texas 78238-5166 c spainter@swri.edu c cc ccc ccc cc ccc ccc ccc c ccccc c ccc cc ccc cc cccc ccc ccc cccc ccc Cc ccc Cc Cc CcC cC cc c VERSION/REVISION HISTORY c \$Id\$ c \$Log\$ c-------------------------- c Date Author(s) Comments/Modifications c-----_-___________________-________ c C c c c c April 97 May 98 February 2000 Peter C. Lichtner Mohan S. Seth Peter C. Lichtner Mohan S. Seth Scott Painter c May 2000 c August 2000 c c c December 2000 c c c c c c c July 2001 c c June 2002 c Initial Implementation Beta Release 1.2 Release V1.2.1 Minor Bug fixes V1.2.2 Fix bug related to water density calculation and phase change test V1.2.3 Fix bug related to dryout in GEM. Also change surface area update in GEM, which was bypassed for secondary minerals. Minor **fix** to printing errors. V1.5 Section 5,7,8 oE V2.0 SRD Also assorted minor fixes SCR351 **4-1** Header
Information ccccccccccccccccccccccccccc ccccccccccccccccccccccccc ccccccccccc(ccCcccccc c DISCLAIMER/NOTICE c This computer code/material was developed as an account of work c performed by the Center for Nuclear Waste Regulatory Analyses (CNWRA) c for the Division of Waste Management of the Nuclear Regulatory c Commission (NRC), an independent agency of the United States c Government. The developer(s) of the code nor any of their sponsors c make any warranty, expressed or implied, or assume any legal c liability or responsibility for the accuracy, completeness, or c usefulness of any information, apparatus, product or process c disclosed, or represent that its use would not infringe on c privately-owned rights. c IN NO EVENT UNLESS REQUIRED BY APPLICABLE LAW WILL THE SPONSORS c OR THOSE WHO HAVE WRITTEN OR MODIFIED THIS CODE, BE LIABLE FOR c DAMAGES, INCLUDING ANY LOST PROFITS, LOST MONIES, OR OTHER SPECIAL, c INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR c INABILITY TO USE (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA c BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY THIRD PARTIES OR A c FAILURE OF THE PROGRAM TO OPERATE WITH OTHER PROGRAMS) THE PROGRAM, c EVEN IF YOU HAVE BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES, c OR FOR ANY CLAIM BY ANY OTHER PARTY. cccccc ccc Cc **ccc cc** ccccccccccc c cc cc cc ccc ccc cc cc Cc cC **cCcc** ccc Ccc C Cc cC ccc **cC** c cc c PURPOSE: c This routine is the main program for the MULTIFLO driver which

c couples METRA and GEM.

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MULTIFLO, 1.5.1 LIST OF FILES, June 21, 2002

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SOFTWARE VALIDATION TEST PLAN FOR MULTIFLO VERSION 1.5.1

Prepared by

Scott Painter

Center for Nuclear Waste Regulatory Analyses San Antonio, Texas

July 2002

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Appendix A Appendix B

1.0 SCOPE OF THE VALIDATION

This software validation is for MULTIFLO V1.5.1 which comprises the METRA and GEM modules. Details of the software and its functioning can be found in the MULTIFLO User Manual. This validation covers the major capabilities of the code that are to be used in regulatory activities. These include:

- (1) Nonisothermal multiphase flow and phase-change phenomena in partially saturated porous media
- (2) Flow in composite fractured/porous media using a dual continuum formulation
- (3) Flow in saturated porous media including compressibility effects
- (4) Advective and diffusive transport of chemicals in the aqueous and gaseous phase
- (5) Equilibrium speciation of aqueous and gaseous phase constituents
- (6) Kinetically controlled mineral formation and dissolution, and resulting effects on porosity, permeability, and flow
- (7) Unstructured grid configuration with arbitrary interblock connectivity

2.0 REFERENCES

J. Bear, Dynamics of Fluids in Porous Media, Dover Publications, New York, 1972.

P. A. Domenico and F. W. Schwartz, Physical and Chemical Hydrogeology, John Wiley, New York, 1990.

C. Doughty and K. Pruess, Journal of Geophysical Research, Vol 97, No. B2, Feb 10, 1992, pg. 1821 -1838.

P.C. Lichtner, Continuum Formulation of Multicomponent-multiphase Reactive Transport in Reactive Transport in Porous Media, Edited by P.C Lichter, C.l. Steefel and E.H. Oelkers, Mineralogical Society of America, 1996.

S. Painter, P.C. Lichtner, and M. S. Seth, MULTIFLO User's Manual: Two-phase Nonisothermal Coupled Thermal-hydrological-chemical Flow Simulator, MULTIFLO Version 1.5, Center for Nuclear Waste Regulatory Analyses, 2001.

B. Sagar, Dispersion in three dimension: Approximate analytical solutions: American Society of Civil Engineers, Journal of Hydraulics Division, v 108, 1982.

E. Wexler, Analytical Solutions for 1-, 2-, and 3-Dimensional Solute Transport in Ground-Water Systems with Uniform Flow, USGS TWRI 3-B7,1991.

3.0 ENVIRONMENT

Validation is to be performed on the SUN server known as Spock, which uses the Solaris 5.8 operating system. The commercial program Mathematica 4.1, running on the Windows NT (version 4) workstation Brahma, will be used for comparisons. No special peripherals are required.

4.0 PREREQUISITES, ASSUMPTIONS AND CONSTRAINTS

Requires the chemical reaction databases, which are controlled as part of the MULTIFLO system.

5.0 TEST CASES

5.1 Multiphase Simulations of Doughty and Pruess

These simulations are designed to test METRA's representation of nonisothermal flow, phase-change phenomena, and heat transport under transient conditions with and without vapor pressure lowering (major capability 1 in Section 1.0). The geometry is one-dimensional cylindrical with a line heat source in the center. These test cases correspond to Figure 6 of Doughty and Pruess (1992) and are described in more detail in Scientific Notebook 282E Vol 4 pg 9 and Scientific Notebook 282E Vol 5 pg 5.

5.1.1 Test Input

Test input files are on the accompanying disk: TestCase1\Run2\novpl.dat and TestCase 1 *1Run21vpl.dat*

5.1.2 Test Procedure

The test procedure is as follows:

- (1) Run the case without vapor pressure lowering by typing MULTIFLO novpl.
- (2) Enter 1 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Plot results for liquid saturation at 1 year versus the dimensionless similarity variable and compare with the "no vapor pressure lowering" case from Figure 6 of Doughty and Pruess (1992).
- (5) Run the case with vapor pressure lowering by typing MULTIFLO vpl.
- (6) Enter 1 at the command prompt.
- (7) Verify that the code runs to completion without error.
- (8) Plot results for liquid saturation and temperature at 1 year versus the dimensionless similarity variable and compare with Figure 6 of Doughty and Pruess.

5.1.2 Expected Results

Calculated temperature should be within 5% of the results shown in Figure 6 of Doughty and Pruess (1992). Saturation profiles should have the same shape as those in Figure 6 of Doughty and Pruess. Exact agreement is not to be expected because of numerical discretization error. Because of minor details between the two models, localized large deviations may also occur in regions where the saturation changes rapidly.

5.2 Infiltration in Dual Permeability Media

These simulations are designed to test METRA's representation of unsaturated flow in dual permeability media (major capability 2 in Section 1.0). The geometry is one dimensional vertical with specified saturation at the top and bottom boundaries. Details of the simulation can be found in Scientific Notebook 282E Vol 1 pg 16 and Scientific Notebook 282E Vol 5 pg 8.

5.2.1 Test Input

Test input file is on the accompanying disk: TestCase2\sstate1.dat

5.2.2 Test Procedure

The test procedure is as follows:

- (1) Execute MULTIFLO by typing MULTIFLO sstate1.
- (2) Enter 1 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Plot steady state results for liquid saturation in fractures and matrix versus depth, and compare with Mathematica solution as calculated by the script Richards-DKM.nb.

5.2.3 Expected Results

Richards equation provides an adequate approximation to the physical situation. A dual-permeability solution to Richard's equation was implemented in Mathematica, as described in Scientific Notebook 282E Vol 1 pg 16. The Mathematica script Richards-DKM.nb is on the attached disk. Small discrepancies (<5%) between the Mathematica and MULTIFLO solutions are to be expected. These may be caused, for example, by numerical discretization error and neglect of the air-phase in Richards approximation.

5.3 Drawdown in Infinite Confined Aquifer

This simulation is designed to test METRA's representation of saturated flow including compressibility effects (major capability 3 in Section 1.0). The geometry is one dimensional radial with specified withdrawal from the center.

5.3.1 Test Input

Details of the simulation can be found in Scientific Notebook 282E Vol 4 pg 12. The test input file is on the accompanying disk: TestCase3\theis.dat.

5.3.2 Test Procedure

The test procedure is as follows:

- (1) Execute MULTIFLO by typing MULTIFLO theis.
- (2) Enter 1 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Extract results for pressure at 1.e-4 years, convert to equivalent drawdown in meters, and compare with the well-known analytical solution of Theis.

5.3.3 Expected Results

This configuration has a well-known analytical solution by Theis (see Domenico and Schwartz, 1990), which is implemented in the Mathematica script (Theis.nb) on the attached disk. Small discrepancies (<10%) between the analytical and MULTIFLO solution are to expected because of numerical discretization error.

5.4 Equilibrium Speciation in GEM

This simulation is designed to test GEM's representation of equilibrium speciation for aqueous and gaseous species at 25 °C (major capability 5 in Section 1.0). The reaction system is written in MULTIFLO form as:

 $\varnothing \Leftrightarrow \text{oh} + \text{h}^+ - \text{h}_2\text{o}$

$$
\emptyset \quad \Leftrightarrow \quad -\operatorname{co}_2(\operatorname{aq}) + \operatorname{h}^+ + \operatorname{hco}_3^- - \operatorname{h}_2\operatorname{o}
$$

 $\emptyset \Leftrightarrow -\cos_3^2 - h^+ + \text{hco}_3^-$

$$
\emptyset \quad \Leftrightarrow \quad -\csc_{3}(aq) + ca^{2+} - h^{+} + hco_{3}^{-}
$$

 $\emptyset \Leftrightarrow -\cos_2(g) + h^+ + h\cos_3 - h_2o$

In addition, the solution is set to be in equilibrium with calcite mineral. The geometry is one dimensional with two blocks. Transport is by diffusion only and the initial and boundary conditions are identical. Under these assumptions, the system is started in steady state and should remain in steady state. The initial and boundary conditions are described in Scientific Notebook 282E, Vol 3, pg 9.

5.4.1 Test Input

The test input file is on the accompanying disk: TestCase4\eq3compare.inp

5.4.2 **Test Procedure**

The test procedure is as follows:

- (1) Execute MULTIFLO by typing MULTIFLO eq3compare.
- (2) Enter 2 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Using the reported initial concentrations and activity coefficients for primary and secondary species, verify by hand calculation that the mass action equations are satisfied and that the activity coefficients as calculated by the extended Debye-Huckel representations are correct.
- (5) Verify that the concentrations for primary and secondary species are identical to the initial concentrations and that the calculated concentrations for interior nodes remain unchanged.

5.4.3 **Expected Results**

Left and right sides of the mass action equation should agree. Activity coefficients should agree with hand calculations. Initial and boundary concentrations should agree. In each case, the quantities should agree to four significant digits. Exact agreement is not to be expected because of finite precision in the MULTIFLO output.

5.5 **Solute Transport in Dual Permeability Media**

This simulation is designed to test GEM's representation of advective/diffusive transport in dual permeability media (major capability 4 in Section 1.0). The configuration involves constant flow in one dimension with flow in both the fractures and matrix. Transport is by advection and diffusion with first order mass exchange between the fracture and matrix system. At t=0, the inlet concentration for matrix and fractures is set to 10 times the initial concentration.

5.5.1 Test Input

The test input file is on the accompanying disk: TestCase5\masin1.inp

5.5.2 Test Procedure

The test procedure is as follows:

- (1) Execute MULTIFLO by typing MULTIFLO masinl.
- (2) Enter 2 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Verify that the concentrations agree with those of the semi-analytical solutions described in Scientific Notebook 282E Vol 1, page 26.

5.5.3 Expected Results

Results should agree with the semi-analytical solution described in Appendix A and Scientific Notebook 282E Vol 1, page 26, which is implemented in the Mathematica notebook FloThruDCM.nb. Small discrepancies (<5%) are to be expected because of discretization error.

5.6 Three-Dimensional Advective/Dispersive Transport in GEM

The test case tests the transport in GEM in three dimensions (major capability 4 in Section 1.0). The velocity field is uniform and constant with flow directed in the x direction. The initial concentration is 0.0008. At t=0, the concentration on a small "patch" at the inlet was increased by a factor of ten, and the system is allowed to evolve for 1 year. A constant darcy velocity of 1 m/yr in the x direction and a diffusion coefficient of 3.15 m²/yr is used. The system size is 40 \times 11×11 cells, with a nonuniform spacing in each direction. More details can be found in Scientific Notebook 282E Vol 1, page 29.

5.6.1 Test Input

The test input file is on the accompanying disk: TestCase6\masin21.inp

5.6.2 Test Procedure

The test procedure is as follows:

- (1) Execute MULTIFLO by typing MULTIFLO masin2l.
- (2) Enter 2 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Verify that the spatial profiles of concentration at 1 year agree with the analytical solution of Sagar (1982), which is described in Scientific Notebook 282E Vol 1, page 29.

5.6.3 Expected Results

Results should agree, to within 10%, with the analytical solution of Sagar (1982) as summarized in Wexler (1991). Exact agreement is not to be expected because of numerical discretization error.

5.7 Fully Coupled Flow/Transport with Mineral Dissolution and Permeability Modification

This simulation tests the coupling between METRA and GEM and kinetically controlled mineral reactions in GEM (major capability 6 in Section 1.0). The geometry is a one-dimensional "flow through" configuration with constant pressure drop across the modeled region. The system contains only quartz initially in equilibrium with $sio₂(aq)$. At t=0, the concentration at the inlet is decreased by a factor of 10. The pressure drop across the system is held constant. As the mineral dissolves, both the permeability and velocity increase as a result. Details can be found in Scientific Notebook 282E Vol 4, pg 14.

Three scenarios are considered. In the first scenario, the simulation ends before the mineral is dissolved fully at the inlet, transport is by a combination of advection and diffusion, and a powerlaw relationship with exponent of 2 is used to relate permeability to porosity, as described in the MULTIFLO Users Manual. The second scenario is similar to the first except that the simulation time is longer, thereby allowing full dissolution of the mineral at the inlet, and transport is by advection only. The third scenario is the same as the second, except that no permeability modification is allowed.

5.7.1 Test Input

The test input file is on the accompanying disk. For Scenario 1, the METRA and GEM input files are TestCase7\multi92.inp and TestCase7\masin92.inp, respectively. For Scenario 2, the input files are TestCase7\multi81.inp and TestCase7\masin81.inp. For Scenario 3, the input files are TestCase7\multi80.inp and TestCase7\masin80.inp.

5.7.2 Test Procedure

The test procedure is as follows:

- (1) Run the first scenario by typing MULTIFLO multi92 masin92.
- (2) Enter 3 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Verify that the spatial profile of quartz volume fraction at 30,000 years agrees with the analytical solution (Scientific Notebook 282E Vol 4, page 14).
- (5) Run the second scenario by typing MULTIFLO multi81 masin81.
- (6) Enter 3 at the command prompt.
- (7) Verify that the code runs to completion without error.
- (8) Verify that the spatial profile of quartz volume fraction at 40,000 years agrees with the analytical solution (Scientific Notebook 282E Vol 4, page 15).
- (9) Run the third scenario by typing MULTIFLO multi8O masin8O
- (10) Enter 3 at the command prompt.
- (11) Verify that the code runs to completion without error.
- (12) Verify that the spatial profile of quartz volume fraction at 40,000 years agrees with the analytical solution (Scientific Notebook 282E Vol 4, page 15).

5.7.3 Expected Results

Results should agree with the analysis described in Appendix B and Scientific Notebook 282E Vol 4, page 14-16, and implemented in the Mathematica notebook, FloThruCoupled.nb, which is on the attached disk. Results should be within 5% of the analytical results for the case with no permeability modification. For the cases with permeability modification, the analytical results are approximate and exact agreement is not to be expected. For these cases, the position of the dissolution front should agree with the analytical model to within 10%.

5.8 Tests of the Unstructured Grid Capability

The test case tests the unstructured grid capability (major capability 7 in Section 1.0). The test case is the same as Test Case 3 of Section 5.3, but implemented as an unstructured grid.

5.8.1 Test Input

The test input file is on the accompanying disk: TestCase8\unstruct.inp

5.8.2 Test Procedure

The test procedure is as follows:

(1) Execute MULTIFLO by typing MULTIFLO unstruct

0

- (2) Enter 1 at the command prompt.
- (3) Verify that the code runs to completion without error.
- (4) Verify that the drawdown agrees with that Test Case 3.

5.8.3 Expected Results

Results should be within 5% of the results of Test Case 3.

APPENDIX A: SEMI-ANALYTICAL SOLUTION FOR TEST CASE 5

The configuration involves constant flow in the z direction with flow in both the fractures and matrix. Transport is by advection and diffusion, with first order mass exchange between the fracture and matrix system. The inlet concentration for matrix and fractures is set to 10 times the initial concentration at t=O.

A semi-analytical approach was developed and implemented in Mathematica. The mass balance equations in one-dimension are,

$$
\frac{\partial}{\partial t} [\varphi SC] = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \alpha [C - C_F]
$$

$$
\frac{\partial}{\partial t} [\varepsilon_F \varphi_F S_F C_F] = D_F \frac{\partial^2 C_F}{\partial x^2} - V_F \varepsilon_F \frac{\partial C_F}{\partial x} + \alpha [C - C_F]
$$

where φ is the porosity, S the saturation, C the concentration, D the diffusion coefficient, and V the velocity. The subscript F denotes the fracture system. The symbol ε refers to the fracture volume fraction. The **E** multiplying the fracture velocity comes from the way block areas are calculated in GEM. The diffusion coefficient for the fracture system is $D_F = \varphi_F S_F \varepsilon_F D_0$ where the tortuosity has been set to 1. The diffusion coefficient for the matrix is the same except that it is missing the ε factor.

The diffusional coupling term is $\alpha \approx \frac{\varphi SD2A}{d}$ where A is the fracture/matrix interfacial area per unit volume, and d is the matrix block size (presumed constant). This expression neglects the fracture aperture relative to the grid block size. For constant properties, this system becomes,

$$
\frac{\partial C}{\partial t} = D_0 \frac{\partial^2 C}{\partial x^2} - \frac{V}{\varphi S} \frac{\partial C}{\partial x} - \alpha_1 [C - C_F]
$$

$$
\frac{\partial C_F}{\partial t} = D_0 \frac{\partial^2 C_F}{\partial x^2} - \frac{V_F}{\varphi_F S_F} \frac{\partial C_F}{\partial x} + \alpha_2 [C - C_F]
$$

$$
\alpha_1 = \frac{2DA}{d}
$$

$$
\alpha_2 = \frac{2DA}{d} \frac{\varphi S}{\varphi_F S_F \varepsilon_F}
$$

Taking the Laplace transform of the above, and applying the initial condition $C(0)=C_F(0)=0.$

$$
\mathbf{s}\hat{C} = D\frac{\partial^2 \hat{C}}{\partial x^2} - V \frac{\partial \hat{C}}{\partial x} - \alpha_1 [\hat{C} - \hat{C}_F]
$$

$$
\mathbf{s}\hat{C}_F = D\frac{\partial^2 \hat{C}_F}{\partial x^2} - V_F^* \frac{\partial \hat{C}}{\partial x} + \alpha_2 [\hat{C} - \hat{C}_F]
$$

with initial conditions $\hat{C}(0) = C_0/s$ and $\hat{C}_F(0) = C_{F0}/s$ and bounded at positive infinity. This system has the solution,

$$
\vec{\xi}(x;s) = c_1 \mathbf{v}_1 \exp[\lambda_1 x] + c_2 \mathbf{v}_2 \exp[\lambda_2 x]
$$

where $\vec{\xi}(x; s) = \left(C\frac{\partial C}{\partial x}, C_F, \frac{\partial C_F}{\partial x}\right)^T$, λ_1 and λ_1 are the negative eigenvalues, and v_1 and v_2 the corresponding eigenvectors of the matrix

 c_1 and c_2 are constants calculated so that the boundary conditions at x=0 are met. The calculation of the eigenvalues and eigenvectors is done using a Mathematica script FloThruDCM.nb. Once the solution is constructed this way in the Laplace domain, a numerical inverse Laplace transform is performed to obtain the solution in the time domain.

APPENDIX B: ANALYTICAL SOLUTION FOR TEST CASE 7

Consider a one-dimensional system with $SiO₂(aq)$ initially in equilibrium with quartz. At t=0, the concentration at the inlet is decreased by a factor of 10. The pressure drop across the system is held constant. As the mineral dissolves, the permeability and velocity increase as a result. Considered two situations where it is possible to get an approximate solution: in Scenario 1, the mineral is not allowed to fully disappear at the inlet, while in Scenario 2, the problem is advection dominated. In both situations, the quasi-stationary state approximation of Lichtner (1996) was used. Specifically, the characteristic time for mineral dissolution was much larger than the time required for the aqueous concentration to reach equilibrium. Thus the aqueous concentration is assumed to be stationary, or more precisely, to be described by a sequence of quasi-stationary states.

The aqueous concentration $C(x,t)$ is governed by the following equation

$$
\frac{\partial}{\partial t}\phi C + v \frac{\partial C}{\partial x} - \phi D \frac{\partial^2 C}{\partial x^2} = -k's (C - C_{eq})H(x - \ell(t))
$$

where ϕ is the porosity, v is the darcy velocity, C_{eq} is the equilibrium concentration, D is the diffusion coefficient, s is the specific surface area, and $k' = k/C_{eq}$, where k is the reaction rate. $H(\cdot)$ is the Heaviside function. Quartz is dissolving and will eventually dissolve fully; $\ell(t)$ is the width of the fully dissolved region.

The mineral volume fraction is given by

$$
\frac{\partial \phi_s}{\partial t} = \overline{V_s} k' s (C - C_{eq}) H (x - \ell(t))
$$

where $\overline{V_s}$ is the molar volume for the mineral. The initial and boundary conditions are:

$$
C(x, t = 0) = C_{eq}
$$

$$
C(x = 0, t) = C_0
$$

$$
\phi_s(x, t = 0) = \phi^0
$$

B.1 Mineral Not Fully Dissolved at Inlet

If the mineral has not fully dissolved at the inlet, $\ell(t) = 0$, the concentration is stationary, and we can neglect the time derivative in the above equation for aqueous concentration. The time required for the mineral to dissolve at the inlet is $\tau_s = \frac{\phi_s^0}{k's\Delta CV_s}$. The equation has solution

$$
C(x,t) - C_{eq} = (C_0 - C_{eq}) \exp \left[-x \frac{v}{2\phi D} \left(\sqrt{1 + \frac{4k' s \phi D}{v^2}} - 1 \right) \right]
$$

if advection dominates, this is approximately

$$
C(x,t)-C_{eq} = (C_0-C_{eq})\exp\biggl[-\frac{k's}{v}x\biggr].
$$

These solutions are found on page 631 of Bear (1972). Note, that Bear's λ is $\frac{k's}{v}$. The minera volume fraction has solution

$$
\phi_S^0(x,t) = \phi_S^0 \left[1 - \exp\left(-qx\right)t/\tau_S\right]
$$

where

$$
q = \frac{v}{2\phi D} \left[\sqrt{1 + \frac{4k' s \phi D}{v^2}} - 1 \right]
$$

If no permeability modification is allowed, the velocity is fixed in the above equation.

If permeability modification is allowed, the velocity will change. The effect of this velocity change can be accounted for in an approximate way by replacing the velocity $|V|$ with a time averaged velocity $\overline{\mathcal{V}}$, which is calculated as follows. At time twe have from the condition of fixed pressure gradient in one dimension,

$$
\frac{v_0}{v} - \frac{1}{L} \int_0^L \left(\frac{\kappa_0}{\kappa}\right) dx = 0
$$

where the latter term on the left is the ratio of initial permeability κ_{0} to effective permeability at time t. The permeability change is calculated from the porosity change according to a power-law (other relationships are possible but are not considered here):

$$
\frac{\kappa_0}{\kappa} = \left(\frac{\phi_0}{\phi}\right)^m = \left[\frac{\phi_R - \phi_S^0}{\phi_R - \phi_S^0 - \exp(-qx)t/\tau_S}\right]^m
$$

where ϕ_R is the reactive volume fraction. The velocity appearing in q here is not $v(t)$ but the time-averaged velocity. This can be approximated as $\bar{v} = (v_0 + v)/2$, which implies $v = 2v - v_0$. Substituting $v = 2v - v_0$ and for $\left(\frac{\kappa_0}{\kappa}\right)$ in the integrand, and then calculating the integral explicitly yields the following equation:

$$
\frac{v_0}{2v - v_0} + \frac{b + (b + a \exp(Lq))\log(b + a \exp(Lq))}{(b + a \exp(Lq))q} - \frac{b + (b + a)\log(b + a)}{(b + a)q} = 0
$$

Here, the effective (time averaged) velocity is to be used in the expression for q, not $v(t)$. This equation is then to be solve for \bar{v} . This result is specific to the situation $m = 2$. Note $a = \phi_R - \phi_S^0$ and $b = t\overline{V_s}k's\Delta C$.

Once the time-averaged velocity is obtained, it is used in place of V in the expression for the mineral volume fraction.

B.2 Advection Dominated Case

The preceding analysis applies when the mineral has not completely disappeared at the inlet. This analysis is difficult to extend to the more general situation of a moving boundary in the general case, but can be extended if we restrict our consideration to the advection dominated case. The solution in this situation is given, for example, on page 51 of Lichtner (1996)

$$
C(x,t) - C_{eq} = (C_0 - C_{eq}) \exp \left[-\frac{k's}{v} (x - \ell(t)) \right].
$$

where

$$
\ell(t) = \frac{\overline{V_s} \nu \Delta C}{\phi_s^0} (t - \tau_s)
$$

Similarly, the mineral volume fraction is given by:

$$
\phi_{S}\left(x,t\right)=\phi_{S}^{0}\left(1-\exp\left[-\frac{k's}{v}\left(x-l(t)\right)\right]\right)
$$

Continuing as before, the following equation is obtained;

$$
\frac{v_0}{2\bar{v}-v_0}-\frac{b+(b+a\exp(L'q))\log(b+a\exp(L'q))}{(b+a\exp(L'q))q}+\frac{b+(b+a)\log(b+a)}{(b+a)q}-\left(\frac{\phi_0}{\phi_R}\right)^m\ell(t)=0
$$

where $q(\overline{v}) = \frac{k \cdot s}{n}$, $b = \overline{V_s} k' s \Delta C$, $L' = L - \ell(t)$, and the time average velocity is to be used in *V* place of V in $\ell(t)$. This equation is to be solved for the time averaged velocity, which

0 ⁰

replaces the velocity in the equation for the mineral volume fraction.

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